Robust adaptive terminal sliding mode control for dynamic positioning of a semi-submersible offshore platform

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Abstract

In this paper, a robust adaptive terminal sliding mode controller is proposed for dynamic positioning of a semi-submersible offshore platform. First, a state feedback controller is designed to stabilize the nominal system. Then, a robust adaptive terminal sliding mode compensator is developed to eliminate the effects of uncertain dynamics and disturbances. It is shown using Lyapunov methods that the tracking error is driven to zero in finite time using the proposed control. The efficacy of the control algorithm is validated using simulation studies and it is shown that recent developments in the domain of robust exact differentiation are very helpful for controller implementation.

Keywords

sliding mode control; dynamic positioning system; semi-submersible offshore platform, state estimation, marine systems

Introduction

Dynamic positioning (DP) of semi-submersible offshore platforms is a key requirement within the oil and gas industry as discussed by [Fossen and Strand(1998)], [Fossen(1994)] and [Mao and Yang(2016)]. The performance of any control system directly determines the stability of the vessel impacting on its safe and efficient operation ([Sϕrensen(2005), Sϕrensen(2011)]).

A classical PID control strategy incorporating a low-pass filter has been traditionally used to offset the effect of environmental impacts on the system. Indeed, since the 1960s this has been the primary approach to the DP control problem as described by [Panagou and Kyriakopoulos(2014), Zhao et al.(2014)]. A self-tuning Kalman filter was designed for the DP controller by [Fung and Grimble(1983)]. A modified LQG control algorithm was proposed by [Balchen(1993)] and extended to a model-based control scheme which can provide both station-keeping and tracking capability ([Sϕrensen et al.(1996)]). A reliability-based control algorithm for DP of floating vessels was proposed by [Leira et al.(2004)]. An adaptive control strategy which can accommodate on-line modification of the controller gains has been developed by [Tannuri et al.(2006)] which can guarantee performance levels across a wide operating range.

Sliding mode control (SMC) is known to provide strong robustness to uncertainty and external disturbances and this property has been validated across a range of application domains including off-shore vessels. The application of sliding mode control to the dynamic positioning of a turret moored FPSO (Floating Production Storage and Offloading) vessel was presented by [Tannuri et al.(2001)]. A sliding-mode control law has also been presented and experimentally tested for trajectory tracking of underactuated autonomous surface vessels by [Ashrafiuon(2008)]. An experimental analysis of sliding mode control was executed by [Tannuri et al.(2010)], which verified the effectiveness of the sliding mode control paradigm for DP control. A second order sliding mode control has been proposed for surface vessels by [Valenciaga(2014)] who

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uses the super twisting method, which can deal with parameter uncertainty very effectively.

It is clear that many results have been obtained. All the above results are under-pinned by asymptotic stability considerations, where an infinite time horizon is required to stabilize the tracking error to zero. To provide rapid convergence, such approaches require high control gains in general, which may lead to problems with control input saturation and/or inefficient use of energy. A finite time control approach [Oza et al. (2015)], on the other hand, has the potential to provide fast convergence, strong robustness and high control precision, as under appropriate conditions error signals become identically zero in a finite time. These characteristics have been shown to be particularly appropriate for mechanical systems. Recent work in [Song et al. (2017)] has considered a class of systems represented by a nominal linear system in the presence of unknown nonlinearity and disturbances where finite time boundedness is guaranteed by a state feedback controller. A finite-time H^{∞} controller is proposed based on L_2-gain analysis in [Xie, Lam and Li (2017)] for a particular continuous-time periodic piecewise linear system representation. Again the controller is implemented based on knowledge of the system state.

Terminal sliding mode control (TSMC) is a particular finite time control strategy that has been shown to provide robustness to system uncertainty and external disturbances. It has been found to be particularly successful in control applications including robotics [Li and Huang(2010)], spacecraft rendezvous and docking [Lee and Vukovich(2016)], control of piezoelectric actuators [Alghanimi et al.(2016)] and control of underactuated autonomous underwater vehicles [Elmokadem et al.(2017)], all of which require very precise position control and high robustness. In [Mobayen and Javadi (2015)] a recursive terminal sliding mode structure for tracking control of third-order chainedform nonholonomic systems in the presence of the unknown external disturbances is developed. Other recent approaches where sliding mode control concepts have been employed with particular canonical forms in order to counteract disturbances include work in [Yin et al. (2017b)] where the stabilization problem for nonlinear Markovian jump systems with output disturbances, actuator and sensor faults was considered. An approach developing an integral sliding mode controller for singular stochastic Markovian jump systems in uncertain environments is presented in [Zhang et al. (2017)].

A key motivation of the current work is to consider the specific dynamics of the semi-submersible offshore platform which cannot be readily expressed in the canonical forms in the above papers without introducing unnecessary conservatism in the controller design problem. A second motivation is to present a straightforward implementation strategy based on measured outputs only; again the particular structure of the model will be seen to suggest a natural solution to this problem. For the semi-submersible offshore platform, accurate position control is required. Finite time stability will ensure the vessel reaches the desired position rapidly and can render the dynamic positioning process more energy-efficient. Disturbance rejection is particularly important for dynamic position control of the semi-submersible offshore platform. Disturbances will include the wind, waves and ocean currents and these will be distributed in the low and high frequency ranges. TSMC prescribes two phases within the dynamic response; the low frequency disturbance can be suppressed effectively during the reaching phase and the high frequency disturbance can be suppressed effectively during the sliding phase.

By appealing to the properties of TSMC, a novel robust adaptive terminal sliding mode control algorithm is developed. The adaptive law is used to update the control gain in order to ensure the magnitude of the available control can accommodate the level of system uncertainty and external disturbances present in the system whilst ensuring an unnecessarily high gain and conservative control strategy does not result. It is shown that the terminal sliding mode control can make the tracking error converge to zero in finite time. The resulting control strategy is based on knowledge of all the system states. It is demonstrated in the paper that results from the domain of robust exact differentiation which have been reviewed by [Shtessel et al.(2014)] and recently implemented in a Matlab toolbox by [Reichhartinger and Spurgeon(2016), Reichhartinger et al. (2017)] can be used very effectively to implement the proposed control. The paper includes a full suite of case studies as well as theoretical results to validate the effectiveness of the proposed approach. To the best of the authors' knowledge, TSMC is applied for the first time to dynamic position control of semi-submersible offshore platforms in this paper.

The paper is organized as follows: the dynamic model of the semi-submersible offshore platform and some preliminary information is presented in Section 2. The robust adaptive terminal sliding mode control is developed and the corresponding stability analysis presented in Section 3. Case studies are used to validate the effectiveness of the proposed approaches in Section 4. Finally, concluding remarks are given in Section 5.

Problem formulation and preliminaries

The kinematic and dynamic model of a semi-submersible offshore platform is taken from [Fossen(2011)]

$$\begin{cases} \dot{\eta} = J(\psi)\nu \\ M\dot{\nu} + D\nu = \tau + d \end{cases} \tag{1}$$

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where $J(\psi)$ is a state-dependent transformation matrix and is nonsingular for all ψ and $J^{-1}(\psi) = J^T(\psi)$. $\eta = [x, y, \psi]^T$ is the position vector in a Geodetic coordinate system. $\nu = [u, v, r]^T$ is the velocity vector in the coordinate system of the platform. These three modes are referred to as the surge, sway and yaw modes of the system, respectively. $M, D \in R^{3\times3}$ represent the inertia and linear damping matrices. $\tau \in R^{3\times3}$ represents the control forces in surge and sway, and moment in yaw. d represents the disturbance on the system due to the wind, wave and currents.

According to (1), v can be written as

$$v = J^{-1}(\psi)\,\dot{\eta} \tag{3}$$

Assumption 1. *It is assumed that η and its differentive $\dot{\eta}$ can be measured.*

Remark 1. *Assumption 1 is reasonable because of the recent development of robust exact differentiators which can be used to obtain $\dot{\eta}$ directly from measurement of η [Shtessel et al.(2014), Reichhartinger and Spurgeon(2016)].*

From (3) it follows that

$$\dot{v} = \dot{J}^{-1}(\psi)\,\dot{\eta} + J^{-1}(\psi)\,\ddot{\eta} \tag{4}$$

By combining (1) and (4), the dynamic equations of the system may be represented as

$$MJ^{-1}(\psi)\,\ddot{\eta} + \left(M\dot{J}^{-1}(\psi) + DJ^{-1}(\psi)\right)\dot{\eta} = \tau + d \tag{5}$$

Let

$$\begin{aligned} P &= MJ^{-1}(\psi) \\ Q &= M\dot{J}^{-1}(\psi) + DJ^{-1}(\psi) \end{aligned} \tag{6}$$

Then (5) becomes

$$P(\eta)\,\ddot{\eta} + Q(\eta, \dot{\eta})\,\dot{\eta} = \tau(t) + d(t) \tag{7}$$

Assumption 2. *Assume d is bounded so that*

$$\|d\left(t\right)\| < d_1 \tag{8}$$

Assumption 3. *Further assume that (7) has some known parts and unknown parts, which can be described by:*

$$\begin{aligned} &P\left(\eta\right) = P_0\left(\eta\right) + \Delta P\left(\eta\right) \\ &Q\left(\eta,\dot{\eta}\right) = Q_0\left(\eta,\dot{\eta}\right) + \Delta Q\left(\eta,\dot{\eta}\right) \end{aligned} \tag{9}$$

where $P_0(\eta)$, $Q_0(\eta)$ represent the known parts and $\Delta M\left(q\right)$, $\Delta Q\left(q,\dot{q}\right)$ are uncertain.

From (9), the dynamic equation (7) can be written in the following form

$$P_0\left(\eta\right)\ddot{\eta} + Q_0\left(\eta,\dot{\eta}\right)\dot{\eta} = \tau\left(t\right) + \rho\left(t\right) \tag{10}$$

where ρ is the lumped system uncertainty defined by

$$\rho\left(t\right) = -\Delta P\left(\eta\right)\ddot{\eta} - \Delta Q\left(\eta,\dot{\eta}\right)\dot{\eta} + d\left(t\right) \tag{11}$$

Then, the nominal system representation can be described by

$$P_0\left(\eta\right)\ddot{\eta} + Q_0\left(\eta,\dot{\eta}\right)\dot{\eta} = \tau_0\left(t\right) \tag{12}$$

Assumption 4. *The lumped system uncertainty ρ is bounded, so that $\|\rho\| \leq \rho_0$, $\rho_0 > 0$.*

Let η_d represent the desired position or trajectory that the system must follow and define the corresponding output tracking error as $\varepsilon = \eta - \eta_d$. The control objective can then be summarized as follows: under Assumptions 1-4, design a robust adaptive control to counteract the lumped system uncertainty and stabilize the tracking error $\varepsilon = 0$ in finite time.

Robust adaptive terminal sliding mode control design

The proposed control algorithm is developed in two steps for the system (7). Firstly, a nominal feedback controller is designed to stabilize the nominal system (12). Secondly, a robust adaptive terminal sliding mode compensator is designed to eliminate the effects of both the uncertain dynamics and the external disturbances so that the output tracking error can converge to zero in finite time.

From the nominal system equation (12), the tracking error equation is given by

$$\dot{e} = Ae + Br \tag{13}$$

where $e = \left[\varepsilon^T, \dot{\varepsilon}^T\right]^T$, $A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$ and r is given by

$$r = P_0^{-1}(\eta)\left(\tau_0 - Q_0(\eta, \dot{\eta})\,\dot{\eta}\right) - \ddot{\eta}_d \tag{14}$$

Lemma 1. *The error dynamics in equation (13) can be stabilized by the following nominal feedback control law*

$$\tau_0 = Q_0(\eta, \dot{\eta})\,\dot{\eta} + P_0(\eta)\left(Ke + \ddot{\eta}_d\right) \tag{15}$$

where $K = [-K_1, -K_2]$, $K_1 \in R^{n\times n}$, $K_2 \in R^{n\times n}$ *and the matrix* K *is designed such that*

$$A_1 = A + BK \tag{16}$$

has stable poles.

Let the control input in (10) have the following form

$$\tau(t) = \tau_0 + \tau_1 + \tau_2 \tag{17}$$

where τ_0 is the nominal control component defined by equation (15) and τ_1 and τ_2 denote elements of the terminal sliding mode control strategy yet to be defined. The error dynamic equation for the closed loop system in the presence of uncertain dynamics and external disturbances can be written in the following form

$$\dot{e} = A_1 e + BP_0^{-1}(\eta)\left(\tau_1 + \tau_2\right) + BP_0^{-1}(\eta)\,\rho(t) \tag{18}$$

In order to design the control signals τ_1 and τ_2 in expression (18) to guarantee convergence of the tracking error to zero in finite time, the following MIMO terminal sliding surface is defined as in [Man and Yu(1997)]

$$S = C\tilde{e} \tag{19}$$

where $C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$, $C_1 = diag\{c_{11}, c_{12}, \cdots, c_{1n}\}$, $C_2 = diag\{c_{21}, c_{22}, \cdots, c_{2n}\}$.

$$\tilde{e} = \left[\varepsilon_1^{p_1/p_2} \cdots \varepsilon_n^{p_1/p_2}\, \dot{\varepsilon}_1 \cdots \dot{\varepsilon}_n\right]^T \tag{20}$$

In general, p_1 and p_2 are selected to be positive odd integers which satisfy the following conditions

$$\begin{aligned} & p_2 = (2m+1)\,, m = 1, 2, \cdots \\ & p_2 > p_1 \end{aligned} \tag{21}$$

The terminal sliding surface can be expressed as

$$S = C\tilde{e} = C\left(e + \Delta\tilde{e}\right) \tag{22}$$

where

$$\tilde{e} = \Delta\tilde{e} + e \tag{23}$$

$$\Delta\tilde{e} = \left[\varepsilon_1^{p_1/p_2} - \varepsilon_1, \cdots, \varepsilon_n^{p_1/p_2} - \varepsilon_n, 0, \cdots, 0\right]^T \tag{24}$$

According to Assumption 4

$$\left\|\rho\left(t\right)\right\| \le \rho_0 \tag{25}$$

where ρ_0 represents the upper bound of the system uncertainties. A robust TSMC is designed with the components τ_1 and τ_2 given by

$$\tau_1 = \begin{cases} \frac{\left(S^T CBP_0^{-1}\right)^T}{\left\|S^T CBP_0^{-1}\right\|^2}\bar{w} & \|S\| \ne 0 \; and \; \varepsilon_i \ne 0 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

where $\bar{w} = -S^T CA_1 e - S^T C\Delta\dot{\tilde{e}} - \left\|S^T\right\| \left\|CBP_0^{-1}\right\| \rho_0$.

$$\tau_2 = -\xi(CB)^{-1} P_0 S \tag{27}$$

where $\xi > 0$.

Remark 2. *When $p_1 = p_2 = 1$, the robust TSMC from equations (26) and (27) corresponds to the case of classical SMC.*

Lemma 2. *For the uncertain dynamic system (18), if the robust TSMC is designed as (15), (26) and (27), then the tracking error will be finite time stable.*

Proof: Select the Lyapunov function

$$V = \frac{1}{2} S^T S \tag{28}$$

Differentiate (28) with respect to time along (18) to yield

$$\begin{aligned}
\dot{V} &= S^T\dot{S} \\
&= S^T\left(CA_1e + CBP_0^{-1}\tau_1 + CBP_0^{-1}\rho\right) + S^TC\Delta\dot{\tilde{e}} \\
&= S^TCA_1e + S^TCBP_0^{-1}\tau_1 + S^TCBP_0^{-1}\rho + \\
&\quad S^TC\Delta\dot{\tilde{e}} \\
&\le S^TCA_1e + S^TCBP_0^{-1}\tau_1 + \left\|CBP_0^{-1}\right\|\|S\|\|\rho\| \\
&\quad +S^TC\Delta\dot{\tilde{e}} \\
&= S^TCA_1e + \left\|CBP_0^{-1}\right\|\|S\|\|\rho\| + S^TC\Delta\dot{\tilde{e}} \\
&\quad +S^TCBP_0^{-1}\left[\frac{\left(S^TCBP_0^{-1}\right)^T}{\left\|S^TCBP_0^{-1}\right\|^2}(-S^TCA_1e\right. \\
&\quad \left.-S^TC\Delta\dot{\tilde{e}} - \|S\|\left\|CBP_0^{-1}\right\|\bar{\rho})\right] \\
&= -\left\|CBP_0^{-1}\right\|(\rho_0 - \|\rho\|)\|S\| - \xi\|S\|^2 \\
&\le -\mu\|S\|
\end{aligned} \tag{29}$$

where $\mu = \left\|CBP_0^{-1}(\eta)\right\|(\rho_0 - \|\rho\|) > 0$. From sliding mode control theory, the sliding mode can be reached in finite time and according to the definition of the chosen terminal sliding surface, the tracking error will converge to zero in finite time.

It should be noted that the control element τ_1 in (26) is a conservative controller as the magnitude of the discontinuous control component is selected based on the worst case uncertainty bound in (25). This control is now modified to include adaptation whereby the upper bound on the uncertainty required to achieve finite error convergence is estimated online. Define $\hat{\rho}_0$ as the estimate of ρ_0 and define the corresponding estimation error as

$$\tilde{\rho}_0 = \rho_0 - \hat{\rho}_0 \tag{30}$$

An adaptive law is designed as

$$\dot{\hat{\rho}}_0 = \kappa_0\left\|CBP_0^{-1}(\eta)\right\|\|S\| \tag{31}$$

where $\kappa_0 > 0$ is an arbitrary positive number. The corresponding adaptive TSMC is defined by

$$\tau_1 = \begin{cases} \frac{\left(S^TCBP_0^{-1}\right)^T}{\left\|S^TCBP_0^{-1}\right\|^2}\hat{w} & \|S\| \neq 0 \; and \; \varepsilon_i \neq 0 \\ 0 & otherwise \end{cases} \tag{32}$$

where $\hat{w} = -S^T C A_1 e - S^T C \Delta\dot{\tilde{e}} - \|S\| \left\|CBP_0^{-1}\right\| \hat{\rho}_0$.

$$\tau_2 = -\xi (CB)^{-1} P_0 S \tag{33}$$

Remark 3. *When $p_1 = p_2 = 1$, the robust adaptive TSMC from equations (31), (32) and (33) corresponds to the case of classical adaptive SMC.*

Theorem 1. *For the uncertain dynamic system (18), if the robust adaptive TSMC is designed as (15), (31)-(33), the tracking error will be finite time stable.*

Proof: Select the Lyapunov function candidate

$$V = \frac{1}{2} S^T S + \frac{1}{2} \kappa_0^{-1} \tilde{\rho}_0^2 \tag{34}$$

Differentiating (34) with respect to time along (18) yields

$$\begin{aligned}
\dot{V} &= S^T \dot{S} - \kappa_0^{-1} \tilde{\rho}_0 \dot{\hat{\rho}}_0 \\
&= S^T C \left[A_1 e + B P_0^{-1} (\tau_1 + \tau_2)\right. \\
&\quad \left. + B P_0^{-1} (\eta) \rho + \Delta \dot{\tilde{e}}\right] - \kappa_0^{-1} (\rho_0 - \hat{\rho}_0) \dot{\hat{\rho}}_0 \\
&= S^T C B P_0^{-1} (\tau_1 + \tau_2) + S^T C B P_0^{-1} \rho \\
&\quad - S^T C A_1 e + S^T C \Delta \dot{\tilde{e}} - \kappa_0^{-1} (\rho_0 - \hat{\rho}_0) \dot{\hat{\rho}}_0 \\
&= - \|S\| \left\|CBP_0^{-1}\right\| \hat{\rho}_0 + S^T C B P_0^{-1} (\eta) \rho \\
&\quad - (\rho_0 - \hat{\rho}_0) \|S\| \left\|CBP_0^{-1}\right\| - \xi \|S\|^2 \\
&\leq - \|S\| \left\|CBP_0^{-1}\right\| \rho_0 \\
&\quad + \left\|CBP_0^{-1} (\eta)\right\| \|S\| \|\rho\| \\
&\leq - \|S\| \left\|CBP_0^{-1}\right\| (\rho_0 - \|\rho\|) \leq -\mu \|S\|
\end{aligned} \tag{35}$$

where $\mu = \left\|CBP_0^{-1}\right\| (\rho_0 - \|\rho\|) > 0$. As in the proof of Lemma 2, the tracking error will be finite time stable.

Algorithm:1

Step 1: Transform the nominal dynamic system (12) representing the semi-submersible offshore platform into the form of the tracking error dynamic equation given in (13). This is achieved by computing the inverse matrix of $P_0(\eta)$

Step 2: Design a state feedback matrix K for the nominal tracking error dynamic equation (13). The minimum requirement is that the closed-loop poles must be stable, but ensuring the dynamics are faster than the fastest dynamics of the open-loop system may be desirable.

Step 3: Define the terminal sliding surface (19) where the elements of the diagonal matrices C_1 and C_2 are selected to be strictly positive.
Step 4: Define the robust adaptive law (31) by selecting a positive gain value κ_0.
Step 5: Define the two control components. The discontinuous element (32) is entirely determined from the results of previous steps and the continuous linear gain term defined by (33) is determined by the nominal system representation, the terminal sliding surface selected in step 3 together with a user selected gain $\xi > 0$.

Remark 4. *To achieve finite time stability, the discontinuous control element τ_1 is used. However, this discontinuous control may lead to chattering. To reduce the chattering effect, a boundary layer method can be used to replace the discontinuous control. In this case the discontinuous control element is defined by*

$$\bar{\tau}_1 = \begin{cases} \frac{\left(S^T CBP_0^{-1}\right)^T}{\left\|S^T CBP_0^{-1}\right\|^2}\hat{w} & \|S\| \geq \delta \text{ and } \varepsilon_i \neq 0 \\ \frac{\left(S^T CBP_0^{-1}\right)^T}{\delta^2}\hat{w} & \|S\| < \delta \text{ and } \varepsilon_i \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

where $\delta > 0$ is a small positive number, $\hat{w} = -S^T CA_1 e - S^T C\Delta\dot{\hat{e}} - \|S\| \left\|CBP_0^{-1}\right\| \hat{\rho}_0$.
The boundary layer method can reduce chattering effectively but this will be at the expense of robustness. In general, a smaller boundary layer will possess stronger robustness properties but chattering may be excited. A larger boundary layer will reduce chattering more effectively but robustness may be compromised. In addition, if there are high frequency disturbances and/or a time varying desired trajectory chattering may occur, particularly if the boundary layer is small.

Remark 5. *The proposed algorithm is not complex. All of the design steps can be performed offline if a nominal system representation is available. The algorithm requires only computation of a stabilising state feedback controller as well as selection of parameters to define C, κ_0 and δ, where the only assumption is that all parameters must be positive.*

Case study

Dynamic positioning of a semi-submersible offshore platform is considered to validate the proposed approach. The parameter matrices in (1) are given by

$$M = \begin{bmatrix} 5.3122 \times 10^6 & 0 & 0 \\ 0 & 8.2831 \times 10^6 & 0 \\ 0 & 0 & 3.7454 \times 10^9 \end{bmatrix} \tag{37}$$

$$D = \begin{bmatrix} 5.0242 \times 10^4 & 0 & 0 \\ 0 & 2.7229 \times 10^5 & -4.3933 \times 10^6 \\ 0 & -4.3933 \times 10^6 & 4.1894 \times 10^8 \end{bmatrix} \tag{38}$$

The performance of the designed controllers will be considered in the presence of the following external disturbance

$$\rho = \begin{bmatrix} 7.5 \times 10^3 \sin(1000t) + 0.9 \times 10^3 \sin(0.01t) \\ 3.5 \times 10^3 \sin(1000t) + 1.2 \times 10^3 \sin(0.01t) \\ 1.5 \times 10^3 \sin(1000t) + 1.5 \times 10^3 \sin(0.01t) \end{bmatrix} \tag{39}$$

Remark 6. *The assumed disturbance is a lumped model representing the effects of the wind, currents and waves. High and low frequency elements which coincide with the real world situation have been used to determine (39).*

By using a Lyapunov function approach, the state feedback matrix K required to stabilize the nominal portion of the dynamics given in (13) is selected as

$$K = \begin{bmatrix} 1 & 0 & 0 & 1.7321 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1.7321 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1.7321 \end{bmatrix} \tag{40}$$

Both set point tracking control and trajectory tracking control scenarios will be considered. In both cases, robust TSMC and robust adaptive TSMC will be tested and benchmarked against robust classical SMC and robust adaptive classical SMC.

Set point control

The desired location is given by $\begin{bmatrix} x_d & y_d & \psi_d \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$. The initial conditions are given by $\eta(t) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ and $\dot{\eta}(t) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$. A robust

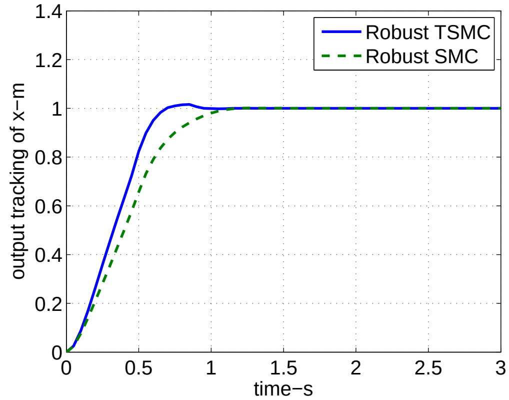

Figure 1. Output tracking of x controlled by robust control in the presence of the lumped system uncertainty.

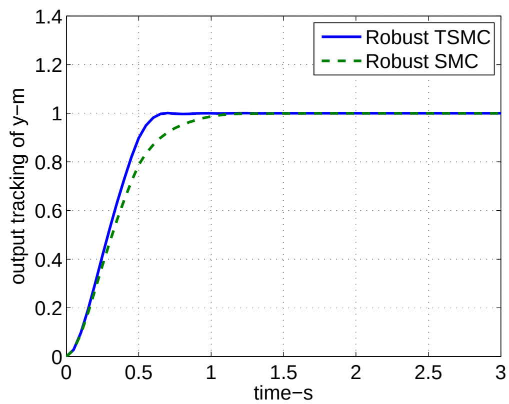

Figure 2. Output tracking of y controlled by robust control in the presence of the lumped system uncertainty.

TSMC is first derived where the sliding surface parameters in (19) and (20) are given by $p_1 = 3$, $p_2 = 5$, $C_1 = diag\{5, 5, 5\}$, $C_2 = diag\{1, 1, 1\}$. The control parameters in (27) and (36) are given by $\xi = 5$, $\delta = 0.05$, $\bar{\rho} = 9000$, $\kappa_0 = 1 \times 10^6$, $\hat{\rho}(0) = 5000$. Comparisons with the conventional SMC described in Remark 2 are presented to validate the proposed approach where $p_1 = 1$, $p_2 = 1$ and all other control parameters are selected as for the robust TSMC to ensure a fair comparison. Fig. 1-3 show the tracking performance of the x, y and ψ states, respectively. The solid lines show the performance of the robust TSMC and

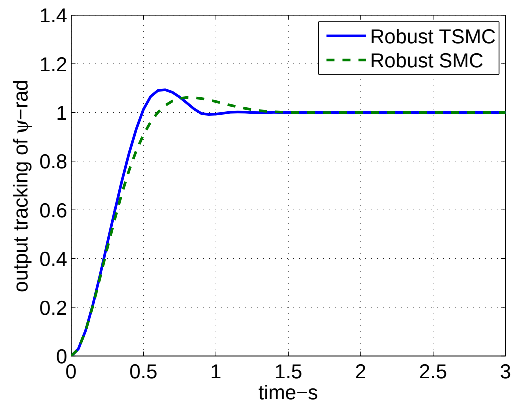

Figure 3. Output tracking of ψ controlled by robust control in the presence of the lumped system uncertainty.

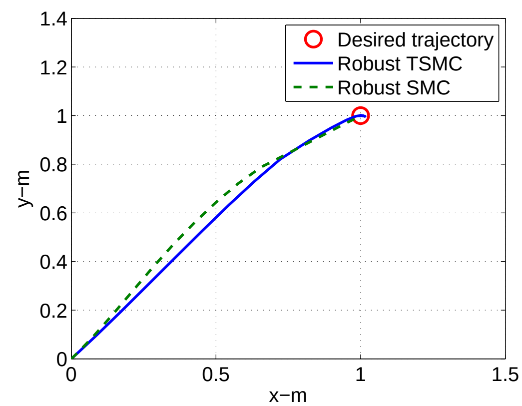

Figure 4. Output tracking performance in $x - y$ plane controlled by robust control in the presence of the lumped system uncertainty.

the dashed lines show the performance of the conventional SMC. Fig. 4 shows the tracking performance in the $x - y$ plane. Fig. 5-7 show the corresponding control inputs. Though both the robust TSMC and conventional SMC are seen to deal effectively with the lumped uncertainty, the proposed robust TSMC has a faster convergence speed, which is desirable for dynamic positioning. The control signals in all cases are smooth and bounded as the boundary layer approach is used for implementation.

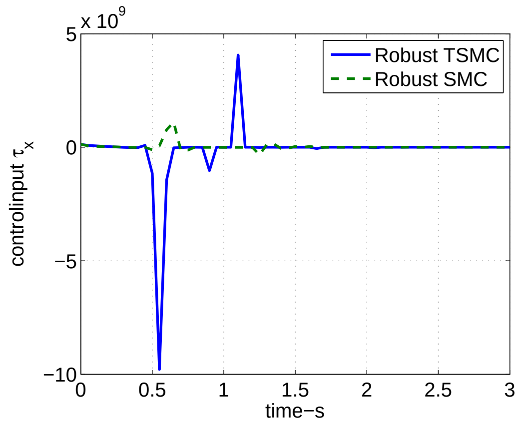

Figure 5. Robust control input τ_x in the presence of the lumped system uncertainty.

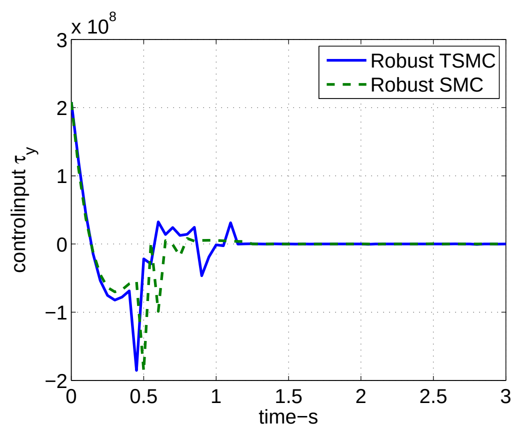

Figure 6. Robust control input τ_y in the presence of the lumped system uncertainty.

Fig. 8-15 show the performance of the robust adaptive TSMC when compared with a classical adaptive sliding mode control. The solid lines show the performance of the robust adaptive TSMC and the dashed lines show the performance of the conventional adaptive SMC. Fig. 8-10 show the tracking performance of the x, y and ψ states, respectively. Fig. 11 shows the tracking performance in the $x - y$ plane. Fig. 12-14 show the corresponding control inputs. Fig. 15 shows the adaptive gain in each case. The robust adaptive TSMC

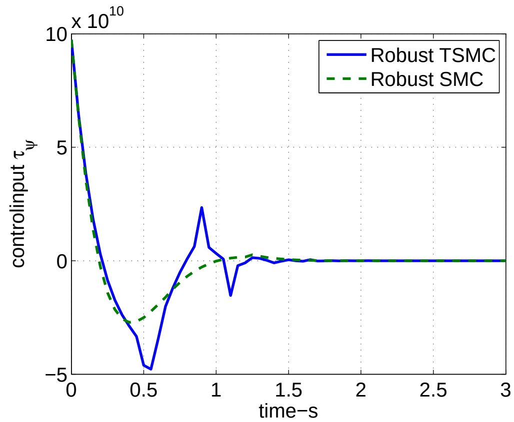

Figure 7. Robust control input τ_ψ in the presence of the lumped system uncertainty.

has a more rapid speed of convergence than the corresponding adaptive SMC. The control inputs are all bounded and smooth. The adaptive law is stable. These simulation results validate the proposed robust adaptive TSMC approach.

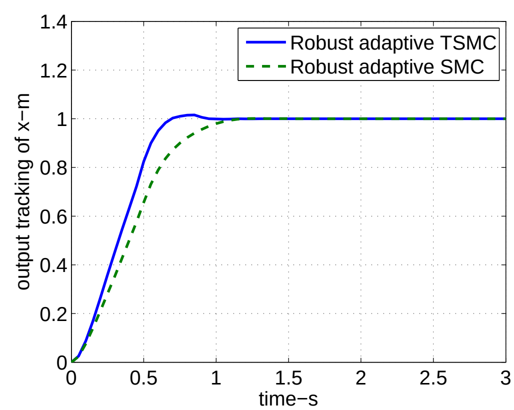

Figure 8. Output tracking of x controlled by robust adaptive control in the presence of the lumped system uncertainty.

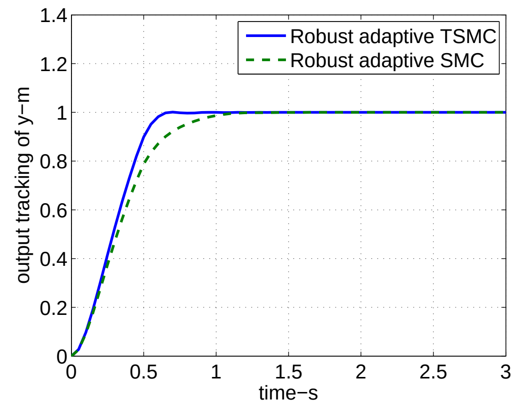

Figure 9. Output tracking of y controlled by robust adaptive control in the presence of the lumped system uncertainty.

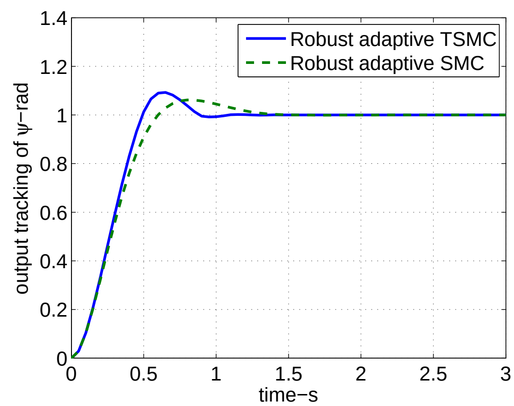

Figure 10. Output tracking of ψ controlled by robust adaptive control in the presence of the lumped system uncertainty.

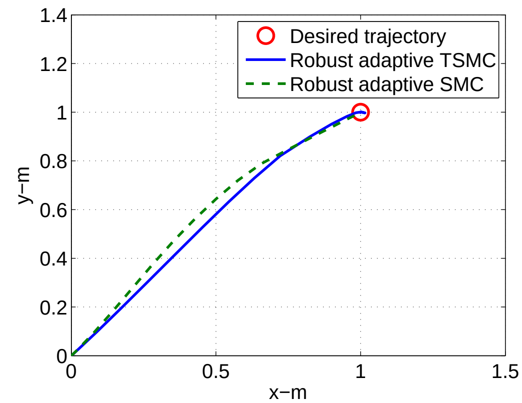

Figure 11. Output tracking performance in $x-y$ plane controlled by robust adaptive control in the presence of the lumped system uncertainty.

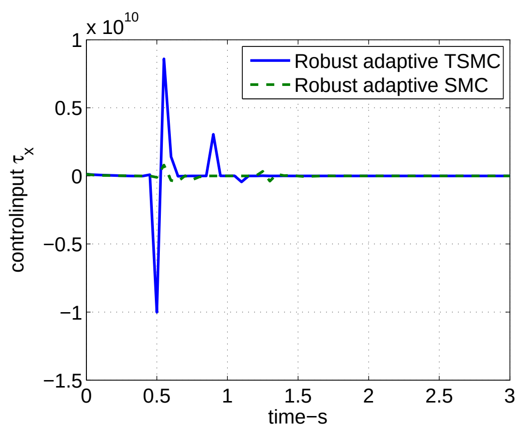

Figure 12. Adaptive robust control input τ_x in the presence of the lumped system uncertainty.

Trajectory tracking control

To further validate the proposed approach, trajectory tracking control is considered to compare the performance of the robust adaptive TSMC and the corresponding classical adaptive SMC. The desired trajectories are given by

$$\begin{cases} x_d = 2\sin(t) \\ y_d = 2\cos(t) \\ \psi_d = \sin(t) \end{cases} \tag{41}$$

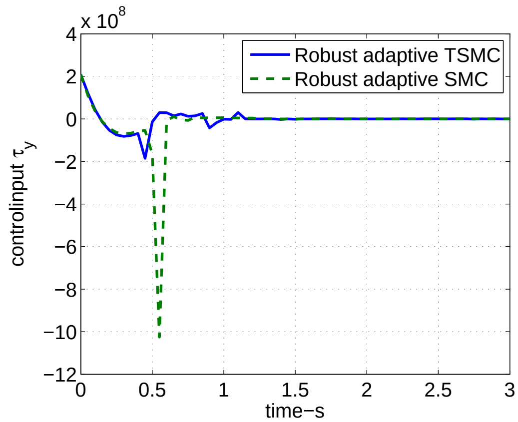

Figure 13. Adaptive robust control input τ_y in the presence of the lumped system uncertainty.

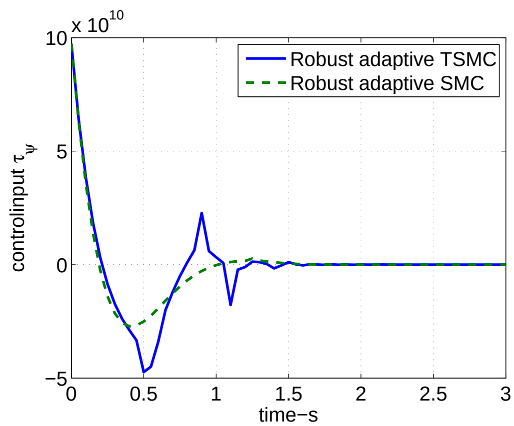

Figure 14. Adaptive robust control input τ_ψ in the presence of the lumped system uncertainty.

The initial conditions are given such by $\eta(0) = \begin{bmatrix} 2.5 & 2.5 & \pi/5 \end{bmatrix}$ and $\dot{\eta}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$. A robust adaptive TSMC is first derived where the sliding surface parameters in (19) and (20) are given by $p_1 = 3$, $p_2 = 5$, $C_1 = diag\{5, 5, 5\}$, $C_2 = diag\{1, 1, 1\}$. The control parameters in (31) , (33) and (36) are given by $\xi = 5$, $\delta = 0.05$, $\kappa_0 = 1 \times 10^6$, $\hat{\rho}(0) = 5000$. Comparisons with the conventional adaptive SMC described in Remark 3 are presented to validate the proposed approach where $p_1 = 1$, $p_2 = 1$ and all other control parameters are selected as for the robust adaptive TSMC to ensure a fair comparison. To further validate

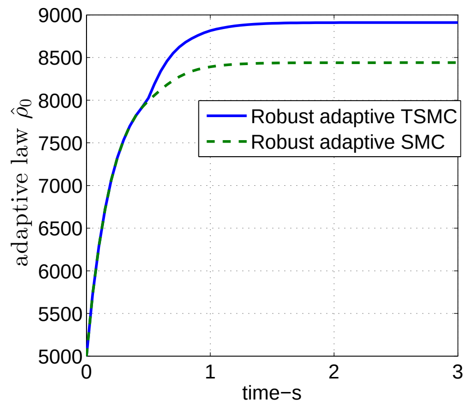

Figure 15. Adaptive law in the presence of the lumped system uncertainty.

the proposed approach, an external impulsive disturbance is also applied to the system given by

$$\begin{cases} f = 5 \times 10^8 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T & 5 \leq t \leq 5.1 \\ 0 & \text{otherwise} \end{cases} \tag{42}$$

Note that the lumped system uncertainty (39) is also applied to the system. For ease of exposition only the tracking performance in the x direction, the tracking in the $x - y$ plane, the control input in the x direction and the response of the adaptive law are given to illustrate the performance. Fig. 16-19 present the corresponding simulation results. From these simulation results, it is seen that the robust adaptive TSMC has a faster convergence speed than the robust adaptive SMC. All the signals are smooth and bounded. The proposed control strategy can alleviate the system response to the impulsive disturbance. Tracking a desired trajectory is another common requirement for a semi-submersible offshore platform. The simulation results show that the proposed approach yields good performance.

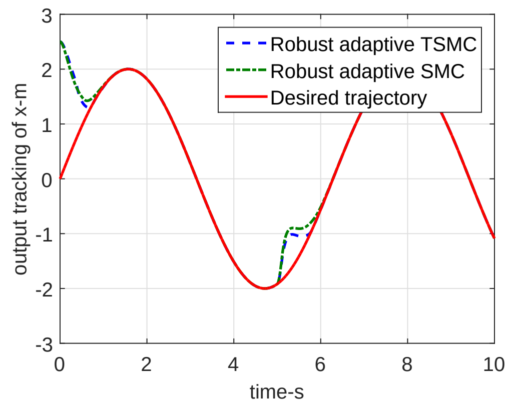

Figure 16. Output tracking of x controlled by robust adaptive control in the presence of the lumped system uncertainty and external impulse disturbance.

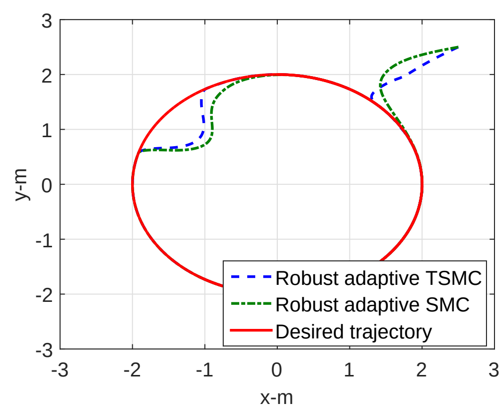

Figure 17. Output tracking performance in $x - y$ plane controlled by robust adaptive control in the presence of the lumped system uncertainty and external impulse disturbance.

All the simulation testing performed thus far has assumed that all the system states are measureable, which may be limiting for practical implementation. In Remark 1 it has been noted that robust exact differentiators can be used to generate rate signals from position signals. It should be noted that other sliding mode approaches could be considered for state estimation such as the recently developed method for both state and unknown input estimation for linear continuous-time switched systems with simultaneous disturbances, sensor and actuator faults as presented in [Yin et al. (2017a)]. However, the particular

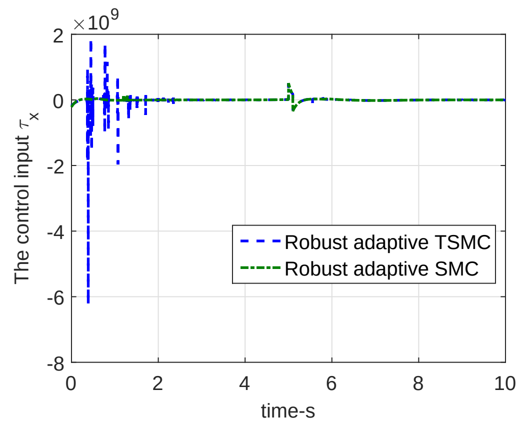

Figure 18. Adaptive robust control input τ_x in the presence of the lumped system uncertainty and external impulse disturbance.

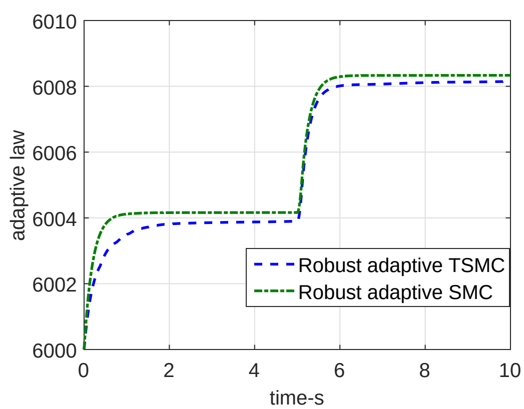

Figure 19. Adaptive law in the presence of the lumped system uncertainty and external impulse disturbance.

structure of the vessel dynamics, the ease of design and implementation of the differentiator as well as the need to only estimate velocity support adopting a differentiator based approach to implementation for this particular system. In the final set of simulation tests, a recently developed robust exact differentiator toolbox is used to construct $\dot{\eta}$ from η. The toolbox used is that described in [Reichhartinger and Spurgeon(2016)] and can be downloaded at www.reichhartinger.at. For each of the three position outputs, a first order differentiator is implemented with the robustness factor selected as 8 when differentiating the signals x and y, and 5 for the signal ψ. The integrator time-step for the differentiator implementation was set as 0.001 for all three differentiators. In this simulation test, the lumped system uncertainty and the impulsive disturbance are present. Figures 20-23 show the tracking performance in the x direction, the tracking in the $x - y$ plane, the control input in the x direction and the response of the adaptive law when the control schemes are implemented using the robust exact differentiator toolbox. Comparing Figures 16-19 with Figures 20-23 it is seen that performance levels achieved using the differentiator toolbox are very similar to the performance achieved when full-state feedback is assumed. There is an initial transient observed whilst the differentiators converge, but otherwise excellent performance levels are maintained even in the presence of unmeasureable states.

To further demonstrate the advantages of the proposed approach over the conventional method, the controller parameters in the robust adaptive SMC are selected so that both the robust adaptive SMC and the robust adaptive TSMC have similar control input ranges. This was achieved by selecting the control parameters in (27) and (36) as $\kappa_0 = 1 \times 10^7$ and $\hat{\rho}(0) = 6000$; all other parameter settings were preserved as for the previous tests. Figures 24-26 demonstrate the results. It can be seen that the proposed approach has a more rapid convergence rate and exhibits greater robustness can be achieved with the conventional method, even when the control gains are selected to achieve similar control effort. The results further validate the advantages of the proposed approach.

From the above case study, it can be seen that the proposed approach can achieve finite time stability while achieving higher control precision and exhibiting greater robustness. The proposed robust adaptive law can estimate the boundary of the lumped uncertainty. The use of the robust differentiator allows the velocity to be estimated online. The use of the differentiator toolbox, which has a very straight forward automated tuning procedure to ensure convergence, greatly simplifies the route to implementation of the controller.

Conclusion

A novel robust adaptive terminal sliding mode tracking control method has been developed for a semi-submersible offshore platform. A formal stability analysis has been undertaken. Extensive simulation testing has employed to validate the resulting algorithm. It is shown that the tracking error can converge to zero in finite time in the presence of lumped system uncertainty as well as impulsive disturbances. The efficacy of a recently developed robust exact differentiator toolbox in robustly implementing the controller has been shown. The proposed approach provides a rapid speed of convergence as well as high robustness and tracking accuracy. However, singularity effects have not been eliminated completely. Future work will focus on experimental testing of the proposed approach and singularity free TMSC.

Acknowledgements

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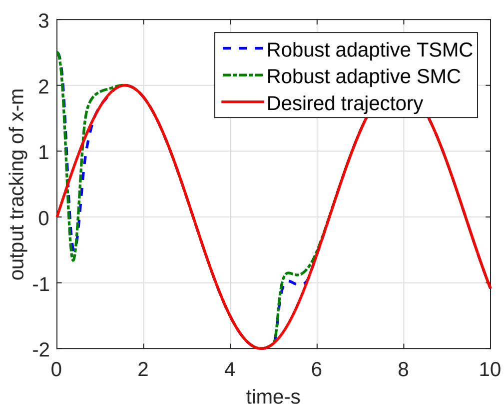

Figure 20. Output tracking of x controlled by robust adaptive sliding mode control in the presence of the lumped system uncertainty and external impulse disturbance. The rate signals are estimated using the robust exact differentiator toolbox [Reichhartinger and Spurgeon(2016)].

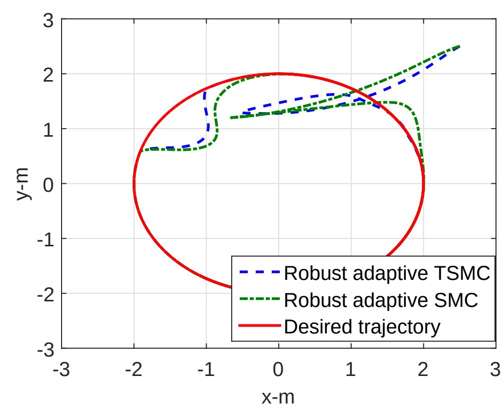

Figure 21. Output tracking performance in $x - y$ plane controlled by robust adaptive sliding mode control in the presence of the lumped system uncertainty and external impulse disturbance.The rate signals are estimated using the robust exact differentiator toolbox described in [Reichhartinger and Spurgeon(2016)]

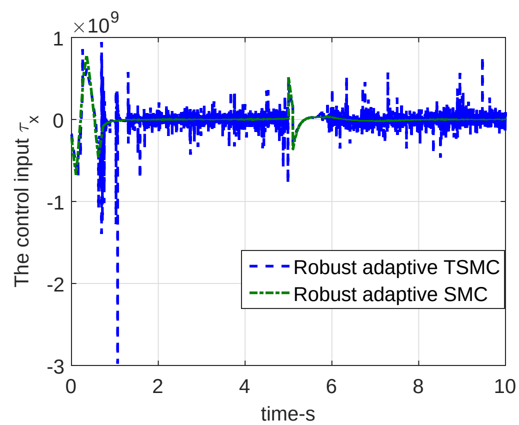

Figure 22. Adaptive robust sliding mode control input τ_x in the presence of the lumped system uncertainty and external impulse disturbance. The rate signals are estimated using the robust exact differentiator toolbox described in [Reichhartinger and Spurgeon(2016)].

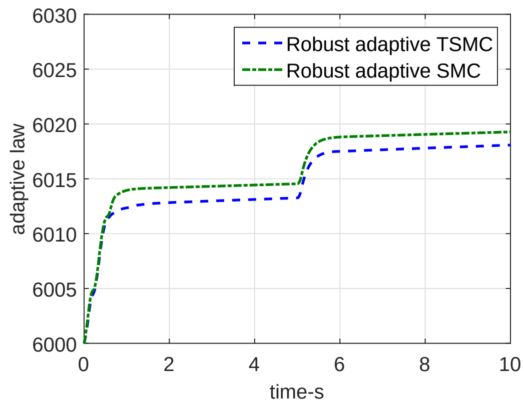

Figure 23. Adaptive law in the presence of the lumped system uncertainty and external impulse disturbance. The rate signals are estimated using the robust exact differentiator toolbox described in [Reichhartinger and Spurgeon(2016)].

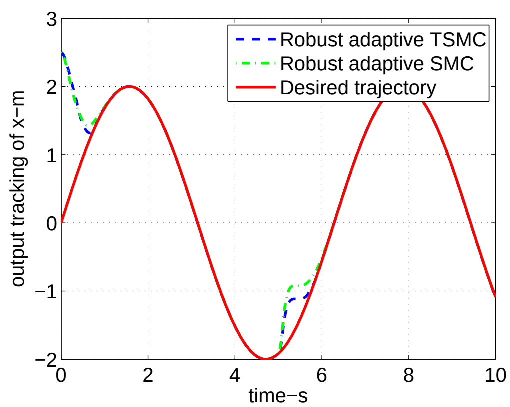

Figure 24. Adaptive law in the presence of the lumped system uncertainty and external impulse disturbance. Both of the controller parameters are selected so as to have similar control input ranges.

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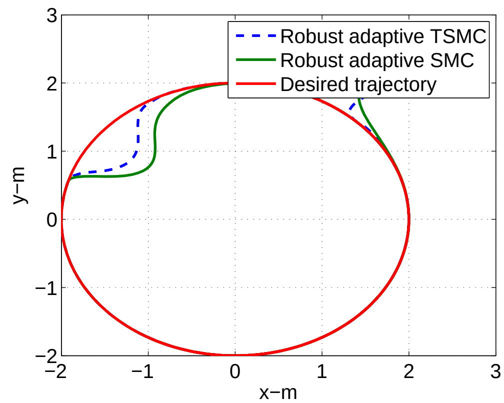

Figure 25. Adaptive law in the presence of the lumped system uncertainty and external impulse disturbance. Both of the controller parameters are selected so as to have similar control input ranges.

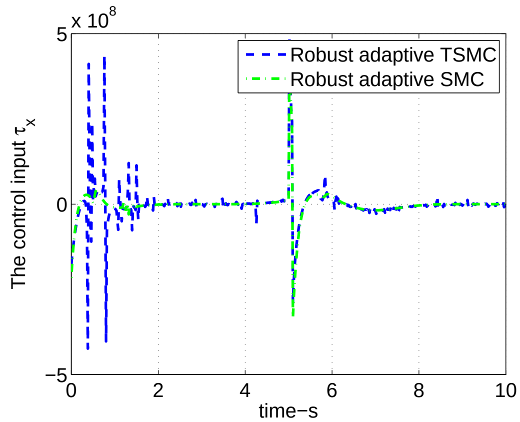

Figure 26. Adaptive law in the presence of the lumped system uncertainty and external impulse disturbance. Both of the controller parameters are selected so as to have similar control input ranges.

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