Short Papers

Robust AN-Aided Secure Beamforming and Power Splitting in Wireless-Powered AF Relay Networks

Hehao Niu, Bangning Zhang, Kai-Kit Wong, Zheng Chu, and Fuhui Zhou

***Abstract*—In this letter, we investigate the secrecy design in a wireless-powered amplify-and-forward relay network, where the relay is energy constrained and powered by the signal from the transmitter. Specifically, by adopting the power splitting (PS) scheme at the relay, we investigate the worst-case secrecy rate maximization by jointly designing the relay beamforming matrix, artificial noise covariance, and the PS ratio. However, the formulated problem is highly nonconvex due to the secrecy rate function and the dynamic relay power constraint. By exploiting the hidden convexity, we transform the original problem to a solvable reformulation via the successive convex approximation and constrained concave–convex procedure, which can provide a high-level approximated beamforming solution. Then, an iterative algorithm is proposed to obtain the solution. Numerical results showed the effectiveness of the proposed robust scheme.**

***Index Terms*—Amplify-and-forward (AF) relay, artificial noise (AN), beamforming (BF), power splitting (PS), wireless powered.**

I. Introduction

Relays are widely used to extend the wireless network coverage. However, given the complex communication environment and the openness of wireless channels, the security of relay networks is facing severe challenges [1]. The physical layer security technique [2] has been proved to be an effective way to improve the security [3]–[7].

Specifically, [3] proposed a low-complexity secrecy beamforming (BF) design in amplify-and-forward (AF) relay networks, [4] proposed a joint design of the BF, jamming, and power allocation, and [5] investigated the secrecy design with considering untrusted relay, respectively, while [6] and [7] investigated the secrecy design with the simultaneous wireless information and power transfer (SWIPT) in one-way AF relay networks and two-way AF relay networks, respectively.

Recently, there has been an urgent concern about the secrecy design in wireless-powered relay networks, e.g., the relay can harvest energy from the radio-frequency signal [8]–[11]. Specifically, [8] analyzed the secrecy capacity in wireless-powered AF relay networks, while [9] investigated the secure relay methods to maximize the achievable secrecy rate. Furthermore, in [10] and [11], the authors investigated the secure relay design to minimize the relay power consumption. However the 2-D search and semidefinite relaxation-based methods in [11] are quiet complex and cannot guarantee to obtain a rank-one BF solution.

Motivated by these observations, in this letter, we investigate the robust secrecy design in wireless-powered AF relay networks, where the relay utilizes power splitting (PS) to extract the information and energy simultaneously. Specifically, we aim to maximize the worst-case secrecy rate by jointly optimizing the relay BF, artificial noise (AN), and PS ratio. Different from related works such as [6] and [7], the signal-to-interference-and-noise ratios (SINRs) for the receivers are jointly determined by the BF matrix, AN covariance, and PS ratio, and the transmit power constraint at the relay is dynamic, thus making the problem highly nonconvex. To address this problem, we find the hidden convexity of the relay power constraint, which is a perspective of the quadratic form function. Then, we proposed a successive convex approximation (SCA) and constrained concave–convex procedure (CCCP)-based iterative algorithm to solve the approximated reformulation, which can provide a high-level BF solution. Finally, numerical results validate our proposed design.

II. System Model

A. System Model

Let us consider a relay wiretap system, in which a transmitter (Alice) sends confidential messages to a legitimate receiver (Bob) with the aid of a wireless-powered AF relay, in the presence of an eavesdropper (Eve). We assume that the relay is equipped with N_r antennas, while the other nodes are equipped with a single antenna. In addition, $\mathbf{f}$, $\mathbf{h}$, and $\mathbf{g}$ denote the channel vectors from Alice to the relay, from the relay to Bob, and from the relay to Eve, respectively. Besides, there is no direct link between Alice and Bob, as well as Eve. Since the relay operates in a half-duplex mode, one transmission round T is composed of two phases.

In the first phase, Alice sends signal s satisfying $\mathbb{E}[|s|^2] = 1$ to the relay, and thus, the received signal at the relay is

$$\mathbf{y}_r = \sqrt{P_s}\mathbf{f}s + \mathbf{n}_r \tag{1}$$

where P_s is the transmit power at Alice and $\mathbf{n}_r$ is the additive noise at the relay with variance σ_r^2. The received signal $\mathbf{y}_r$ is divided into two parts. Denoting $\rho (0 \le \rho \le 1)$ as the PS ratio at the relay for information decoding (ID), thus the received signal for ID is

$$\mathbf{y}_r^{ID} = \sqrt{\rho P_s}\mathbf{f}s + \sqrt{\rho}\mathbf{n}_r + \mathbf{n}_p \tag{2}$$

where $\mathbf{n}_p$ is the relay processing noise with variance σ_p^2.

On the other hand, the total harvested energy at the relay can be expressed as

$$E = \frac{T}{2}\eta\left(1-\rho\right)\left(P_s\|\mathbf{f}\|^2 + \sigma_r^2\right) \tag{3}$$

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where η denotes the energy conversion efficiency. Without loss of generality, we assume that $\eta = 1$.

Accordingly, the maximum available power at the relay is

$$P_{\max} = \frac{E}{T/2} + P_0 = (1-\rho)\left(P_s\|\mathbf{f}\|^2 + \sigma_r^2\right) + P_0 \tag{4}$$

where P_0 denotes the initial power at the relay, as done in [11].

In the second phase, the relay forwards $\mathbf{y}_r^{\mathrm{ID}}$ by a BF matrix $\mathbf{W}$. In addition, AN is added to interfere Eve. Thus, the transmitted signal by the relay can be expressed as

$$\mathbf{x}_r = \mathbf{W}\mathbf{y}_r^{ID} + \mathbf{z} \tag{5}$$

where $\mathbf{z}$ is the AN vector with $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$.

Thus, the received signals at Bob and Eve are given by

$$y_b = \sqrt{\rho P_s}\mathbf{h}^H\mathbf{W}\mathbf{f}s + \sqrt{\rho}\mathbf{h}^H\mathbf{W}\mathbf{n}_r + \mathbf{h}^H\mathbf{W}\mathbf{n}_p + \mathbf{h}^H\mathbf{z} + n_b \tag{6a}$$

$$y_e = \sqrt{\rho P_s}\mathbf{g}^H\mathbf{W}\mathbf{f}s + \sqrt{\rho}\mathbf{g}^H\mathbf{W}\mathbf{n}_r + \mathbf{g}^H\mathbf{W}\mathbf{n}_p + \mathbf{g}^H\mathbf{z} + n_e \tag{6b}$$

respectively, where n_b and n_e are the additive noises at Bob and Eve, with variance σ_b^2 and σ_e^2, respectively.

According to the received signals in (6), the SINRs at Bob and Eve are, respectively, given by

$$\Gamma_b = \frac{\rho P_s\left|\mathbf{h}^H\mathbf{W}\mathbf{f}\right|^2}{\rho\sigma_r^2\|\mathbf{h}^H\mathbf{W}\|^2 + \sigma_p^2\|\mathbf{h}^H\mathbf{W}\|^2 + |\mathbf{h}^H\mathbf{z}|^2 + \sigma_b^2} = \frac{\mathbf{w}^H\mathbf{Q}_b\mathbf{w}}{\mathbf{w}^H\mathbf{R}_b\mathbf{w} + t\mathbf{w}^H\mathbf{U}_b\mathbf{w} + t\mathbf{h}^H\mathbf{\Sigma}\mathbf{h} + t\sigma_b^2} \tag{7a}$$

$$\Gamma_e = \frac{\rho P_s\left|\mathbf{g}^H\mathbf{W}\mathbf{f}\right|^2}{\rho\sigma_r^2\|\mathbf{g}^H\mathbf{W}\|^2 + \sigma_p^2\|\mathbf{g}^H\mathbf{W}\|^2 + |\mathbf{g}^H\mathbf{z}|^2 + \sigma_e^2} = \frac{\mathbf{w}^H\mathbf{Q}_e\mathbf{w}}{\mathbf{w}^H\mathbf{R}_e\mathbf{w} + t\mathbf{w}^H\mathbf{U}_e\mathbf{w} + t\mathbf{g}^H\mathbf{\Sigma}\mathbf{g} + t\sigma_e^2} \tag{7b}$$

where $\mathbf{w} = \mathrm{vec}(\mathbf{W})$, $\mathbf{Q}_b = P_s\mathbf{f}^\dagger\mathbf{f}^T \otimes \mathbf{h}\mathbf{h}^H$, $\mathbf{R}_b = \sigma_r^2\mathbf{I}\otimes\mathbf{h}\mathbf{h}^H$, $\mathbf{U}_b = \sigma_p^2\mathbf{I}\otimes\mathbf{h}\mathbf{h}^H$, $\mathbf{Q}_e = P_s\mathbf{f}^\dagger\mathbf{f}^T\otimes\mathbf{g}\mathbf{g}^H$, $\mathbf{R}_e = \sigma_r^2\mathbf{I}\otimes\mathbf{g}\mathbf{g}^H$, $\mathbf{U}_e = \sigma_p^2\mathbf{I}\otimes\mathbf{g}\mathbf{g}^H$, and $t = 1/\rho$.

On the other hand, the transmit power of the relay is

$$P_r = \rho P_s\|\mathbf{W}\mathbf{f}\|^2 + \rho\sigma_r^2\|\mathbf{W}\|^2 + \sigma_p^2\|\mathbf{W}\|^2 + \mathrm{Tr}(\mathbf{\Sigma}) \tag{8}$$

which is constrained by

$$\mathrm{Tr}\left(\mathbf{w}^H\mathbf{C}\mathbf{w}\right) + \mathrm{Tr}(\mathbf{\Sigma}) \le (1-\rho)\left(P_s\|\mathbf{f}\|^2 + \sigma_r^2\right) + P_0 \tag{9}$$

where $\mathbf{C} = \left(\rho\left(P_s\mathbf{f}\mathbf{f}^H + \sigma_r^2\mathbf{I}\right) + \sigma_p^2\mathbf{I}\right)^T \otimes \mathbf{I}$.

By substituting $t = 1/\rho$ into (9), (9) can be equivalently rewritten as

$$P_s\|\mathbf{f}\|^2 + \sigma_r^2 + P_0 \ge \frac{\mathbf{w}^H\left(\left(P_s\mathbf{f}\mathbf{f}^H + \sigma_r^2\mathbf{I}\right)^T\otimes\mathbf{I}\right)\mathbf{w}}{t} + \frac{1}{t}\left(P_s\|\mathbf{f}\|^2 + \sigma_r^2\right) + \mathbf{w}^H\left(\sigma_p^2\mathbf{I}\otimes\mathbf{I}\right)\mathbf{w} + \mathrm{Tr}(\mathbf{\Sigma}). \tag{10}$$

The introduced variable t is a key transformation to handle the dynamic relay power constraint and is beneficial to formulate a convex problem, which will be further confirmed by the following analysis.

B. Problem Statement

Similar to [3] and [11], we assume that only imperfect Eve's CSI is available and the imperfect CSI is modeled as

$$\mathcal{G} = \{\mathbf{g}\,|\mathbf{g} = \bar{\mathbf{g}} + \Delta\mathbf{g}, \|\Delta\mathbf{g}\| \le \epsilon\} \tag{11}$$

where $\bar{\mathbf{g}}$ denotes the estimate of $\mathbf{g}$; $\Delta\mathbf{g}$ denotes the channel uncertainty; and ϵ is the size of the bounded error region.

Accordingly, the worst-case secrecy rate is

$$R_s = \frac{1}{2}\log(1+\Gamma_b) - \max_{\forall\mathbf{g}\in\mathcal{G}}\frac{1}{2}\log(1+\Gamma_e). \tag{12}$$

Now, we aim to maximize the worst-case secrecy rate by jointly optimizing the BF matrix, the AN covariance, and the PS ratio. Mathematically, our problem is formulated as

$$\max_{\mathbf{w},\mathbf{\Sigma}\succeq\mathbf{0},t\ge 1} \quad R_s \tag{13a}$$

$$\text{s.t.} \quad (10). \tag{13b}$$

Notably, (13) is hard to handle due to the nonconvex (13a) and (10). Based on this, we will design an effective method for (13) based on the SCA and CCCP in the following section.

III. SCA- and CCCP-Based Robust Design

In order to make (13) tractable, we first present the following proposition for (10).

Proposition 1: Equation (10) is jointly convex w.r.t. $\mathbf{w}$, t, and $\mathbf{\Sigma}$.

Proof. Since for any positive semidefinite matrix $\mathbf{A}\succeq\mathbf{0}$, the quadratic form $\mathbf{w}^H\mathbf{A}\mathbf{w}$ is convex w.r.t. $\mathbf{w}$. Furthermore, for any $t > 0$, $\frac{\mathbf{w}^H\mathbf{A}\mathbf{w}}{t}$ is the perspective of $\mathbf{w}^H\mathbf{A}\mathbf{w}$ [12]. Since the perspective operation preserves convexity, $\frac{\mathbf{w}^H\mathbf{A}\mathbf{w}}{t}$ is jointly convex w.r.t. $\mathbf{w}$ and t [12]. Besides, $(P_s\|\mathbf{f}\|^2 + \sigma_r^2)/t$ is convex w.r.t. t. Therefore, Proposition 1 holds. ■

Next, we turn our attention to (13a). By introducing auxiliary variables $\{\tau, x_1, x_2, y_1, y_2\}$, (13a) can be equivalently rewritten as

$$\min_{\substack{\tau,x_1,y_1,x_2,y_2,\\ \mathbf{w},\mathbf{\Sigma}\succeq\mathbf{0},t\ge 1}} \quad \tau \tag{14a}$$

$$\text{s.t.} \quad e^{x_2+y_2-x_1-y_1} \le \tau \tag{14b}$$

$$\mathbf{w}^H\left(\mathbf{Q}_b + \mathbf{R}_b + t\mathbf{U}_b\right)\mathbf{w} + t\mathbf{h}^H\mathbf{\Sigma}\mathbf{h} + t\sigma_b^2 \ge e^{x_1} \tag{14c}$$

$$\mathbf{w}^H\left(\mathbf{R}_b + t\mathbf{U}_b\right)\mathbf{w} + t\mathbf{h}^H\mathbf{\Sigma}\mathbf{h} + t\sigma_b^2 \le e^{x_2} \tag{14d}$$

$$\mathbf{w}^H\left(\mathbf{R}_e + t\mathbf{U}_e\right)\mathbf{w} + t\mathbf{g}^H\mathbf{\Sigma}\mathbf{g} + t\sigma_e^2 \ge e^{y_1} \tag{14e}$$

$$\mathbf{w}^H\left(\mathbf{Q}_e + \mathbf{R}_e + t\mathbf{U}_e\right)\mathbf{w} + t\mathbf{g}^H\mathbf{\Sigma}\mathbf{g} + t\sigma_e^2 \le e^{y_2}. \tag{14f}$$

Furthermore, (13) can be rewritten as follows:

$$\min_{\substack{\tau,x_1,x_2,y_1,y_2,\mathbf{w},\mathbf{\Sigma}\succeq\mathbf{0},\\ t\ge 1,\alpha,\beta,\chi,\delta,\theta,\varphi,\mu,\vartheta}} \quad \tau \tag{15a}$$

$$\text{s.t.} \quad e^{x_2+y_2-x_1-y_1} \le \tau, (10) \tag{15b}$$

$$\mathbf{w}^H\left(\mathbf{Q}_b + \mathbf{R}_b\right)\mathbf{w} \ge \alpha, \mathbf{w}^H\mathbf{U}_b\mathbf{w} + \mathbf{h}^H\mathbf{\Sigma}\mathbf{h} + \sigma_b^2 \ge \beta \tag{15c}$$

$$\mathbf{w}^H\mathbf{R}_b\mathbf{w} \le \chi, \mathbf{w}^H\mathbf{U}_b\mathbf{w} + \mathbf{h}^H\mathbf{\Sigma}\mathbf{h} + \sigma_b^2 \le \delta \tag{15d}$$

$$\mathbf{w}^H\mathbf{R}_e\mathbf{w} \ge \theta, \mathbf{w}^H\mathbf{U}_e\mathbf{w} + \mathbf{g}^H\mathbf{\Sigma}\mathbf{g} + \sigma_e^2 \ge \varphi \tag{15e}$$

$$\mathbf{w}^H\left(\mathbf{Q}_e + \mathbf{R}_e\right)\mathbf{w} \le \mu \tag{15f}$$

$$\mathbf{w}^H\mathbf{U}_e\mathbf{w} + \mathbf{g}^H\mathbf{\Sigma}\mathbf{g} + \sigma_e^2 \le \vartheta \tag{15g}$$

$$\alpha + t\beta \ge e^{x_1}, \theta + t\varphi \ge e^{y_1} \tag{15h}$$

$$\chi + t\delta \le e^{x_2}, \mu + t\vartheta \le e^{y_2} \tag{15i}$$

where $\{\alpha,\beta,\chi,\delta,\theta,\varphi,\mu,\vartheta\}$ are auxiliary variables.

Equation (15) is highly nonconvex due to (15c), (15e), (15h), and (15i). Next, we will utilize CCCP to find a proper approximation of (15h). It is noted that $4t\beta = (t+\beta)^2 - (t-\beta)^2$, by denoting

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$f(t,\beta,\tilde{t},\tilde{\beta}) = 2(\tilde{t}+\tilde{\beta})(t+\beta-\tilde{t}-\tilde{\beta}) + (\tilde{t}+\tilde{\beta})^2$ as the first-order Taylor expansion of the term $(t+\beta)^2$ around given point $\{\tilde{t},\tilde{\beta}\}$, we have $t\beta \geq \frac{1}{4}f(t,\beta,\tilde{t},\tilde{\beta}) - \frac{1}{4}(t-\beta)^2$. Thus, (15h) can be approximated as

$$\alpha + \frac{1}{4}f\left(t,\beta,\tilde{t},\tilde{\beta}\right) - \frac{1}{4}(t-\beta)^2 \geq e^{x_1} \tag{16a}$$

$$\theta + \frac{1}{4}f\left(t,\varphi,\tilde{t},\tilde{\varphi}\right) - \frac{1}{4}(t-\varphi)^2 \geq e^{y_1} \tag{16b}$$

where $f(t,\varphi,\tilde{t},\tilde{\varphi}) = 2(\tilde{t}+\tilde{\varphi})(t+\varphi-\tilde{t}-\tilde{\varphi}) + (\tilde{t}+\tilde{\varphi})^2$.

Then, we turn to (15i). By denoting $g(t,\delta,\tilde{t},\tilde{\delta}) = -2(\tilde{t}-\tilde{\delta})(t-\delta-\tilde{t}+\tilde{\delta}) - (\tilde{t}-\tilde{\delta})^2$ as the first-order Taylor expansion of the term $-(t-\delta)^2$ around given point $\{\tilde{t},\tilde{\delta}\}$, (15i) can be approximated as

$$\chi + \frac{1}{4}(t+\delta)^2 + \frac{1}{4}g\left(t,\delta,\tilde{t},\tilde{\delta}\right) \leq e^{\tilde{x}_2}\left(x_2 - \tilde{x}_2 + 1\right) \tag{17a}$$

$$\mu + \frac{1}{4}(t+\vartheta)^2 + \frac{1}{4}g\left(t,\vartheta,\tilde{t},\tilde{\vartheta}\right) \leq e^{\tilde{y}_2}\left(y_2 - \tilde{y}_2 + 1\right) \tag{17b}$$

where $g(t,\vartheta,\tilde{t},\tilde{\vartheta}) = -2(\tilde{t}-\tilde{\vartheta})(t-\vartheta-\tilde{t}+\tilde{\vartheta}) - (\tilde{t}-\tilde{\vartheta})^2$. Besides, the exponential functions are approximated by the first-order Taylor expansions around given point $\{\tilde{x}_2,\tilde{y}_2\}$.

Nextly, we focus on (15c). Since (15c) is nonconvex w.r.t. $\mathbf{w}$, we handle these constraints by neglecting the quadratic terms. Specifically, denoting $\Delta\mathbf{w} = \mathbf{w} - \tilde{\mathbf{w}}$, where $\tilde{\mathbf{w}}$ is a given point, we obtain the following two approximations:

$$\tilde{\mathbf{w}}^H\left(\mathbf{Q}_b + \mathbf{R}_b\right)\tilde{\mathbf{w}} + 2\Re\left\{\Delta\mathbf{w}^H\left(\mathbf{Q}_b + \mathbf{R}_b\right)\tilde{\mathbf{w}}\right\} \geq \alpha \tag{18a}$$

$$\tilde{\mathbf{w}}^H\mathbf{U}_b\tilde{\mathbf{w}} + 2\Re\left\{\Delta\mathbf{w}^H\mathbf{U}_b\tilde{\mathbf{w}}\right\} + \mathbf{h}^H\mathbf{\Sigma}\mathbf{h} + \sigma_b^2 \geq \beta. \tag{18b}$$

Similarly, (15e) can be approximated as

$$\tilde{\mathbf{w}}^H\mathbf{R}_e\tilde{\mathbf{w}} + 2\Re\left\{\Delta\mathbf{w}^H\mathbf{R}_e\tilde{\mathbf{w}}\right\} \geq \theta \tag{19a}$$

$$\tilde{\mathbf{w}}^H\mathbf{U}_e\tilde{\mathbf{w}} + 2\Re\left\{\Delta\mathbf{w}^H\mathbf{U}_e\tilde{\mathbf{w}}\right\} + \mathbf{g}^H\mathbf{\Sigma}\mathbf{g} + \sigma_e^2 \geq \varphi. \tag{19b}$$

To this end, the remaining task is to deal with the CSI uncertainty in (15f), (15g), and (19). First, to deal with (19), we introduce the following S-lemma.

Lemma 1 (S-Lemma [13]): Define the function

$$f_j\left(\mathbf{x}\right) = \mathbf{x}^H\mathbf{A}_j\mathbf{x} + 2\mathrm{Re}\left\{\mathbf{b}_j^H\mathbf{x}\right\} + c_j, \quad j = 1,2$$

where $\mathbf{A}_j = \mathbf{A}_j^H \in \mathbb{C}^{n\times n}$, $\mathbf{b}_j \in \mathbb{C}^{n\times 1}$, and $c_j \in \mathbb{R}$. The implication $f_1(\mathbf{x}) \leq 0 \Rightarrow f_2(\mathbf{x}) \leq 0$ holds if there exists $\lambda \geq 0$ such that

$$\lambda\begin{bmatrix}\mathbf{A}_1 & \mathbf{b}_1\\ \mathbf{b}_1^H & c_1\end{bmatrix} - \begin{bmatrix}\mathbf{A}_2 & \mathbf{b}_2\\ \mathbf{b}_2^H & c_2\end{bmatrix} \succeq \mathbf{0}$$

provided that there exists a point $\mathbf{x}_0$ such that $f_1(\mathbf{x}_0) < 0$.

By using $\mathrm{vec}(\mathbf{B})^H(\mathbf{A}^T\otimes\mathbf{C})\mathrm{vec}(\mathbf{D}) = \mathrm{Tr}(\mathbf{A}\mathbf{B}^H\mathbf{C}\mathbf{D})$, we obtain

$$\begin{aligned}\tilde{\mathbf{w}}^H\mathbf{R}_e\tilde{\mathbf{w}} &= \tilde{\mathbf{w}}^H\left(\sigma_r^2\mathbf{I}\otimes\mathbf{g}\mathbf{g}^H\right)\tilde{\mathbf{w}}\\ &= \mathrm{Tr}\left(\sigma_r^2\tilde{\mathbf{W}}^H\mathbf{g}\mathbf{g}^H\tilde{\mathbf{W}}\right) = \sigma_r^2\mathbf{g}^H\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{g}\end{aligned} \tag{20}$$

where $\tilde{\mathbf{W}} = \mathrm{vec}^{-1}(\tilde{\mathbf{w}})$.

Thus, (19a) can be transformed into

$$\sigma_r^2\mathbf{g}^H\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{g} + 2\Re\left\{\sigma_r^2\mathbf{g}^H\tilde{\mathbf{W}}\Delta\mathbf{W}^H\mathbf{g}\right\} \geq \theta \tag{21}$$

where $\Delta\mathbf{W} = \mathrm{vec}^{-1}(\Delta\mathbf{w})$.

With the help of Lemma 1, (21) can be rewritten as the following linear matrix inequality (LMI):

$$\begin{bmatrix}\zeta\mathbf{I} + \mathbf{\Theta} & \mathbf{\Theta}\bar{\mathbf{g}}\\ \bar{\mathbf{g}}^H\mathbf{\Theta} & -\zeta\epsilon^2 - \theta + \bar{\mathbf{g}}^H\mathbf{\Theta}\bar{\mathbf{g}}\end{bmatrix} \succeq \mathbf{0} \tag{22}$$

where $\mathbf{\Theta} = \sigma_r^2\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H + \sigma_r^2\tilde{\mathbf{W}}\Delta\mathbf{W}^H + \sigma_r^2\Delta\mathbf{W}\tilde{\mathbf{W}}^H$ and $\zeta \geq 0$ is an auxiliary variable.

Similarly, (19b) can be rewritten as

$$\begin{bmatrix}\upsilon\mathbf{I} + \mathbf{\Xi} & \mathbf{\Xi}\bar{\mathbf{g}}\\ \bar{\mathbf{g}}^H\mathbf{\Xi} & -\upsilon\epsilon^2 - \varphi + \sigma_e^2 + \bar{\mathbf{g}}^H\mathbf{\Xi}\bar{\mathbf{g}}\end{bmatrix} \succeq \mathbf{0} \tag{23}$$

where $\mathbf{\Xi} = \sigma_p^2\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H + \sigma_p^2\tilde{\mathbf{W}}\Delta\mathbf{W}^H + \sigma_p^2\Delta\mathbf{W}\tilde{\mathbf{W}}^H + \mathbf{\Sigma}$ and $\upsilon \geq 0$ is an auxiliary variable.

Next, we handle the CSI uncertainty in (15f) and (15g), and we introduce the following Nemirovski lemma.

Lemma 2 (Nemirovski Lemma) [14]: For a given set of matrices $\mathbf{A} = \mathbf{A}^H$, $\mathbf{B}$, and $\mathbf{C}$, the following LMI is satisfied:

$$\mathbf{A} \succeq \mathbf{B}^H\mathbf{X}\mathbf{C} + \mathbf{C}^H\mathbf{X}^H\mathbf{B},\ \|\mathbf{X}\| \leq t$$

only and only if there exists a $a \geq 0$ such that

$$\begin{bmatrix}\mathbf{A} - a\mathbf{C}^H\mathbf{C} & -t\mathbf{B}^H\\ -t\mathbf{B} & a\mathbf{I}\end{bmatrix} \succeq \mathbf{0}.$$

Via Schur's complement [12], (15f) can be rewritten as

$$\begin{bmatrix}\mu & \mathbf{g}^H\mathbf{W}\mathbf{\Omega}\\ \mathbf{\Omega}^H\mathbf{W}^H\mathbf{g} & \mathbf{I}\end{bmatrix} \succeq \mathbf{0} \tag{24}$$

where $\mathbf{\Omega} = (P_s\mathbf{f}\mathbf{f}^H + \sigma_r^2\mathbf{I})^{1/2}$.

Furthermore, from (24), we obtain the following relationship:

$$\begin{aligned}(24) \Rightarrow &\begin{bmatrix}\mu & \bar{\mathbf{g}}^H\mathbf{W}\mathbf{\Omega}\\ \mathbf{\Omega}^H\mathbf{W}^H\bar{\mathbf{g}} & \mathbf{I}\end{bmatrix}\\ &\succeq -\begin{bmatrix}\mathbf{0}\\ \mathbf{\Omega}^H\mathbf{W}^H\end{bmatrix}\Delta\mathbf{g}\left[1,\mathbf{0}\right] - \begin{bmatrix}1\\ \mathbf{0}\end{bmatrix}\Delta\mathbf{g}^H\left[\mathbf{0},\mathbf{W}\mathbf{\Omega}\right].\end{aligned} \tag{25}$$

Via Lemma 2, we transformed the aforementioned relationship into the following LMI:

$$\begin{bmatrix}\mu - \omega & \bar{\mathbf{g}}^H\mathbf{W}\mathbf{\Omega} & \mathbf{0}\\ \mathbf{\Omega}^H\mathbf{W}^H\bar{\mathbf{g}} & \mathbf{I} & \epsilon\mathbf{\Omega}^H\mathbf{W}^H\\ \mathbf{0} & \epsilon\mathbf{W}\mathbf{\Omega} & \omega\mathbf{I}\end{bmatrix} \succeq \mathbf{0} \tag{26}$$

where $\omega \geq 0$ is an auxiliary variable.

Finally, (15g) holds if the following relationships hold:

$$\begin{bmatrix}\varsigma - \pi & \sigma_p\bar{\mathbf{g}}^H\mathbf{W} & \mathbf{0}\\ \sigma_p\mathbf{W}^H\bar{\mathbf{g}} & \mathbf{I} & \epsilon\sigma_p\mathbf{W}^H\\ \mathbf{0} & \epsilon\sigma_p\mathbf{W} & \pi\mathbf{I}\end{bmatrix} \succeq \mathbf{0} \tag{27a}$$

$$\begin{bmatrix}\varpi\mathbf{I} - \mathbf{\Sigma} & -\mathbf{\Sigma}\bar{\mathbf{g}}\\ -\bar{\mathbf{g}}^H\mathbf{\Sigma} & -\varpi\epsilon^2 + \gamma - \bar{\mathbf{g}}^H\mathbf{\Sigma}\bar{\mathbf{g}}\end{bmatrix} \succeq \mathbf{0} \tag{27b}$$

$$\varsigma + \gamma + \sigma_e^2 \leq \vartheta \tag{27c}$$

where $\{\varsigma,\pi \geq 0,\gamma,\varpi \geq 0\}$ are introduced auxiliary variables.

According to these steps, we obtain the following approximated problem:

$$\min_{\substack{\tau,x_1,x_2,y_1,y_2,\mathbf{w},\\ \mathbf{\Sigma}\succeq\mathbf{0},t\geq 1,\alpha,\beta,\chi,\\ \delta,\theta,\varphi,\mu,\vartheta,\varsigma,\gamma}} \quad \tau \tag{28a}$$

$$\text{s.t. } (15\text{b}), (15\text{d}), (16), (17), (18), (22), (23), (26), (27) \tag{28b}$$

$$\zeta \geq 0, \upsilon \geq 0, \omega \geq 0, \pi \geq 0, \varpi \geq 0 \tag{28c}$$

around given point $\{\tilde{\mathbf{w}},\tilde{t},\tilde{x}_2,\tilde{y}_2,\tilde{\beta},\tilde{\delta},\tilde{\varphi},\tilde{\vartheta}\}$, which can be effectively solved by the toolbox CVX [16].

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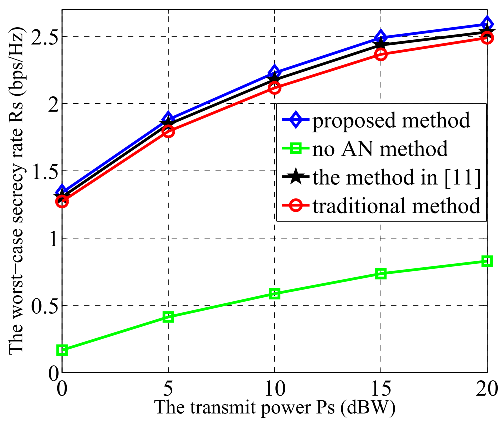

Fig. 1. Worst-case secrecy rate versus the transmit power.

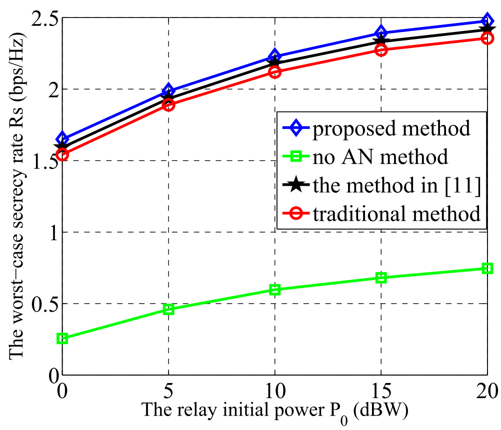

Fig. 2. Worst-case secrecy rate versus the relay initial power.

The convergency of our algorithm can be guaranteed by the method in [15]. In addition, by following a similar way as [5] and [16], we conclude that the complexity for an κ-optimal solution to (28) is about $O((2N_r^2 + 16)N_r^7 \ln(1/\kappa))$.

IV. Simulation Result

In this section, we evaluate the performance of our design through Monte Carlo simulations. The simulation settings are assumed as follows: $N_r = 4$, $P_s = 10$ dBW, $P_0 = 10$ dBW, and $\sigma_r^2 = \sigma_p^2 = \sigma_b^2 = \sigma_e^2 = -50$ dBm. Both channel fading and pathloss is considered, e.g., each entry of $\mathbf{f}, \mathbf{h}$, and $\bar{\mathbf{g}}$ is randomly generated by $\mathcal{CN}(0, 10^{-3})$ and the CSI uncertainty is $\epsilon^2 = 10^{-6}$. In addition, we compare our algorithm with the following methods: 1) the traditional secrecy AF method, e.g., setting $\rho = 1$; 2) the no-AN method, e.g., setting $\mathbf{\Sigma} = \mathbf{0}$; and 3) the method in [11]. The three methods are labeled as "traditional method," "no-AN method," and "the method in [11]," respectively.

First, we show the worst-case secrecy rate versus the transmit power P_s in Fig. 1. From Fig. 1, we can see that our proposed design outperforms the three other methods. Both our design and the design in [11] outperform the traditional secrecy AF relay method, which suggests the effect of PS on the security, since PS allows the relay to use more power to transmit the confidential signal and emit the AN. Besides, the no-AN method is the worst design, which suggests the significant effect of the AN on the security.

Second, we show the worst-case secrecy rate versus the initial power at the relay P_0 in Fig. 2. From Fig. 2, we can see that the worst-case secrecy rate increases with the increasing of P_0. In addition, by jointly comparing the curves in Figs. 1 and 2, we find that the slope of the curves in Fig. 1 is steeper, which means that P_s has more impact on the worst-case secrecy rate, since increasing P_s not only enhances the signal strength, but also increases the available power of the relay.

V. Conclusion

In this letter, we have investigated an AN-aided robust BF and PS design for a secure wireless-powered AF relay network. Specifically, we formulated the worst-case secrecy rate maximization design with dynamic relay power constraint. By analyzing the hidden convexity, we proposed an SCA- and CCCP-based iterative algorithm to solve this highly nonconvex problem. Numerical results validated the effectiveness of our proposed scheme.

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