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Participatory decision-support model in the context of building structural design embedding BIM with QFD



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ABSTRACT

The design and optimisation of building structures is a complex undertaking that requires the effective collaboration of various stakeholders and involves technical and non-technical expertise. The paper investigated an integrated decision-support framework using Quality Function Deployment (QFD) in structural design optimisation. The aim of the study was to develop and test a systematic participatory model that utilises Building Information Modelling (BIM)-enabled technologies for data collection and group decision-making theory. The uncertainties associated with the decision-makers' preferences were computed using Evidential Reasoning (ER) algorithms in the QFD house of quality. An actual decision scenario was used to test the proposed framework and investigate its capabilities in the context of reinforced concrete buildings. The study demonstrated how the proposed QFD model could effectively enhance decision-making by managing the diversity of stakeholders' preferences via design integration, enhanced communication and shared domain knowledge.

1. Introduction

The complexity and poor decision practices of construction projects often lie in the inconsistencies occurred during the early design stages when engineers, clients, architects, and contractors formalise multiple priorities and preferences [1]. In structural engineering problems where the project team can significantly influence the final design decisions, it is important to consider both quantitative (analysis) and qualitative (preferences) aspects when outlining a decision-making framework [2]. Group decision-making processes are suitable for the selection of engineering design priorities when there is a need to satisfy several conflicting opinions. However, obtaining consensus within design teams remains a significant challenge even though in recent years it has received considerable attention amongst researchers and practitioners [3]. Furthermore, the selection of appropriate decision-making methods is not easy as it involves understanding of different decision criteria, process analyses and domain requirements [4]. Various methods have been applied to different decision-making problems in product development [5] and infrastructure management [6], seismic retrofit of structures [7,8], building [9], envelope designs [10,11] and energy conservation [12]. Additional work is still required in the domain of structural optimisation to support more robust and practical decision-making procedures. The optimisation of building systems such as the structural systems requires (1) clear organisation of all the information provided by the various stakeholders against project specific decision criteria [13] and (2) assessment methods that offer a better understanding of the final design selection [14].

In the context of building structures most of the optimisation problems can be represented using multi-objective algorithms that involve more than one objective function. In multi-objective optimisation trade-off solutions between conflicting design objectives are identified and represented in the Pareto front. However, a good approximation of the Pareto front does not necessarily mean that the optimisation procedure is finalised [15]. Further analysis of the resulting solutions is necessary to complete the decision-making. The reason for that is because the human factor is always responsible for the final approval and selection of a design solution [16]. One way to increase the acceptance levels of the solutions associated with the Pareto front by the design team members was formulated by Grierson [17], which involves the manipulation of Pareto data to identify designs based on a trade-off analysis.

The evaluation of the Pareto front solutions could be performed using Multi-Criteria Decision-Making (MCDM) approaches that are structured using specific decision criteria articulated by a group of decision-makers. The current paper explores a new approach to specify decision criteria that are necessary in MCDM [18] for the final selection of solutions obtained from stuctural optimisation analysis with

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conflicting objective functions. To achieve that a decision-support model for the specification of such decision criteria based on group decision-making was investigated. The proposed decision-support model was based on the Quality Function Deployment (QFD). QFD is a strategic methodology that allows companies and multi-disciplinary teams to recognise clients' needs and translate them into technical attributes during the product development phases [19].

QFD was originally applied in product and services development in Japan [20]. In the construction industry, QFD approaches have been widely used as their implementation could be particularly effective in addressing specific project needs [21]. In addition, QFD approach could effectively lead designers to a better understanding of stakeholders' requirements, whilst it could enhance the communication between the users and the designers [22]. Previous applications involve early design stage development [21-23], bridge design [24,25] and maintenance [25], energy efficiency measures in office [26] and residential buildings [10] and sustainable performance assessments [27,25,28]. However, more focused efforts are still required to effectively integrate QFD in the optimisation of building structures where design decisions often tend to be unsystematic and fragmented. If QFD is implemented early in the design development it can be particularly helpful with the: (1) prioritisation of project requirements, (2) articulation of design criteria, (3) efficient resources management (quality, construction delays, materials waste, etc.) [23,25], (4) information transfer between disciplines [10]. Furthermore, it was reported that early interactions of decision-makers in QFD could increase the feasibility and adoption of design solutions [26]. QFD could enhance the decision-making processes associated with structural design by: (1) Analysing clients' and users' needs, (2) Specifying functional and technical performance assessments, (3) Ranking engineering criteria [29]. According to Eldin and Hikle [23], QFD can significantly reduce design costs and time. Yang et al. [30] have recognised that the integration of QFD with quantitative methods such as Analytic Hierarchy Process (AHP) [31], Neural Networks [32] or Fuzzy Set Theory [31] could provide intelligent information systems that support the development of structured decision-making processes.

An important consideration this paper puts forward is the integration of QFD with Building Information Modelling (BIM) technologies. BIM applications have gained a lot of attention in various structural engineering operations (programme improvements, cost estimations, data documentation and interoperability, design team communication), as reported in an extensive review by Eleftheriadis et al. [33]. BIM can effectively address the dynamic nature of structural optimisation and decision-making problems as suggested by [34-36] advocating that several synergies and new development opportunities might exist. Antucheviciene et al. [37] have recognised the possibility of enhancing the current BIM-based software applications with decision support systems that assist existing multi-criteria problem solvers. Albukhari [38] proposed a BIM based decision support framework for the automatic evaluation of window submittals based on performance criteria such as life cycle cost and implementing Multi-Attribute Utility Theory (MAUT) and AHP. Significant time savings and reductions in decision subjectivity are the main advantages of this methodology.

The proposed decision support model utilises BIM technologies with QFD to amplify decision-making evaluation of design solutions obtained from structural optimisation with conflicting objective functions. The general concepts and computational components of the proposed model are presented in Section 2. In Section 3 the application and practical extensions of the decision-support model in the case of reinforced concrete structures are illustrated. Finally, in Section 4 the robustness of the proposed method is assessed and the main contributions are discussed. The paper concludes in Section 5.

2. Research framework

A BIM-based decision support model which integrates QFD is proposed in this study to assist engineers identify relevant decision criteria.

The implementation of the decision support model in an actual optimisation and MCDM scenario is outside the scope of the current paper but it is investigated in future research. The specification of a post-processing workflow for the proposed decision support in future work is described in Section 3.4. Thus, the main objective of this paper is to build and validate the BIM-based QFD decision-support model for the identification of decision criteria by translating detailed project requirements into structural design parameters using stakeholders' expert knowledge. The project requirements may vary between different projects which means that the engineering requirements will need to be adapted accordingly. However, the study provides a general method on how this decision support model can be implemented in various practical decision circumstances.

2.1. Decision support model

The development of the QFD normally follows a four-stage approach, in which each one corresponds to a stage of a product's development: (1) Planning phase (clients' attributes), (2) Design phase (technical requirements), (3) Operational phase (component characteristics), (4) Control phase (process steps) [39,30]. The planning phase of QFD is formalised through a conceptual matrix called the House of Quality (HOQ) which helps prioritise technical attributes early in the project development by relating client needs (WHATs) into engineering design attributes (HOWs) [40,41]. Fig. 1 shows a general representation of the HOQ.

For the purposes of this study the general HOQ was adapted so that the client WHATs are converted into project requirements (PR) to address building relevant needs, whereas the client HOWs were translated to structural design requirements (DR) that address the project requirements. The preparation of the HOQ relies on the inclusion of data in the form of expert opinions for the specification of the: (Step 1) Project requirements and importance matrix, (Step 2) Engineering requirements and relationship matrix, and (Step 3) Interrelationship matrix of engineering requirements.

In most new building projects, design teams work collectively in a shared BIM model. This offers many opportunities to use BIM technologies as a platform to exchange and record design preferences and decision criteria. This is particularly useful during the early design stages when different design options are investigated. The current research utilises BIM to collate necessary data for the computation and delivery of

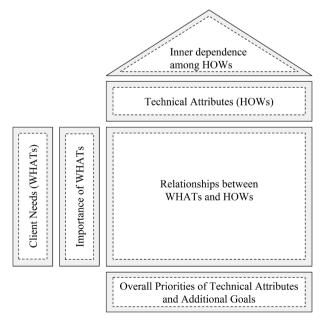


Fig. 1. House of quality (adapted from [19]).

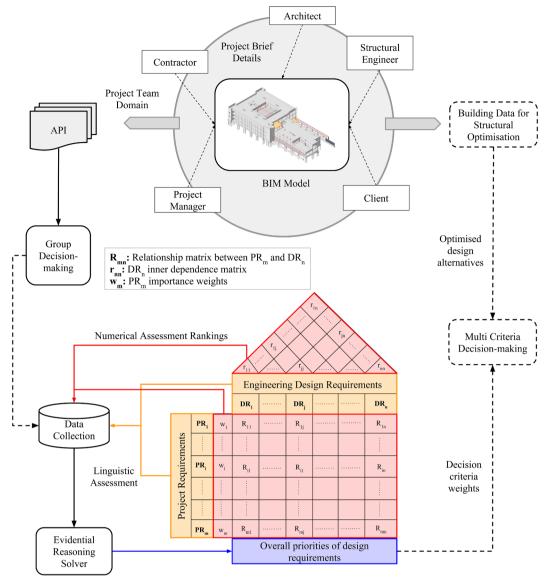


Fig. 2. Structure of the decision support model.

the HOQ in the QFD model. The structure of the BIM component within the decision support model is presented in Fig. 2. The shared BIM model is accessed by team members such as architects, structural engineers, contractors or subcontractors to add or amend building information. This functionality is enhanced with a custom application within BIM that is used for the data collection of stakeholders' preferences.

In this study Autodesk Revit Application Programming Interface (API) was used to develop the application for the collection of HOQ data from the project team. The application could be loaded as an add-on at the beginning of every project. The application is responsible for the specification of the three HOQ steps identified previously: the project team not only can record PR based on the current project brief but it also provides numerical assessment based on their preference and experience. The DR are then recognised by the team of engineers as well as the relationships between the PR and the DR. Finally, the interrelationship between DR is used to identify the final DR importance ratings. All necessary computations for the specification of DR priorities are performed using an Evidential Reasoning (ER) algorithm solver.

The final perceived decision priorities from the decision support model could be subsequently used for the assessment of design alternatives obtained from structural optimisation studies that utilise building data and information directly from the BIM model like the ones proposed in [34–36]. This assembly is not covered in the current study but it is addressed in future work as it is expected to create more efficient design and decision workflows within BIM's virtual environment enhancing the collaboration between project team members.

2.2. Numerical assessment

Fig. 3 shows a snapshot of the BIM tool with the corresponding tabs which are associated to the three HOQ steps. To conduct the numerical assessment with the stakeholders' preferences the rating scales from Chin et al. [42] were implemented within the BIM tool to identify the corresponding PR importance weights w_{mn} {9 = Extremely Important, 7 = Very Important, 5 = Moderately Important, 3 = Weakly Important, 1 = Very Weakly Important, 0 = Not Important}, the relationships between PR and DR R_{mn} {9 = Very Strong Relationship, 7 = Strong Relationship, 5 = Moderate Relationship, 3 = Weak Relationship, 1 = Very Weak Relationship, 0 = No Relationship} and the correlation of DR, r_{mn} {9 = Very Strong Correlation, 7 = Strong Correlation, 5 = Moderate Correlation, 3 = Weak Correlation, 1 = Very Weak Correlation, 0 = No Correlation}.

At Step 1, the numerical assessment (w_{mn}) for the PR is computed, whereas the relationships (R_{mn}) between the PR and the DR takes place

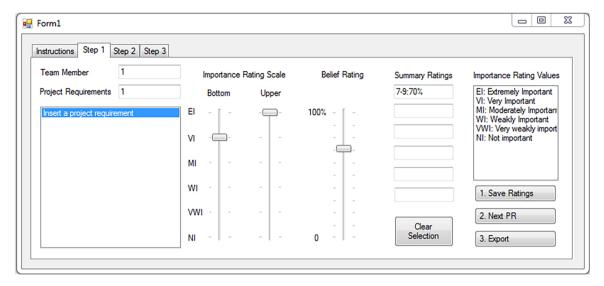


Fig. 3. BIM application interface panel with different functionalities for data collection.

at Step 2. Finally, the inner relationships between the DR (r_{mn}) are assessed at Step 3. The interface components for Step 2 and Step 3 follow a similar logic with the one shown for Step 1 and thus were not fully displayed in Fig. 3. Background information about the QFD methodology and user guidelines are summarised in the instructions tab.

An important contribution of the proposed decision support model is the ability to address the uncertainties associated with the decision makers' opinions. This is a significant limitation of the traditional QFD models and in this study, is addressed utilising Evidential Reasoning (ER) algorithms. ER is embedded within the QFD component to tackle two main types of uncertainties: (1) vagueness and ambiguity of data input, (2) incomplete, imprecise data. Typically, most methods address the first type of QFD uncertainties, which involves the vagueness of input data using fuzzy logic [43,44]. However, fuzzy logic approaches embedded in the commonly used AHP or ANP cannot deal with incomplete, or missing information (ignorance) in QFD which is often inevitable in human being's subjective judgement [42]. The most common way to address this kind of data limitation is using ER logic.

ER uses Dempster-Shaffer's theory of evidence [45] which deals with multiple attribute decision analysis problems that include both quantitative and qualitative features with various types of uncertainties [46]. The methodology allows decision-makers to express their subjective judgement using belief structures which means they can provide more than one perceived ratings if they are not feeling confident about a single selection [42].

In addition, this approach allows incomplete ratings to be considered in the model if decision-makers are uncertain about their selection. Simple examples to show how these properties are modelled are shown below. One decision-maker can assess one of the project requirements (PR_i) as being 50% very important and 50% moderately important (total 100% belief).

This is modelled as $\{(7, 50\%), (5, 50\%)\} = 7 \times 50\% + 5 \times 50\%$. This means that the total rating is 6. Another decision-maker could assess the same criterion as 60% between moderately important and extremely important and only 20% as weakly important. This preference is modelled as $\{([5-9], 60\%), (3, 20\%)\} = [5-9] \times 60\% + 3 \times 20\% + [0-9] \times 20\%$. The reason the component $[0-9] \times 20\%$ is added into the model is because the initial belief structure is incomplete (80% < 100%). In that case the remaining belief degree represents the probability that has not been assigned to any of the other ratings [0-9]. This means that the rating would be interval ranging from 3.6 (lower bound) to 7.8 (upper bound).

2.3. Data processing and computations

Once all team members have completed their numerical assessment using the rating scales and the belief structures described earlier, the data are exported to calculate the HOQ and the final priorities of the structural design requirements. The solver in this study is a Microsoft Excel worksheet that uses the data collected from the BIM application (csv data format). The overall computational workflow developed in the current study is shown in Fig. 4.

Chin et al. [42] have introduced the implementation of the ER functions within QFD. Mehrabi-Kandsar et al. [47] have applied Chin et al.'s [42] ER-based QFD in the compressor manufacturing industry but no applications in the context of structural engineering have been found in the literature. The importance ratings of PR were computed and normalised first using the project team's preferences followed by the computation of the relationship matrix between DR and PR using structural engineers' preferences.

Finally, the interrelationships between DR were incorporated and the final DR priorities defined. Because the PR ratings could be intervals

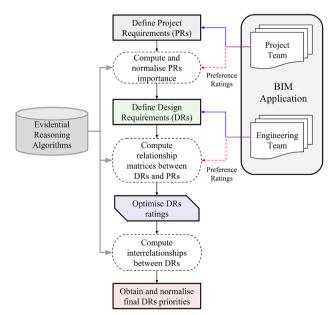


Fig. 4. Computational workflow and HOQ stages.

(ranges and not crisp numbers) the DR rating might need to be optimised accordingly to identify its corresponding lower and upper bounds. The detailed algorithmic functions for all the computational components including the DR optimisation models are described in Appendix A.

2.3.1. PR computations

To compute the importance ratings for each of the project requirements (PR), a relative weight is assigned first to all the project team members to resemble their influence in the decision procedure. Each weight takes any value between 0 and 1 with the condition that the sum of all the team members' weights is 1. Subsequently, each member of the team gives a perceived score to all the specified project requirements based on their expertise and domain knowledge. The team members' scores are called belief structures and are represented linguistically using the rating scales described in Section 2.2. The belief structure can be represented as a single rating (e.g. "Extremely Important") if there is no ambiguity in the team members' preferences or more frequently as a range when the participants are unsure about their particular selection (e.g. "Very Important - Extremely Important"). Each linguistic rating is computed using a corresponding numerical representation as summarised in Section 2.2. The numerical assessment for each of the importance ratings is multiplied by a confidence percentage (0-100%) which describes how confident the participants are with their selections. The sum of the team members' ratings multiplied by their corresponding influence weights gives the weighted average ratings for each of the PR. Typically, the weighted average ratings would be represented in a numerical range format. The normalised ratings are computed last using the average weights calculated previously.

2.3.2. PR and DR computations

Belief structures were also used to compute the relationship matrix between the PR and the DR. In this case, instead of project team members, a group of structural engineers was responsible for the specification of the relationships to ensure that each decision scenario is analysed in detail. Similarly to the PR computations, each of the structural engineers is assigned an influence weight (from 0 to 1) typically based on their role in the design. The project engineers have more influence on design decisions compared to the team of structural engineers that only support the analysis. The sum of all relative weights also needs to be equal to 1. All structural engineers give a perceived relationship score using the linguistic expressions from Section 2.2 based on their intuition and valuable project experience. After computing the belief structures for all the DR and PR relationships, they were then combined for each of the DR by converting them into probability masses using the ER algorithms and the PR ratings previously computed. The probability masses for each of the two PR were considered as two pieces of evidence and thus they needed to be

combined. All the probability masses for the DR and PR were combined in a recursive process that combined all the pieces of evidence. However, because the weights for the PR were typically interval ratings (range between two values), the combination process needed to be repeated for all the interval ratings. Once this process was completed, the minimum and maximum ratings for each of the DR were generated.

2.3.3. DR interrelationship computations

To compute the final DR priorities, the interrelationships between the various design requirements were incorporated to ensure the engineering interactions are adequately considered. This step is significant as the DR relationships can influence how the final rankings are prioritised. The interrelationship matrix was developed by the same team of structural engineers who analysed the DR and PR relationships using the numerical assessment outlined in Section 2.2. Once the detailed DR relationship ratings are recognised for all the design requirements, the initial DR importance ratings are multiplied with the interrelationship matrix. Once this step is completed, the normalised weighted DR ratings are finally computed.

3. Project application and assessment

To test the proposed decision-support model a practical example in the context of structural design was analysed in this section. A particular stage of the design development was investigated which involved the necessary decisions for the prioritisation of structural design parameters. Prioritising design parameters helps structural engineers recognise optimisation opportunities. This means that the structural typology and system for a building have already been identified and agreed by the project team in advance. The design domain of multistorey reinforced concrete (RC) structures was covered in this paper. However, the proposed method is not limited to the structural material and applications in other structural domains such as steel or timber structures could also be investigated in a similar manner.

3.1. Domain description and background

The design and optimisation of reinforced concrete structures in multi-storey buildings involves several design components that are often interrelated. Understanding these relationships can be time-consuming involving not only engineering analysis but also the assessment of whole building interactions and construction requirements. Material properties, column grids, slab thickness, columns sizes and reinforcement detailing are a few of the parameters structural engineers should consider whilst liaising with the design team.

Fig. 5 shows representative building examples that were used to test the proposed framework. The tested buildings were organised in two main blocks that shared the majority of their structural design characteristics and specifications (all buildings were residential and the



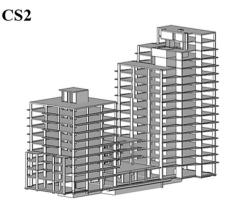


Fig. 5. BIM models of building scenarios under investigation.

structure consisted of flat slab floors supported by in situ columns which is a typical and widely used solution in the UK). The findings of this study are not limited to the selection of buildings as the participants were asked to define generalised design and project characteristics using their expert knowledge and domain experience. Similar design procedures should be expected for buildings that fall into the two main groups of buildings. In addition, the proposed framework could be adapted to any building context and could be used by any project team. The general design priorities were determined through the participatory QFD model to satisfy typical project requirements.

Senior project team members were advised and the background research, briefing, surveying and data collection took place in two decision experiments during June and July 2017. Each one lasted approximately 2 h. Detailed programme guidelines for the sessions were sent to the participants beforehand without disclosing any specific information for the building scenarios. The reason for this was to capture the participants' spontaneous opinions and preferences. The first experiment was organised with a multidisciplinary design team (architects, contractors, etc.) to specify the project related characteristics, whereas in the second experiment only structural engineers were involved to analyse the engineering design characteristics. During each experiment the project (PR) and engineering requirements (DR) were identified first (linguistic assessment) followed by the specification of their corresponding weights (numerical assessment) using the BIM application.

3.2. Project and design requirements specification

3.2.1. Project requirements

To investigate project requirements five qualified members of an existing project team were invited to participate in the study. The team members included the architect (Team Member 1), the client (Team Member 2), the contractor (Team Member 3) and the project manager (Team Member 4) and the structural engineer (Team Member 5). A briefing session was organised to answer any questions and explain the motivations of the study. After the briefing session, the participants were given access to the BIM application to develop and assess usual project requirements based on their expertise in the design and delivery of multi-storey reinforced concrete structures similar to the ones shown in Fig. 5. The project team agreed that for RC building structures the most relevant project requirements are associated with time and cost factors.

Six project requirements were analysed in this study by the project team's initial time-cost hierarchy. These are listed in Table 1. Delays in the construction works should be eliminated as they can have significant cost implications and interrupt the completion of site works (PR₁). In addition, for the timely completion of the project, the structural designs should be rationalised (PR₆) and the construction processes should be simplified (PR₂) as much as possible. Complicated and expensive solutions should be avoided unless they are absolutely necessary. At the same time, the overall quality of the resulting design should be high to eliminate future maintenance needs which would add to the total costs of the structure (PR₅). The successful integration of the structure with other building systems can significantly reduce the

design development time which means that the construction works could start earlier (PR_4). Costly design decisions and other relevant design risks could also be effectively managed by a project team that has experience in the specific building typology and structural system (PR_3).

3.2.2. Design requirements

To meet the PR presented in the previous section, four structural engineering practitioners of variable seniority were asked to specify and assess relevant design requirements using the BIM application. Two senior and two project engineers with extensive experience in RC structures participated in the study. The structural engineers were asked to draw from their expertise and list design parameters related to the superstructure that effectively address the project requirements. The parameters could involve any engineering component of the structure that is typically specified at this stage of the design development. The participants focused primarily on the structural floors and the column supports as according to their feedback "minimal changes can be achieved in the design of the structural cores". Table 2 summarises the DRs specified by the four structural engineers and a brief description for each one of the DR. As expected all DRs are quantifiable components in a typical building structure. This mean that for any given design configuration the seven DR can be directly obtained from the structural BIM model which significantly helps the post-processing and the final decision assessment. The structural engineers recognised that most of the specified design requirements are interconnected and thus detailed interrelationships assessment is conducted in Section 3.3.3.

3.3. Numerical evaluation

After the development of the project and design requirements the numerical assessment is conducted in the following sections implementing the equations from Appendix A. The PR importance weights matrix is calculated first (WHATs) followed by the relationship and interrelationship matrices (HOWs).

3.3.1. Importance weights matrix

The importance matrix of WHATs is computed in this section. Once all the PRs (WHATs) are identified, the corresponding importance weights were computed using the belief structures obtained from the five project team members via the BIM application. Table 3 demonstrates the assessment information for the six PR as obtained from the participants' data entries. In the tested case, it is assumed that all project members have the same relative weight $(\lambda_1 = \lambda_2 = \cdots = \lambda_5 = 0.2)$ which might not be always the case in real projects. For simplicity, the participants received equal treatments. The weights could be adjusted based on project specific needs.

The final normalised ratings were computed and the results are shown in Table 3. It is observed that the proposed QFD method with ER can effectively compute the ambiguity in the data collected from the design team members. All team members except Team Member 5 have provided their perceived importance ratings using a range scale. This clearly verifies that the selection of the ER in the QFD model was a

Table 1
Project requirements (WHATs) specified by the design team.

Project Requirements	Description
PR ₁ – Construction Speed	It is the time that is required to complete the assembly and the construction of the structure on site
PR ₂ – Buildability	If the construction complexity of the proposed engineering system is reduced the whole construction programme will be faster and cheaper
PR ₃ - Expertise	If the design team is experienced the overall construction and design quality can be improved whilst cost could be reduced
PR ₄ – Integration between disciplines	The integration between the structure and other systems such as architectural layouts and services
PR ₅ – Quality PR ₆ – Design Standardisation	If the overall design and construction delivery is improved the quality assurance of the project is increased Rationalisation of design elements help reduce errors, speed up the completion of works on site and reduce overall project costs

Table 2Design Requirements (HOWs) specified by the structural engineers associated with the main components of the superstructure.

Design Requirements	Description
DR ₁ – Column Grid	The column grid is given in column spacing. Larger column spacing help architects plan the internal layout of the building. However, at the same time the decision of a structural grid could make the whole structure more inefficient due to larger spans
DR ₂ – Slab thickness	The slab thickness is normally defined in mm and follows a set of discrete options. The selection of slab thickness is related to both the column grid and column sizes
DR ₃ – Slab reinforcement	The reinforcement rate in the slab is given in kg of steel per m ³ of concrete. The slab thickness and the load cases significantly affect the reinforcement rate
DR ₄ – Weight of structure	The total weight of the structural system is given in tonnes. The structural weight is a combination of the weight of the concrete and the reinforcement. Lighter structure could reduce construction time and waste and foundation costs
DR ₅ – Slab reinforcement spacing	The spacing of reinforcement is given in mm and it is defined as the distance between reinforcement bars in the slab. Engineers try to reduce the total number of reinforcement spacing to ease the construction process
DR_6 – Structure reinforcement schedule	The number of different bar diameters used in the slab and the columns significantly influences buildability and detailing efficiency. Typically, engineers try to reduce the number of different bar diameters in the structure
DR ₇ – Column Sizes	The area of the columns can affect the design of the slab. In addition, architects try to integrate columns within partitions which can create design challenges for the engineers

necessary functionality. The results for the normalised importance ratings of the PR suggest that design standardisation is the most important project requirement for the structure according to the participants followed by the buildability and systems integration. The results are not surprising as these are common constraints that structural engineers should address when they specify a structural system. These results could be attributed to the opinion that simpler structural systems are easier and cheaper to build. These are not findings that can be applied to any structural system but provide a good indication of what is perceived as common practice in the case of RC structures.

Similar observations were also put forward in a study conducted by Moynihan and Allwood [48] where they identified that rationalisation and poor buildability are the main reasons for the low structural efficiencies observed in 23 real steel structures they analysed. Finally, the results obtained suggest that if structural engineers want to optimise a structural system they should be aware of the implications on the buildability of the optimised designs. This could be achieved by integrating more constructability constraints in the formulation of optimisation procedures.

A sensitivity analysis was performed to investigate the impact of different relative importance weights (λ_i) of each design team member in the normalised PR importance ratings and to better analyse the previous results. This is because for the assessment presented in Table 3 equal weights (20%) between the five design team members were assumed. To conduct the sensitivity analysis, various weight distributions were computed for each of the λ_i weights using 5% increments with minimum range being 0% and maximum range being 100%. This means that a project team member could have no power (0%) in the project decisions or could be entirely in charge of the relevant decisions (100%). In total 10,686 weight combinations were identified using a

Python script for each of the five λ_i and applied in the model to calculate the different PR importance ratings. The lower bound and the upper bound value of the importance ratings were calculated. The results from the sensitivity analysis were used to calculate Pearson correlation (r) [49], which is the covariance of the team weights and the resulting PR ratings divided by the product of their standard deviations.

Fig. 6 shows the results from the sensitivity analysis of all the six PR ratings showing the corresponding positive and negative correlation coefficients for the five λ_i weights. It is observed that the upper and lower bounds of the PR ratings have different sensitivity to the λ_i weights. The sensitivity analysis also shows that the construction speed (PR₁) is related to the structural engineers', the architects' and the project managers' decisions whereas the buildability (PR2) is related mainly to the contractors' decisions. The technical expertise (PR₃) required for the effective delivery of the project's structure should be supported by the structural engineers' experience. The system integration (PR_4) is a main concern for the architects and the clients, whilst the overall project quality (PR₅) is important for the client and the project manager. Finally, the design standardisation (PR₆) is related to most of the stakeholders as it requires close coordination between the different disciplines. These findings are clearly associated with the numerical assessment provided by the participants in the study (Table 3). However, after analysing the results it becomes evident that they could be reasonably generalised and effectively used in similar decision procedures in RC projects as they offer a sensible method to distribute team roles and priorities amongst the project team members.

3.3.2. Relationship matrix

In this section, the relationship matrix between WHATs and HOWs was computed. The four structural engineers who participated in the

Table 3Importance ratings for the six project requirements (WHATs).

	Team Member 1 (20%)	Team Member 2 (20%)	Team Member 3 (20%)	Team Member 4 (20%)	Team Member 5 (20%)	Weighted average ratings	Normalised importance ratings
PR ₁	5:60% 0–9:40%	7–9:90% 0–9:10%	9:90% 0–9:10%	9	1	5.48-6.92	0.145–0.195
PR_2	7:80% 0-9:20%	5–7:85% 0–9:15%	9:90% 0-9:10%	7	5	5.99–7.14	0.158-0.203
PR ₃	5:50% 0-9:50%	5	5	3:50% 5:50%	7	4.70–5.60	0.123-0.160
PR ₄	9:90% 0-9:10%	5	7	7	5	6.42–6.60	0.165-0.193
PR ₅	7:80% 0-9:20%	7	3	5	3	4.72–5.08	0.122-0.148
PR ₆	9:80% 0–9:20%	5–7:90% 0–9:10%	9:90% 0-9:10%	7	7	6.76–7.84	0.177-0.223

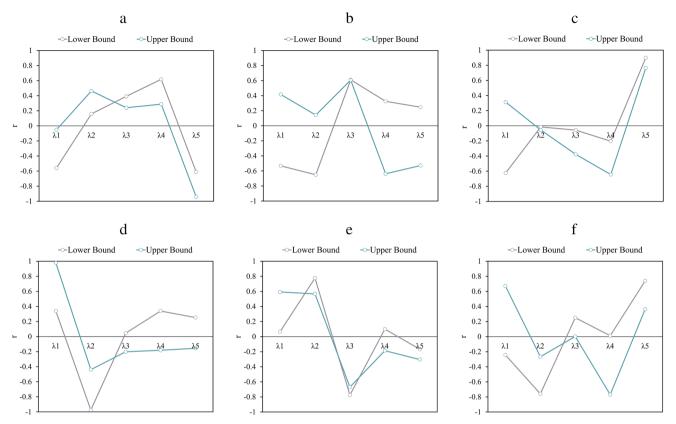


Fig. 6. Sensitivity analysis results for PR importance ratings (a) PR₁ Construction Speed, (b) PR₂ Buildability, (c) PR₃ Expertise, (d) PR₄ Integration between disciplines, (e) PR₅ Quality, (f) PR₆ Design Standardisation, where λ_i are the relative influence weights associated with the five members of the design team.

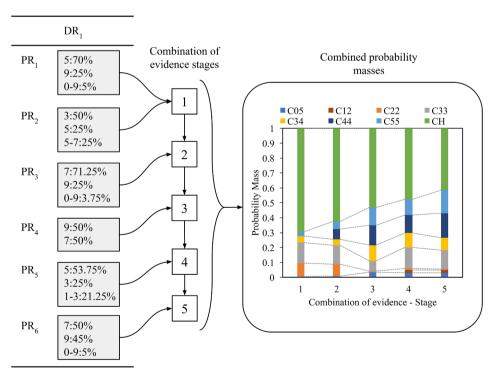


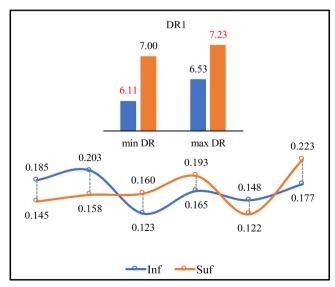
Fig. 7. Combination of evidence for DR_1 and computed probability masses at each stage.

study provided the numerical assessment of the relationship matrix. The complete assessment of the relationship between DR and PR is shown in the Appendix B (Table B.1) using the data the engineers provided through the BIM application. In this study, it is assumed that all engineers have equal relative weights $(\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0.25)$.

Similarly to the computation of the project team's weights presented in the previous section the intention here was also to obtain an objective understanding of the model's behaviour and its ability to compute group behaviour. Often in a real project condition, the project engineer could influence more the design decisions over a design engineer and

this can be accurately reflected in the weights to model individual's power. Once the weighted and averaged belief relationship between DR and PR are subsequently converted into probability masses. Fig. 7 shows an example of how the model combines probability masses from the six pieces of evidence (PR) to create joint probability masses. The figure demonstrates that the sum of probability masses on all five combination stages is 1, which is a good indication about the accuracy of the performed computations. In addition, it can be observed how the distribution of the individual masses changes with the addition of new information on every stage. For example, at stage 4 C₁₂ which corresponds to belief relationship 1-3:21.25% is added and maintained until the whole combination is complete. The same process is repeated until all the combined probability masses for the seven DR are computed. Because the obtained importance ratings PR in Table 3 are interval numbers (ranges) the importance ratings for the DR cannot be uniquely identified. To improve this condition, the two optimisation models are implemented. The first model searches for the combination of PR ratings from Table 3 for which the DR importance ratings are minimised. On the other hand, the second model identifies the PR ratings combinations that maximise the DR importance ratings. This means that 14 optimisation simulations (using Eqs. (A.15) and (A.16)) were performed (2 for each of the DR) to obtain the lower and the upper bounds of the initial DR importance ratings. They are called initial importance ratings because the interrelationships between the DR are not incorporated yet.

Fig. 8 shows an example of the optimisation models' results as computed for DR₁. The resulting lower bound is 6.11 and it was calculated using the following combination of PR weights as represented in the inf curve $\{w_1 = 0.185, w_2 = 0.203, w_3 = 0.123, w_4 = 0.165,$ $w_5 = 0.148$, $w_6 = 0.177$ }, whereas the upper bound is 7.23 with corresponding weights in the sup curve $\{w_1 = 0.145, w_2 = 0.158,$ $w_3 = 0.160$, $w_4 = 0.193$, $w_5 = 0.122$, $w_6 = 0.223$ }. It is observed that the computations are sensitive to the normalised PR importance ratings as small changes in their weights can significantly change the results for the DR. The results also validate the initial hypothesis as the DR₁ upper bound for the weights in the inf curve is 7 which is not the maximum value whereas the DR₁ lower bound for the weights in the suf curve is 6.53 which is not the minimum value either. The lower and upper bounds for the rest of the DR are computed in a similar manner. The results from the entire optimisation procedure are summarised in Table 4.



PR1 (w1) PR2 (w2) PR3 (w3) PR4 (w4) PR5 (w5) PR6 (w6)

Fig. 8. Optimisation results obtained from Eqs. (A.15) and (A.16) for DR_1 importance ratings.

Table 4Importance ratings for DR excluding the correlation matrix.

	DR_1	DR_2	DR_3	DR ₄	DR ₅	DR_6	DR ₇
Lower bound	6.108	4.992	2.066	3.516	4.059	4.298	5.380
Upper bound	7.226	7.032	3.220	4.873	5.041	5.254	5.908

Table 5
Correlation matrix of DR from Table B.4.

	DR_1	DR_2	DR_3	DR ₄	DR_5	DR_6	DR ₇
DR1	9	8.9–9	8.1625–9	7.95–8.9	0	0	8.05
DR2	8.9-9	9	8.55-9	8.775-9	0	0	4.625-5.3
DR3	8.1625-9	8.55-9	9	0.725 - 1.2	0	0	0.45 - 1.4
DR4	7.95-8.9	8.775-9	0.725 - 1.2	9	0	0	7.1-7.6
DR5	0	0	0	0	9	0	0
DR6	0	0	0	0	0	9	0
DR7	8.05	4.625-5.3	0.45-1.4	7.1–7.6	0	0	9

3.3.3. Interrelationship matrix

The internal relationships of HOWs are calculated herein. As previously observed there are certain interactions between the design requirements and therefore it is necessary to consider these interactions in the final computations of the importance rankings. For that reason, an additional assessment of the interrelationship matrix was calculated herein using the domain knowledge from the participating structural engineers. To compute the final importance DR ratings the interrelationship matrix is integrated in the results from Table 4. The information assessment for the interrelationships between DR is shown in Table B.3. The engineers' numerical assessment for the interrelationships is weighted and averaged and the results are used as the transformed correlation matrix which is summarised in Table 5. The correlation matrix from Table 5 is used to calculate and normalise the final importance ratings of DR. The final normalised importance ratings and rankings are shown in Table 6 and visualised in Fig. 9(a) showing the average as well as the upper and lower bounds ratings.

Using the probability scheme of Eq. (A.21) the final degree of preference for the DR is calculated. The results from the numerical assessment of this decision scenario suggest that the column grid (DR₁) and slab thickness (DR₂) are the most important decision parameters. On the other hand, the design requirements that received the lowest rankings are the reinforcement spacing (DR₅) and the reinforcement bars (DR₆). Comparing the final results with the previous analysis without the interrelationship in Fig. 9(b) it can be seen how necessary is to consider the interrelationship matrix, especially for the importance ratings of DR₃, DR₄, DR₅ and DR₆ where the largest discrepancies were observed

A significant advantage of the proposed decision-support model is that it can effectively capture what is perceived as the most important parameters from an engineering specification standpoint as both the structural grid and the slab thickness are design components that are specified early in RC projects according to the participant engineers. The results can also be organised into two main categories:

- The first one involves the design parameters that received higher rankings and are associated with the main sizing elements of the structure such as the structural grid, the slab thickness, the columns area.
- On the other hand, parameters associated with structural detailing such as the reinforcement parameters of the slab and the columns comprise the second category that received the lowest rankings in the model.

These findings provide a good indication of how the design development occurs is such structures and it can ultimately help structural

Table 6Final importance ratings and ranking order.

		DR_1	DR_2	DR_3	DR ₄	DR ₅	DR ₆	DR ₇
Importance Ratings	Lower Bound Upper Bound	187.546 248.271	172.708 232.509	116.115 171.451	163.717 220.243	36.538 36.538	38.683 38.683	146.584 190.174
Normalised Importance Ratings	Lower Bound Upper Bound	0.1741 0.2691	0.1602 0.2523	0.1073 0.1869	0.1514 0.2398	0.0321 0.0424	0.0339 0.0449	0.1339 0.2100
	Average	0.2216	0.2062	0.1471	0.1956	0.0373	0.0394	0.1720
Ranking		1	2	5	3	7	6	4

engineers identify optimisation opportunities within these two categories.

3.4. Post-processing specification

In the preceding section, the numerical assessment for the decision criteria that developed from the BIM-integrated QFD process was computed. Following on from this process the main question that emerges is "How this numerical data can be used in practical terms by structural engineers to conduct more informed design decisions?". All the obtained design requirements (DR) represent quantitative qualities or parameters of a structure that can be identified by the structural engineers from the corresponding BIM structural models. For instance, the total weight of the structure and the reinforcement rates can be obtained directly from the material schedules (BIM quantity take-offs) whilst the rest of the sizing parameters such as slab thicknesses, column areas, column grids are found in the BIM families' properties. This is significant for the post-processing of the OFD results as the DR normalised importance ratings can be directly used within a MCDM workflow for the assessment of different structural design alternatives. A post processing workflow is proposed herein as illustrated in Fig. 10.

Assume there is a structural design scenario with a conflicting set of optimisation objectives as described in the literature section with the optimised results represented in a graph similar to the one shown in Fig. 11. There will be a set of structural alternatives on the Pareto front that will require further assessment to identify a single solution. For that purpose a MCDM model that combines quantitative data from the BIM structural models and the DR weights from the BIM-based QFD model could be used. The two sets of data are aggregated within the MCDM model and the final design rankings of the different structural options could be identified. Thus, the proposed decision-support model becomes an integral component of a comprehensive decision workflow that utilises structural optimisation analysis, BIM technologies and QFD

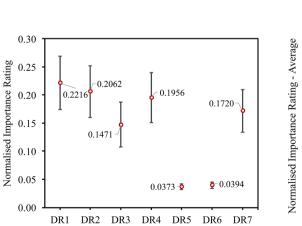
data combined with MCDM procedures. The main benefit of such a workflow is that structural optimisation can be effectively integrated with decision processes which could ultimately enhance the use of such procedures in practice.

3.5. Shared domain knowledge

A follow up session with the project team took place and the results obtained from the analysis of the OFD model were presented whilst future guidelines were also reviewed. After analysing the results all participants agreed that the proposed method have significantly helped them synthesise the main project priorities associated with the buildings structure. Furthermore, the structural engineers verified that through the QFD approach they managed to get a better understanding of what is important for the project team. Both these observations are particularly useful as domain knowledge can be effectively shared through the QFD model allowing a formal and systematic decision structure. In addition, this suggests that decision-makers may be receptive to new decision hierarchies that help them create a shared understanding of the decision problem whilst collectively establish domain knowledge necessary for the final decision-making. Generally, shared domain knowledge can make the whole decision process and design development more efficient. Finally, a few recommendations for further development were also provided by the participants: BIM knowledge-based systems that embed the results from the QFD model could further engage the design team and help them assess the outputs from the entire process.

4. Discussion

By enabling knowledge and information transfer between disciplines, decision-making practices could effectively capture domain knowledge and offer more integrated solutions to complex structural



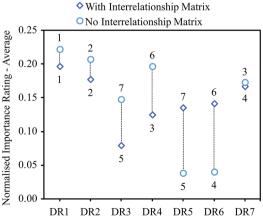


Fig. 9. (a) Final ranking of the engineering design requirements – HOWs (average values with lower and upper boundaries), (b) comparison of normalised importance ratings with and without interrelationship matrix.

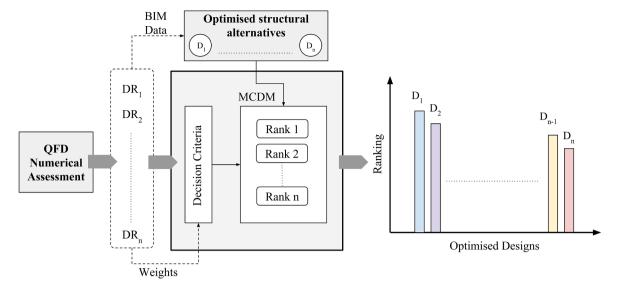


Fig. 10. Workflow for post processing of the results.

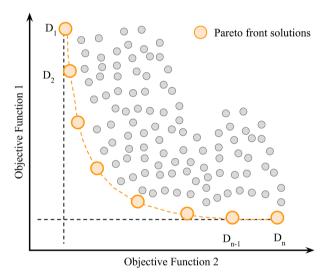


Fig. 11. Optimised structural alternatives for conflicting objective functions.

design optimisation problems. By identifying the decision priorities for the structural system, it is anticipated that structural engineers would be able to effectively use them when assessing optimised designs.

4.1. Method evaluation

The methodology of the proposed decision-support model was assessed against Ullman's [50] hierarchy for robust decision-making. Ullman [51] suggested that decision-making models utilise three fundamental categories of information comprising data, models and knowledge. To obtain the knowledge necessary to make a decision the relationships between the data are firstly analysed through explicit model representations followed by understanding and evaluating the behaviour of such models. Specifically for the development of decision criteria which is the main focus of this paper, Ullman recommended QFD concepts to define and measure the important features of a decision problem. The method in this paper effectively builds on Ullman's hierarchy: numerical and linguistic data are implemented to identify project and design requirements through a participatory QFD model whilst decision insights are obtained through the analysis of the QFD model using integrated evidential reasoning algorithms. The study aims to fill the gap in the knowledge regarding the use of BIM in the

optimisation and decision-making of the buildings structures. The use of BIM technologies brought together the different features of the hierarchy and improved the decision procedures in three main directions: (1) Design Integration, (2) Enhanced Communication, (3) Decision-based Optimisation Delivery.

4.2. Design integration

The design integration opportunities were recognised as the main advantage of the proposed participatory decision process against conventional practices by the participants of the study. An illustrative design situation was highlighted during the experiment: on most instances the column grids are specified by the architects to accommodate room layouts and planning requirements. This leaves structural engineers with limited options when it comes to the optimisation of the column grid. This approach could restrict the design efficiency or even worse could result in uneconomic or inefficient structural solutions. Following on from the pilot study the team architect seemed to be open to more explicit structural engineering advice especially during the early design stages when the selection of column grid takes place. Similar design integration opportunities with the structural systems could be enabled via the proposed BIM application. Decision priorities that are collectively planned and identified by the project team are translated in detailed design characteristics which are subsequently can be stored in the shared BIM models and relevant knowledge-based systems. Individual team members could access these design priorities and review whether trade-off solutions that improve design efficiencies are obtainable. Further synergies could be identified between structural engineers and architects (column grids and element sizes), structural engineers and contractors (construction sequencing), structural engineers and M&E engineers (services-structures integration).

4.3. Enhanced communication

Extending the capabilities of BIM applications to further integrate them with decision-making processes and enhance the communication amongst the project team members especially during the early stages of the design development was perceived as a positive transformation by the participants of this study. As it was observed in the decision workflow in Section 3.4 BIM applications create interactions between structural optimisation, decision-making procedures and project teams. This is particularly important as the adoption of such decision support models are expected to enhance the collaboration of the decision-

makers through interactive BIM applications and possibly BIM-enabled knowledge-based systems. Overall, the intention of the proposed research is not only to challenge the current decision practice in the structural engineering domain but also to encourage further integration of quality-based decision support models in the construction industry as a whole. Similar research efforts in other structural domains such as steel or timber construction could be realised through similar BIM-based decision procedures.

4.4. Decision-based optimisation

The numerical example presented in Section 3.3 exhibited how the decision support model could be implemented in real optimisation occurrences in the context of RC structures. The results from the study clearly indicated that the decision-makers value buildability and constructability as the most important parameters for the structural systems. This is significant as it can inform future policies on structural optimisation procedures. New decision-based optimisation models could be formulated to incorporate decision-makers' feedback. In addition, new computational paradigms that incorporate constructability functions as algorithmic constraints would be required to increase the adoption of structural optimisation procedures in practical design problems. BIM-enabled structural optimisation frameworks would have a primary role in the development of decision-based optimisation models as they can be directly integrated in a decision workflow as the one shown in Section 3.4 to generate structural optimised alternatives. In that way, every aspect associated with the decision making of building structures will be facilitated and managed with BIM related applications.

5. Conclusions

The design optimisation and decision-making analysis of building structures could be complicated due to the plethora of stakeholders that often have conflicting requirements. This could inherently create a lot of inefficient designs as structural engineers cannot easily specify key design optimisation parameters in a project brief. To address this drawback, a novel decision support model that assists structural engineers and other stakeholders prioritise design criteria based on project specific requirements was proposed in this paper as part of a comprehensive decision-based optimisation model. The different project and design requirements were modeled using a participatory QFD method which was adapted for this purpose.

Active stakeholder involvement was facilitated via a custom BIM application which was used to collect data relevant to the QFD model. Evidential reasoning algorithms under uncertainty were implemented within the QFD model to effectively process the data collected from the BIM application and create the final numerical assessment of the

engineering design priorities. The real value of the BIM integration is that it allows different design teams to easily repeat the decision process at the beginning of every project. In this paper, the proposed decision support model was tested and verified in decision experiments analysing the interactions of real design teams.

The decision experiments provided new insights and suggestions on how the proposed participatory decision model can be practically used as an effective design prioritisation framework and optimisation guidance in the context of reinforced concrete building structures by recognising and computing project and design engineering requirements. In the conducted decision experiments, the (1) Construction speed, (2) Buildability, (3) Expertise, (4) Integration between disciplines, (5) Quality and (6) Design standardisation were recognised as the main project characteristics relevant to the structural design. Between those six project requirements for the structure, the most important one was the design standardisation followed by the buildability and systems integration.

Furthermore, seven common structural design requirements that address the aforementioned project requirements were identified during the conducted experiments: (1) Column grid, (2) Slab thickness, (3) Slab reinforcement, (4) Structural weight, (5) Slab reinforcement spacing, (6) Structure reinforcement schedule, and (7) Column sizes. After analysing the relationships between the project and design requirements as well as the interrelationships between the design requirements, it was concluded that the column grid, the slab thickness, and the structural weight were the most important design characteristics of the tested structural typologies and they should be prioritised early in the design process.

Overall, the study suggested that by enabling knowledge and information transfer between the different disciplines involved in the decision-making processes of building structures, domain knowledge could be effectively captured, whilst more integrated solutions to complex structural design optimisation problems could be articulated. Therefore, by identifying decision priorities for the structural system early in the project development, it is anticipated that structural engineers would be able to effectively use them when assessing designs obtained from optimisation procedures.

Acknowledgements

The authors would like to thank the structural engineers and the project team who participated in the decision experiment. This research has been made possible through funding provided by the Engineering and Physical Sciences Research Council (EPSRC) and from Price & Myers LLP via the UCL EngD Centre (Grant number: EP/G037159/1) in Virtual Environments, Interaction and Visualisation and this is gratefully acknowledged here.

Appendix A. Evidential reasoning algorithms within QFD was implemented from [42,47]

 $E(S_i^{(l)})$ is the expected score obtained from the belief structure of a decision-maker l when assessing the relative importance of a project requirement PRi. The total score is the sum of the expected scores from all project team members L and it can be represented as:

$$E(S_i) = \sum_{l=1}^{L} \lambda_l E(S_i^{(l)}), \quad l = 1, \dots, L; \quad i = 1, \dots, m$$
(A.1)

where λ_l is the relative weight of the team member l. Based on this equation the relative importance of PRi can be defined as:

$$w_{i} = \left[\frac{E^{L}(S_{i})}{E^{L}(S_{i}) + \sum_{i \neq j} E^{U}(S_{i})}, \frac{E^{U}(S_{i})}{E^{U}(S_{i}) + \sum_{i \neq j} E^{L}(S_{i})} \right], \quad i = 1, \dots, m; \quad \sum_{i=1}^{m} w_{i} = 1; \quad L = \text{lowerbound}, \quad U = \text{upperbound}$$
(A.2)

The relationship matrix $(R_{ij}, i = 1,...,m, j = 1,...,m)$ between PR and DR is computed using the structural engineers' belief structures. Assuming that M structural engineers are participating in this study each of them is given a weight $\theta_k > 0$ $(k = 1 \cdots, M)$ with $\sum_{k=1}^{M} \theta_k = 1$. The belief structures recorded by the k-th team member is represented by $\{(H_{pq}, \beta_{pq}^{(k)}), p = 0 \cdots, N; q = 0 \cdots, N\}$ where $\beta_{pq}^{(k)}$ are the belief degrees to which the relationship R_{ij} is evaluated to the interval rating H_{pq} .

The crisps ratings defined for the relationship assessment are H_{pp} for p = 0..., N whereas the interval ratings are H_{pq} for p = 0..., N and q = p + 1..., N. The matrix in Eq. (A.3) shows the 21 possible relationships that can exist between the PR and the DR. Only six crisp ratings exist in the matrix consisting of H_{00} , H_{11} , H_{22} , H_{33} , H_{44} and H_{55} . The remaining 15 ratings are interval.

The expected belief degree for all the M team members regarding every relationship between PR and DR is computed using the following

$$\beta_{pq} = \sum_{k=1}^{M} \theta_k \beta_{pq}^{(k)}, \quad p = 0, \dots, N, \quad q = p, \dots, N$$
(A.4)

The interrelationship matrix uses belief structures from the engineering team and it is computed using Equation (A.5)

$$r_{jk} = \sum_{k=1}^{M} \vartheta_{k}\{(H_{pq}, a_{pq}^{(k)}), p = -N, \dots, N; q = p, \dots, N\} = \left\{ \left(H_{pq}, \sum_{k=1}^{M} \vartheta_{k} a_{pq}^{(k)} \right), p = -N, \dots, N; q = p, \dots N \right\}, j, k = 1, \dots, n$$
(A.5)

where $a_{pq}^{(k)}$ is the belief degree to which r_{jk} is assessed to the interval H_{pq} .

After computing the belief structures for all the relationships R_{ij} they are combined for each of the DR. For two belief structures $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ and $R_{i,j} = \{(H_{pq}, \beta_{pq}(R_{i,j})), p = 0, \dots, N, q = p, \dots, N\}$ requirements PR_{i1} and PR_{i2} related to a design requirement and w_{i1} and w_{i2} are their normalised weights, the belief structures are converted into probability masses using the equations below:

$$m_{pq} = w_{i_1} \beta_{pq}(R_{i_1 j}), \quad p = 0, \dots, N; \quad q = p, \dots, N$$
 (A.6)

$$m_{H} = 1 - \sum_{p=0}^{N} \sum_{q=p}^{N} w_{i_{1}} \beta_{pq}(R_{i_{1}j}) = 1 - w_{i_{1}} \sum_{p=0}^{N} \sum_{q=p}^{N} \beta_{pq}(R_{i_{1}j}) = 1 - w_{i_{1}}$$
(A.7)

$$n_{pq} = w_{i_2} \beta_{pq}(R_{i_2j}), \quad p = 0, \dots, N; \quad q = p, \dots, N$$
 (A.8)

$$n_{H} = 1 - \sum_{p=0}^{N} \sum_{q=p}^{N} w_{i_{2}} \beta_{pq}(R_{i_{2}j}) = 1 - w_{i_{2}} \sum_{p=0}^{N} \sum_{q=p}^{N} \beta_{pq}(R_{i_{2}j}) = 1 - w_{i_{2}}$$
(A.9)

The above probability masses are combined using Eqs. (A.10), (A.11) and (A.12). The procedure to provide a set of joint probability masses is based on the evidential reasoning approach and Dempster-Shafer theory of evidence which uses two pieces of evidence to create joint probability masses C_{pq} $(p = 0, \dots, N, q = p, \dots, N)$ and C_H [47] using the following equations:

$$C_{pq} = \frac{1}{1 - K} \left[\sum_{s=0}^{p} \sum_{t=q}^{N} \left(m_{st} n_{pq} + m_{pq} n_{st} \right) + \sum_{s=0}^{p-1} \sum_{t=q+1}^{N} \left(m_{sq} n_{pt} + m_{pt} n_{sq} \right) \right] + \frac{1}{1 - K} [m_{H} n_{pq} + m_{pq} n_{H} - m_{pq} n_{pq}], \quad p = 0, \dots, N; \quad q = p, \dots, N$$
(A.10)

$$K = \sum_{p=0}^{N} \sum_{q=p}^{N} \sum_{s=0}^{p-1} \sum_{t=s}^{p-1} (m_{st} n_{pq} + m_{pq} n_{st})$$
(A.11)

$$C_H = \frac{m_H n_H}{1 - K} \tag{A.12}$$

Final normalised combined probability mass is computed using:

$$\delta_{pq} = \frac{x_{pq}}{1 - x_H}, \quad p = 0, \dots, N; \quad q = p, \dots, N$$
(A.13)

The overall assessment is computed by an expected interval which represents the importance rating of the DR:

$$E(DR_{j}) = \sum_{p=0}^{N} \sum_{q=p}^{N} \delta_{pq} H_{pq} = \left[\sum_{p=0}^{N} \sum_{q=p}^{N} \delta_{pq} H_{p}, \sum_{p=0}^{N} \sum_{q=p}^{N} \delta_{pq} H_{q} \right]$$
(A.14)

Because the importance weights of PR that are computed by Eqs. (A.1) and (A.2) could be intervals the importance rating of DR obtained by Eq. (A.14) cannot be uniquely identified. To improve this drawback the following two optimisation models that consider the PR as decision variable are used to identify the lower and upper limits for all the DR.

Lower bound:

$$min \ E^{L}(DR_{j}) = \sum_{p=0}^{N} \sum_{p=q}^{N} \delta_{pq} H_{p}$$
 (A.15)

Subject to:
$$w_i^L \le w_i \le w_i^U$$
, $i = 1, \dots, m$ and $\sum_{i=1}^m w_i = 1$

Upper bound:

$$\max \ E^{U}(DR_{j}) = \sum_{p=0}^{N} \sum_{p=q}^{N} \delta_{pq} H_{q}$$
(A.16)

Subject to:
$$w_i^L \le w_i \le w_i^U$$
, $i = 1, \dots, m$ and $\sum_{i=1}^m w_i = 1$

where δ_{pq} is identified by Eq. (A.13). The models in Eqs. (A.15) and (A.16) are computed using the Excel Solver for each of the DR and the final weights are used in the next step where the interrelationship matrix between the DR is integrated.

The most common way to incorporate the interrelationship matrix is shown below after [42]:

$$R'_{ij} = \sum_{k=1}^{n} R_{ik} r_{kj}, \quad i = 1, \dots, m; \quad j = 1, \dots, n$$
(A.17)

where R'_{ij} is the adjusted relationship between PR and DR and η_{kj} is the interrelationship between DR. Using Eq. (A.17) the importance rating of DR can be obtained from the equation:

$$DR'_{j} = \sum_{i=1}^{m} w_{i} R'_{ij} = \sum_{i=1}^{m} w_{i} \left(\sum_{k=1}^{n} R_{ik} \eta_{kj} \right) = \sum_{k=1}^{n} \left(\sum_{i=1}^{n} w_{i} R_{ik} \right) \eta_{kj} = \sum_{k=1}^{n} DR_{k} \eta_{kj}, \quad j = 1, \dots, n$$
(A.18)

Where DR'_j and DR_j are the importance ratings with and without the interrelationship matrix. Assuming that $E(\eta_{ij})$ is the expected score matrix obtained from Eq. (A.5) and $E(DR_k) = [E_k^L, E_k^U]$, k = 1, ..., n is the initial DR importance ratings obtained by solving Eqs. (A.15) and (A.16) the final importance rating is computed from Eq. (A.18) as:

$$DR'_{j} = \sum_{k=1}^{n} E(DR_{k})E(\eta_{k}), \qquad j = 1, \dots, n$$
(A.19)

The last equation computes the technical importance of the design requirements however due to the uncertainties associated with the interval rankings the results need to be normalised. The equation below is used to normalise the design importance ratings:

$$DR_{j} = \frac{DR_{j}^{'}}{\sum_{i=1}^{n} DR_{i}^{'}} = \left[\frac{(DR_{j}^{'})^{L}}{(DR_{j}^{'})^{L} + \sum_{i \neq j} (DR_{i}^{'})^{U}}, \frac{(DR_{j}^{'})^{U}}{(DR_{j}^{'})^{U} + \sum_{i \neq j} (DR_{i}^{'})^{L}} \right], \quad j = 1, \dots, n$$
(A.20)

where $(DR_i^{'})^L$ and $(DR_i^{'})^U$ are the lower and upper boundaries of $DR_i^{'}$.

Once the normalised weights for the design requirements have been obtained their priority rankings can been estimated. In this study the equation proposed by Wang et al [52] is used to compute the priority degree as shown below:

$$P(a > b) = \frac{max(0, a_2 - b_1) - max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)}$$
(A.21)

where $a = [a_1, a_2]e$ and $b = [b_1, b_2]$ are two positive interval numbers.

Appendix B. Comprehensive preference data collected from the BIM application

See Tables B.1-B.4.

Table B.1Assessment on the relationships between DR and PR.

		DR_1	DR_2	DR_3	DR_4	DR_5	DR_6	DR_7
PR ₁	TM ₁ (25%)	5:80%	5	5	5	7	9	7
	TM ₂ (25%)	5	Unknown	Unknown	Unknown	9	7	7
	TM ₃ (25%)	5	3	5	5	7	7	3–5
	TM ₄ (25%)	9	5	7	3	5	5-7: 80%	5:70%
							9:20%	7:30%
PR_2	TM ₁ (25%)	5	7:90%	3	5	7	9	7
	TM ₂ (25%)	3	Unknown	3–5	7–9	9	7	7
	TM ₃ (25%)	3	5	7:75%	Unknown	7	5:85%	3–5
				5:25%			7:15%	
	TM ₄ (25%)	5–7	5	7	5	5–7	7	3
PR_3	TM ₁ (25%)	7	7	0	3	1	1	7
	TM ₂ (25%)	9	5:90%	0	1-3	0-1	0-1	5
	TM ₃ (25%)	7	5	0	5	0-1	0-1	3–5
	TM ₄ (25%)	7:85%	7	0	5	3	3	7
PR_4	TM ₁ (25%)	9	7	0	3	1	1	5

(continued on next page)

Table B.1 (continued)

		DR_1	DR_2	DR_3	DR_4	DR ₅	DR ₆	DR ₇
	TM ₂ (25%)	7	9	0	1–3	1	1	3
	TM ₃ (25%)	7	Unknown	0	5	0-1	0-1	5–7
	TM ₄ (25%)	9	9	0	3	3	3	7
PR_5	TM ₁ (25%)	5	3	0	3	1	1	5
	TM ₂ (25%)	5	1	0	Unknown	0–1	1	3
	TM ₃ (25%)	1–3:85% 5:15%	1	0	3	0–1	0–1	5:80% 3:20%
	TM ₄ (25%)	3	Unknown	0	1-3:80%	3	5	5
PR_6	TM ₁ (25%)	7	7:90%	3	5	7	7	7
	TM ₂ (25%)	9:80%	9	3	3	9	9	7
	TM ₃ (25%)	7	5	5	5	5	7	5–7
	TM ₄ (25%)	9	9	5–7	5	9	9	7

Table B.2Belief relationship matrix between DR and PR.

	DR_1	DR_2	DR_3	DR_4	DR_5	DR_6	DR ₇
PR ₁	5:70%	3:25%	5:50%	3:25%	5:25%	7:50%	7:57.5%
	9:25%	5:50%	7:25%	5:50%	7:50%	9:30%	5:17.5%
	0-9:5%	0-9:25%	0-9:25%	0-9:25%	9:25%	5–7:20%	3-5:25%
PR_2	3:50%	5:50%	3:25%	5:50%	7:50%	9:25%	7:50%
	5:25%	7:22.5%	5:6.25%	7-9:25%	9:25%	7:53.75%	3:25%
	5-7:25%	0-9:27.5	7:43.75%	0-9:25%	5-7:25%	5:21.25%	3-5:25%
			3-5:25%				
PR_3	7:71.25%	7:25%	0	3:25%	1:25%	1:25%	7:50%
	9:25%	9:50%		5:50%	0-1:50%	0-1:50%	5:25%
	0-9:3.75%	0-9:25%		1-3:25%	3:25%	3:25%	3-5:25%
PR ₄	9:50%	9:50%	0	3:50%	1:50%	1:50%	5:25%
	7:50%	7:25%		5:25%	3:25%	3:25%	3:25%
		0-9:25%		1-3:25%	0-1:25%	0-1:25%	7:25%
							5-7:25%
PR_5	5:53.75%	3:25%	0	3:50%	3:25%	1:50%	5:70%
	3:25%	1:50%		1-3:20%	1:25%	0-1:25%	3:30%
	1-3:21.25%	0-9:25%		0-9:30%	0-1:50%	5:25%	
PR_6	7:50%	9:50%	3:50%	5:75%	9:50%	9:50%	7:75%
	9:45%	7:22.5%	5:25%	3:25%	7:25%	7:50%	5-7:25%
	0-9:5%	5:25%	5-7:25%		5:25%		
		0-9:2.5%					

Table B.3 Assessment on the interrelationships between DR.

		DR_1	DR_2	DR_3	DR ₄	DR_5	DR ₆	DR ₇
DR ₁	TM1 (25%)	9	9	9:85%	9:80%	0	0	9
					7:20%			
	TM2 (25%)	9	9:80%	9	9:80%	0	0	9
	TM3 (25%)	9	9	9	7–9	0	0	7:90%
								9:10%
	TM4 (25%)	9	9	7–9	9	0	0	
DR_2	TM1 (25%)	9	9	9:80%	9	0	0	5:80%
	TM2 (25%)	9:80%	9	9	9	0	0	5
	TM3 (25%)	9	9	9	9	0	0	5
	TM4 (25%)	9	9	9	9:90%	0	0	5:90%
DR_3	TM1 (25%)	9:85%	9:80%	9	1:90%	0	0	1
	TM2 (25%)	9	9	9	1	0	0	0-1
	TM3 (25%)	9	9	9	1	0	0	0-1
	TM4 (25%)	7–9	9	9	0–1	0	0	1:80%
DR_4	TM1 (25%)	9:80%	9	1:90%	9	0	0	7:80%
		7:20%						9:20%
	TM2 (25%)	9:80%	9	1	9	0	0	7
	TM3 (25%)	7–9	9	1	9	0	0	7

(continued on next page)

Table B.3 (continued)

		DR_1	DR_2	DR_3	DR ₄	DR_5	DR_6	DR ₇
	TM4 (25%)	9	9:90%	0–1	9	0	0	7–9
DR_5	TM1 (25%)	0	0	0	0	9	0	0
	TM2 (25%)	0	0	0	0	9	0	0
	TM3 (25%)	0	0	0	0	9	0	0
	TM4 (25%)	0	0	0	0	9	0	0
DR_6	TM1 (25%)	0	0	0	0	0	9	0
	TM2 (25%)	0	0	0	0	0	9	0
	TM3 (25%)	0	0	0	0	0	9	0
	TM4 (25%)	0	0	0	0	0	9	0
DR_7	TM1 (25%)	7	5:80%	1	7:80%	0	0	9
					9:20%			
	TM2 (25%)	9	5	0–1	7	0	0	9
	TM3 (25%)	9	5	0–1	7	0	0	9
	TM4 (25%)	7:90%	5:90%	1:80%	7–9	0	0	9
		9:10%						

Table B.4Belief interrelationship matrix between DR.

	DR_1	DR_2	DR_3	DR_4	DR ₅	DR_6	DR ₇
DR ₁	9	9:95% 7–9:5%	9:71.25% 7–9:25% 0–9:3.75%	9:65% 7:5% 7-9:25% 0-9:5%	0	0	9:52.5% 7:47.5%
DR_2	9:95% 7–9:5%	9	9:95% 0–9:5%	9:97.5% 0–9:2.5%	0	0	5:92.5% 0–9:7.5%
DR ₃	9:71.25% 7–9:25% 0–9:3.75%	9:95% 0–9:5%	9	1:72.5% 0–1:25% 0–9:2.5%	0	0	1:45% 0–9:5% 0–1:50%
DR ₄	9:65% 7:5% 7–9:25% 0–9:5%	9:97.5% 0–9:2.5%	1:72.5% 0–1:25% 0–9:2.5%	9	0	0	7:70% 9:5% 7–9:25%
DR_5	0	0	0	0	9	0	0
DR_6	0	0	0	0	0	9	0
DR ₇	9:52.5% 7:47.5%	5:92.5% 0-9:7.5%	1:45% 0-9:5% 0-1:50%	7:70% 9:5% 7–9:25%	0	0	9

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