Experimental Demonstration of Geometrically-Shaped Constellations Tailored to the Nonlinear Fibre Channel

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Abstract *A geometrically-shaped 256-QAM constellation, tailored to the nonlinear optical fibre channel, is experimentally demonstrated. The proposed constellation outperforms both uniform and AWGN-tailored 256-QAM, as it is designed to optimise the trade-off between shaping gain, nonlinearity and transceiver impairments.*

Introduction

Constellation shaping is a coded modulation method which has recently been widely adopted in optical fibre communications to improve spectral efficiency[1,2] and flexibility to vary the modulation format. However, in the vast majority of recent demonstrations, constellations have been shaped to maximise noise tolerance in the linear additive white Gaussian noise (AWGN) channel, and hence may be suboptimal in the presence of optical fibre nonlinearities.

The nonlinear distortion is often approximated as nonlinear interference (NLI) noise, with the NLI a function of the transmitted modulation format. In particular, the NLI power can be approximated[3,4] as

$$\eta_{\text{tot}} P^3 \approx (\eta_1 + \eta_2 \mathfrak{K})\, P^3, \tag{1}$$

with real values η_1 and η_2, total NLI coefficient η_{tot}, channel launch power P and the excess kurtosis

$$\mathfrak{K} \triangleq \frac{\mathbb{E}\left[|X|^4\right]}{\mathbb{E}\left[|X|^2\right]^2} - 2, \tag{2}$$

of the complex constellation. Neglecting the impact of transceiver noise, the change of signal-to-noise ratio (SNR) at optimum launch power between any two constellation A and B is given by

$$\frac{\text{SNR}_{\text{opt,A}}}{\text{SNR}_{\text{opt,B}}} = \left(\frac{1 + c\mathfrak{K}_{\text{B}}}{1 + c\mathfrak{K}_{\text{A}}}\right)^{\frac{1}{3}}, \tag{3}$$

with $c \triangleq \frac{\eta_2}{\eta_1}$. Applying shaping to conventional QAM formats typically consists of making the constellation more Gaussian-like, increasing its excess kurtosis. Although resulting in shaping gain, high excess kurtosis reduces the optimum SNR due to NLI, as shown by (3). Consequently, an optimum balance between shaping gain and nonlinear penalty must be found to maximise system throughput.

In our previous work on probabilistic shaping[5], we obtained a simple distribution for the nonlinear fibre channel, that outperforms the optimal distribution for the AWGN channel. The benefit of geometric shaping optimised for the nonlinear channel was also experimentally confirmed in[6].

In this paper, we describe the design of a *geometrically*-shaped constellation, specifically tailored to the nonlinear fibre model. Our proposed signal shaping is straightforward to implement, being based on a square constellation with each dimension being optimised independently. We present the results of transmission experiments demonstrating that the proposed constellation outperforms the corresponding geometrically-shaped constellation tailored to the AWGN channel.

Constellation design

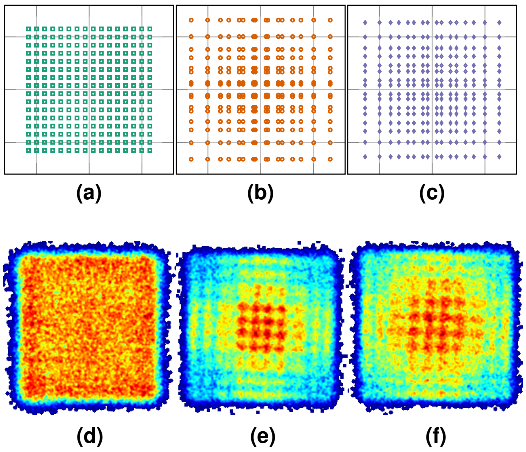

Fig. 1: Geometrically-shaped constellation diagrams for 256-QAM, normalised to unit power: (a) uniform, (b) tailored to the AWGN channel, and (c) tailored to the nonlinear fibre channel. Their respective received constellations after transmission over 160 km are shown in (d, e, f).

Using a quasi-Newton algorithm, the constellations are optimised for GMI. For the nonlinearity-

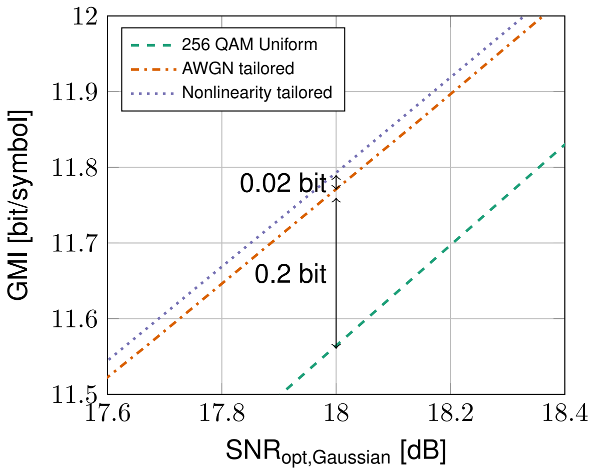

Fig. 2: Geometrically-shaped performance.

tailored constellations, the SNR was changed according to Eq. (3). In-phase quadrature (IQ) independent (1D) geometric shaping was performed, as it gives four advantages: the smaller search space, its easier implementation with practical digital-to-analogue converters (DACs), the conservation of the square shape allows the usage of conventional blind DSP and the straightforward use of Gray coding.

Uniform square 256-QAM was used as the baseline reference format, and to obtain the system parameters of the experimental single-span 160 km long link for the central of three 35.2 GHz-spaced 35 GBd channels. Using the experimental setup, a sweep of the launch power was performed for the uniform 256-QAM. The noise figure of the amplifiers was extracted from a low launch power measurement. The nonlinear coefficient of the fibre was 1.2 W^{-1}km^{-1}. The optimum SNR for Gaussian modulation was approximately 18 dB and for the nonlinearity-tailored constellation the ratio $c = \frac{\eta_1}{\eta_2} = 0.55$ was used. The resulting constellation design is shown in Fig. 1.

The gains are shown in Fig. 2, where the constellation is optimised for different SNR values. At 18 dB SNR, a 0.2 bit/symbol throughput increase can be expected for the AWGN-tailored constellation, and an additional 0.02 bit/symbol increase for the nonlinearity-tailored constellation.

Experimental Setup

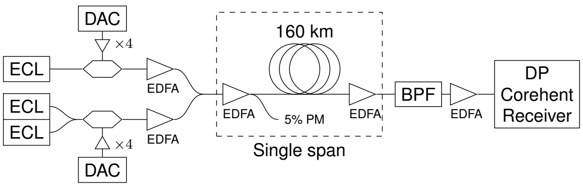

Fig. 3: The experimental setup, with a 3×33 GBd transmitter, single span and single channel receiver.

The experimental setup is shown in Fig. 3. The 3×35 GBd superchannel was transmitted over a single span of 160 km of standard single mode fibre (SSMF) to focus on the impact of NLI. The launch power into the fibre span was swept to investigate the nonlinear tolerance of the constellations designed.

The transmitter consisted of the channel under test in which a single 100 kHz-linewidth external cavity laser (ECL) was modulated using a dual polarisation (DP) IQ modulator. Two additional ECLs were modulated by a separate modulator to form the aggressor channels. Eight 8-bit DAC channels operating at 87.5 GS/s, with ENOB of ∼4 bits at 17.5 GHz were use to generate independent channels. Both signal paths are amplified using an Erbium doped fibre amplifier (EDFA) and combined by a 50/50 coupler.

Before the span, an EDFA and variable optical attenuator (VOA) followed by a 5% power tap with a power meter were used to control the launch power into the span. The 160 km span of SSMF was followed by an EDFA, the output of which was passed to the receiver.

The receiver had a band-pass filter and EDFA followed by a DP coherent receiver with a separate ECL as local oscillator. After 160 GS/s analogue-to-digital converters (ADCs), offline digital signal processing (DSP) was used. After electronic dispersion compensation, a radially directed equaliser and a decision directed carrier phase estimator were used to recover the symbols. The SNR and generalised mutual information (GMI) were extracted from the received symbols.

Experimental results

The received constellations at optimum launch powers are shown in Fig. 1 (d,e,f). In contrast to the uniform QAM scatter plot, the constellations tailored for the AWGN channel, shown in Fig. 1 (e), and for the nonlinear fibre channel, shown in Fig. 1 (f), set the lower energy points closer together, making these points more pronounced. The denser points should not be mistaken for probabilistically shaped constellations; all constellation points are equiprobable, the overlapping noise distributions around these points resulting in increased density of samples within the central area. Furthermore, it can be clearly seen that, as a result of fibre nonlinearity,the relative phase rotation between the central and outer points is higher in the AWGN tailored constellation than in the nonlinearity tailored constellation.

The experimentally-measured GMIs and SNRs are shown in Fig. 4 and Fig. 5, respectively. The markers are experimentally obtained and the lines are from the model. The back-to-back (BtB) SNR and η_{tot} derived from the model are shown in Tab. 1.

Tab. 1: Back-to-back SNR and η_{tot} for the constellations.

	BtB SNR [dB]	η_{tot} [dB]
Uniform	22.78	27.61
AWGN tailored	21.63	28.23
Nonlinearity tailored	22.01	28.08

The uniform QAM exhibited the highest BtB SNR and the lowest η_{tot}, resulting in the highest SNR at optimal launch power. However, this modulation format has no shaping gain and consequently has the lowest GMI of the three constellation formats evaluated, with an optimum value of 11.6 bit/symbol.

The AWGN tailored format has the lowest SNR across all launch powers, but outperforms uniform QAM (i.e. higher GMI) at launch powers below the optimal. At higher launch powers the model does not predict any gains, while the experimentally observed performance was marginally higher.

Due to lower excess kurtosis, the nonlinearity tailored constellation has a η_{tot} lower than the AWGN tailored constellation as predicted by Eq. (1). Additionally, the lower excess kurtosis results in a higher BtB SNR because of the lighter tailed distributions of the constellation and resulting reduced quantisation noise. The nonlinearity tailored constellation offers a trade-off between shaping gain and the impact of the nonlinear interference. It achieved a GMI of 11.7 bit/symbol after transmission over the 160 km link, a $>$ 0.1 bit/symbol increase over the other two constellations.

Conclusions

We experimentally demonstrated a nonlinearity tailored distribution that outperforms the distribution tailored to the AWGN channel by $>$ 0.1 bit/symbol, a result of a trade-off between fibre nonlinearity, transceiver limitations and shaping gain. The work indicates that all three aspects should be taken into consideration to maximise the system performance. However, empirically we have shown that, by reducing the excess kurtosis of a modulation format, the tolerance to both fibre nonlinearity and transceiver limitations can be simultaneously improved.

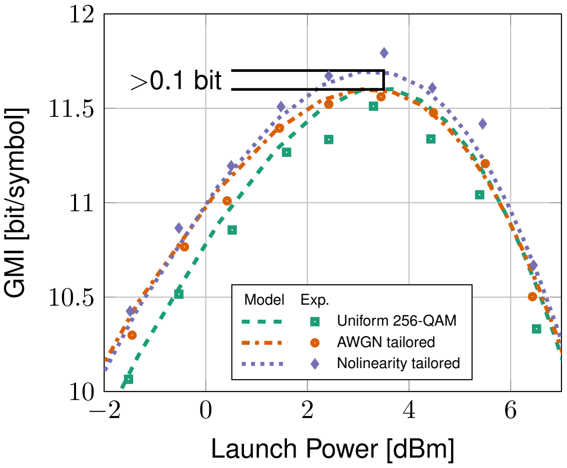

Fig. 4: The GMI versus the launch power for all constellations. The model is shown with the lines and markers are experimental results.

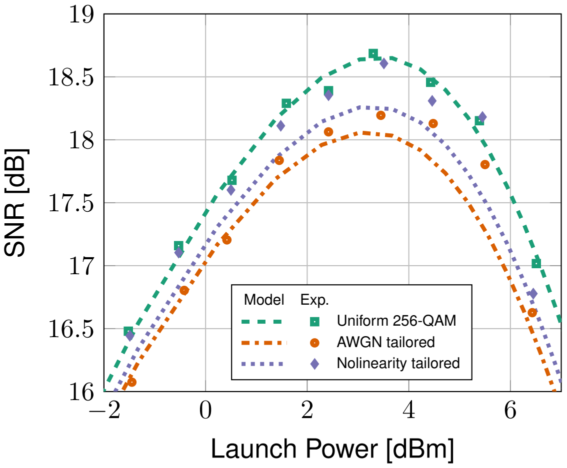

Fig. 5: The SNR versus the launch power for all constellations. The model is shown with the lines and markers are experimental results.

Acknowledgements

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