IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received November 10, 2018, accepted December 7, 2018, date of publication December 13, 2018, date of current version January 7, 2019.

Digital Object Identifier 10.1109/ACCESS.2018.2886607

Cognitive Radio Made Practical: Forward-Lookingness and Calculated Competition

JIE REN[1] AND KAI-KIT WONG[2], (Fellow, IEEE)
[1]School of Electronics and Information Engineering, Beijing Jiaotong University, Beijing 100044, China
[2]Department of Electronic and Electrical Engineering, University College London, London WC1E 7JE, U.K.

Corresponding author: Kai-Kit Wong (kai-kit.wong@ucl.ac.uk)

The work is supported in part by EPSRC under grant EP/K015893/1.

ABSTRACT Cognitive radio is more than just radio environment awareness, and more importantly, has the ability to interact with the environment in the best way possible. Ideally, cognitive radios will form a self-regulating society of mobile radios achieving maximum spectrum utilization. However, challenges arise as mobile radios tend to compete with one another for spectrum, generating harmful interference, and damaging performance individually and for the network as a whole. In this paper, we present a framework that allows competing radios to teach and learn from each other's action so that a desirable equilibrium can be reached. The heart of cognition to establish this is the *forward-looking* ability, which enables competing radios to see beyond the present time, negotiate and optimize their actions toward a more agreeable equilibrium. Technically speaking, we adopt a belief-directed game where each mobile radio, regarded as player, formulates a belief function to project how the radio environment as a whole would respond to any of its action. This model facilitates engineering of the equilibrium by different choices of the players' belief functions. Under this model, players will negotiate naturally through a sequence of calculated competition (i.e., cycles of teaching and learning with each other). We apply this methodology to a cognitive orthogonal frequency-division multiple-access radio network where mobile users are free to access any of the subcarriers and thus compete for radio resources to maximize their rates. The results reveal that the proposed negotiation-by-forward-looking competition mechanism guides users to converge to an equilibrium that benefits not only the individual users but the entire network approaching the maximum achievable sum-rate.

INDEX TERMS Cognitive radios, negotiation mechanism, noncooperative game theory, OFDMA.

I. INTRODUCTION

It has been nearly 20 years since Mitola and Maguire's seminal work on cognitive radio in 1999 [1]. Since then, there have been huge amount of efforts in bringing advances in enabling technologies for software defined radio (SDR) including radio frequency, analog to digital conversion and digital signal processing, see [2], and the visionary paper [3] looking forward to the next 20 years of SDR. Recent advances have further witnessed cognitive radio being applied to massive multiple-input multiple-output (MIMO) antenna systems [4], Internet-of-Things (IoT) [5], [6] and non-orthogonal multiple-access (NOMA) systems [7].

Cognitive radio, the ideal software radio or Mitola radio, indeed goes much beyond the anticipated hardware flexibility and possesses the intelligence to access the spectrum anytime anywhere according to the environment and its need. According to Mitola's description [8], cognitive radio is:

"The point in which wireless personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to detect user communications needs as a function of use context, and to provide radio resources and wireless services most appropriate to those needs."

Giving cognitive radio the required intelligence appears to be the obstacle. The authors of the famous article [9] commented that the fact that an incumbent not fully willing to cooperate with cognitive radio has been the reason that forbids bringing cognitive radio to fruition. This suggests that some form of cooperation between mobile radios be necessary, if they are to coexist and occupy the same spectrum. Cooperation unfortunately implies overhead, cost and worse, restrictions as well as rigidity. There is a growing opinion that cognitive radio may never reach the required level of intelligence to coexist without one form of cooperation or another, dampening the prospect of the concept. This school of thought is clear from a vast amount of literature, see the most recent papers, e.g., [10]–[17].

In [10], the primary spectrum owner needs to run an auction to lease out its idle channels to the secondary users (SUs). Later, the work in [11] looked at self-coexistence for several secondary systems and adopted a congestion-averse game for a decentralized approach after a number of approximations and simplifications. The approach is also not entirely decentralized because a pricing model is assumed to exist for use of the resources. In enhancing the environment awareness, [12] proposed to deploy sensor nodes at different locations and employ deep learning to optimise the transmit power of a cognitive radio given the sensing information. Relying on the presence of sensor nodes to aid cognitive radio is however unrealistic, let alone the bandwidth needed for communication between sensor nodes and the cognitive radio. In [13], historical behavior of the primary users (PUs) was utilized to maximize the throughput of a cognitive SU network but a centralized coordinator was needed to maintain a reliable historical record for the behavior of the PUs. On the other hand, [14] considered a slotted ALOHA system and proposed a fair medium access control (MAC) protocol for cognitive radio networks but a common control channel was assumed to be available to all SUs. Setting up a control channel for the SUs was also recently investigated in [15]. Moreover, deep neural network (DNN) was used to address the resource allocation problem for cognitive radio networks but DNN had to be operated in a centralized fashion [16]. A framework for cooperative spectrum sensing and resource allocation for cognitive IoT systems was presented in [17].

Despite the lack of intelligence harmonizing competition, game theory remains the most used mathematical tool to design autonomous access control methods for cognitive radio networks, as it is the tool to analyze the outcome from a group of competing and rational players, as is the case for cognitive radio environments composed of competing interference-generating radios [18]–[34]. In interference channels like cognitive radio networks, users are decentralized and uncoordinated by default, compete and interfere with each other. How a cognitive radio gathers sufficient network-wide information and optimize itself to benefit not only its own but the entire network is an open challenge.

There are deadlocks that prevent existing game-theoretic results from making practical impacts [18]. The first deadlock is the low efficiency of Nash equilibrium (NE) where players are *myopic* and harmfully compete with each other [19]–[22]. NE is also often not unique and its performance can be unpredictable. To outperform NE, Stackelberg equilibrium (SE) arises where there is a *foresighted* player, referred to as *leader*, by knowing full information about the environment and the strategies of all other myopic users (a.k.a. *followers*). Considering the orthogonal frequency-division multiple-access (OFDMA) model, [23] studied the use of SE for autonomous spectrum sharing for cognitive radios, revealing a gain over NE.

Unfortunately, there are operational obstacles in achieving SE. First of all, to be qualified as a leader, the player should possess tremendous cognition power, although it could be learned from the environment by conjectures [24].[1] Worse, a bi-level game is required for reaching SE,[2] where there is a strict order of how the game proceeds to reach the prescribed equilibrium. For cognitive radio networks, this is either too restrictive or unrealistic for the very distributed nature of users in the environment.

The challenges do not end here. There is strong desire to extend the SE framework to the case where multiple leaders coexist [24], [25], [35]. DeMiguel and Xu [35] studied the extension and in their definition of K-SE (K leaders out of all players), leaders are myopic with respect to (w.r.t.) other leaders, but only foresighted to followers. To approach the K-SE, usual challenges in SE remain. Furthermore, according to their definition, if all players are leaders, the game degenerates to an NE, losing the foresightedness advantages of the leader. In [25], K-SE was employed for optimizing femtocell communications in which strong assumptions on leaders such as full channel state information (CSI)[3] and exhaustive optimization, apply. Notably, despite the gain of SE over NE as reported in [24] and [25], there is still a huge gap from the centralized optimal solutions [36]–[38] and the existing methods are far from satisfactory. Meanwhile, there are emerging applications of game theory for device-to-device (D2D) communications and heterogenous networks that addressed related but different network resource optimization problems [26]–[34].

The problem of game theory in this type of applications is that a rational player by default is selfish and only cares about its own reward, which depends on not only its own strategy but also competitors' responses. Ideally, a player should optimize its strategy, not based on its immediate reward but the final reward after others' strategies all settle. This will require the player to possess cognition power to think beyond the present time and into the future. However, game theory in its traditional form does not have the mechanism to permit such optimization, and reckless competition will lead to undesirable equilibria. This may be why game-theoretic approaches have been largely ineffective in cognitive radios.

To remedy this, in this paper, our novelty is to introduce each player's cognition by a belief function which quantifies how the environment as a whole would respond to any of

[1] Note that for SE, although conjectural equilibrium (CE) can be used to make the leader learn the necessary network-wide information, it was acknowledged in [24] that extending it to K-player SE scenarios is a topic for future investigation.

[2] To achieve SE via a bi-level game, the leader will start his action first, wait for all the followers to play a sub-game to reach an NE, then revise his action, and the whole process repeats until convergence. A bi-level game is needed even for CE.

[3] In this paper, *full* CSI refers to the CSI knowledge of every transmitter-receiver link of the entire network, in contrast to *local* CSI which we refer to it as the CSI only from a transmitter to its intended receiver.

its action [39]. By doing so, the players (cognitive radios in our application) are empowered with *forward-looking*[4] ability and we propose to use a form of Taylor series to define the players' reward functions embracing the interactive nature of the game via their beliefs. The outcome is transformative, since optimization of the equilibrium resulting from the players' strategies becomes possible by different choices of belief functions. In this paper, we apply this concept to the cognitive OFDMA radio network where the aim is to optimize, *autonomously*, every cognitive radio's power allocation strategy based on only *local* CSI for maximizing its rate. We also consider a hierarchical cognitive radio network where cognitive radios or SUs compete for the spectrum holes left out by the PUs but for any busy subcarrier they have no knowledge whether it is occupied by a PU or another SU, in the latter case of which the SUs should be allowed to compete and negotiate the subcarriers among themselves. In both scenarios, simulation results illustrate promising results that the proposed belief-directed game [39] gives the cognition power or brain that cognitive radios need in order to achieve perfect interference avoidance while maximizing their achievable rates.

It is worth emphasizing that this paper is not a simple application of the belief-directed game in [39] to cognitive radio networks. In particular, we need to choose judiciously the belief function of the cognitive radios to make it work before they compete. Choosing an inappropriate belief function will disable any possible negotiation that could have happened between the cognitive radios, giving rise to harmful competition, and a suitable belief function will make all the difference and allow cognitive radios to naturally negotiate the spectrum resources via a sequence of calculated forward-looking competitions. To this end, we generalize the water-filling (WF) optimization of one's strategy to take into account of the interactive nature of the OFDMA game, enabling forward-looking WF (FWF).[5,6] Then we study the equilibrium resulting from the FWF competition of cognitive radios, and derive key properties that the belief function needs to satisfy in order to converge to a desirable equilibrium.

In summary, the proposed cognitive approach has the following major advantages:

- First, the proposed method is highly distributed as it requires each cognitive radio only to know the CSI from its intended receiver and does not need any global knowledge such as the crosstalk CSI, the number of coexisting cognitive radios, the PUs' activity and any historical behavioural data.
- Secondly, it follows from the standard model of free competition in games. This means that there is no specific order the competition of the players should take place, unlike in Stackelberg and conjectural games which are bi-level, and no synchronization among the participants is needed. Additionally, being in a game allows cognitive radios to respond to changes naturally by revising their actions. Hidden terminal problems may also be fixed through interaction between the radios.
- More importantly, the proposed approach provides a new platform for cognitive radios to negotiate the spectrum resources over the air by simply competing in a calculated way via a judicious choice of belief functions. Remarkably, results illustrate that the negotiation-by-forward-looking-competition works extraordinarily well, and that in the cognitive OFDMA case, all cognitive radios manage to avoid each other and settle on their respective good subcarriers free of interference. On the other hand, in the interweave case where cognitive radios can access any spectrum holes left vacant by the PUs, our results show that the forward-looking game empowers the cognitive radios to identify the PUs' occupied subcarriers and avoid them autonomously without knowing a-priori which subcarriers are being used by the PUs, while negotiating with other cognitive radios by competing with them normally, according to their carefully chosen beliefs, for the other subcarriers.
- Simulation results reveal that the proposed approach achieves nearly the sum-rate as the *centralized near-optimal* iterative spectrum balancing (ISB) method in [37] exploiting full *global* CSl.
- This paper uncovers that tragedy of the commons is not an inevitable outcome for an interference channel with competing radios, and can be avoided if the radios are cognitive with forward-looking ability. Besides, cognitive radios are incentivised to negotiate with each other in order to get their best rewards, while at the same time have their actions benefiting the entire network. Importantly, an "average" cognitive radio in the proposed game outperforms significantly the leader in the SE, showing that it is much preferred to have all the competitors forward-looking, than myopic.

The rest of this paper is organized as follows. Section II introduces the OFDMA network model and the game-theoretic model for cognitive radios. Section III presents the forward-looking OFDMA game and the FWF power allocation for the cognitive radios. Then Section IV uses the forward-looking game as the framework to interpret NE and SE before developing key results for a desirable equilibrium. Convergence analysis is also provided. Section V considers the use of forward-lookimg game for a hierarchical cognitive OFDMA network. Section VI presents simulation results and we conclude the paper in Section VII.

[4]There is clear distinction between being forward-looking and foresighted. The former refers to the attempt of taking into account the possible or likely consequences of a given action, whereas the latter corresponds to the case of knowing perfectly the final outcome of a given action. Also, being forward-looking may not imply any accurate prediction of the outcome.

[5]In [40], an interference price, defined as the negative derivative of a user's utility function w.r.t. the total interference power it receives, was introduced into a resource allocation game for taking into account the interactive nature of the players. However, the artificial "prices" one has to pay for causing interference to others and the *crosstalk CSI* need to be known.

[6]It is worth noting that in our proposed scheme, the FWF solution takes into account the likely reaction of other players in one's strategy optimization, and is fundamentally different from the conventional WF and the modified WF in [41].

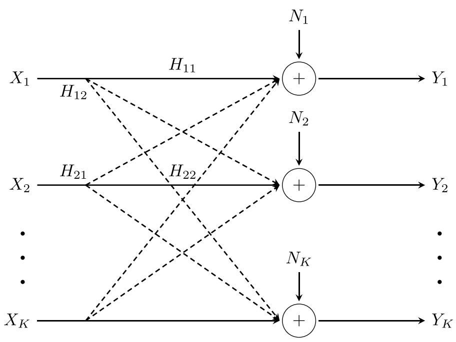

FIGURE 1. A multi-user interference channel model.

II. PRELIMINARIES

Cognitive radio is all about accessing the best channels while controlling its interference to others to an acceptable level. In this paper, the dynamic spectrum access problem is viewed as a resource self-allocation problem, modelled as finding the self-optimizing strategy for an OFDMA interference channel. OFDMA is the simplest model to study the problem, with its result easily extendable to more realistic models.

A. OFDMA INTERFERENCE CHANNELS

Consider a K-user OFDMA interference network, as shown in Fig. 1, where each user (or any cognitive transmitter-receiver link) is free to occupy any of the $N(\geq K)$ orthogonal frequency subcarriers for communications. The users operate in a non-cooperative manner and there exists no central spectrum manager. For user k, the total transmitted power is constrained by

$$\sum_{n=1}^{N} p_k[n] \leq P_k, \quad \forall k \in \{1, 2, \ldots, K\}, \tag{1}$$

where $p_k[n] \geq 0$ denotes the power allocated for the nth subcarrier by user k and P_k denotes the power budget for user k. We write $\mathsf{p}_k \triangleq \{p_k[1], p_k[2], \ldots, p_k[N]\}$ as the power allocation pattern for user k, which is drawn from some strategy $\mathcal{P}_k$ satisfying (1), or it can be written as $\mathsf{p}_k \in \mathcal{P}_k$.

Let $H_{ij}[n]$ denote the flat-fading channel coefficient from transmitter i to receiver j which is regarded as static under which the achievable rate is evaluated, and $N_k[n]$ denote the noise power density for the complex additive white Gaussian noise (AWGN) at receiver k on the nth subcarrier. The achievable rate for user k using single-user decoding is therefore given by

$$R_k \equiv \sum_{n=1}^{N} R_k[n] = \sum_{n=1}^{N} \log_2\left(1 + \frac{p_k[n]}{\sigma_k[n] + I_k[n]}\right) \tag{2}$$

$$= \sum_{n=1}^{N} \log_2\left(1 + \frac{p_k[n]}{c_k[n]}\right), \tag{3}$$

where $\sigma_k[n] \triangleq \frac{N_k[n]}{|H_{kk}[n]|^2}$ is the normalized noise power on subcarrier n, $I_k[n] = \sum_{i=1, i\neq k}^{K} p_i[n]\theta_{ik}[n]$ is the total interference power on subcarrier n for user k, with $\theta_{ik}[n] \triangleq \frac{|H_{ik}[n]|^2}{|H_{kk}[n]|^2}$ denoting the normalized (by user k) channel power gain from transmitter (or interference) i to receiver k, and $c_k[n] \triangleq \sigma_k[n]+I_k[n]$ corresponds to the overall "noise" on subcarrier n for user k.

Under this model, users compete on their individual achievable rates. In particular, a cognitive transmitter may choose to allocate more power on its own good subcarriers to boost its rate but will interfere other users more on these subcarriers. Channel allocation will be automatically achieved through power allocation. That is, if $p_k[n] = 0$ for some n, then the nth subcarrier is not assigned to user k.

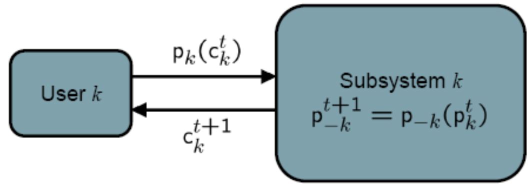

FIGURE 2. The power game subsystem with user k interacting with the environment.

B. A GAME-THEORETIC SUBSYSTEM MODEL

Consider the OFDMA interference channel above. Let $\mathcal{P} \triangleq \{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_K\}$ denote the set of collection for all users' power allocation strategies, with $\mathsf{p}_k \in \mathcal{P}_k$ $\forall k$ and define

$$\mathsf{p}_{-k} \triangleq \{\mathsf{p}_1, \ldots, \mathsf{p}_{k-1}, \mathsf{p}_{k+1}, \ldots, \mathsf{p}_K\}. \tag{4}$$

We also let $\mathsf{BP}_k \in \mathcal{P}_k$ be user k's best power allocation strategy in response to the interference pattern seen by that user $\mathcal{I}_k \triangleq \{I_k[1], I_k[2], \ldots, I_k[N]\}$. Then mathematically, BP_k is given by

$$\mathsf{BP}_k = \arg\max_{\mathsf{p}_k \in \mathcal{P}_k} R_k. \tag{5}$$

Fig. 2 shows a power game subsystem which depicts how users interact with each other in a competitive way. Given the interference seen by user k, it optimizes its power allocation to give $\mathsf{p}_k = \mathsf{BP}(\mathsf{I}_k)$ which in turn changes the interference patterns of other users $\{\mathsf{I}_\ell\}_{\ell\neq k}$ giving rise to their new power allocation strategies $\{\mathsf{BP}(\mathsf{I}_\ell)\}_{\ell\neq k}$. The power $\{\mathsf{BP}(\mathsf{I}_\ell)\}_{\ell\neq k}$ will become the cause of the changes in the interference pattern seen by user k, I_k. That is,

$$\mathsf{I}_k = \mathsf{I}_k\left(\{\mathsf{BP}(\mathsf{I}_\ell(\mathsf{p}_k))\}_{\ell\neq k}\right). \tag{6}$$

For convenience, we abuse our notation slightly and define

$$\mathsf{BP}_{-k}(\mathsf{p}_k) \triangleq \{\mathsf{BP}(\mathsf{I}_1), \ldots, \mathsf{BP}(\mathsf{I}_{k-1}), \mathsf{BP}(\mathsf{I}_{k+1}), \ldots, \mathsf{BP}(\mathsf{I}_K)\}. \tag{7}$$

The interference pattern for user k, $\mathsf{I}_k(\mathsf{BP}_{-k}(\mathsf{p}_k))$, is recognized to be a function of its own power allocation strategy p_k. Depending on how cognitive transmitters react to change of the environments and how the power allocation game runs, if the system converges, it will converge to some equilibrium.

Throughout, we will use the superscript $*$ to denote the parameters at the equilibrium so that at the equilibrium, the power allocation strategy is $\mathsf{P}^* = \{\mathsf{p}_k^*, \mathsf{p}_{-k}^*\}$.

In this setup, users or cognitive radio transmitters are all uncoordinated individuals, each of which allocates its power over the subcarriers in order to maximize its own rate based on its observation of the environment, $\{c_k[n]\}$ for all n (which is referred to as the local CSI possessed by user k), and its belief on how the environment would react to its action. The environment user k observes can change because other users may alter their strategies to respond to user k's action. It is a dynamic process where users all interact and may compromise to an equilibrium, if their interactions converge over time.

To model this interaction, from user k's viewpoint, we can regard other users as a subsystem (or the environment seen by user k), as shown in Fig. 2, which takes its action at time t as inputs (i.e., $\mathsf{p}_k^t = \{p_k^t[n]\}$) and produces a new interference pattern at time $t+1$ as outputs (i.e., $\mathsf{c}_k^{t+1} = \{c_k^{t+1}[n]\}$). The subsystem reacts by an overall response from the actions of all other users, $\mathsf{p}_{-k}^t \triangleq \{\mathsf{p}_1^t, \ldots, \mathsf{p}_{k-1}^t, \mathsf{p}_{k+1}^t, \ldots, \mathsf{p}_K^t\}$.

Based on the rate (3), the distributed optimization problem for the OFDMA channel is

$$\max_{\mathsf{p}_k \in \mathcal{P}_k} R_k, \quad \forall k. \tag{8}$$

The challenge to the above optimization is that R_k is a time-varying function (via c_k) which changes, according to the action or reaction of the remaining users. One way of characterizing the interactive process is to use the framework of the Nash game with forward-looking players [39].

III. A GAME OF FORWARD-LOOKING PLAYERS

A. DEFINITIONS

Players' cognition is key to defining the resulting equilibrium of a game. We adopt the framework of the Nash game with forward-looking players [39]. In particular, a player is said to be forward-looking if it uses a belief function to quantify the future reaction of the environment according to its action at present. Before presenting the formal definition of such game, the following concepts are important:

- *Environmental function*—In a competition process, the reward for player k depends not only on its own strategy x_k but also others' strategies $\boldsymbol{x}_{-k}$ at any given time instant t. We use the environmental function $r_k(\boldsymbol{x}_{-k}^t)$ to quantify the influence of other players' strategies onto player k's reward.
- *Belief function*—A player's understanding on its environmental function reflects its cognition about the competition in the game. In a belief-directed game, it is considered that player k possesses the knowledge of a belief function, which is denoted as $r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^t)$, where $\boldsymbol{x}_{-k}^t$ denotes the strategies from all the players at time instant t except player k, and clearly, $r_k^{\mathsf{B}}(x_k^t, \boldsymbol{x}_{-k}^t) = r_k(\boldsymbol{x}_{-k}^t)$. The belief function may be constructed using some form of Taylor series expansion, e.g.,

$$r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^t) = r_k(\boldsymbol{x}_{-k}^t) + \varphi_k^t \times (x_k - x_k^t), \tag{9}$$

where $\varphi_k^t \triangleq \frac{\partial r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^t)}{\partial x_k}$ is regarded as the *interference derivative* that predicts the amount of future reaction from the environment due to present change in strategy. The selection of the interference derivative φ_k^t by the player permits engineering of the equilibrium of the game. Moreover, the belief function $r_k^{\mathsf{B}}(\cdots)$ can be understood as player k's cognition on what the environment function $r_k(\cdots)$ would be, given a strategy x_k and the present state $\boldsymbol{x}_{-k}^t$. Note that φ_k^t is only a belief for player k and whether the derivative $\frac{\partial r_k}{\partial x_k}$ actually exists or not in the game is irrelevant.
- *Predicted reward*—The predicted reward function, $f_k(x_k, r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^t))$, gives the amount of reward player k believes to achieve by the strategy, x_k, given other players' strategies at time t, $\boldsymbol{x}_{-k}^t$, and the belief function $r_k^{\mathsf{B}}(\cdots)$ that player k uses to predict the future impact in the environment.

Based on the above definitions, we now present the NE with forward-looking players whose cognition is modelled in the form of some belief functions. Mathematically, this is written as

$$f_k(x_k^*, r_k^{\mathsf{B}}(x_k^*, \boldsymbol{x}_{-k}^*)) \geq f_k(x_k, r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^*)), \quad \forall k, \tag{10}$$

where $r_k^{\mathsf{B}}(\cdot)$ is the belief function reflecting player k's cognition ability and $f_k(\cdot)$ is the predicted reward function for player k. Note that (10) can be rewritten as

$$f_k(x_k^*, \boldsymbol{x}_{-k}^*) \geq f_k(x_k, r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^*)), \quad \forall k. \tag{11}$$

This can be explained by recognizing the fact that according to (9), $r_k^{\mathsf{B}}(x_k^*, \boldsymbol{x}_{-k}^*) = r_k(\boldsymbol{x}_{-k}^*)$ and as such at the equilibrium, we have

$$f_k(x_k^*, r_k^{\mathsf{B}}(x_k^*, \boldsymbol{x}_{-k}^*)) = f_k(x_k^*, r_k(\boldsymbol{x}_{-k}^*)) = f_k(x_k^*, \boldsymbol{x}_{-k}^*). \tag{12}$$

In this model, the belief function $r_k^{\mathsf{B}}(\cdot)$ can be chosen arbitrarily and it only serves to indicate player k's understanding about the competition in the environment. In fact, (11) embraces the conventional NE in which players are myopic and have the belief function $r_k^{\mathsf{B}}(x_k, \boldsymbol{x}_{-k}^t) = r_k(\boldsymbol{x}_{-k}^t)$ which effectively treats the environment player k observes at any present time instant t as fixed and constant and ignores the subsequent changes in other players' strategies triggered by player k's new strategy.

B. OFDMA WITH FORWARD-LOOKING RADIOS

For the system in Section II, the reward function is the achievable rate R_k in (3) and the strategy for player k (or cognitive radio transmitter k) is the power allocation strategy p_k. Also, the environmental function is the overall interference resulting from other users' strategies $\mathsf{c}_k(\mathsf{p}_{-k}^t)$.

Under the belief-directed game, we can form the belief function to enable player k's cognition by the Taylor series

$$c_k^{\mathsf{B}}[n] \triangleq c_k^t[n] + \sum_{\ell=1}^{\infty} \varphi_k^{(\ell)^t}[n](p_k[n] - p_k^t[n])^{\ell}, \quad (13)$$

where $\varphi_k^{(\ell)^t}[n]$ denotes the ℓth order derivative of the overall noise w.r.t. the strategy $p_k[n]$ at time t. The derivatives $\{\varphi_k^{(\ell)^t}[n]\}$ can be chosen freely to give a different belief function for the player (which will result in a different game) and hence play the role of the engineering parameters for the game. Also, we write $\mathsf{c}_k^{\mathsf{B}}(\mathsf{p}_k, \mathsf{p}_{-k}^t) = \{c_k^{\mathsf{B}}[1], c_k^{\mathsf{B}}[2], \ldots, c_k^{\mathsf{B}}[N]\}$, which may be viewed as a prediction to $\mathsf{c}_k(\mathsf{p}_{-k})$.

Based on the belief function $\mathsf{c}_k^{\mathsf{B}}$, user k has the predicted reward function (or predicted rate)

$$f_k(\mathsf{p}_k, \mathsf{c}_k^{\mathsf{B}}(\mathsf{p}_k, \mathsf{p}_{-k}^t)) \triangleq \sum_{n=1}^{N} \log_2\left(1 + \frac{p_k[n]}{c_k^{\mathsf{B}}[n]}\right), \quad (14)$$

which is the achievable rate in (3) after replacing $c_k[n]$ by $c_k^{\mathsf{B}}[n]$. Note that $f_k(\cdots)$ provides a predictive rate given the strategy p_k and the present state p_{-k}^t at time instant t.

Mathematically, at the NE with forward-looking players, we have

$$f_k(\mathsf{p}_k^*, \mathsf{c}_k^{\mathsf{B}}(\mathsf{p}_k^*, \mathsf{p}_{-k}^*)) \geq f_k(\mathsf{p}_k, \mathsf{c}_k^{\mathsf{B}}(\mathsf{p}_k, \mathsf{p}_{-k}^*)), \quad \forall \mathsf{p}_k \in \mathcal{P}_k \text{ and } \forall k. \quad (15)$$

NE is a special case of the equilibrium of the belief-directed game and this can be seen when $\varphi_k^{(\ell)*}[n] = 0 \ \forall \ell \geq 1$ and $\forall n, k$. In this case, we have $\mathsf{c}_k^{\mathsf{B}} = \mathsf{c}_k^t$ and (15) is therefore degenerated to

$$f_k(\mathsf{p}_k^*, \mathsf{c}_k(\mathsf{p}_{-k}^*)) \geq f_k(\mathsf{p}_k, \mathsf{c}_k(\mathsf{p}_{-k}^*)), \quad \forall \mathsf{p}_k \in \mathcal{P}_k \text{ and } \forall k. \quad (16)$$

The above states that at the equilibrium, the rewards for all cognitive radios are maximized, giving no incentive for them to deviate from it, and they all settle on their strategies $\{\mathsf{p}_k^*\}$.

Note that the predicted rate does not need to be accurate for the game to work, though the general sense is that an accurate prediction should help users find better strategies. Also, NE is a result from an unrealistic prediction, which is based on the assumption of a static environment.

C. FWF

For the conventional NE, each user attempts to maximize its rate as if the interference c_k is fixed regardless of its applied strategy. However, this is flaw and due to the interactive nature of the environment, c_k will depend on user k's strategy. Ignoring this fact will mean that users do not intentionally control their interference to others while attempting to enhance their rates [19]. In this paper and in contrast to previous work, the interactiveness of the game is taken into account, and the users become forward-looking. For the power allocation of the OFDMA type problem, the solution usually has an interpretation of WF over the subcarriers and this is no different even if the users are forward-looking.

In the OFDMA belief-directed game, at time t, user k aims to find the strategy for time $t+1$ by

$$\mathsf{p}_k^{t+1} = \arg\max_{\mathsf{p}_k \in \mathcal{P}_k} f_k(\mathsf{p}_k, \mathsf{c}_k^{\mathsf{B}}(\mathsf{p}_k, \mathsf{p}_{-k}^t)). \quad (17)$$

In order to make (17) solvable, it is advisable to choose $\{\varphi_k^{(\ell)^t}[n]\}$ such that

$$c_k^{\mathsf{B}}[n] > 0, \quad (18a)$$

$$\frac{\partial f_k(\mathsf{p}_k, \mathsf{c}_k^{\mathsf{B}})}{\partial p_k[n]} > 0, \quad (18b)$$

$$\frac{\partial^2 f_k(\mathsf{p}_k, \mathsf{c}_k^{\mathsf{B}})}{\partial (p_k[n])^2} < 0, \quad (18c)$$

so that (17) is a convex optimization problem. Note that (13) is a very general form to describe the belief function, and how to utilize it to design the most efficient OFDMA belief-directed game is unknown. In this paper, for mathematical tractability, we propose to choose $\{\varphi_k^{(1)^t}[n]\}$ appropriately to optimize the OFDMA resource allocation game while setting $\varphi_k^{(\ell)^t}[n] = 0$ for $\ell \geq 3$ and $\forall t, n$. Also,

$$\varphi_k^{(2)^t}[n] > -\frac{c_k^t[n] + \varphi_k^t[n](p_k[n] - p_k^t[n])}{(p_k[n] - p_k^t[n])^2} \quad (19)$$

can be chosen to ensure that $c_k^{\mathsf{B}}[n] > 0$ is satisfied, where $\varphi_k^t[n] = \varphi_k^{(1)^t}[n]$ for notational brevity.

Based on the definition (15), at time instant t, forward-looking user k aims to solve

$$\mathsf{p}_k^{t+1} = \arg\max_{\mathsf{p}_k \in \mathcal{P}_k} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_k[n]}{c_k^{\mathsf{B}}(\mathsf{p}_k, \mathsf{p}_{-k}^t)[n]}\right). \quad (20)$$

Theorem 1: The optimal strategy update for user k in (20) is given by

$$p_k^{t+1}[n] = \left(\frac{w_k^t x_k^t[n] - (x_k^t[n])^2}{w_k^t y_k^t[n] + x_k^t[n]}\right)^+, \quad (21)$$

where $(a)^+ = \max\{0, a\}$, $x_k^t[n] \triangleq c_k^{\mathsf{B}}(\mathsf{p}_k^{t+1}, \mathsf{p}_{-k}^t)[n]$, $y_k^t[n] \triangleq \left.\frac{\partial c_k^{\mathsf{B}}[n]}{\partial p_k[n]}\right|_{p_k[n]=p_k^{t+1}[n]}$ and $w_k^t (> 0)$ is regarded as the water-level that ensures the equality of the power constraint (1) for the maximization.

Proof: See Appendix A. □

The power allocation (21) is based on the maximization of a forward-looking or predicted rate (via the belief function $\mathsf{c}_k^{\mathsf{B}}$) and therefore dubbed FWF solution. Note that its WF interpretation follows exactly the same way as the conventional WF with a water-level w_k^t. As $t \to \infty$, if all users' strategies converge to a stable state, then from (13), we have

$$\begin{cases} x_k^*[n] = \left. c_k^{\mathsf{B}*}[n] \right|_{p_k[n]=p_k^*[n]} = c_k^*[n], \\ y_k^*[n] = \left. \frac{\partial c_k^{\mathsf{B}*}[n]}{\partial p_k[n]} \right|_{p_k[n]=p_k^*[n]} = \varphi_k^*[n], \end{cases} \quad (22)$$

and as a consequence, at the equilibrium, we obtain the users' strategies

$$p_k^*[n] = \left(\frac{w_k^* c_k^*[n] - (c_k^*[n])^2}{w_k^* \varphi_k^*[n] + c_k^*[n]}\right)^+, \quad \forall k, n. \tag{23}$$

The intuition is that although there can be many parameters $\{\varphi_k^{(m)^t}[n]\}_{\forall m,n}$ that can be adjusted in a player's belief, the power allocation strategy at the equilibrium depends only on $\varphi_k^*[n]$ (the first derivative but not the higher-order ones), or the choice of $\{\varphi_k^{(m)^t}[n]\}_{\forall m\geq 2}$ plays no role in defining the equilibrium. This justifies setting the higher-order derivatives in the belief function to zeros earlier.

A common structure in the OFDMA resource allocation game, regardless of the chosen belief functions and the resulting equilibrium, is to solve (5), which can be tackled by the Lagrangian multiplier method. In particular, this is done by defining

$$\mathcal{L} = \sum_{n=1}^{N} \log_2\left(1 + \frac{p_k[n]}{\sigma_k[n] + I_k[n]}\right) - \lambda\left(\sum_{n=1}^{N} p_k[n] - P_k\right), \tag{24}$$

where λ denotes the Lagrange multiplier, and finding p_k that maximizes $\mathcal{L}$. The Karush-Kuhn-Tucker (KKT) conditions for optimality can be derived as

$$\frac{\partial \mathcal{L}}{\partial p_k[n]} \begin{cases} = 0 & \text{for } p_k[n] > 0, \\ \leq 0 & \text{for } p_k[n] = 0. \end{cases} \tag{25}$$

Moreover, with (26), as shown at the top of the next page, and knowing $\varphi_k[n] \triangleq \frac{\partial c_k[n]}{\partial p_k[n]} = \frac{\partial I_k[n]}{\partial p_k[n]}$, it can be simplified as

$$\frac{\partial \mathcal{L}}{\partial p_k[n]} = \frac{1}{\frac{c_k^2[n] + \varphi_k[n] p_k^2[n]}{c_k[n] - \varphi_k[n] p_k[n]} + p_k[n]} - \lambda \equiv \frac{1}{\eta_k[n] + p_k[n]} - \lambda. \tag{27}$$

As a consequence, the optimal power allocation (or best resource allocation response) for user k has the well-known WF interpretation and is given by

$$p_k[n] = (w_k - \eta_k[n])^+, \tag{28}$$

where

$$w_k = \frac{(\sigma_k[n] + I_k[n])^2 + \varphi_k[n] p_k^2[n]}{\sigma_k[n] + I_k[n] - \varphi_k[n] p_k[n]} + p_k[n], \tag{29}$$

is the "water-level" set to satisfy the power constraint $\sum_n p_k[n] \leq P_k$. Although this version of WF solution may look complicated (as both w_k and $\eta_k[n]$ are functions of $p_k[n]$), this is much more general and facilitates the design of algorithms converging to different equilibria by obtaining an appropriate (belief) function $\varphi_k[n]$ to model various level of cognition ability of the cognitive radio user. The introduction of the interference derivative $\varphi_k[n]$ is significant because this provides the parameter that, if properly chosen, can guide the negotiation process between the users to reach a desirable equilibrium. This is the form we use to carry out the FWF allocation if the interference derivative $\varphi_k[n]$ is known.

For NE as an example, $\varphi_k[n] = 0$ is considered (because the user believes that $c_k^{\mathsf{B}}[n]$ is fixed and does not depend on the strategy $p_k[n]$ or $\varphi_k[n] = \frac{\partial c_k^{\mathsf{B}}[n]}{p_k[n]} = 0$) and (28) can therefore be greatly simplified as

$$p_k[n] = (w_k - \sigma_k[n] - I_k[n])^+. \tag{30}$$

IV. EQUILIBRIUM VERSUS BELIEF

A. NE

At NE, $\mathsf{p}_k^* = \mathsf{BP}(\mathsf{I}_k^*)$ and $\mathsf{p}_{-k}^* = \mathsf{BP}_{-k}(\mathsf{p}_k^*)$, so the strategy profile is written as $\mathsf{P}^* = \{\mathsf{BP}(\mathsf{I}_k^*), \mathsf{BP}_{-k}(\mathsf{p}_k^*)\}$. Additionally, NE can be formulated formally as

$$R_k(\mathsf{p}_k^*, \mathsf{BP}_{-k}(\mathsf{p}_k^*)) \geq R_k(\mathsf{p}_k, \mathsf{BP}_{-k}(\mathsf{p}_k^*)), \quad \forall \mathsf{p}_k \in \mathcal{P}_k \text{ and } \forall k \in \{1, 2, \ldots, K\}. \tag{31}$$

User k's data rate, R_k^*, is therefore given by

$$R_k^* = \max_{\mathsf{p}_k \in \mathcal{P}_k} \sum_n \log_2\left(1 + \frac{p_k[n]}{\sigma_k[n] + I_k^*[n]}\right) \tag{32}$$

$$= \max_{\mathsf{p}_k \in \mathcal{P}_k} \sum_n \log_2\left(1 + \frac{p_k[n]}{\sigma_k[n] + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^*))[n]}\right). \tag{33}$$

Note that the above maximization is done under a fixed interference pattern I_k^*. As mentioned before, achieving NE will have $\varphi_k[n] = \frac{\partial I_k^*[n]}{\partial p_k[n]} = 0$ and

$$\mathsf{p}_k^* = \mathsf{BP}(\mathsf{I}_k(\mathsf{BP}_{-k}(\mathsf{p}_k^*))) \quad \forall k. \tag{34}$$

This can be achieved by the iterative (multiuser) WF algorithm as follows:

$$p_k^t + \sigma_k + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^{t-1})) = w_k^t, \tag{35a}$$

$$p_k^{t+1} + \sigma_k + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^t)) = w_k^{t+1}, \tag{35b}$$

$$\vdots$$

$$p_k^* + \sigma_k + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^*)) = w_k^*. \tag{35c}$$

In the above, the subcarrier index $[n]$ has been omitted for convenience. The iterations (35) follow largely from the single-user WF solution in (30) but the only difference is that the interference pattern seen by user k changes due to a new response from other users before reaching to the steady state.

Every user in the equilibrium is deemed self-optimal giving the highest rate, w.r.t. other users' steady-state power allocation. Thus, a user does not gain by deviating from the equilibrium. However, NE can be overly competitive leading to a very suboptimal power game equilibrium.

B. SE

It is possible to reach a different equilibrium that to some user allows better reconciliation between users to outperform NE. One such equilibrium is SE [23] where one user is regarded as the Stackelberg leader, say user κ, who can maximize its own rate, R_κ, by finding its optimal strategy knowing that other

$$\frac{\partial \mathcal{L}}{\partial p_k[n]} = \frac{1}{\sigma_k[n]+I_k[n]+p_k[n]}\left(\frac{\sigma_k[n]+I_k[n]-p_k[n]\frac{\partial I_k[n]}{\partial p_k[n]}}{\sigma_k[n]+I_k[n]}\right)-\lambda \tag{26}$$

users (regarded as followers) are myopic and will respond by their (Nash-like) best power allocation.

Mathematically, we have

$$\begin{cases} R_\kappa(\mathsf{p}_\kappa^*, \mathsf{BP}_{-\kappa}(\mathsf{p}_\kappa^*)) \\ \quad \geq R_\kappa(\mathsf{p}_\kappa, \mathsf{BP}_{-\kappa}(\mathsf{p}_\kappa)), \quad \forall \mathsf{p}_\kappa \in \mathcal{P}_\kappa, \\ R_k(\mathsf{p}_k^*, \mathsf{BP}_{-k}(\mathsf{p}_k^*)) \\ \quad \geq R_k(\mathsf{p}_k, \mathsf{BP}_{-k}(\mathsf{p}_k^*)), \quad \forall \mathsf{p}_k \in \mathcal{P}_k \text{ and } k \neq \kappa. \end{cases} \tag{36}$$

As a result, user κ's data rate, R_κ^*, can be formulated as

$$R_\kappa^* = \sum_n \log_2\left(1+\frac{p_\kappa^*[n]}{\sigma_\kappa[n]+I_\kappa^*[n]}\right) \tag{37}$$

$$= \max_{\mathsf{p}_\kappa \in \mathcal{P}_\kappa} \sum_n \log_2\left(1+\frac{BP(\mathsf{I}_\kappa(\mathsf{BP}_{-\kappa}(\mathsf{p}_\kappa)))[n]}{\sigma_\kappa[n]+I_\kappa(\mathsf{BP}_{-\kappa}(\mathsf{p}_\kappa))[n]}\right). \tag{38}$$

In the literature, when solving (38), a brute-force optimization is usually needed, which is not only prohibitively complex but also that no known method exists to play the leader to reach SE. Using the belief-directed game as a tool, however, one could choose $\varphi_\kappa^t[n]$ appropriately to achieve that. Note that $\varphi_\kappa^t[n]$ is, by no means, the actual interference derivative of the game (which is usually impossible to know) but, when chosen, it carries the operational meaning to force the game to behave a certain way. In this paper, our interpretation is that through $\varphi_\kappa^t[n]$, users compete in a calculated fashion.

Theorem 2: At the equilibrium of SE, the leader, user κ, satisfies

$$-\frac{c_\kappa^*[n]}{2c_\kappa^*[n]+p_\kappa^*[n]} \leq \varphi_\kappa^*[n] \leq \frac{c_\kappa^*[n]}{p_\kappa^*[n]}. \tag{39}$$

Proof: See Appendix B. □

The following theorem studies the uniqueness of SE in the belief-directed game.

Theorem 3: If $\varphi_\kappa^t[n]$ exists and it satisfies (39) for all time $t > 0$, then SE is unique and user κ achieves the maximum achievable rate given that all other users are myopic.

Proof: See Appendix C. □

The result of Theorem 2 suggests that the belief for the leader κ in the form of interference derivative be chosen to satisfy (39). Note that there are likely many choices of $\varphi_\kappa^t[n]$ that can all permit the users to converge to SE. Nevertheless, we choose

$$\varphi_\kappa^t[n] = -\frac{c_\kappa^t[n]}{2c_\kappa^t[n]+p_\kappa^t[n]} \quad \forall n. \tag{40}$$

There are a few observations we can make to understand why this choice is good. First of all, to have a judicious belief on $\varphi_\kappa^t[n]$, the interference derivative should be negative, or $\varphi_\kappa^t[n] \leq 0$ because it then has a strong tendency to invest power on a good subcarrier. On the contrary, if $\varphi_\kappa^t[n] > 0$, this will imply that the more the power it allocates for a subcarrier the more the interference it gets, thus ending up occupying a poor subcarrier at the equilibrium. On the other hand, it has been shown in Theorem 3 that as long as $\varphi_\kappa^*[n]$ exists and (39) is true, the equilibrium is unique and therefore, any value in this interval could be a sensible choice for the interference derivative $\varphi_\kappa^t[n]$. However, it should also be noted that the magnitude of $\varphi_\kappa^t[n]$ reflects the forward-looking capability, or if $\varphi_\kappa^t[n] = 0$, the user is myopic and not forward-looking. As a result, (40) is expected to be a proper choice for maximizing user κ's rate. In addition, setting (40) in the FWF solution of the leading user κ for all time $t > 0$ will force the game to behave as if the actual interference derivative of the game satisfies (39) even though it may actually be different.

Specifically, to reach the leader-followers SE, the leader (i.e., user κ) can therefore play the FWF power allocation (28) with appropriately chosen $\{\varphi_\kappa[n]\}$ given by (40) while the followers (i.e., the remaining users) respond by the traditional WF (30) with $\varphi_k[n] = 0$. As such, the following iterative FWF algorithm can be used to let the users compete to converge to the SE:

$$\text{Leader } \kappa \begin{cases} p_\kappa^t + \eta_\kappa^t(\mathsf{p}_\kappa^{t-1}) = w_\kappa^t \\ p_\kappa^{t+1} + \eta_\kappa^t(\mathsf{p}_\kappa^t) = w_\kappa^{t+1} \\ \qquad \vdots \\ \quad p_\kappa^* + \eta_\kappa^* = w_\kappa^* \end{cases} \tag{41}$$

and for all followers $k \neq \kappa$, we have

$$\begin{cases} p_k^t + \sigma_k + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^{t-1})) = w_k^t, \\ p_k^{t+1} + \sigma_k + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^t)) = w_k^{t+1}, \\ \qquad \vdots \\ \quad p_k^* + \sigma_k + I_k(\mathsf{BP}_{-k}(\mathsf{p}_k^*)) = w_k^*. \end{cases} \tag{42}$$

Note that $\eta_\kappa^t[n] \triangleq \frac{(c_\kappa^t[n])^2+\varphi_\kappa^t[n](p_\kappa^t[n])^2}{c_\kappa^t[n]-\varphi_\kappa^t[n]p_\kappa^t[n]}$ (which was defined in (27) earlier) using (40).

C. EQUILIBRIUM FROM ALL FORWARD-LOOKING RADIOS

We have now shown that both NE and SE can be achieved autonomously using belief-directed games as each individual user simply needs to perform its FWF power allocation and their simultaneous interactions, once converged, will lead to the results. Therefore, spectrum sharing is done in a self-organising and autonomous way. However, so far, only the Stackelberg leader is forward-looking, while the Nash users are all myopic. In order to let all users become forward-looking, all $\{\varphi_k[n]\}$ need to be set non-zero.

Mathematically, the equilibrium for an all forward-looking cognitive radio game can be written as

$$R_k(\mathsf{p}_k^*, \mathsf{BP}_{-k}(\mathsf{p}_k^*)) \geq R_k(\mathsf{p}_k, \mathsf{BP}_{-k}(\mathsf{p}_k)), \quad \forall \mathsf{p}_k \in \mathcal{P}_k \text{ and } k = 1, 2, \ldots, K. \quad (43)$$

We refer to this equilibrium as forward-looking equilibrium (FE). It is possible to derive similar results like (40) in SE for FE. We present such results in the following theorems.

Theorem 4: The interference derivatives for FE, $\{\varphi_k^*[n]\}$, satisfy

$$\varphi_k^*[n] = \sum_{\substack{i=1\\i\neq k}}^{K} \frac{\mathsf{sgn}(p_i^*[n])\theta_{ik}[n]}{\varphi_i^*[n]} \frac{\partial I_i^*[n]}{\partial p_k^*[n]}. \quad (44)$$

Proof: First, by definition, we have

$$\varphi_k^*[n] = \frac{\partial I_k^*[n]}{\partial p_k^*[n]} = \sum_{\substack{i=1\\i\neq k}}^{K} \mathsf{sgn}(p_i^*[n])\theta_{ik}[n] \frac{\partial p_i^*[n]}{\partial p_k^*[n]} \quad (45)$$

$$= \sum_{\substack{i=1\\i\neq k}}^{K} \mathsf{sgn}(p_i^*[n])\theta_{ik}[n] \frac{\partial p_i^*[n]}{\partial I_i^*[n]} \frac{\partial I_i^*[n]}{\partial p_k^*[n]}, \quad (46)$$

where $\frac{\partial p_i^*[n]}{\partial I_i^*[n]} = \left(\frac{\partial I_i^*[n]}{\partial p_i^*[n]}\right)^{-1} = (\varphi_i^*[n])^{-1}$, which directly yields (44) and completes the proof. □

In the above proof, the physical meaning to have $\frac{\partial p_i^*[n]}{\partial I_i^*[n]} = \left(\frac{\partial I_i^*[n]}{\partial p_i^*[n]}\right)^{-1} = (\varphi_i^*[n])^{-1}$ is that since users are all forward looking, their strategies are to reverse any changes deviating from the equilibrium to maintain the equilibrium (the most beneficial outcome). Theorem 4 provides a key property to facilitate exchange of network-wide information between the forward-looking cognitive radios through competition.

Before we derive the belief, i.e., $\varphi_k^t[n]$, to approach FE, the following proposition is useful.

Proposition 1: To achieve FE, we propose to construct a game to satisfy the physical property:

$$\sum_{\substack{i=1\\i\neq k}}^{K} \mathsf{sgn}(p_i^*[n])\mathsf{sgn}(p_k^*[n])\theta_{ik}[n]\theta_{ki}[n] \leq \frac{c_k^*[n]}{2c_k^*[n] + p_k^*[n]}. \quad (47)$$

Proof: See Appendix D. □

Proposition 2: To achieve FE, the interference derivative can be made to satisfy

$$\varphi_k^*[n] \geq -\sqrt{\frac{c_k^*[n]}{2c_k^*[n] + p_k^*[n]}} \quad \forall k, n. \quad (48)$$

Proof: See Appendix F. □

It is worth noting that (48) is a *weaker* property for FE, a result deduced from the original property (44). Satisfying (48) does not necessarily imply (44). Nonetheless, it does bring the possibility of network-wide forward-looking-ness by competition between cognitive radios based on local channel observation and will therefore make self-optimization realizable by the following proposition.

Proposition 3: A self-optimizing OFDMA cognitive radio network approaching to FE can be constructed by FWF such that the power allocation for user k at time t is updated by

$$p_k^t[n] = \left(w_k^t - \frac{(c_k^t[n])^2 + \varphi_k^t[n]\left(p_k^{t-1}[n]\right)^2}{c_k^t[n] - \varphi_k^t[n]p_k^{t-1}[n]}\right)^+,$$

$$\text{where } \varphi_k^t[n] = -\sqrt{\frac{c_k^t[n]}{2c_k^t[n] + p_k^{t-1}[n]}} \quad \forall k. \quad (49)$$

The above self-optimization follows the same rationale behind that for achieving SE described before. In particular, the operational interference derivative for carrying out FWF is chosen to satisfy the environmental interference derivative property of FE (48). Again, being a self-optimization, the equilibrium can be achieved by simultaneous FWF in any order. The following theorem studies its uniqueness.

Theorem 5: If $\varphi_k^t[n] > -\frac{c_k^t[n]}{4c_k^t[n]+p_k^t[n]}$ $\forall k, n$ such that $p_k^t[n] > 0$ and this is true for all time $t > 0$, then the game has a unique FE and the FWF in (49) achieves the unique equilibrium and is also rate-optimal.

Proof: See Appendix G. □

D. CONVERGENCE ANALYSIS

The cognitive power-interference OFDMA game can be generally analyzed by recognizing that

$$\begin{cases} \Delta p_k^t[n] = \rho_k^t[n]\Delta I_k^t[n], \quad \text{for some } \rho_k^t[n] \leq 0 \\ \quad \text{(FWF reoptimization—observe and act)}, \\ \Delta I_k^{t+1}[n] = \varphi_k^t[n]\Delta p_k^t[n], \quad \text{for some } \varphi_k^t[n] \leq 0 \\ \quad \text{(environmental response—act and react)}, \end{cases} \quad (50)$$

where $\rho_k[n] \triangleq \frac{\partial p_k[n]}{\partial I_k[n]}$ is defined as the derivative for the power allocation w.r.t. the interference pattern.

In what follows, we can write

$$\Delta I_k^{t+1}[n] = \varphi_k^t[n]\Delta p_k^t[n] = \sum_{i\neq k} \Delta p_i^t[n]\theta_{ik}[n], \quad (51)$$

$$\begin{aligned}\Delta p_k^{t+1}[n] &= \rho_k^{t+1}[n]\Delta I_k^{t+1}[n] \\ &= \rho_k^{t+1}[n]\varphi_k^t[n]\Delta p_k^t[n] \\ &= \rho_k^{t+1}[n]\sum_{i\neq k} \Delta p_i^t[n]\theta_{ik}[n]. \end{aligned} \quad (52)$$

Convergence may take place in two possible cases: (*i*) $|\Delta p_k^{t+1}[n]| < |\Delta p_k^t[n]|$ for all $t \geq t_0$ for some $t_0 > 0$, and (*ii*) $|\Delta p_k^{t+1}[n]| > |\Delta p_k^t[n]|$ for all $t \geq t_0$ for some $t_0 > 0$, the latter of which belongs to the case of a strong interference channel. To proceed, we define $|\overline{\Delta p}^t[n]| \triangleq \max_k |\Delta p_k^t[n]|$, $|\underline{\Delta p}^t[n]| \triangleq \min_k |\Delta p_k^t[n]|$, $|\overline{\rho}^t[n]| \triangleq \max_k |\rho_k^t[n]|$, $|\underline{\rho}^t[n]| \triangleq \min_k |\rho_k^t[n]|$, $|\overline{\theta}[n]| \triangleq \max_{i\neq j} |\theta_{ij}[n]|$ and $|\underline{\theta}[n]| \triangleq \min_{i\neq j} |\theta_{ij}[n]|$. Our objective here is to show that it is possible to obtain sufficient conditions for

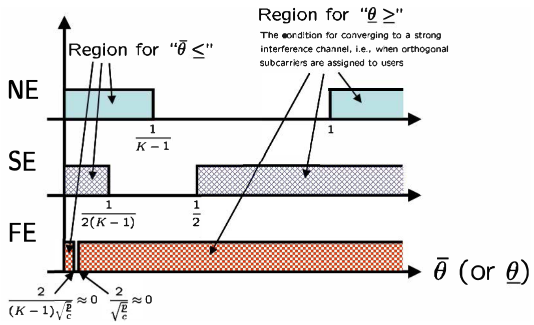

FIGURE 3. **The sufficient convergence regions according to Lemma 1.**

convergence in terms of the channel parameters $|\overline{\theta}[n]|$ and $|\underline{\theta}[n]|$. To see this, we first consider (*i*) by

$$|\overline{\Delta p^{t+1}}[n]| \overset{(a)}{=} \max_k \left| \rho_k^{t+1}[n] \sum_{i\neq k} \Delta p_i^t[n]\theta_{ik}[n] \right| \quad (53)$$

$$\leq |\overline{\rho^{t+1}}[n]||\overline{\Delta p^t}[n]|\overline{\theta}[n](K-1), \quad (54)$$

where (*a*) is due to (52). Therefore, a sufficient condition for convergence is to have

$$|\overline{\rho^{t+1}}[n]|\overline{\theta}[n](K-1) < 1 \Rightarrow \overline{\theta}[n] < \frac{1}{(K-1)|\overline{\rho^{t+1}}[n]|} \quad (55)$$

because this will guarantee $|\overline{\Delta p^{t+1}}[n]| < |\overline{\Delta p^t}[n]|\ \forall k$, leading to $\Delta p_k^{\infty}[n] = 0\ \forall k$.

On the other hand, for (*ii*) $|\Delta p_k^{t+1}[n]| > |\Delta p_k^t[n]|$, this will converge as well since $|\Delta p_k^t[n]|$ is finite due to the total power constraint, P_k. This can be interpreted as the scenario where a user, say k, decides to allocate all its power to given subcarriers (if $\Delta p_k^t[n] > 0$) or to withdraw all its power (if $\Delta p_k^t[n] < 0$). The former illustrates the case that user k wants to take over the subcarriers and others will have a strong tendency to leave the subcarriers due to severe interference caused by user k. The consequence is that those subcarriers will be occupied by user k *only*, and hence, $\varphi_k^t[n] = 0$, resulting the strong interference channel studied in [42], [43]. To obtain a sufficient condition for such convergence, we consider

$$|\Delta p_k^{t+1}[n]| = \left| \rho_k^{t+1}[n] \sum_{i\neq k} \Delta p_i^t[n]\theta_{ik}[n] \right|$$

$$\geq |\underline{\rho^{t+1}}[n]||\underline{\Delta p^t}[n]|\underline{\theta}[n] \quad \forall k, \quad (56)$$

where we used the fact that $\{\Delta p_i^t[n]\}_{\forall i}$ are all of the same sign (all positive or all negative). As such, it will converge to a strong interference channel if

$$|\underline{\rho^{t+1}}[n]|\underline{\theta}[n] > 1 \Rightarrow \underline{\theta}[n] > \frac{1}{|\underline{\rho^{t+1}}[n]|}. \quad (57)$$

Fig. 3 illustrates the convergence regions based on the two sufficient conditions above.

The convergence behaviors for various equilibria can thus be analyzed via $\rho_k^t[n]$ (for $p_k^t[n] > 0$):

$$\rho_k^t[n] = \frac{\Delta p_k^t[n]}{\Delta I_k^t[n]} = \frac{\Delta(w_k^t - \eta_k^t[n])^+}{\Delta I_k^t[n]}$$

$$= -\frac{\Delta \eta_k^t[n]}{\Delta I_k^t[n]} = -\frac{\Delta \eta_k^t[n]}{\Delta c_k^t[n]}. \quad (58)$$

As a result, we have the following lemma for evaluating $\rho_k^t[n]$ of NE, SE and FE.

Lemma 1: Based on (58), we have (59) (see next page).

Proof: See Appendix H. □

For NE, $|\rho_k^t[n]| = 1$ and a sufficient convergence condition is $\overline{\theta}[n] < \frac{1}{K-1}$, as reported in [20], [44]–[46]. Also, our analysis shows that $\underline{\theta}[n] > 1$ will ensure convergence and in this case a given subcarrier will be occupied by one user only, which aligns with the result for the strong interference channel in [42], [43]. Since $|\rho_k^t[n]| = 2$ for SE, it has a similar, but slightly better, convergence behavior than NE.

For FE, as opposed to NE and SE, $|\rho_k^t[n]|$ can take very different values depending on $\frac{p_k^{t-1}[n]}{c_k^t[n]}$. If $\frac{p_k^{t-1}[n]}{c_k^t[n]}$ is large, then $|\rho_k^t[n]|$ will be very large and thus convergence will tend to occur on the second region $\underline{\theta}[n] \gtrsim 0$, in which case according to our analysis users at the equilibrium will be made orthogonal.

V. FE FOR HIERARCHICAL COGNITIVE OFDMA

Here, we apply the FE approach (realized by FWF in (49)) for autonomous resource allocation of a group of SUs who opportunistically access the spectrum holes left vacant by the PUs of the spectrum.

A. MODEL AND THE COGNITIVE FE GAME

Our model assumes an N-subcarrier OFDMA system serving M PUs in the presence of K SUs (each with a total power constraint P_k). The SUs have no prior knowledge of which subcarriers are used by the PUs and which by any other SUs. Let $q_m[n]$ and $p_k[n]$ denote, respectively, the power allocated by the mth PU and that by the kth SU on the nth subcarrier. Also, we denote the channel between PU m and SU k on subcarrier n as $\tilde{H}_{mk}[n]$, while the channel between SU k and SU j is denoted by $H_{kj}[n]$. With $N_k[n]$ being the noise power at SU k on subcarrier n, the achievable rate for SU k is

$$R_k^{\mathsf{SU}} = \sum_{n=1}^{N} \log_2 \left(1 + \frac{p_k[n]}{\sigma_k[n] + I_k[n] + \tilde{I}_k[n]} \right) \quad (60)$$

where $\sigma_k[n]$ and $I_k[n]$ are defined as before, and $\tilde{I}_k[n] \triangleq \sum_{m=1}^{M} q_m[n]\tilde{\theta}_{mk}[n]$ denotes the interference caused by the PUs on subcarrier n seen by SU k with $\tilde{\theta}_{mk}[n] \triangleq \frac{|\tilde{H}_{mk}[n]|^2}{|H_{kk}[n]|^2}$. Also, we define the overall noise as

$$c_k[n] \triangleq \sigma_k[n] + I_k[n] + \tilde{I}_k[n]. \quad (61)$$

Similarly, we also have the total interference on subcarrier n seen by PU m caused by SUs as $\tilde{\tilde{I}}_m[n] = \sum_{k=1}^{K} p_k[n]\tilde{\theta}_{km}[n]$.

$$|\rho_k^t[n]| = \left|\frac{\Delta\eta_k^t[n]}{\Delta c_k^t[n]}\right| = \begin{cases} 1 & \text{if } \varphi_k^t[n] = 0 \text{ (NE)}, \\ 2 & \text{if } \varphi_k^t[n] = -\frac{c_k^t[n]}{2c_k^t[n]+p_k^{t-1}[n]} \text{ (SE with user } k \text{ as leader)}, \\ \frac{1}{2}\sqrt{\frac{p_k^{t-1}[n]}{c_k^t[n]}} & \text{if } \varphi_k^t[n] = -\sqrt{\frac{c_k^t[n]}{2c_k^t[n]+p_k^{t-1}[n]}} \text{ when } \frac{p_k^{t-1}[n]}{c_k^t[n]} \text{ is large (FE)}, \end{cases} \tag{59}$$

Ideally, for those $n \in \mathcal{Q} \triangleq \{n|q_m[n] > 0\}$, i.e., the subcarriers used by PUs, the SUs should have $p_k[n] = 0$. We assume that the number of subcarriers unused by the PUs is greater than K.

FE is a realization that all users adapt their strategies knowing how others respond based on the belief established from the property (49). Therefore, the equilibrium for the optimization of SUs is given as

$$R_k^{\mathsf{SU}}(\mathsf{p}_k^*, \mathsf{BP}_{-k}(\mathsf{p}_k^*)) \geq R_k^{\mathsf{SU}}(\mathsf{p}_k, \mathsf{BP}_{-k}(\mathsf{p}_k)), \quad \forall \mathsf{p}_k \in \mathcal{P}_k \text{ and } k=1,2,\ldots,K, \tag{62}$$

which means that any SU will not gain (in rate) by deviating from the FE based on the belief.

Proposition 3 is used for the SUs to carry out FWF simultaneously to approach the equilibrium.

B. WATER-LEVEL ANALYSIS AS PROTECTION INDICATOR FOR PUS

The water-level at the steady-state is the only parameter that decides if an SU stays or leaves any given subcarriers. Here, we analyze the water-level of the cognitive FE game.

Corollary 1: If $c_k[n] \geq w_k^*$, then $p_k[n] = 0$, meaning that subcarrier n will not be taken by SU k.

Proof: See Appendix I. □

For each subcarrier, in FWF, $\eta_k[n]$ is compared with the water-level and if it is smaller than the water-level, the subcarrier will be taken by SU k; otherwise, it will be left out. It is worth pointing out that $\eta_k[n]$ has the meaning of being a "strategic" or "interactive" noise because it couples the overall channel noise with the responses from the environment (i.e., $\varphi_k[n]$) due to the power allocation by that user. For the best interest of the PUs, the water-level for SU k, w_k^*, should be kept as low as possible. Ideally, if the water-level is the noise floor, then SU k will only use spectrum holes.

In Section V-D, it is found that users in the OFDMA FE game will be made orthogonal or converge to a strong interference channel [42], [43] if the given channel satisfies, see (57) and Lemma 1,

$$\min_{\substack{j,k,n \\ j\neq k}} \theta_{jk}[n] > \left(\min_{k,n}\left|\frac{\partial \eta_k[n]}{\partial c_k[n]}\right|\right)^{-1} \approx \frac{2}{\min_{k,n}\sqrt{\frac{p_k[n]}{c_k[n]}}}. \tag{63}$$

As explained before, it is of interest to consider the case that every subcarrier chosen by a SU has a good quality channel for that SU, i.e., with high SNR. If this is the case, then the right-hand-side of (63) will be a very small number, and as such, for a given channel realization $\{\theta_{jk}[n]\}$, (63) will likely be met, leading FE to allocate orthogonal subcarriers to the SUs (hence, $I_k[n] = 0$ for those chosen subcarriers). This makes sense and is indeed possible in our setting because the number of subcarriers available for the SUs is greater than the number of competing SUs. It is also anticipated that for maximizing the SU rates, as required by FE, users should be harmonized, resulting in an interference-free channel.

Proposition 4: At the steady-state of FE, the water-level for SU k can be approximated by

$$w_k^* \approx \sqrt{c_k[n]p_k[n]} \text{ for some } n \text{ s.t. } \frac{p_k[n]}{c_k[n]} \gg 1. \tag{64}$$

Proof: See Appendix J. □

Proposition 5: The water-level of FE can be estimated as

$$w_k^* \approx N_0\sqrt{\frac{P_0}{N_0}\frac{K}{\tilde{N}}} \leq N_0\sqrt{\frac{P_0}{N_0}}, \tag{65}$$

where $N_k[n] = N_0, \forall k, n$, $P_k = P_0, \forall k$ and $\tilde{N}$ denotes the number of subcarriers leftover by the PUs. The water-level is independent of k as users are assumed statistically independent and identical.

Proof: See Appendix K. □

Note that the water-level of FE for the SUs is a scaled version of the noise floor, see (65).

Proposition 6: SU k at FE will avoid the PUs completely if

$$\min_{n\in\mathcal{Q}} \frac{\tilde{I}_k[n]}{N_0} > \sqrt{\frac{P_0}{N_0}\frac{K}{\tilde{N}}} - 1. \tag{66}$$

Proof: See Appendix L. □

This states that if the PUs are causing substantial interference, $\tilde{I}_k[n']$, to the SUs, then the SUs will tend to prevent those subcarriers because the PUs do not play the FE game and those interference cannot be mitigated by the SU strategies. As a matter of fact, the SUs have a very good sensitivity in protecting the PUs. For instance, with the average per-subcarrier SNR for an SU, $\frac{P_0}{N_0}\frac{K}{\tilde{N}} = 100$, as long as the PU-to-SU interference to noise ratio, $\frac{\tilde{I}_k[n']}{N_0} > 9$ (i.e., > 9.5dB), it will be sufficient to protect the PUs. This can be easily achieved if the PUs transmit high power for their active subcarriers. Intriguingly, it is also possible to impose an upper limit on $\frac{P_0}{N_0}$ (a channel-independent average SNR) for the SUs so that a higher protection (making (66) easier to meet) for the PUs can be achieved.

VI. SIMULATION RESULTS

In this section, we provide simulation results to evaluate the self-optimizing FWF algorithm for cognitive radios. The iterative spectrum balancing (ISB) method in [37], which is

TABLE 1. Number of times for divergence in 1000 independent simulations.

(K, N)	$(3, 9)$			$(4, 16)$			$(5, 15)$		
Equilibrium	NE	SE	FE	NE	SE	FE	NE	SE	FE
$x = 5$	0	0	0	5	2	0	9	7	0
$x = 0.8$	78	58	0	310	251	0	386	323	0
$x = 0.5$	23	25	0	115	122	0	200	203	0
$x = 0.2$	0	0	6	0	0	39	0	0	48

TABLE 2. Average number of iterations for convergence where convergence is said to achieve if the change in strategy for every subcarrier is smaller than 10^{-3}, with $x = 0.4$, $P_k = 100$, $N_k[n] = 0.01$ and $\frac{N}{K} = 3$.

(K, N)	$(2, 6)$	$(3, 9)$	$(4, 12)$	$(5, 15)$
NE	4.011	3.799	3.538	3.464
SE	9.169	7.978	6.545	5.428
FE	62.856	61.058	59.222	57.532

TABLE 3. Average users' sum-rates for the 3-user 9-subcarrier interference channel.

Sum-rate	User 1	User 2	User 3
NE	13.9944	13.8966	13.8021
SE	16.5275	15.5493	15.4743
FE	32.6641	32.8518	33.0399

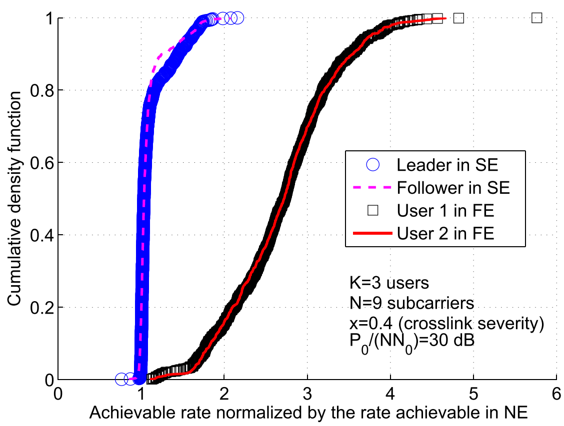

FIGURE 4. The cumulative density functions for the rate-ratios. Results for only two users are shown.

a near-optimal *centralized* algorithm requiring full network CSI, is used as a performance upper bound, if provided. In the simulations, we model each subcarrier by an equal-power four-ray frequency selective Rayleigh fading channel [47]

$$|H_{ij}[n]|^2 = |h_{ij}^{(1)}[n]|^2 + |h_{ij}^{(2)}[n]|^2 + |h_{ij}^{(3)}[n]|^2 + |h_{ij}^{(4)}[n]|^2 \tag{67}$$

and it is assumed that $\mathbb{E}[|H_{ij}[n]|^2] = x$ and $\mathbb{E}[|h_{ij}^{(l)}[n]|^2] = 0.25x$ $\forall l$, for $i \neq j$. For $i = j$, we set $\mathbb{E}[|H_{kk}[n]|^2] = 1$ and $\mathbb{E}[|h_{kk}^{(l)}[n]|^2] = 0.25$ $\forall \ell$. Hence, to help with the discussion, we use the parameter x to measure the relative severity of the interference channel. If x is larger, the interference is more and vice versa.

A. CONVERGENCE

Table 1 provides the likelihood results of convergence for different FWF intending for different equilibria with

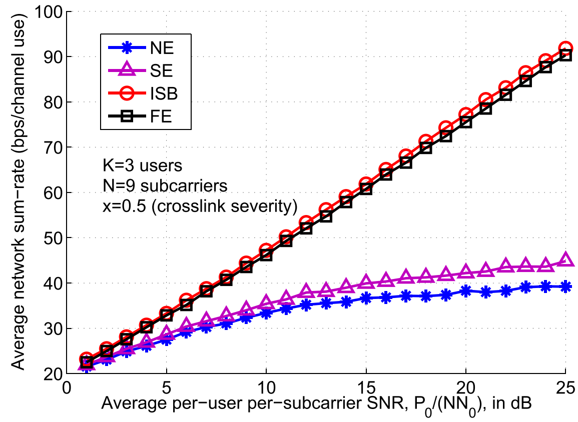

FIGURE 5. The average sum-rate comparison against the SNR.

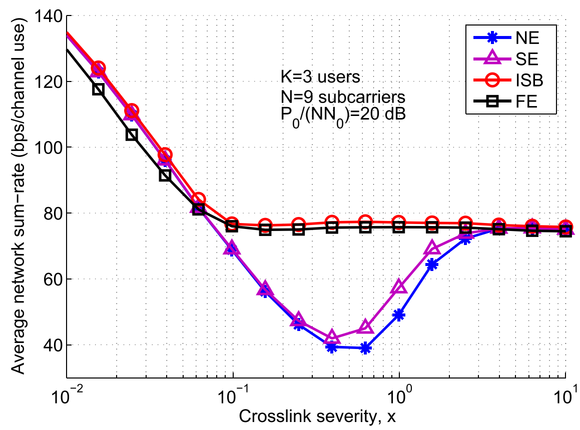

FIGURE 6. The average system sum-rate comparison for various x.

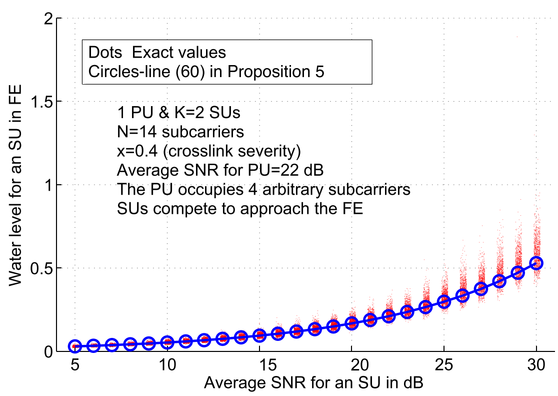

FIGURE 7. The water-level at FE against the SU SNR.

$P_k = 100$ $\forall k$ and $N_k[n] = 0.01$ $\forall k, n$. As shown, there is a small percentage of cases where NE and SE diverge and as the number of users increases, this will become more problematic. In contrast, FE always converges except when x is small meaning that the channel has only very weak crosstalk. This is because for small x, noise becomes the dominant effect which does not react to the players' actions but users in FE tend to interpret the overall noise as interference due to other users. As such, there will be a small percentage of times

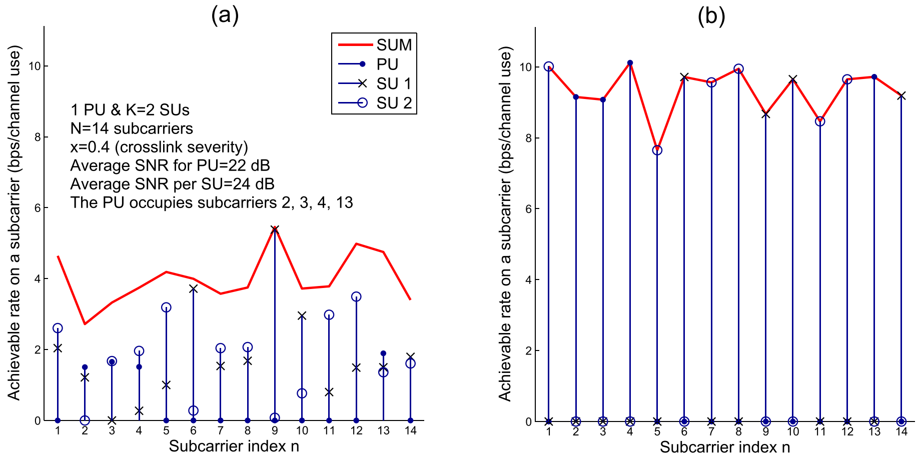

FIGURE 8. **Rate allocation comparison for NE and FE. (a) NE. (b) FE.**

(less than 5%) for FE to diverge. In terms of convergence speed, nevertheless, FE generally would take longer to converge than NE and SE, see the results in Table 2.

B. RATE PERFORMANCE

In Table 3, results are provided for the average users' sum-rates for a 3-user 9-subcarrier interference channel, with $P_k = 100\ \forall k$, $N_k[n] = 0.01\ \forall k, n$ and $x = 0.4$. Results show that there is a considerable gain in the sum-rates using FE over NE and SE. The cumulative density functions for the sum-rate ratios between FE and NE users and between SE and NE users are given in Fig. 4. We see that for FE it is possible to have the sum-rate 5 times greater than what is achieved by NE. Furthermore, we compare the rate performance between FE and ISB [37] in Fig. 5 for $x = 0.5$ and various SNR which is defined as $\frac{P_0}{NN_0}$ where $P_k = P_0\ \forall k$, $N_k[n] = N_0\ \forall k, n$. Results show that FE achieves nearly the same average sum-rate as ISB and significantly outperforms SE and NE in terms of rates for the entire range of SNR.

To provide a more complete comparison between NE, SE, FE and ISB, in Fig. 6, we provide the average system sum-rate results for various x (various severity of interference between the users). A 3-user 9-subcarrier channel with $P_k = 100\ \forall k$, $N_k[n] = 0.01\ \forall k, n$ and x from 0.001 to 10 is considered. We discuss the results by inspecting three separate regions: (1) $x \leq 0.06$, (2) $0.06 < x \leq 4$ and (3) $x > 4$. In Region (1), x is so small that the channel is reduced to K parallel single-user OFDMA channels. In this case, users do not interfere with each other and NE, SE and FE all achieve sum-rate performance close to the centralized ISB, with FE being slightly inferior than NE and SE. Basically, all the game-theoretic approaches operate under an assumption that the players' strategies affect the interference patterns the users see but for small x, this is no longer true or at least not necessary. For this reason, since FE users are using an overly aggressive strategy in playing, they are slightly inferior than NE and SE users.

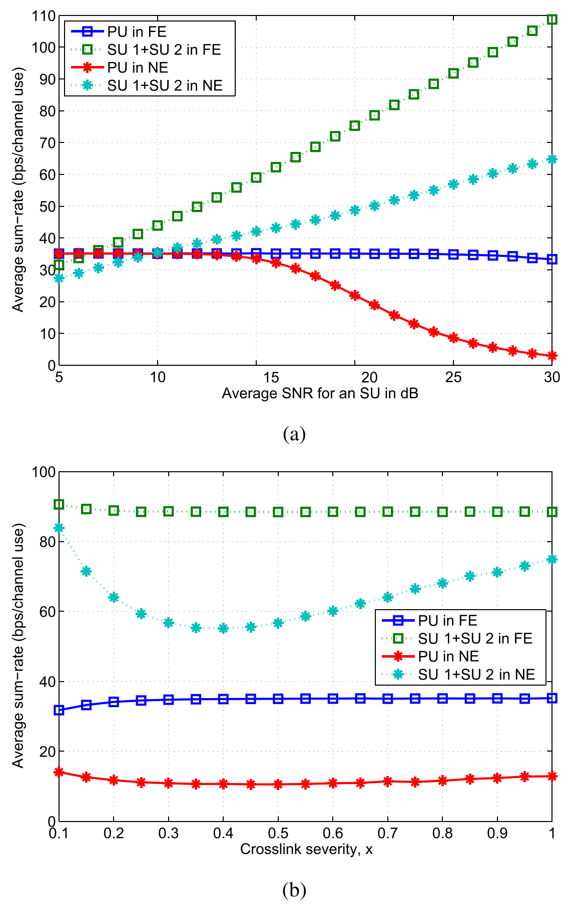

FIGURE 9. **The average sum-rate results comparison: (a) Rate against the SNR per SU in dB with $x = 0.4$ and (b) rate against the crosslink severity x with the SNR per SU being 24 dB.**

Region (2) covers the most typical scenarios of interference channels. In this region, the performance differences between ISB and game-theoretic equilibria become much more significant. In particular, the average sum-rates of NE and SE are considerably compromised, while FE is able to achieve the sum-rate close to that of ISB. Finally, in Region (3), NE, SE and FE all appear to converge to the same performance which is also very close to that for ISB. This is because when x is very large (i.e., the interference links are much stronger than the desired links), this forces to completely avoid sharing any subcarriers, even for NE and SE [42], [43]. Consequently, they all perform equally and as well as ISB.

C. PU-SU COEXISTENCE

Next, we evaluate the performance of the proposed FE game for cognitive OFDMA in the presence of PUs. In the simulations, we considered 14 subcarriers (i.e., $N = 14$) and four randomly chosen subcarriers were occupied by a PU with a fixed transmit power $q[n] = 5$ on each such subcarrier. Two SUs ($K = 2$) were seeking to access the subcarriers. They do not have prior knowledge on who is occupying a particular subcarrier, but should ideally compete only with each other but not the PU. For convenience, we assumed that $P_k = P_0\ \forall k\ N_k[n] = N_0\ \forall k, n$ and as before modeled each crosstalk link by an equal-power four-ray Rayleigh channel with $\mathbb{E}[|H_{jk}[n]|^2] = \mathbb{E}[|\tilde{H}_{mk}[n]|^2] = x$, for $j \neq k$. The SNR for an SU is defined as $\frac{P_0}{NN_0}$.

Fig. 7 shows the numerical values of many independent runs of FE for various SNR assuming $x = 0.4$ and $N_0 = 0.01$. Results illustrate that our estimate for the water-level by (65) is accurate for the entire range of the SU SNR. On the other hand, results in Fig. 8 provide an example of rate allocation for an arbitrary channel realization, which shows that the SUs in FE manage to avoid the subcarriers used by the PU and each other automatically, whereas the SUs in NE mostly collide with each other and jam the PU. Fig. 9 further compares the FE and NE as the SU strategies in terms of the average sum-rates for both the PU and the SUs. As we can see, FE can achieve a much higher sum-rate for the SUs than NE, while at the same time having a much stronger protection for the PU. In particular, the PU sum-rate is unaffected by the SUs in FE but can be severely penalized by the SUs in NE for high SNRs.

VII. CONCLUSION

This paper introduced brain power to cognitive radios by way of some belief on the interference derivative of the resource allocation game, which in turn allows them to project their achievable rates after competition w.r.t. their actions before they actually compete. This new formulation has led to WF power allocation at the cognitive radio that is forward looking, hence referred to as FWF. Based on this formulation, we studied the properties of the interference derivative for the rate-maximizing equilibrium and proposed to tune the belief of cognitive radios based on these properties. Our interpretation for the FWF power allocation using the chosen belief is that with the forward-looking ability, cognitive radios can gain network-wide information to negotiate the spectrum resources among themselves by a sequence of calculated competitions, thereby converging to a much more efficient equilibrium. Simulation results have illustrated some promises of this approach and in particular, an autonomous OFDMA using the proposed FE approach, with each cognitive radio possessing only local CSI, can achieve nearly the same network sum-rate as that for the centralized ISB solution requiring full network CSI. We have also applied this method to the hierarchical cognitive radio network where SUs compete among themselves for the spectrum holes left vacant by the PUs. Results also have revealed that forward-looking SUs can settle on their good subcarriers to maximize their rates while completely avoiding each other and the PUs, even not knowing which subcarriers are occupied by the PUs at the beginning. As a side product, we have also shown that we can interpret NE and SE using the FE framework with different choices of belief on the interference derivative. This may provide routes to discover new understanding of those games. The last concluding remark we wish to make is that although the proposed FE approach is believed to have made a big step towards making cognitive radio practical, there are indeed challenging issues remained to be addressed. One is the relatively slow convergence speed to FE that needs improving. Another is the performance fluctuation for all users before convergence for any game-theoretic approaches. During the cycles of updating the strategies from SUs before convergence, the PUs are not protected. It is hoped that this paper can spark more interest on these issues.

APPENDIX A
PROOF OF THEOREM 1

The problem (20) can be solved by introducing the Lagrange multiplier λ, using a Lagrangian multiplier formulation

$$\mathcal{L} = \sum_{n=1}^{N} \ln\left(1 + \frac{p_k[n]}{c_k^{\mathsf{B}}(\mathsf{p}_k, \mathsf{p}_{-k}^t)[n]}\right) - \lambda\left(\sum_{n=1}^{N} p_k[n] - P_k\right). \tag{68}$$

To proceed, we first obtain (69), as shown at the top of the next page. Then by setting $\left.\frac{\partial \mathcal{L}}{\partial p_k[n]}\right|_{p_k[n]=p_k^{t+1}[n]} = 0$ (which is the necessary condition for the maximization), and using the definitions $x_k^t[n] \triangleq c_k^{\mathsf{B}}(\mathsf{p}_k^{t+1}, \mathsf{p}_{-k}^t)[n]$ and $y_k^t[n] \triangleq \left.\frac{\partial c_k^{\mathsf{B}}[n]}{\partial p_k[n]}\right|_{p_k[n]=p_k^{t+1}[n]}$, we get

$$p_k^{t+1}[n] = \left(\frac{\frac{1}{\lambda}x_k^t[n] - (x_k^t[n])^2}{\frac{1}{\lambda}y_k^t[n] + x_k^t[n]}\right)^+, \tag{70}$$

where $(\cdot)^+$ ensures the positiveness of the power. Finally, renaming $\frac{1}{\lambda}$ as w_k^t yields the desired result.

APPENDIX B
PROOF OF THEOREM 2

Given that we have p_κ^* at the SE, we consider a power allocation strategy, $\tilde{\mathsf{p}}_\kappa$ that deviates from p_κ^* such that $\tilde{p}_\kappa[l] = p_\kappa^*[l]\ \forall l \neq m, n$, but $\tilde{p}_\kappa[n] = p_\kappa^*[n] + \Delta p > 0$ and $\tilde{p}_\kappa[m] = p_\kappa^*[m] - \Delta p > 0$ for some $1 \leq m \neq n \leq N$ and small $\Delta p > 0$. Denote the interference patterns for p_κ^* and $\tilde{\mathsf{p}}_\kappa$, respectively, as I_κ and $\tilde{\mathsf{I}}_\kappa$. Then we can write $\tilde{I}_\kappa[n] = I_\kappa[n] + \Delta I[n]$ and

$$\frac{\partial \mathcal{L}}{\partial p_k[n]} = \frac{1}{1+\frac{p_k[n]}{c_k^{\mathsf{B}}(\mathsf{p}_k,\mathsf{p}_{-k}^t)[n]}}\left(\frac{c_k^{\mathsf{B}}(\mathsf{p}_k,\mathsf{p}_{-k}^t)[n] - p_k[n]\frac{\partial c_k^{\mathsf{B}}(\mathsf{p}_k,\mathsf{p}_{-k}^t)[n]}{\partial p_k[n]}}{(c_k^{\mathsf{B}}(\mathsf{p}_k,\mathsf{p}_{-k}^t)[n])^2}\right) - \lambda \quad (69)$$

$$\log_2\left(1+\frac{p_\kappa^*[n]+\Delta p}{c_\kappa^*[n]+\Delta I[n]}\right) - \log_2\left(1+\frac{p_\kappa^*[n]}{c_\kappa^*[n]}\right) + \log_2\left(1+\frac{p_\kappa^*[m]-\Delta p}{c_\kappa^*[m]+\Delta I[m]}\right) - \log_2\left(1+\frac{p_\kappa^*[m]}{c_\kappa^*[m]}\right) \le 0 \quad (71)$$

$$\left(\frac{c_\kappa^*[n]+\Delta I[n]+p_\kappa^*[n]+\Delta p}{c_\kappa^*[n]+\Delta I[n]}\right)\left(\frac{c_\kappa^*[n]}{c_\kappa^*[n]+p_\kappa^*[n]}\right)\left(\frac{c_\kappa^*[m]+\Delta I[m]+p_\kappa^*[m]-\Delta p}{c_\kappa^*[m]+\Delta I[m]}\right)\left(\frac{c_\kappa^*[m]}{c_\kappa^*[m]+p_\kappa^*[m]}\right) \le 1 \quad (72)$$

$\tilde{I}_\kappa[m] = I_\kappa[m] + \Delta I[m]$ for some small $\Delta I[n]$, $\Delta I[m] \neq 0$. As user κ's rate is maximized by p_κ^* at SE, we therefore have $R_\kappa(\tilde{\mathsf{p}}_\kappa) - R_\kappa(\mathsf{p}_\kappa^*) \le 0$ implying (71), as shown at the top of this page. This can further be simplified to (72), as shown at the top of this page, which can be rewritten as

$$\left(1+\frac{\Delta p + \Delta I[n]}{c_\kappa^*[n]+p_\kappa^*[n]}\right)\left(1-\frac{\Delta p+\Delta I[m]}{c_\kappa^*[m]+p_\kappa^*[m]}\right) \le \left(1+\frac{\Delta I[n]}{c_\kappa^*[n]}\right)\left(1-\frac{\Delta I[m]}{c_\kappa^*[m]}\right). \quad (73)$$

Now, using (73) and noting that it is valid also after swapping m and n because subcarrier indices m and n are arbitrary, it can be easily shown after some manipulations that

$$\left(1-\frac{(\Delta I[n]+\Delta p)^2}{(c_\kappa^*[n]+p_\kappa^*[n])^2}\right)\left(1-\frac{(\Delta I[m]+\Delta p)^2}{(c_\kappa^*[m]+p_\kappa^*[m])^2}\right) \le \left(1-\frac{\Delta I[n]^2}{(c_\kappa^*[n])^2}\right)\left(1-\frac{\Delta I[m]^2}{(c_\kappa^*[m])^2}\right) \quad \forall m,n. \quad (74)$$

Therefore, we must have

$$\frac{|\Delta p + \Delta I[n]|}{c_\kappa^*[n]+p_\kappa^*[n]} \ge \frac{|\Delta I[n]|}{c_\kappa^*[n]} \quad \forall n. \quad (75)$$

Since $\Delta p > 0$, we analyze the following three possible cases:

Case 1: If $\Delta I[n] > 0$, then we have

$$\frac{\Delta p+\Delta I[n]}{c_\kappa^*[n]+p_\kappa^*[n]} \ge \frac{\Delta I[n]}{c_\kappa^*[n]} \Rightarrow \frac{c_\kappa^*[n]}{p_\kappa^*[n]} \ge \frac{\Delta I[n]}{\Delta p} > 0. \quad (76)$$

Case 2: If $\Delta I[n] \le 0$ and $\Delta p + \Delta I[n] \ge 0$, then

$$\frac{\Delta p+\Delta I[n]}{c_\kappa^*[n]+p_\kappa^*[n]} \ge -\frac{\Delta I[n]}{c_\kappa^*[n]} \Rightarrow 0 \ge \frac{\Delta I[n]}{\Delta p} \ge -\frac{c_\kappa^*[n]}{2c_\kappa^*[n]+p_\kappa^*[n]}. \quad (77)$$

Case 3: If $\Delta I[n] \le 0$ and $\Delta p + \Delta I[n] < 0$, then

$$\frac{-\Delta p-\Delta I[n]}{c_\kappa^*[n]+p_\kappa^*[n]} \ge -\frac{\Delta I[n]}{c_\kappa^*[n]} \Rightarrow (+\text{ve})\,\frac{c_\kappa^*[n]}{p_\kappa^*[n]} \le \frac{\Delta I[n]}{\Delta p}\,(-\text{ve}), \quad (78)$$

which is impossible, so this case is prohibited.

Taking $\varphi_\kappa^*[n] = \lim_{\Delta p\to 0}\frac{\Delta I[n]}{\Delta p}$ and summarizing the above, we get the desired result of (39).

APPENDIX C
PROOF OF THEOREM 3

From our FWF analysis, it can be seen that at any time (with the time index omitted for brievity)

$$\frac{\partial \mathcal{L}}{\partial p_\kappa[n]} = \frac{c_\kappa[n]-\varphi_\kappa[n]p_\kappa[n]}{c_\kappa[n](c_\kappa[n]+p_\kappa[n])} - \lambda. \quad (79)$$

Together with the condition $\varphi_\kappa[n] \le 0 < \frac{c_\kappa[n]}{p_\kappa[n]}$ in Theorem 2, $\exists\lambda > 0$ such that $\frac{\partial \mathcal{L}}{\partial p_\kappa^*[n]} = 0$. On the other hand, for $p_\kappa[n] > 0$, we have $w_\kappa = \eta_\kappa[n] + p_\kappa[n]$, giving

$$w_\kappa = \frac{(c_\kappa[n])^2 + \varphi_\kappa[n](p_\kappa[n])^2}{c_\kappa[n]-\varphi_\kappa[n]p_\kappa[n]} + p_\kappa[n]. \quad (80)$$

In [24, Proposition 1], it is known that in SE, $\varphi_\kappa^*[n]$ is a constant and hence $\frac{\partial \varphi_\kappa^*[n]}{\partial p_\kappa[n]} = 0$. If this is true not just at SE but at any time, then we get (81), as shown at the top of the next page. From the condition in Theorem 2, we have $c_\kappa[n] - \varphi_\kappa[n]p_\kappa[n] \ge 0$ and also $\varphi_\kappa[n] \ge -\frac{c_\kappa[n]}{2c_\kappa[n]+p_\kappa[n]}$. Therefore,

$$\frac{\partial w_\kappa}{\partial p_\kappa[n]} \ge \frac{\left(-\frac{c_\kappa[n]}{2c_\kappa[n]+p_\kappa[n]}\right)(2c_\kappa[n]+p_\kappa[n]) + c_\kappa[n]}{(+\text{ve})} = 0. \quad (82)$$

Knowing that $\frac{\partial \mathcal{L}}{\partial p_\kappa[n]} = \frac{1}{w_\kappa} - \lambda$, we have

$$\frac{\partial^2 \mathcal{L}}{\partial (p_\kappa[n])^2} = -\left(\frac{1}{w_\kappa}\right)^2 \frac{\partial w_\kappa}{\partial p_\kappa[n]} \le 0, \quad (83)$$

which shows that $\mathcal{L}$ is a concave function in $\{p_\kappa[n]\}$, after considering the interaction from other users via the interference derivative of SE (39) and thus has a unique maximum.

APPENDIX D
PROOF OF PROPOSITION 1

We start by writing the per-interaction interference derivative,[7] $\lim_{\Delta p_k^*[n]\to 0}\frac{\Delta I_k^*[n]}{\Delta p_k^*[n]}$, as (84) (see next page), where (a) is due to $\lim_{\Delta p_k^*[n]\to 0}\frac{\Delta I_i^*[n]}{\Delta p_k^*[n]} = \theta_{ki}[n]$ because high-order interactions are ignored, (b) is because the derivative is

[7] In FE, $\varphi_k^*[n] \neq \lim_{\Delta p\to 0}\frac{\Delta I[n]}{\Delta p}$ (in Theorem 1) because the higher-order interactions need to be included in $\varphi_k^*[n]$.

$$\begin{aligned}\frac{\partial w_\kappa}{\partial p_\kappa[n]} &= \frac{(c_\kappa[n]-\varphi_\kappa[n]p_\kappa[n])(2c_\kappa[n]\varphi_\kappa[n]+2p_\kappa[n]\varphi_\kappa[n])-((c_\kappa[n])^2+\varphi_\kappa[n](p_\kappa[n])^2)(\varphi_\kappa[n]-\varphi_\kappa[n])}{(c_\kappa[n]-\varphi_\kappa[n]p_\kappa[n])^2}+1\\ &= \frac{2c_\kappa[n]\varphi_\kappa[n]+\varphi_\kappa[n]p_\kappa[n]+c_\kappa[n]}{c_\kappa[n]-\varphi_\kappa[n]p_\kappa[n]} \end{aligned} \tag{81}$$

$$\begin{aligned}\lim_{\Delta p_k^*[n]\to 0}\frac{\Delta I_k^*[n]}{\Delta p_k^*[n]} &= \lim_{\Delta p_k^*[n]\to 0}\sum_{\substack{i=1\\ i\neq k}}^{K}\mathsf{sgn}(p_i^*[n])\theta_{ik}[n]\frac{\Delta p_i^*[n]}{\Delta p_k^*[n]}\\ &= \lim_{\Delta p_k^*[n]\to 0}\sum_{\substack{i=1\\ i\neq k}}^{K}\mathsf{sgn}(p_i^*[n])\theta_{ik}[n]\frac{\Delta p_i^*[n]}{\Delta I_i^*[n]}\frac{\Delta I_i^*[n]}{\Delta p_k^*[n]}\\ &\overset{(a)}{\approx} \lim_{\Delta p_k^*[n]\to 0}\sum_{\substack{i=1\\ i\neq k}}^{K}\mathsf{sgn}(p_i^*[n])\mathsf{sgn}(p_k^*[n])\theta_{ik}[n]\theta_{ki}[n]\frac{\Delta p_i^*[n]}{\Delta I_i^*[n]}\\ &\overset{(b)}{\geq} -\lim_{\Delta p_k^*[n]\to 0}\sum_{\substack{i=1\\ i\neq k}}^{K}\mathsf{sgn}(p_i^*[n])\mathsf{sgn}(p_k^*[n])\theta_{ik}[n]\theta_{ki}[n]\left|\frac{\Delta p_i^*[n]}{\Delta I_i^*[n]}\right|\\ &\overset{(c)}{\geq} -\lim_{\Delta p_k^*[n]\to 0}\sum_{\substack{i=1\\ i\neq k}}^{K}\mathsf{sgn}(p_i^*[n])\mathsf{sgn}(p_k^*[n])\theta_{ik}[n]\theta_{ki}[n] \end{aligned}\tag{84}$$

negative (see the discussion before Proposition 1), (c) is due to the belief that $\left|\frac{\Delta p_i^*[n]}{\Delta I_i^*[n]}\right| \leq 1$, and (84) gives a lower bound of the per-interaction environmental derivative. To justify the belief (c), we infer that ideally for subcarrier n on which user k occupies, i.e., $p_k[n] > 0$, this should not be chosen by other users $i \neq k$, or $p_i[n] \approx 0\ \forall i \neq k$. In this case, the strategic noise becomes $\eta_i[n] \approx c_i[n]$, so user i effectively employs an NE approach on that subcarrier and the belief is true according to the result in Appendix E. Using Theorem 2 to ensure user k achieving the highest rate, the per-interaction interference derivative must satisfy (39) and this can be achieved by making the lower bound in (84) satisfy

$$\begin{aligned}-\lim_{\Delta p_k^*[n]\to 0}\sum_{\substack{i=1\\ i\neq k}}^{K}\mathsf{sgn}(p_i^*[n])\mathsf{sgn}(p_k^*[n])\theta_{ik}[n]\theta_{ki}[n]&\\ \geq -\frac{c_k^*[n]}{2c_k^*[n]+p_k^*[n]},&\end{aligned}\tag{85}$$

which directly gives the result (47).

APPENDIX E
JUSTIFICATION OF (C) IN THE PROOF OF PROPOSITION 1

To begin, we find it useful to define $L_i \triangleq \sum_{m=1}^{N} 1(w_i > I_i[n]+\sigma_i[n])$ and $\bar{L}_i \triangleq \sum_{m=1}^{N} 1(w_i \geq I_i[n]+\sigma_i[n]) \geq L_i$, where $1(A)$ returns 1 if the event A is true or 0 otherwise, so for some user i, we have

$$P_i = L_i w_i - \sum_{m=1}^{N}(I_i[n]+\sigma_i[n])1(w_i > I_i[n]+\sigma_i[n]), \tag{86}$$

$$P_i = \bar{L}_i w_i - \sum_{m=1}^{N}(I_i[n]+\sigma_i[n])1(w_i \geq I_i[n]+\sigma_i[n]). \tag{87}$$

We first present the following lemma before we present the main result in the next lemma.

Lemma 2: The rate of change of the water-level w.r.t. the interference pattern satisfies

$$0 \leq \frac{\partial w_i}{\partial I_i[n]} \leq \frac{1}{L_i}. \tag{88}$$

Proof: To show the result, we study what will happen following an infinitesimal change in the interference pattern on subcarrier n under different cases. In our proof, we use the fact that the change in the interference pattern is infinitesimally small and as such the subcarriers whether user i occupies or not will not change except those subcarriers where the water-level equals the overall noise before the change.

Case 1: If $w_i < I_i[n]+\sigma_i[n]$ before the change, then it was not used by user i. After the change $\Delta I_i[n](\to 0)$, subcarrier n will remain unused by user i and thus the water-level is unchanged, or $\frac{\partial w_i}{\partial I_i[n]} = 0$.

Case 2: If $w_i > I_i[n]+\sigma_i[n]$ before the change, then subcarrier n was used by user i. After an increase in the interference $\Delta I_i[n] > 0$ (but $\to 0$), the water-level w_i will increase and those subcarriers, $m \neq n$, that previously satisfied $w_i = I_i[m]+\sigma_i[m]$ will then be used. Hence, we can use (87) to get

$$\frac{\partial w_i}{\partial I_i[n]_+} = \frac{1}{\bar{L}_i}, \tag{89}$$

where the notation $\partial I_i[n]_+$ specifies that the change is positive (or increasing).

Case 3: If $w_i > I_i[n] + \sigma_i[n]$ before the change, then as in Case 2 subcarrier n was used by user i. After a decrease in the interference $\Delta I_i[n] < 0$ (but $\to 0$), the water-level w_i will decrease and whether the subcarriers are occupied or not by user i will not change. Hence, we use (86) to get

$$\frac{\partial w_i}{\partial I_i[n]_-} = \frac{1}{L_i}, \tag{90}$$

where the notation $\partial I_i[n]_-$ specifies that the change is negative (or decreasing).

Case 4: If $w_i = I_i[n] + \sigma_i[n]$ before the change, it was not used by user i. After an increase in the interference $\Delta I_i[n] > 0$ (but $\to 0$), subcarrier n will remain unused and the water-level unchanged, or $\frac{\partial w_i}{\partial I_i[n]_+} = 0$.

Case 5: If $w_i = I_i[n] + \sigma_i[n]$ before the change, it was not used by user i. After a decrease in the interference $\Delta I_i[n] < 0$ (but $\to 0$), subcarrier n will then be used by user i while the status of other subcarriers remain the same. Therefore, using (86), we have

$$\frac{\partial w_i}{\partial I_i[n]_-} = \frac{1}{L_i + 1}. \tag{91}$$

Summarizing the results for all five cases gives the desired result in (88). □

Lemma 3: The rate of change of the power allocation for Nash user i w.r.t. the interference satisfies

$$-1 \le \frac{1}{L_i} - 1 \le \frac{\partial p_i[n]}{\partial I_i[n]} = \frac{\partial p_i^{\mathsf{NE}}[n]}{\partial I_i[n]} \le 0. \tag{92}$$

Proof: With user i being Nash-like, its power allocation is given by

$$p_i[n] = p_i^{\mathsf{NE}}[n] = (w_i - I_i[n] - \sigma_i[n])^+. \tag{93}$$

Case 1: If $w_i < I_i[n] + \sigma_i[n]$ before the change, then it was not used by user i or $p_i[n] = 0$. After the change $\Delta I_i[n] (\to 0)$, subcarrier n will remain unused and $p_i[n] = 0$ is unchanged so $\frac{\partial p_i[n]}{\partial I_i[n]} = 0$.

Case 2: If $w_i > I_i[n] + \sigma_i[n]$ before the change, then $p_i[n] = w_i - I_i[n] - \sigma_i[n]$. Therefore, we have

$$\frac{\partial p_i[n]}{\partial I_i[n]} = \frac{\partial w_i}{\partial I_i[n]} - 1. \tag{94}$$

After an increase in the interference $\Delta I_i[n] > 0$ (but $\to 0$), we have (89) so (94) becomes

$$\frac{\partial p_i[n]}{\partial I_i[n]_+} = \frac{1}{\bar{L}_i} - 1. \tag{95}$$

Case 3: If $w_i > I_i[n] + \sigma_i[n]$ before the change, then as in Case 2 we have (94). After a decrease in the interference $\Delta I_i[n] < 0$ (but $\to 0$), we have (90) which gives

$$\frac{\partial p_i[n]}{\partial I_i[n]_-} = \frac{1}{L_i} - 1. \tag{96}$$

Case 4: If $w_i = I_i[n] + \sigma_i[n]$ before the change, $p_i[n] = 0$. After an increase in the interference $\Delta I_i[n] > 0$ (but $\to 0$), subcarrier n will remain unused and $p_i[n] = 0$ is unchanged, or $\frac{\partial p_i[n]}{\partial I_i[n]_+} = 0$.

Case 5: If $w_i = I_i[n] + \sigma_i[n]$ before the change, it was not used by user i. After a decrease in the interference $\Delta I_i[n] < 0$ (but $\to 0$), subcarrier n will be used by user i. Therefore, using (94) and (97) yields

$$\frac{\partial p_i[n]}{\partial I_i[n]_-} = \frac{1}{L_i + 1} - 1. \tag{97}$$

Summarizing the results for all five cases gives the desired result. □

APPENDIX F
PROOF OF PROPOSITION 2

Consider the case with $p_k^*[n] > 0$ because otherwise the subcarrier is not used and $p_k^*[n] = 0$. Using (44) and summing over $k = 1, 2, \ldots, K$ on both sides, we get

$$\begin{aligned}\sum_{k=1}^{K} \varphi_k^*[n] &\overset{(a)}{=} \sum_{k=1}^{K}\sum_{i=1}^{K} \frac{\mathsf{sgn}(p_i^*[n])}{\varphi_i^*[n]} \theta_{ik}[n] \frac{\partial I_i^*[n]}{\partial p_k^*[n]} \\ &\overset{(b)}{=} \sum_{k=1}^{K} \frac{1}{\varphi_k^*[n]} \sum_{i=1}^{K} \mathsf{sgn}(p_k^*[n])\mathsf{sgn}(p_i^*[n])\theta_{ki}[n] \\ &\quad \times \frac{\partial I_k^*[n]}{\partial p_i^*[n]} \\ \varphi_k^*[n] &\overset{(c)}{=} \frac{1}{\varphi_k^*[n]} \sum_{i=1}^{K} \mathsf{sgn}(p_k^*[n])\mathsf{sgn}(p_i^*[n])\theta_{ki}[n] \frac{\partial I_k^*[n]}{\partial p_i^*[n]}\end{aligned} \tag{98}$$

$$\left(\varphi_k^*[n]\right)^2 \overset{(d)}{=} \sum_{i=1}^{K} \mathsf{sgn}(p_k^*[n])\mathsf{sgn}(p_i^*[n])\theta_{ki}[n]\theta_{ik}[n] \tag{99}$$

where (a) is due to (44) and by defining $\theta_{ij}[n] \triangleq 0$ for $i = j$, (b) is due to swapping the indices k and i, (c) is enforced by equating the terms under the summations on both sides, which allows us to decouple the interdependence of $\{\varphi_k^*[n]\}_{\forall k}$ and thus makes possible a concise property to be derived, and (d) is due to the subsystem response to the subject user k from user i, i.e., $\frac{\partial I_k^*[n]}{\partial p_i^*[n]} = \theta_{ik}[n]$.

Now, using (47) on (99), we then get

$$\begin{aligned}\left(\varphi_k^*[n]\right)^2 &\le \frac{c_k^*[n]}{2c_k^*[n] + p_k^*[n]} \\ &\Rightarrow -\sqrt{\frac{c_k^*[n]}{2c_k^*[n] + p_k^*[n]}} \le \varphi_k^*[n] \le 0.\end{aligned} \tag{100}$$

The negative is chosen to give (48) because $\varphi_k^*[n] \le 0$.

APPENDIX G
PROOF OF THEOREM 5

We omit the time index t for brevity. To begin with, we find it useful to define $\gamma_k[n] \triangleq \sqrt{\frac{2c_k[n]+p_k[n]}{c_k[n]}}$ so that $\varphi_k[n] = -\frac{1}{\gamma_k[n]}$

due to the FWF in (49) and $p_k[n] = (\gamma_k[n])^2 c_k[n] - 2c_k[n]$. Then, for those k, n such that $p_k[n] > 0$, we have

$$w_k[n] = \frac{(c_k[n])^2 + c_k[n]p_k[n]}{c_k[n] - \tilde{\varphi}_k[n]p_k[n]} = \frac{c_k[n]\gamma_k[n](\gamma_k[n]+1)}{\gamma_k[n]+2}. \tag{101}$$

To see how $w_k[n]$ varies w.r.t. $p_k[n]$, we first obtain (102) (see top of next page). Therefore, $\gamma_k[n]$ increases as $p_k[n]$ increases. On the other hand, $\frac{\partial\left(\frac{\gamma_k[n]+1}{\gamma_k[n]+2}\right)}{\partial\gamma_k[n]} = \frac{1}{(\gamma_k[n]+2)^2} > 0$ and hence if $p_k[n]$ increases, then $\gamma_k[n]$ will increase and therefore $\frac{\gamma_k[n]+1}{\gamma_k[n]+2}$ will increase accordingly.

It remains to show that $c_k[n]\gamma_k[n]$ is also increasing with $p_k[n]$. To do so, we note that $c_k[n]\gamma_k[n] = \sqrt{2(c_k[n])^2 + p_k[n]c_k[n]}$. As a consequence, if $\varphi_k[n] > -\frac{c_k[n]}{4c_k[n]+p_k[n]}$ is satisfied, we obtain (103) (see top of next page).

As a result, under the aforementioned condition, $w_k[n]$ is increasing with $p_k[n]$. Then following the same argument as in the proof of Theorem 3 in Appendix C, FE is unique, and the equilibrium is also optimal in maximizing the user's rate, which completes the proof.

APPENDIX H
PROOF OF LEMMA 1

In this proof, for convenience, we will omit the iteration index t and assume that the change in the water-level w_k is negligible or the derivative in w_k w.r.t. the change in c_k is zero. Also, note that $|\rho_k[n]| = \frac{\partial\eta_k[n]}{\partial I_k[n]} = \frac{\partial\eta_k[n]}{\partial c_k[n]}$. For NE, $\eta_k[n] = c_k[n]$ and therefore $|\rho_k[n]| = 1$.

In the case of SE, using (27) and the corresponding definition of φ in SE, we have

$$\eta_k[n] = \frac{c_k^2[n] + \left(-\frac{c_k[n]}{2c_k[n]+p_k[n]}\right)p_k^2[n]}{c_k[n] + \frac{c_k[n]}{2c_k[n]+p_k[n]}p_k[n]} = c_k[n] - \frac{p_k[n]}{2}. \tag{104}$$

Then we have $p_k[n] = w_k - \eta_k[n] = 2(w_k - c_k[n]) \Rightarrow \frac{\partial p_k[n]}{\partial c_k[n]} = -2$ and

$$\frac{\partial\eta_k[n]}{\partial c_k[n]} = 1 - \frac{1}{2}\frac{\partial p_k[n]}{\partial c_k[n]} = 2 \;\Rightarrow\; |\rho_k[n]| = 2. \tag{105}$$

For the case of FE, we start by expressing (106) (see next page). On the other hand, we can also examine $\frac{\partial\varphi_k[n]}{\partial c_k[n]}$ to obtain

$$\frac{\partial\varphi_k[n]}{\partial c_k[n]} = \frac{1}{2\varphi_k[n]}\frac{c_k^2[n]}{(2c_k[n]+p_k[n])^2}\frac{p_k[n]}{c_k^2[n]} = \frac{1}{2}\varphi_k^3[n]\frac{p_k[n]}{c_k^2[n]}. \tag{107}$$

Substituting this result into (106) then gives (108) (see top of next page). For large $\frac{p_k[n]}{c_k[n]}$,[8] $\varphi_k[n] = -\sqrt{\frac{c_k[n]}{2c_k[n]+p_k[n]}} \approx -\sqrt{\frac{c_k[n]}{p_k[n]}}$. Consequently, $|\rho_k[n]| \approx \frac{1}{2}|\varphi_k[n]|\frac{p_k[n]}{c_k[n]} = \frac{1}{2}\sqrt{\frac{p_k[n]}{c_k[n]}}$.

[8] As an approximate analysis, as far as FE is concerned, it makes more sense to consider the case with high signal-to-noise ratio (SNR) on a chosen subcarrier because the user should choose only good subcarrier(s), or with high SNR, to operate.

This completes the proof and knowing $|\rho_k[n]|$ will be useful in understanding the convergence behavior.

APPENDIX I
PROOF OF COROLLARY 1

At the convergence of FE, it is easy to see that

$$w_k^* \overset{(a)}{=} \frac{c_k^2[n] + c_k[n]p_k[n]}{c_k[n] - \varphi_k[n]p_k[n]} \tag{109}$$

$$\overset{(b)}{=} \frac{c_k^2[n] + c_k[n]p_k[n]}{c_k[n] + \sqrt{\frac{c_k[n]}{2c_k[n]+p_k[n]}}p_k[n]} > 0, \tag{110}$$

where (a) is due to the assumption that $p_k[n] \geq 0$ and $w_k = p_k[n] + \eta_k[n]$ and (b) is due to (49).

Now, considering the case $c_k[n] \geq w_k^*$, we then get

$$c_k[n] \geq w_k^* = \frac{c_k^2[n] + c_k[n]p_k[n]}{c_k[n] + \sqrt{\frac{c_k[n]}{2c_k[n]+p_k[n]}}p_k[n]} \tag{111}$$

implying that

$$c_k[n]\sqrt{\frac{c_k[n]}{2c_k[n]+p_k[n]}}p_k[n] \geq c_k[n]p_k[n], \tag{112}$$

which is only true if $p_k[n] = 0$. The proof is completed.

APPENDIX J
PROOF OF PROPOSITION 4

Using (110), for those used subcarriers, the water-level of AFE can be expressed as

$$w_k^* = \frac{c_k[n]\left(1 + \frac{p_k[n]}{c_k[n]}\right)}{1 - \varphi_k[n]\frac{p_k[n]}{c_k[n]}}. \tag{113}$$

For high SNR, $\varphi_k[n] \approx -\sqrt{\frac{c_k[n]}{p_k[n]}}$. Plugging this into (113), we obtain the desired result (64).

APPENDIX K
PROOF OF PROPOSITION 5

Using Proposition 4, we have

$$(w_k^*)^2 \approx c_k[n]p_k[n]\mathsf{sgn}(p_k[n]) = \frac{p_k[n]\mathsf{sgn}(p_k[n])}{1/c_k[n]}, \tag{114}$$

which can then be rewritten as

$$(w_k^*)^2 \approx \frac{\sum_n p_k[n]\mathsf{sgn}(p_k[n])}{\sum_n \frac{1}{c_k[n]}\mathsf{sgn}(p_k[n])} = \frac{\sum_n p_k[n]\mathsf{sgn}(p_k[n])\frac{1}{\sum_n \mathsf{sgn}(p_k[n])}}{\sum_n \frac{1}{c_k[n]}\mathsf{sgn}(p_k[n])\frac{1}{\sum_n \mathsf{sgn}(p_k[n])}} \approx \frac{\mathbb{E}[p_k[n]]}{\mathbb{E}\left[\frac{1}{c_k[n]}\right]}. \tag{115}$$

Without loss of generality, we set $\mathbb{E}[|H_{kk}[n]|^2] = 1, \forall k, n$. Based on the anticipation that FE will make the users orthogonal (to be verified by simulation results in Section VII), we therefore have $\mathbb{E}[\frac{1}{c_k[n]}] = \mathbb{E}[\frac{|H_{kk}[n]|^2}{N_0}] = \frac{1}{N_0}$. In addition, $\mathbb{E}[p_k[n]] = P_0\frac{K}{N}$. This completes the proof.

$$\frac{\partial \gamma_k[n]}{\partial p_k[n]} = \frac{1}{2}\frac{1}{\sqrt{2+\frac{p_k[n]}{c_k[n]}}}\frac{\partial\left(\frac{p_k[n]}{c_k[n]}\right)}{\partial p_k[n]} = \frac{1}{2}\frac{1}{\sqrt{2+\frac{p_k[n]}{c_k[n]}}}\frac{1}{c_k[n]}\left(1-\frac{p_k[n]\varphi_k[n]}{c_k[n]}\right) \geq 0 \ (\because \varphi_k[n] \leq 0) \tag{102}$$

$$\frac{\partial c_k[n]\gamma_k[n]}{\partial p_k[n]} = \frac{1}{2}\frac{1}{\sqrt{2(c_k[n])^2 + p_k[n]c_k[n]}}[(4c_k[n]+p_k[n])\varphi_k[n]+c_k[n]] > 0 \tag{103}$$

$$\frac{\partial \eta_k[n]}{\partial c_k[n]} = \frac{c_k^2[n] - 2c_k[n]\varphi_k[n]p_k[n] + c_k[n]\frac{\partial \varphi_k[n]}{\partial c_k[n]}p_k^2[n] - \varphi_k[n]p_k^2[n] + c_k^2[n]\frac{\partial \varphi_k[n]}{\partial c_k[n]}p_k[n]}{(c_k[n]-\varphi_k[n]p_k[n])^2} \tag{106}$$

$$\frac{\partial \eta_k[n]}{\partial c_k[n]} = \frac{c_k^2[n] - 2c_k[n]\varphi_k[n]p_k[n] + \frac{1}{2}\varphi_k^3[n]\frac{p_k^3[n]}{c_k[n]} - \varphi_k[n]p_k^2[n] + \frac{1}{2}\varphi_k^3[n]p_k^2[n]}{(c_k[n]-\varphi_k[n]p_k[n])^2} \tag{108}$$

APPENDIX L
PROOF OF PROPOSITION 6

From Corollary 1, it is known that $p_k[n] = 0$ when $c_k[n] > w_k^*$. Therefore, SU k will avoid the subcarrier, say n', occupied by the PUs if

$$c_k[n'] = N_0 + \tilde{I}_k[n'] > w_k^*. \tag{116}$$

Then, using (65) in Proposition 5 and considering all subcarriers used by the PUs, we get (66).

REFERENCES

[1] J. Mitola and G. Q. Maguire, Jr., "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13–18, Apr. 1999.

[2] J. Mitola, P. Marshall, K.-C. Chen, M. Mueck, and Z. Zvonar, "Software defined radio—20 years later: Part 1 [guest editorial]," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 22–23, Sep. 2015.

[3] V. Sagar, R. Chandramouli, and K. P. Subbalakshmi, "Software defined access for HetNets," *IEEE Commun. Mag.*, vol. 54, no. 1, pp. 84–89, Jan. 2016.

[4] H. Xie, B. Wang, F. Gao, and S. Jin, "A full-space spectrum-sharing strategy for massive MIMO cognitive radio systems," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 10, pp. 2537–2549, Oct. 2016.

[5] A. Aijaz and A. H. Aghvami, "Cognitive machine-to-machine communications for Internet-of-Things: A protocol stack perspective," *IEEE Internet Things J.*, vol. 2, no. 2, pp. 103–112, Apr. 2015.

[6] A. A. Khan, M. H. Rehmani, and A. Rachedi, "Cognitive-radio-based Internet of Things: Applications, architectures, spectrum related functionalities, and future research directions," *IEEE Wireless Commun.*, vol. 24, no. 3, pp. 17–25, Jun. 2017.

[7] Y. Liu, Z. Ding, M. Elkashlan, and J. Yuan, "Nonorthogonal multiple access in large-scale underlay cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 12, pp. 10152–10157, Dec. 2016.

[8] J. Mitola, "Cognitive radio—An integrated agent architecture for software defined radio," Ph.D. dissertation, Dept. Teleinformat., KTH Royal Inst. Technol., Stockholm, Sweden, 2000.

[9] F. Boccardi, R. W. Heath, Jr., A. Lozano, T. L. Marzetta, and P. Popovski, "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 74–80, Feb. 2014.

[10] C. Yi and J. Cai, "Ascending price progressive spectrum auction for cognitive radio networks with power-constrained multiradio secondary users," *IEEE Trans. Veh. Technol.*, vol. 67, no. 1, pp. 781–794, Jan. 2018.

[11] H. Maloku, E. Hamiti, Z. L. Fazliu, V. P. Lesta, A. Pitsillides, and M. Rajarajan, "A decentralized approach for self-coexistence among heterogeneous networks in TVWS," *IEEE Trans. Veh. Technol.*, vol. 67, no. 2, pp. 1302–1312, Feb. 2018.

[12] X. Li, J. Fang, W. Cheng, H. Duan, Z. Chen, and H. Li, "Intelligent power control for spectrum sharing in cognitive radios: A deep reinforcement learning approach," *IEEE Access*, vol. 6, pp. 25463–25473, 2018.

[13] S. K. Gottapu, N. Kapileswar, P. V. Santhi, and V. K. R. Chenchela, "Maximizing cognitive radio networks throughput using limited historical behavior of primary users," *IEEE Access*, vol. 6, pp. 12252–12259, 2018.

[14] Z.-H. Wei and B.-J. Hu, "A fair multi-channel assignment algorithm with practical implementation in distributed cognitive radio networks," *IEEE Access*, vol. 6, pp. 14255–14267, 2018.

[15] Y.-C. Chang, C.-S. Chang, and J.-P. Sheu, "An enhanced fast multi-radio rendezvous algorithm in heterogeneous cognitive radio networks," *IEEE Trans. Cogn. Commun. Netw.*, to be published.

[16] W. Lee, "Resource allocation for multi-channel underlay cognitive radio network based on deep neural network," *IEEE Commun. Lett.*, vol. 22, no. 9, pp. 1942–1945, Sep. 2018.

[17] W. Ejaz and M. Ibnkahla, "Multiband spectrum sensing and resource allocation for IoT in cognitive 5G networks," *IEEE Internet Things J.*, vol. 5, no. 1, pp. 150–163, Feb. 2018.

[18] K. Akkarajitsakul, E. Hossain, D. Niyato, and D. I. Kim, "Game theoretic approaches for multiple access in wireless networks: A survey," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 3, pp. 372–395, Mar. 2011.

[19] O. Popescu, C. Rose, and D. C. Popescu, "Simultaneous water-filling in mutually interfering systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 1102–1113, Mar. 2007.

[20] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 5, pp. 1105–1115, Jun. 2002.

[21] L. Lai and H. E. Gamal, "The water-filling game in fading multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 54, no. 5, pp. 2110–2122, May 2008.

[22] G. Scutari, D. P. Palomar, and S. Barbarossa, "Competitive design of multiuser MIMO systems based on game theory: A unified view," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1089–1103, Sep. 2008.

[23] Y. Su and M. Van Der Schaar, "A new perspective on multi-user power control games in interference channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2910–2919, Jun. 2009.

[24] Y. Su and M. Schaar, "Conjectural equilibrium in multiuser power control games," *IEEE Trans. Signal Process.*, vol. 57, no. 9, pp. 3638–3650, Sep. 2009.

[25] S. Guruacharya, D. Niyato, D. I. Kim, and E. Hossain, "Hierarchical competition for downlink power allocation in OFDMA femtocell networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 4, pp. 1543–1553, Apr. 2013.

[26] X. Kang, R. Zhang, and M. Motani, "Price-based resource allocation for spectrum-sharing femtocell networks: A stackelberg game approach," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 538–549, Apr. 2012.

[27] C. H. Yu, K. Doppler, C. Ribeiro, and L. Tirkkonen, "Resource sharing optimization for device-to-device communication underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 57–72, Aug. 2011.
[28] H. Min, W. Seo, J. Lee, S. Park, and D. Hong, "Reliability improvement using receive mode selection in the device-to-device uplink period underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 413–418, Feb. 2011.
[29] S. Parsaeefard, M. van der Schaar, and A. R. Sharafat, "Robust power control for heterogeneous users in shared unlicensed bands," *IEEE Trans. Wireless Commun.*, vol. 13, no. 6, pp. 3167–3182, Jun. 2014.
[30] T. Zhang, W. Chen, Z. Han, and Z. Cao, "Hierarchic power allocation for spectrum sharing in OFDM-based cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 4077–4091, Oct. 2014.
[31] Z. Liu, L. Hao, Y. Xia, and X. Guan, "Price bargaining based on the Stackelberg game in two-tier orthogonal frequency division multiple access femtocell networks," *IET Commun.*, vol. 9, no. 1, pp. 133–145, 2015.
[32] R. Yin, C. Zhong, G. Yu, Z. Zhang, K. K. Wong, and X. Chen, "Joint spectrum and power allocation for D2D communications underlaying cellular networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 4, pp. 2182–2195, Apr. 2016.
[33] N. Sharma, D. Badheka, and A. Anpalagan, "Multiobjective subchannel and power allocation in interference-limited two-tier OFDMA femtocell networks," *IEEE Syst. J.*, vol. 10, no. 2, pp. 544–555, Jun. 2016.
[34] H. Zhang, C. Jiang, X. Mao, and H. H. Chen, "Interference-limited resource optimization in cognitive femtocells with fairness and imperfect spectrum sensing," *IEEE Trans. Veh . Technol.*, vol. 65, no. 3, pp. 1761–1771, Mar. 2016.
[35] V. DeMiguel and H. Xu, "Stochastic multiple-leader Stackelberg model: Analysis, computation, and application," *Oper. Res.*, vol. 57, no. 5, pp. 1220–1235, Sep./Oct. 2009.
[36] R. Cendrillon, W. Yu, M. Moonen, J. Verliden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, May 2006.
[37] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.
[38] Z. Han, Z. Ji, and K. J. R. Liu, "Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalitions," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1366–1376, Aug. 2005.
[39] J. Ren, K. K. Wong, and J. Hou, "A forward-looking Nash game and its application to achieving Pareto-efficient optimization," *Appl. Math.*, vol. 4, no. 12, pp. 1609–1615, Dec. 2013.
[40] D. Schmidt, C. Shi, R. Berry, M. Honig, and W. Utschick, "Distributed resource allocation schemes: Pricing algorithms for power control and beamforming design in interference networks," *IEEE Signa Process. Mag.*, vol. 26, no. 5, pp. 53–63, Sep. 2009.
[41] W. Yu, "Multiuser water-filling in the presence of crosstalk," in *Proc. Inf. Theory Appl. Workshop*, La Jolla, CA, USA, Jan./Feb. 2007, pp. 414–420.
[42] I. Maric, R. D. Yates, and G. Kramer, "The capacity region of the strong interference channel with common information," in *Proc. 39th Asilomar Conf. Conf. Signal Syst. Comput. Rec.*, 2005, pp. 1737–1741.
[43] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 3, pp. 581–586, Mar. 2004.
[44] G. Scutari, D. P. Palomar, and S. Barbarossa, "Asynchronous iterative waterfilling for Gaussian frequency-selective interference channels: A unified framework," in *Proc. Inf. Theory App. Workshop*, 2007, pp. 349–358.
[45] S. T. Chung, S. J. Kim, J. Lee, and J. M. Cioffi, "A game-theoretic approach to power allocation in frequency-selective Gaussian interference channels," in *Proc. IEEE Int. Symp. Inf. Theory*, Jun./Jul. 2003, p. 316.
[46] J. Huang, R. Cendrillon, M. Chiang, and M. Moonen, "Autonomous spectrum balancing (ASB) for frequency selective interference channels," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2006, pp. 610–614.
[47] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1996.

JIE REN received the master's and Ph.D. degrees from the School of Electronics and Information Engineering, Beijing Jiao Tong University, in 2008 and 2015, respectively. His studies center around radio wave propagation, game theoretical wireless networks, and multiuser communications theory.

KAI-KIT WONG (M'01–SM'08–F'16) received the B.Eng., M.Phil., and Ph.D. degrees in electrical and electronic engineering from the Hong Kong University of Science and Technology, Hong Kong, in 1996, 1998, and 2001, respectively. After graduation, he took up academic and research positions at the University of Hong Kong, Lucent Technologies, Bell-Labs, Holmdel, the Smart Antennas Research Group of Stanford University, and the University of Hull, U.K. He is the Chair of Wireless Communications at the Department of Electronic and Electrical Engineering, University College London, U.K.

His current research centers around 5G and beyond mobile communications, including topics such as massive MIMO, full-duplex communications, millimeter-wave communications, edge caching and fog networking, physical layer security, wireless power transfer and mobile computing, V2X communications, and of course cognitive radios. There are also a few other unconventional research topics that he has set his heart on, including, for example, fluid antenna communications systems, remote ECG detection, and so on. He was a co-recipient of the 2013 IEEE Signal Processing Letters Best Paper Award and the 2000 IEEE VTS Japan Chapter Award at the IEEE Vehicular Technology Conference in Japan, in 2000, and a few other international best paper awards.

Dr. Wong is a Fellow of IET and is also on the editorial board of several international journals. He has served as a Senior Editor for the IEEE Communications Letters, since 2012, and also for the IEEE Wireless Communications Letters, since 2016. He has been an Area Editor for Wireless Communication Theory and Systems I of the IEEE Transactions on Wireless Communications, since 2018. He had also previously served as an Editor for the IEEE Transactions on Wireless Communications, from 2005 to 2011, and as an Associate Editor for the IEEE Signal Processing Letters, from 2009 to 2012. He was also a Guest Editor of IEEE JSAC SI on Virtual MIMO, in 2013, and is currently a Guest Editor of the IEEE JSAC SI on physical layer security for 5G.