FROM 'FROWNS AND GROANS' TO 'ASTONISHMENT AND DELIGHT': SEEKING INDICATORS OF A MATHEMATICS TEACHERS IDENTITY

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This paper reports on a research project based on designing and teaching in-service courses for Non-Specialist Teachers of Mathematics (NSTM). An NSTM is a school teacher who qualified to teach in a subject other than mathematics, yet teaches mathematics in secondary school (11-16 year old students). While the overall aim of our research was to describe what constitutes a trajectory towards a mathematics teacher identity for a NSTM, in this paper we explain how we sought indicators of a mathematics teacher identity. We do so by first describing how we adapted Wenger's notion of identity, then advanced our 'Modes of Belonging' Mathematics Teacher Identity framework. After that we exemplify how we used our framework to locate indicators of mathematics teacher identity in the data from a narrative of NSTMs working on a particular piece of mathematics.

INTRODUCTION

In England, the shortage of mathematics teachers is well-recognised with the demand far outstripping supply. The latest available statistics on teacher supply gathered by the Department for Education revealed that "79.8 per cent of mathematics lessons taught to students in year groups 7-13 were taught by teachers with a relevant qualification; a decrease from 82.7 per cent in 2013" and "75.8 per cent of teachers of mathematics to year groups 7-13 held a relevant post A level qualification (down from 77.6 per cent in 2013)" (Ross 2015, p. 13). The crisis in teacher supply means that subjects like mathematics have to be covered by teachers who are not specialists in these subjects.

The current on-going need for specialist mathematics teachers is not unique to England, but well recognised at international level, too. In Eire, for example a national survey found that 48% of teachers of mathematics at post-primary schools were not mathematics qualified, while in Germany, research on 'fachfremd' (meaning 'non-specialist' in German) teachers of mathematics, includes Bosse's (2014) findings that these teachers enjoyed teaching mathematics even though they viewed mathematics as if it was the mathematics of elementary school and they had had little professional development in mathematics teaching. In the United States, the NSTMs teachers are referred to as teaching 'out-of-field' (e.g., Ingersoll & Curran 2004), while in Australia, Hobbs (2013) found that teachers who were 'teaching across specialisations' (TAS) experienced discontinuities which can impact negatively on their confidence and efficiency as a teacher of the new subject.

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At the first Teaching Across Specialisation (TAS) Collective convened in August 2014, presentations from countries across the world indicated the wide spread of the TAS phenomenon and a call for "Research is needed to establish the key features of effective professional development that leads to such transformation in identity and practice for out-of-field teachers." (Hobbs & Törner 2014, p. 46) was launched.

OUR IN-SERVICE COURSE DESIGN

The design principle of our in-service mathematics courses for NSTMs was informed both by our research that showed that learning to teach a subject without a background in either content or teaching approaches requires focused re-training (Crisan and Rodd 2014) and also an appreciation of the fact that it can actually be quite difficult to teach out-of-field (du Plessis, Carroll & Gillies 2015; Hobbs 2013).

There are two key practices for teachers of mathematics: engaging in (school) mathematics and being a teacher. In the case of our NSTMs, the latter is an established practice, while the former is the practice they are developing. On our in-service mathematics courses, we brokered these two key practices, enabling connections between them, through explicit teaching of school mathematics content within discourses familiar to school teachers.

Our view that effective secondary mathematics teaching is founded on sound subject knowledge, together with a thorough, interconnected, knowledge of the curriculum and sympathetic understandings of students' needs and interests informed the design of our in-service courses. In these courses, there was emphasis on revisiting and teaching school mathematics. This served not only to develop the NSTMs' technical fluency of some of the more challenging topics taught at different levels of school education, but also to promote modes of mathematical enquiry such as generalisation, abstraction, reasoning and proof. We also emphasised precision in mathematical language, as well as recognition of conceptual structures within mathematics. Discussions of pedagogical nature, such as common students' misconceptions, multiple representations of a concept, or different teaching approaches, were integral to course delivery.

OUR RESEARCH INTEREST

A prompt for our research came from the NSTMs themselves. One of our NSTMs (trained to teach humanities), who was applying for a promoted mathematics teacher post, told us that she cried when she saw simultaneous equations and, when that topic came up, always asked a colleague to teach it for her. On one hand, this teacher wanted to be thought of as an expert mathematics teacher, while on the other hand, she was not able either to fluently solve problems on this standard topic in the mathematics curriculum within our class or to contemplate teaching the topic to her students in school.

Such a disjunction confirmed our thinking that issues of identity were relevant to our work with NSTs. We became particularly interested in how to make sense of our NSTMs' mathematics teacher identities formation and development over the duration of the course. The research presented in this paper is part of a larger research study roo-

ted in our teaching of four cohorts of NSTMs over the past four years in London, UK with an overall aim of answering our research question 'What constitutes a trajectory towards a mathematics teacher identity for a NSTMs on an in-service course?'.

However, in order to answer this research questions, we first needed to be able to recognise indicators of a mathematics teacher identity and in this paper we offer an insight into how we engaged with theory and our data in seeking such indicators. We thus proceed to firstly explain how we adapted Wenger's notion of identity to mathematics teacher identity, then we put forward a framework accounting for the three interlinked 'Modes of Belonging: engagement, imagination and alignment' (Wenger, 1998, p. 174) in order to make sense of identity formation in the two key practices of our NSTMs: learning mathematics and being a teacher and lastly, we illustrate how we explicitly sought indicators of a mathematics teacher identity through our engagement with the framework and data from a narrative of NSTMs working on a particular piece of mathematics.

ENGAGEMENT WITH WENGER'S PERSPECTIVE ON IDENTITY

While a variety of frameworks have been employed by researchers to describe teachers' identity development in mathematics teacher in-service courses (e.g., Boaler 2001; Fennema & Nelson 1997), Graven & Lerman (2003) argued that Wenger's (1998) social practice perspective of learning is a suitable framework to use to analyse the process of becoming a teacher of mathematics.

Hence we engaged with Wenger's "Social ecology of identity" (Wenger 1998, p. 190) and adapted it and operationalised it as an analytic tool in the following way: the general illustrative examples in the table on page 190 (*ibid*.) were replaced by mathematics education-specific examples of indicators of aspects of identity, by drawing on our own teaching experiences at secondary school level and expertise in research informed teaching of prospective and practicing teachers. In this way, Wenger's notion of identity was adapted to mathematics teacher identity by interpreting the three interlinked "Modes of Belonging: engagement, imagination and alignment" (Wenger 1998, p. 174) in the two key practices of doing mathematics (Identification with school mathematics) and being a teacher (Negotiability in mathematics teaching) as indicated in Table 1 below.

OUR 'MODES OF BELONGING' MATHEMATICS TEACHER IDENTITY FRAMEWORK

In our study, *Identification with school mathematics* refers to how the NSTMs constructed identities as learners of mathematics during our in-service course. Identification through engagement, imagination, and alignment refers to how the NSTMs invested themselves in learning about and doing school mathematics topics, how they constructed images about how students learn mathematics and how their views converged towards an increasing connection with how the mathematics teaching community views mathematics as a practice.

Negotiability in mathematics teaching through engagement, imagination, and alignment refers to how the NSTMs negotiated their ways in the mathematics teaching community, how the NSTMs constructed images of themselves as potential specialist mathematics teachers and how their views converged towards an increasing connection with the mathematics teaching community.

MATHEMATICS TEACHER IDENTITY				
Identification with (school) mathematics			Negotiability in mathematics teaching	
Identities of participation	Identities of non-participation	MODE	Identities of participation	Identities of non-participation
e.g. Enjoy thinking about the mathematics to be taught.	e.g. Avoid mathematical activity.	Engagement	e.g. Do in-service courses; facilitate students' presenting partial proofs which are discussed	e.g. Rely on text book or on downloaded powerpoint resources.
e.g. Find new ideas in standard topics.	e.g. Act as if there was only 'one correct method'; avoid thinking about alternative approaches.	Imagination	e.g. Share ideas, applications, etc. about mathematics with students; imagine self as a mathematics teacher.	e.g. When being asked by a student 'why are we doing this?' reply 'you need it for exam'.
e.g. Want to understand why, expect proof, work detail.	e.g. Routinely get answers to mathematics problems from internet/ elsewhere; make errors.	Alignment	e.g. Discuss, with students, what progression they have made in mathematics.	e.g. Only show methods in exam mark scheme; want certification of maths specialism without engagement.

Table 1: 'Modes of Belonging' Mathematics Teacher Identity framework

SEEKING INDICATORS OF A MATHEMATICS TEACHER IDENTITY

Data

Throughout the delivery of the four year-long in-service courses we collected biographical data: routes into teaching; subject specialism of their teacher training; teaching

experience: of mathematics, if any, or of their subject specialism; mathematics-related material (written diagnostic assessment of mathematics subject knowledge and capacity to diagnose students' errors/misconceptions; collection of on-going mathematical work); and written reflections (done during and at the end of their course and essay assignments) from all participating teachers as an integral part of their respective course. We also conducted interviews and carried out school observations specifically for this research.

Data analysis

In the following we first explain how we interpreted and hence allocated data from a narrative related to a particular piece of mathematics as indicators of *Identities of participation* in both *Identification with (school) mathematics* and *Negotiability in mathematics teaching* in the table above.

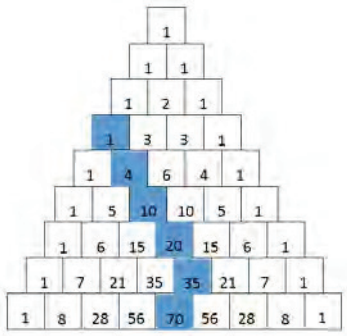
Identification with (school mathematics): Identities of participation-Engagement: One of the activity we designed was intended to give the NSTMs opportunities to investigate number patterns in Pascal's triangle, at the same time facilitating for opportunities to identify for themselves patterns with which they were already familiar. In each cohort there were expressions of astonishment that there was so much mathematical content represented in 'Pascal's triangle', for instance: "how did he ['Pascal'] manage to fit it all in such a simple format?" (Lech, session discussion).

When looking at the mathematics within the Pascal triangle, the teachers were amazed to discover 'in the triangle' many mathematics topics they had previously studied. "It's all in there!" exclaimed one participants in disbelief.

The teachers experienced joy and surprise at noticing connections between different topics, starting to see mathematics in a new light, more than just a set body of independent knowledge and skills, clearly expressed by one other participant: "I actually quite like that. I couldn't grasp it and I can only just touch it – but I really like the fact that it's connected in different ways and we talk about…for example, Pascal's triangles here, there and then!".

Identification with (school mathematics): Identities of participation-Alignment: More advanced mathematics topics, such as binomial coefficients and combinatorial identities were also introduced using Pascal's triangle. However, when the identity $\sum_{i=0}^{k}\binom{n+i}{i}=\binom{n+k+1}{k}\ n,k\in N,\ n>k$ was projected on the board, NSTMs' responses expressed non-participation–mathematics-engagement through their **frowns and groans** and comments of 'frightening', 'scary', 'illegible'.

Our role as tutors on the course included helping the participants overcome the negative affect, and so we introduced the hockey sticks visually (as shown in the diagram) and encouraged the participants to investigate hockey sticks of different sizes.



The NSTMs then described the sticks numerically (e.g., 1 + 4 + 10 + 20 + 35 = 70) and noticed a pattern emerging.
The hockey sticks were then represented using the binomial coefficient notation:
$\binom{3}{0}+\binom{4}{1}+\binom{5}{2}+\binom{6}{3}+\binom{7}{4}=\binom{8}{4}$.

Table 2: Pascal's triangle and the Hockey Stick Theorem

The NSTMs were then able to write down a generalization of the hockey sticks patterns, thus describing the Hockey Stick Theorem with the very expression that 'scared' them when shown earlier, namely $\sum_{i=0}^{k}\binom{n+i}{i}=\binom{n+k+1}{k}$. The NSTMs were **astonished** to have arrived at this concise representation of the identity themselves and expressed **delight** at being able to 'see' the identity in all these formats!

Identification with (school mathematics): Identities of participation–Imagination: Being able to make sense of an abstract mathematical expression, as above, contributed to NSTMs' identity of belonging to the mathematics community through their participation in doing mathematics and alignment with the mathematics that specialist teachers know and do. For example, when yet another emergence of Pascal's triangle got the whole class excited, we classified this as the NSTMs participating mathematically by noticing connections between different mathematical topics, and also as an instance of "joy and satisfaction in undertaking mathematical practices" (Grootenboer & Zvenberger 2008, p. 246) that helped to create a positive group atmosphere, important in building a community of practice within the class of NSTMs and helping, through development of positive affect, the NSTMs participate in other communities of mathematics.

DISCUSSION

In this paper we described the first stage in our data analysis, namely that of explicitly seeking indicators of identity as conceptualised from interpreting Wenger's structure and adapted to mathematics teacher identity by us.

Data from the mathematical episode has been classified in terms of indicating participation in the three different 'Modes of Belonging' – engagement, imagination and alignment' as a result of engaging with the framework tool we put forward in Table 1. Having provided examples of how data were allocated to some of the cells in the Table 1, during the conference presentation we will engage further with our framework and we will exemplify how data collected from one NSTM participant was analysed and a narrative produced, once allocated to the table. In our research project, a similar exercise was applied to all the data we collected from our NSTMs, and this enabled us to

produce a narrative and describe our NSTMs positioning on (different) trajectories towards a mathematics teachers identity.

By considering practices central to being a secondary mathematics teacher, namely *Identification with school mathematics* and *Negotiability in mathematics teaching,* we have offered a way of thinking about mathematics teacher development. In Wenger's terms, our NSTMs were newcomers to the mathematics teaching community and as such they negotiated their trajectories towards becoming a mathematics teachers in their own ways and their individual 'Table 1's looked different from each other's and were different at different points in time. Using our framework at different points in an in-service course provided a way to evidence how mathematics teacher identities emerged and developed.

Graven (2005) points to identity transformation seldom being the focus of in-service courses and the researcher proposes that identity interacts with teachers learning and thus should be a focus of the design and provision of any in-service training. As such, our contribution to knowledge is in drawing attention to how mathematical knowledge is realised within non-specialist teachers' mathematics teacher identity and in developing understandings of non-specialist teachers' experience on an in-service course.

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