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Stochastic Free Vibration Analysis of RC Buildings

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Abstract

Background/Objectives: Free vibration response of RC structures is random in nature due to the uncertainties exist in geometry, material properties and loading. Stochastic analysis methods can represent this randomness in responses. Methods: The Monte Carlo Simulation is a widely accepted method for stochastic structural analysis but the computational effort and cost associated with it is a limitation and hence in the present study, it is used as a method for the comparison and verification of the results obtained by other metamodel based approaches such as the response surface method. The number of analysis samples required depends on the type of approach adopted. **Findings:** Three different design of experiments approaches, Central Composite Design, Box Behnken Design and Full Factorial Design, where used in response surface modelling. The present study is an evaluation of these metamodel based approaches. The natural frequencies obtained by these methods of analysis were comparable with the results from Monte Carlo Simulation. However, the latter required one million analyses, making it computationally cumbersome. The Central Composite Design proved to be the most efficient method as it yielded the most accurate results even though the number of runs were marginally more than the 62 required for Box Behnken Design. **Improvements:** These response surface based metamodel approaches can be further applied to nonlinear stochastic analysis of structures where the cost and effort of analysis is significantly higher.

Keywords: Free Vibration, Metamodel, Monte Carlo simulation, Response Surface Method, Stochastic Analysis

1. Introduction

Structural behavior cannot be predicted deterministically in the case of disastrous loading such as earthquakes, hurricanes, etc. where there is uncertainty associated with both the nature of the loading and structural resistance[1].

Stochastic methods adopted in the analysis can, to an extent, address these uncertainties associated with the structural response to predict the random responses.

The previous studies carried out in the field of stochastic analysis include the use of Statistical Approaches (SA) like Monte Carlo Simulation, MCS[2].

This method was further improved by implementing different sampling techniques like Latin Hypercube sampling[3], stratified sampling[4]. Apart from these SA, Non-Statistical Approach (NSA) is also adopted where the evaluation of the response of the structure is done at some particular set of values of the random parameters. These values of parameters are obtained by methods of Design Of Experiments (DOE). An input-output relation, namely a metamodel, is developed from this set of values. This metamodel can be efficiently used to represent the structural responses[5]

The Response Surface Method is a popular method in which the response surface forms the metamodel[6]. In the present work an effort is made to obtain the random natural frequency of a symmetric RC bare framed building using different Response Surface Methods (RSM). The natural frequency of a building is an important response parameter, which is used to design for dynamic loading such as earthquake and wind. The most economical and accurate RSM technique can be determined by comparing the random responses obtained from the metamodels with that of the popular SA like the Monte Carlo Simulation. The most efficient method may be further adopted in the nonlinear random seismic responses of buildings. The present study is an attempt to develop metamodels that describe the random frequency (output) of an RC frame in terms

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of the random variables representing the material and geometric properties using sampling based on selected design of experiments. The focus of the present study is to evaluate the accuracy of the various methods of design of experiments.

2. Methodology

The response surface (RS) metamodel can be represented by its general form as in Eq. 1.

$$y = f(x_i) + \varepsilon \quad y = f(x_i) + \varepsilon \tag{1}$$

Here, you represent the response (output), xi represents the input variable and ε represent the error in estimation.

The error term can be neglected in the case of computer analysis[7]. The response surface input variables are the parameters whose uncertainty or randomness can cause an uncertainty in the output or response. The response function is modelled by a polynomial function. For a linear system, the first order polynomial can be used, whereas polynomials of higher orders are required to represent the systems with curvature[8]. A second order or quadratic function is selected to represent the free vibration response with considerable accuracy. The form of such a function is shown in Eq. 2.

$$y = \beta 0 + \sum_{i=1}^{k} \beta ixi + \sum_{i=1}^{k} \beta iixi + \sum_{i=1}^{k-1} \sum_{j=1}^{k} \beta ijxij \tag{2}$$

where,

y = response

xi, xj = random variables

β = Unknown coefficients or constants

k = Total number of input

In order to determine the unknown constants, a certain set of values for the random variables is chosen and analysis or experiment is carried out at the chosen points to obtain the response. With a specific set of inputs and outputs, the unknown constants in the polynomials are estimated to obtain the metamodel. Thus the functional relationship is established between the random input parameters and output responses.

The specific input values or design points for the metamodel formulation are determined by using DOE. Depending upon the type of polynomial function selected, a variety of DOE can be used. Each method gives a particular controlled combination of the input variables. The different design or sampling methods adopted in this work are Central Composite Design, CCD[9], Box-Behnken Design, BBD[10] and Full Factorial Designs, FFD[11].

3. Description of the Structure

In the current study a symmetric RC building having four stories and two bays is considered. The building is designed according to Indian Standard code[12] using M25 concrete and Fe415 steel. The details of the building plan, elevation and reinforcement details of the beams and columns are shown in Figure 1. The building has a storey height of 3.0 m and bay width of 4.0 m. The base of the building is considered as fixed. In addition to self-weight of the beams and the columns, the dead and live load (1.5kN/m2) due to the slab is also considered in the design.

4. Modelling of Uncertainty

The uncertainties in the random structural properties are modelled by considering the most significant parameters as random variables. The input variables that can affect the output, which is the natural frequency of the building, are identified. A total of seven variables which affect the output response are selected as shown in Table 1. The probability distributions and its statistical parameters of these variables are taken from the previous studies[13-15]. The structure is modelled in OpenSees[16] as elastic beam column elements.

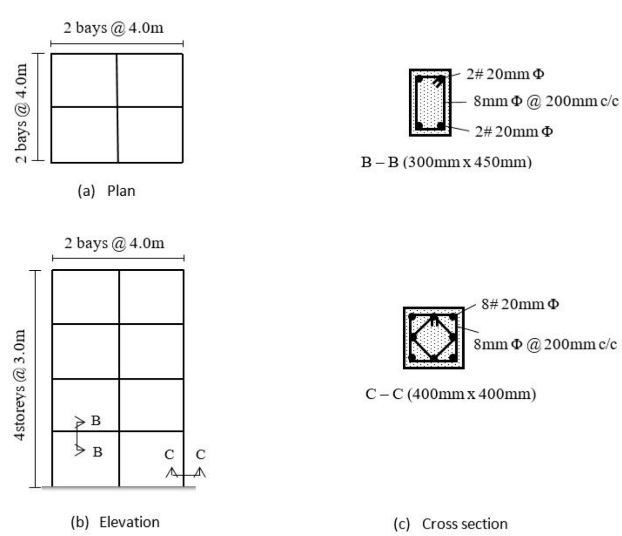

Figure 1. Plan, Elevation and Cross section of the building.

Table 1. Random variables considered in the present study.

No	Property	Mean	COV (%)	Distribution	Source
1	Concrete compressive strength	25 MPa	13	Lognormal	[13]
2	Live Load	1.5 kN/m2	10	Normal	[14]
3	Storey Height	3 m	8	Lognormal	[15]
4	Beam Depth	0.45m	1.5	Lognormal	[13]
5	Beam Width	0.3 m	3	Lognormal	[13]
6	Column Depth	0.4 m	1.5	Lognormal	[13]
7	Column Width	0.4 m	3	Lognormal	[13]

5. Metamodel

The metamodel is a polynomial functional relationship between the structural response (the natural frequency) and the random variables that define the structure. The assumed polynomial type selected in this study contains up to the second order terms of each variable excluding the coperative terms.

The interaction effect of the variables has not considered since the variables that affect the natural frequency are independent. Once the metamodel is represented, the values of the constant terms are computed from the known output values at the selected sampling points (design points). Thus, different RS metamodels are obtained for each DOE method).

6. Discussion

The accuracy of responses using the metamodels (obtained from each DOE's) is verified with respect to that from Monte Carlo Simulations conducted on one million samples. Figures 2, 3 and 4 shows the comparison of the responses (at the input values considered for MCS) obtained from the three DOE methods, CCD, BBD and FFD respectively with that of MCS. It can be inferred from these plots that the metamodel response surface obtained from DOE's are accurate enough to use it for further random simulations.

Simulations are carried out using each metamodels to obtain the natural frequencies at each value of random input variables.

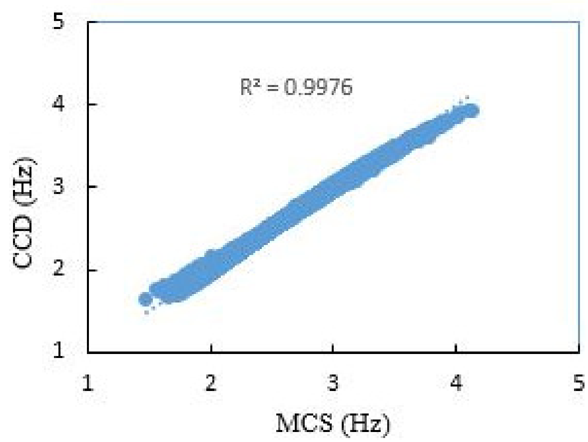

Figure 2. Natural frequency from CCD and MCS.

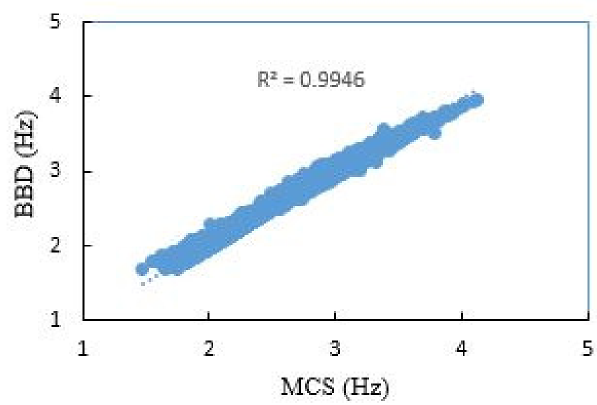

Figure 3. Natural frequency from BBD and MCS.

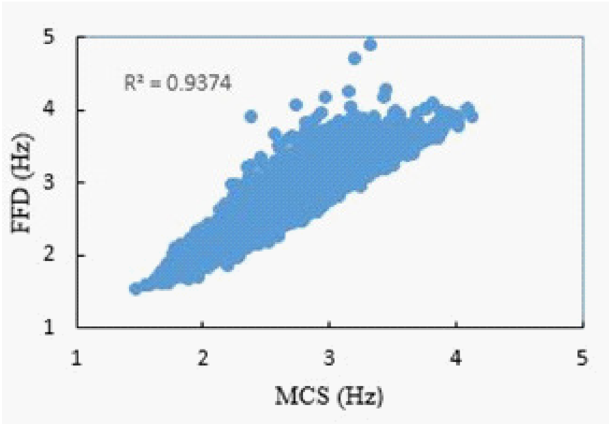

Figure 4. Natural frequency from FFD and MCS.

The obtained natural frequencies are converted to probability distribution curve to check the accuracy compared to that of MCS. The figure 5 shows the comparison of the probability distributions of the responses from the MCS and the DOE metamodel methods. It can be seen

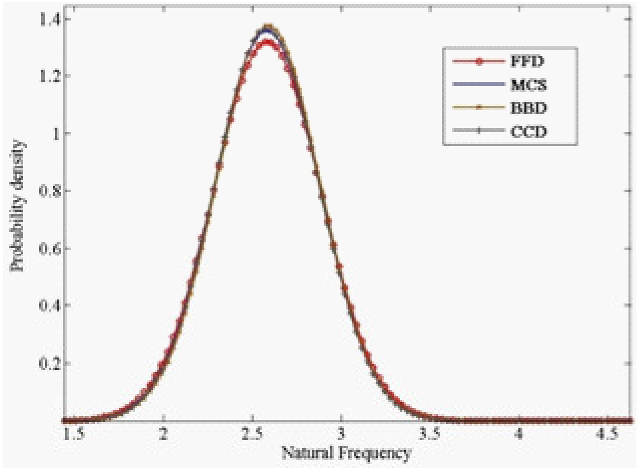

Figure 5. Proabability distribution of natural frequencies from MCS and selected DOE methods.

Table 2. Comparison of frequency from different methods

Method	Mean (Hz)	Std. Dev. (Hz)	Samples Required
MCS	2.5784	0.2937	100000
BBD	2.5867	0.2896	62
CCD	2.5799	0.2923	79
FFD	2.5805	0.3017	128

that the probability distribution curves obtained for natural frequency using CCD almost coincides with that of MCS.

The Table 2 presents the mean and standard deviations of the natural frequencies and the number of experiments/analysis required for each DOE case. Though the BBD requires only 62 number of analyses, the CCD is found to be more accurate with 79 numbers of analysis.

7. Conclusions

Uncertainties in the material properties and geometry makes the free vibration response of RC structures as random. A stochastic analysis is conducted in the present study to obtain the random dynamic responses of a four storeyed RC frame designed as per Indian Seismic Code. Metamodels are developed for representing the output frequency response in terms of various random input parameters sampled using various methods of design of experiments such as such as Central Composite Design, Box Behnken Design and Full Factorial Design. The frequency responses using the metamodels are found to be fairly matching with the responses from the accurate MCS. Although, all the selected design of experiments are able to yield reasonably accurate results with less number of computations, Central Composite Design is found to be marginally superior than other methods.

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