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# Simulating and investigating compressible flows interaction with fractal structures

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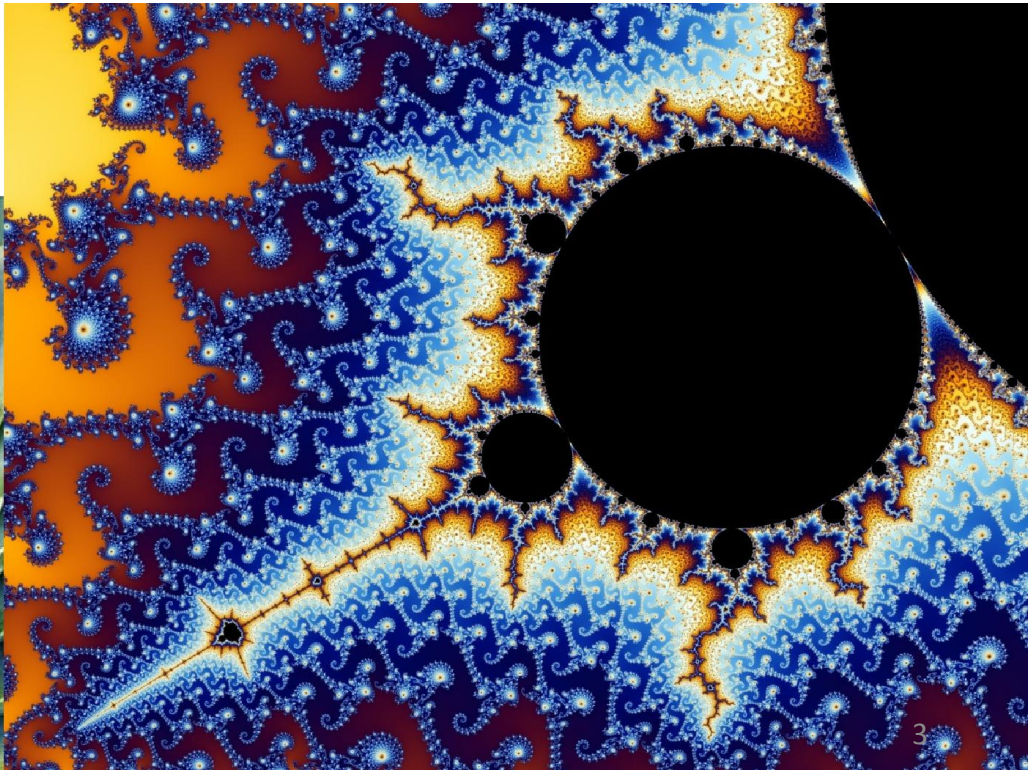
# □ Outline

- Introduction
  - What is a fractal?
  - Motivation
  - Wake generators / fractal plates considered in this study
- Problem formulation and numerical algorithm
  - Scaling / non-dimensionalization
  - Numerical framework
  - Immersed boundary method
- Results & discussion
- Conclusion

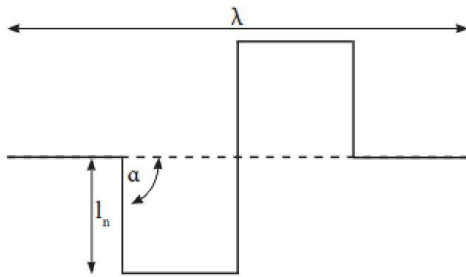
# □ Introduction

## ○ What is a fractal?

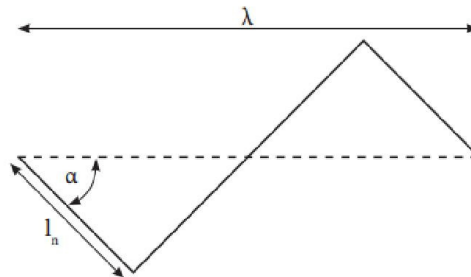
- A fractal is a detailed, recursive, and infinitely self-similar mathematical set that exhibits similar patterns at increasingly small scales.
- In other words and in the most basic sense, fractals are objects that display self-similarity over a wide range of scales.
- Introduced by Mandelbrot to extend the concept of theoretical fractional dimensions to geometric patterns found in nature.



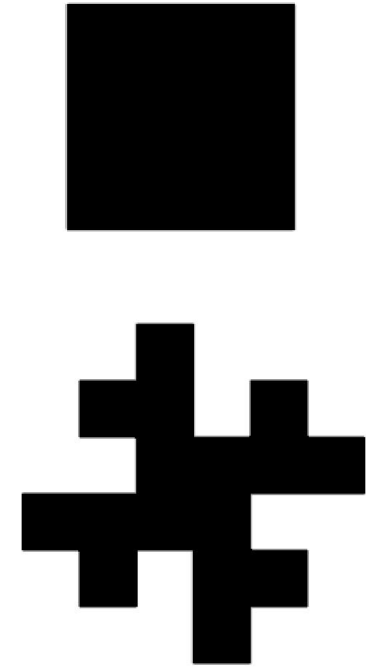
- Example:** Consider a straight line segment of length  $\lambda$ . For the first iteration ( $n=1$ ) with a square pattern ( $\alpha = 90^\circ$ ),  $\lambda$  is replaced by  $d = 8$  segments of length  $l_1 = \lambda / r$ . For  $n$  iterations, the length of the segment is  $l_n = l_{(n-1)}/r$ .



(a)  $D_f = 1.5$



(b)  $D_f = 1.3$



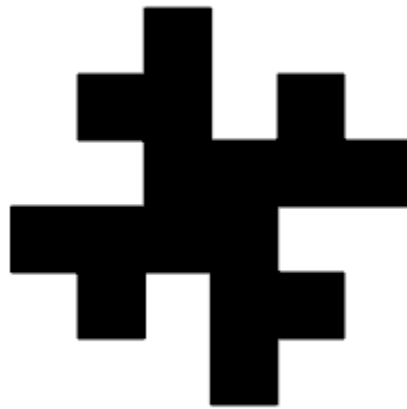
## ○ Motivation

- Previous studies
- Incompressible
- Result in higher turbulence intensities and a more enhanced turbulent mixing.
- Reduce the impact of the recirculation region around aircraft parts, e.g. spoilers, and hence the low-frequency noise.
- Significantly changes the near-field structure of the jet (by breaking up the large-scale coherent structures) responsible for the low-frequency noise.
- The mathematical properties of some fractals.
- Area conservation

- Wake generators / fractal plates considered in this study



(a) Square



(b)  $D_f 1.5(1)$



(c)  $D_f 1.5(2)$

# □ Governing equations & numerical framework

## ○ Scaling / non-dimensionalization

- All dimensional spatial coordinates are normalized by the reference length  $D$  associated with the fractal geometry.

$$(x, y, z) = \frac{(x^*, y^*, z^*)}{D}$$

- The velocity is scaled by the freestream velocity magnitude  $V_\infty^*$

$$(u, v, w) = \frac{(u^*, v^*, w^*)}{V_\infty^*}$$

- The pressure and temperature are non-dimensionalized, respectively, by the freestream dynamic pressure  $\rho_\infty^* V_\infty^{*2}$  and temperature  $T_\infty^*$ .

$$M_a = \frac{V_\infty^*}{a_\infty^*}, \quad Re_\lambda = \frac{\rho_\infty^* V_\infty^* D}{\mu_\infty^*}, \quad Pr = \frac{\mu_\infty^* C_p}{k_\infty^*}$$

- where  $a_\infty^*$ ,  $\mu_\infty^*$ ,  $k_\infty^*$  stand for, respectively, the freestream speed of sound, dynamic viscosity and thermal conductivity,  $C_p$  the specific heat at constant pressure.

- Full compressible Navier-Stokes equations in generalized curvilinear coordinates



## ○ Numerical framework

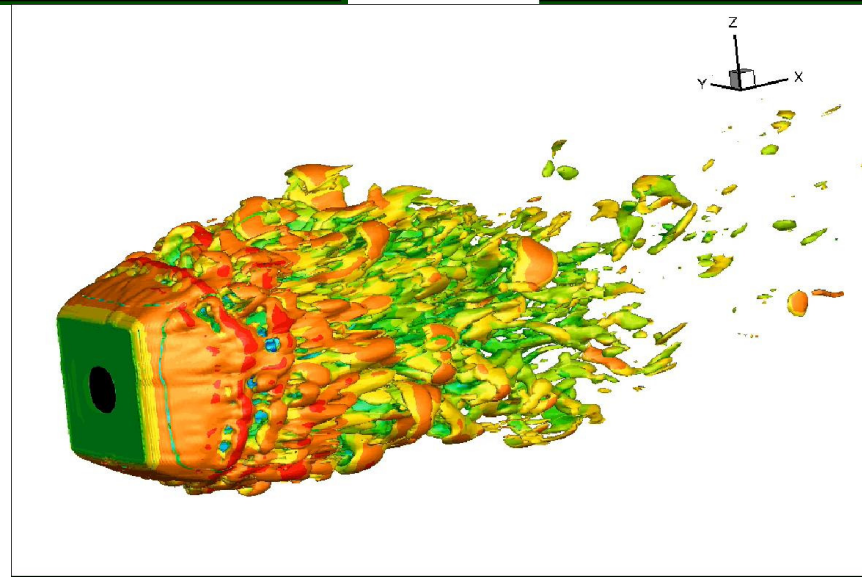
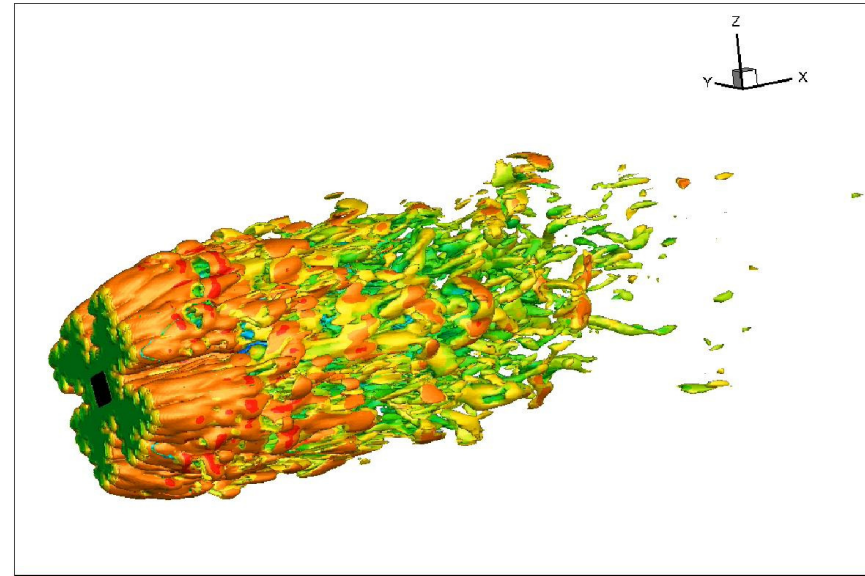
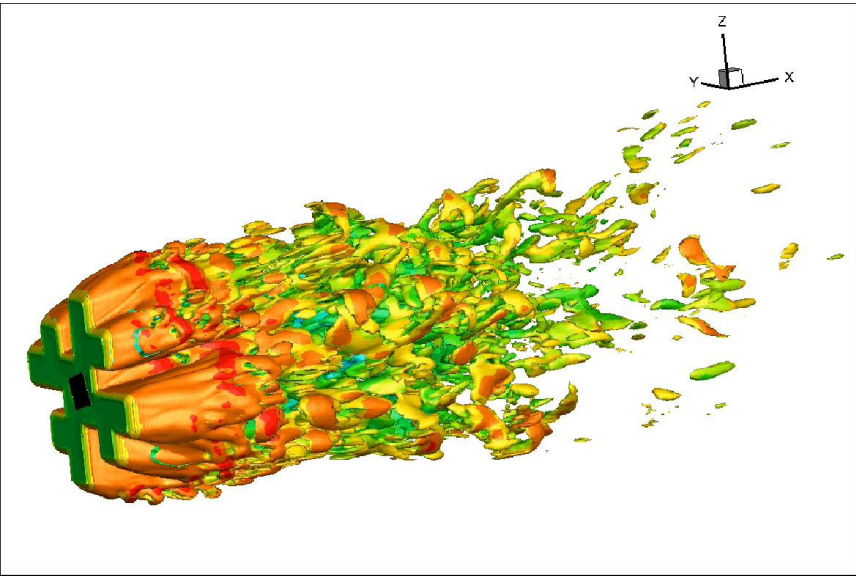
- **Implicit large eddy simulations**, where numerical filtering is applied to account for the missing sub-grid scale energy.
- The numerical algorithm uses high-order **finite difference** approximations for the spatial derivatives and **explicit** time marching.
- The time integration is performed using a **third order TVD Runge-Kutta** method.

## ○ Immersed boundary method

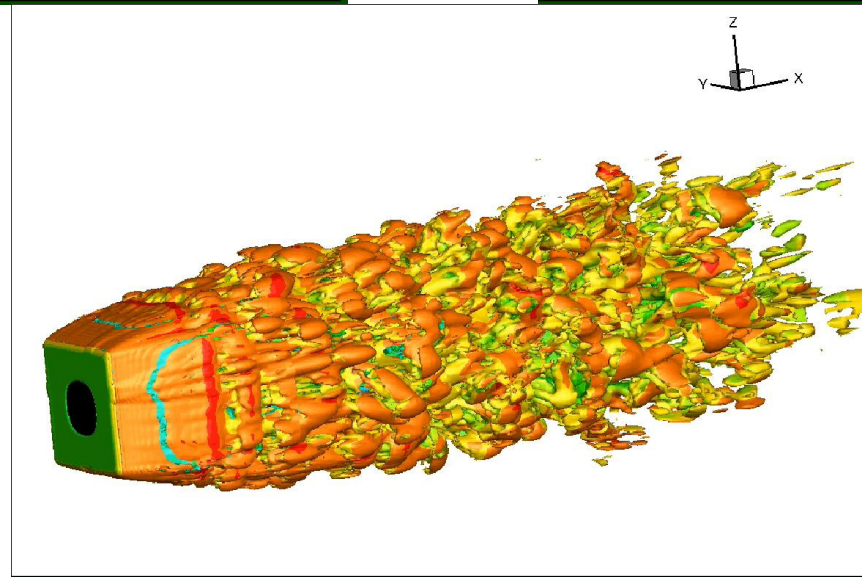
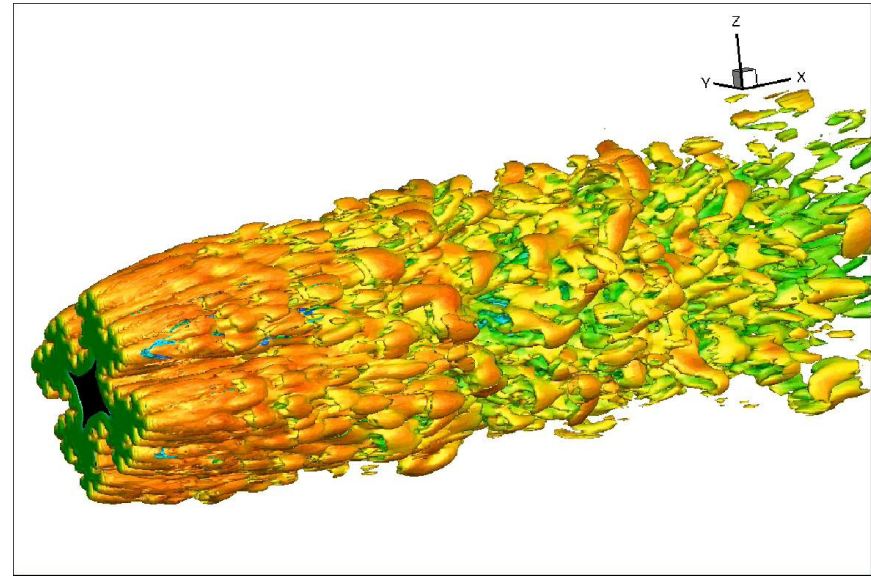
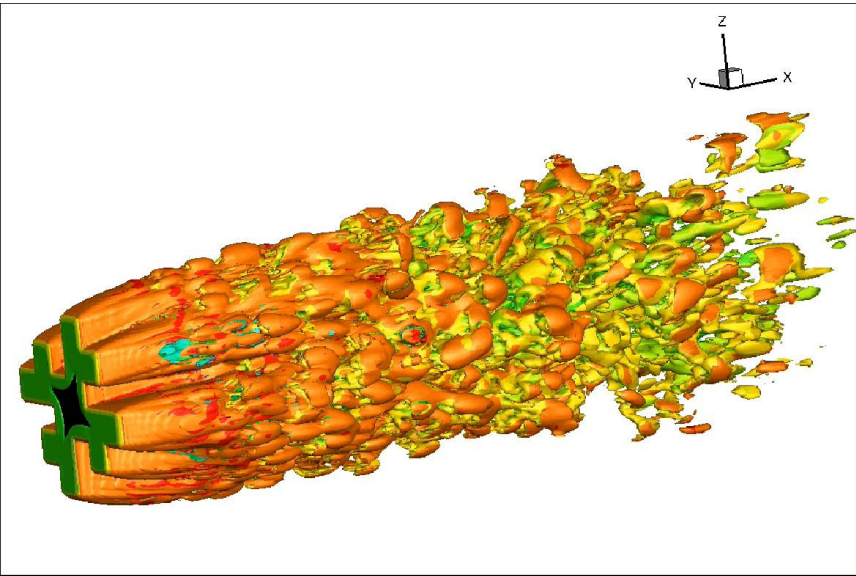
- the construction of the solid geometry inside the **Cartesian grid** is achieved by adding a forcing term **f** to the momentum equations that represents the impermeability of the fractal geometry to the governing equations.
- The fractal objects are obtained by multiple geometrical constrains. The forcing term consists of a penalty factor  $\sigma$  multiplied by the difference between the conserved variables  $\rho$ ,  $\rho u_i$ , and  $E$  and the imposed ones  $\rho_{imp}$ ,  $\rho u_{i,imp}$ ,  $E_{imp}$ .

# □ Results

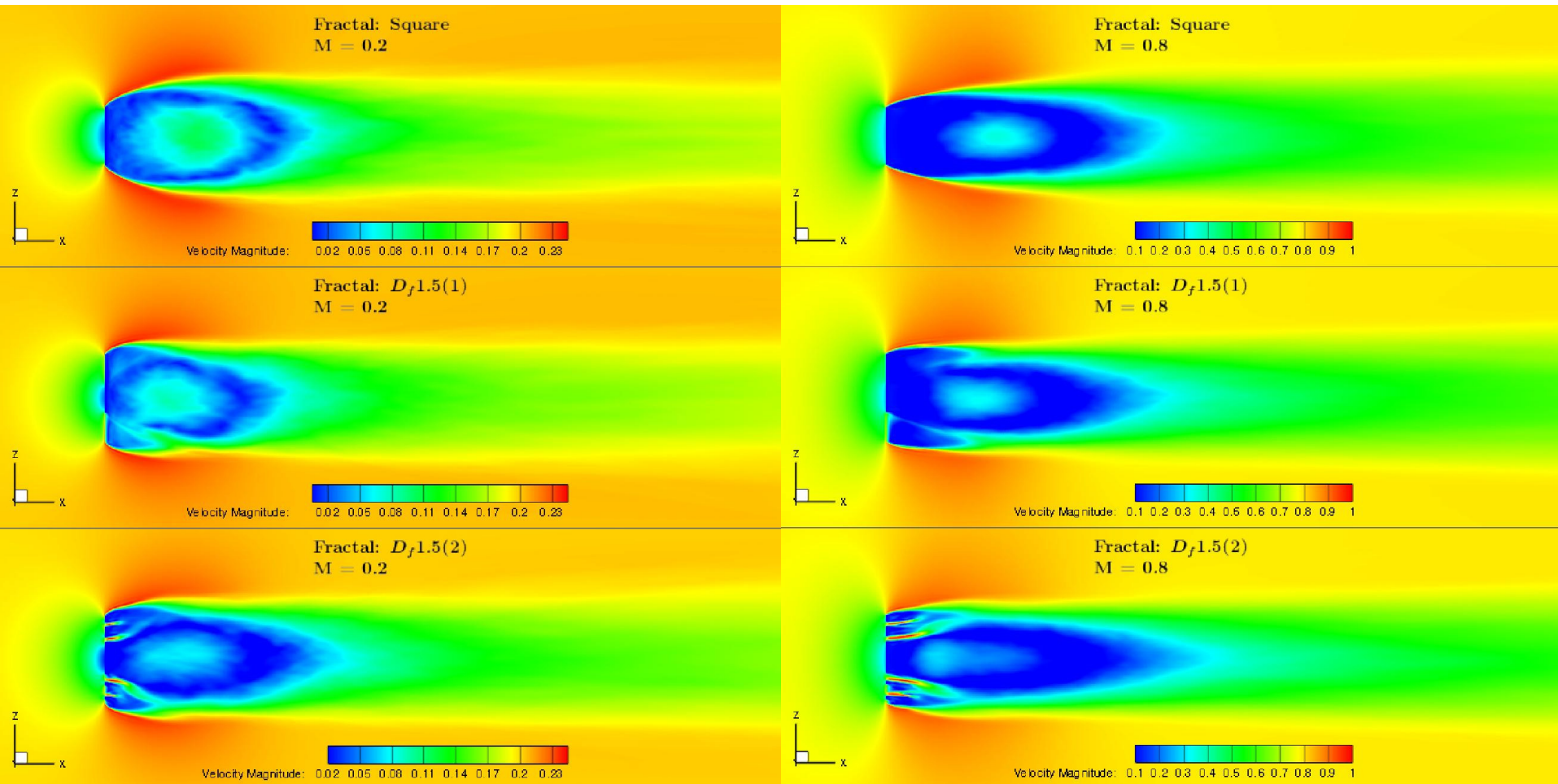
**$M = 0.2$  Iso-surfaces of the vorticity magnitude colored by the velocity magnitude**



**$M = 0.8$  Iso-surfaces of the vorticity magnitude colored by the velocity magnitude**



# Contour plots of the XZ-plane





# U mean velocity

## Square plate

## $D_f 1.2(1)$ fractal plate

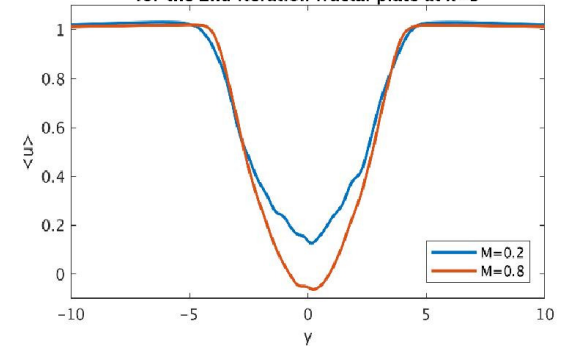
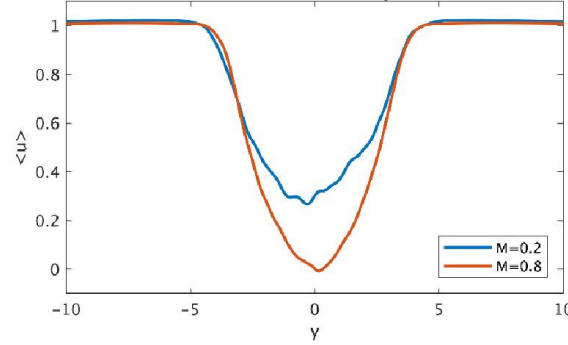
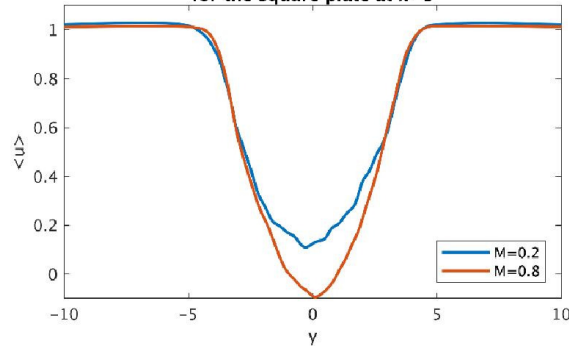
## $D_f 1.2(2)$ fractal plate

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the square plate at  $x=3$

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the 1st iteration fractal plate at  $x=3$

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the 2nd iteration fractal plate at  $x=3$

$X=3$

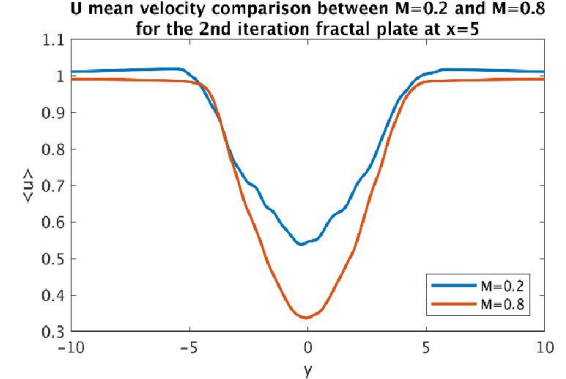
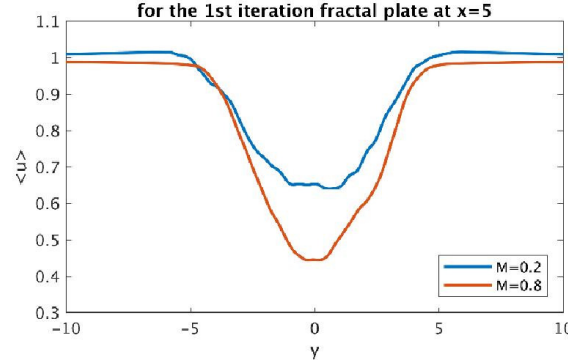
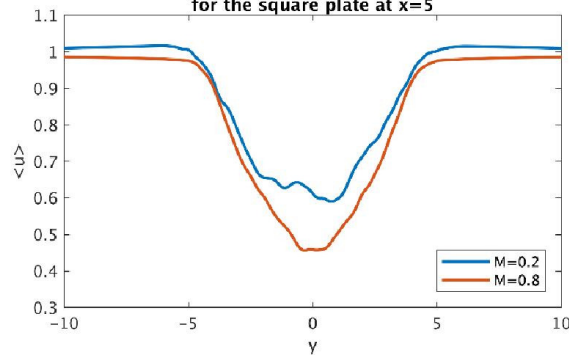


U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the square plate at  $x=5$

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the 1st iteration fractal plate at  $x=5$

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the 2nd iteration fractal plate at  $x=5$

$X=5$

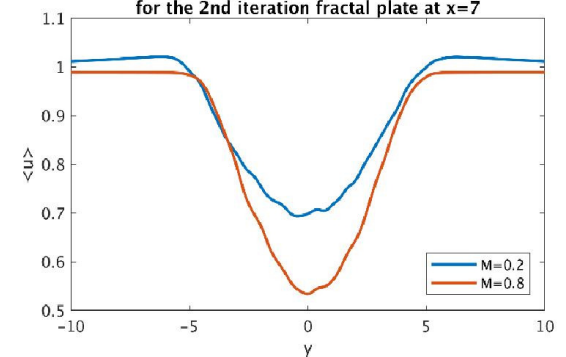
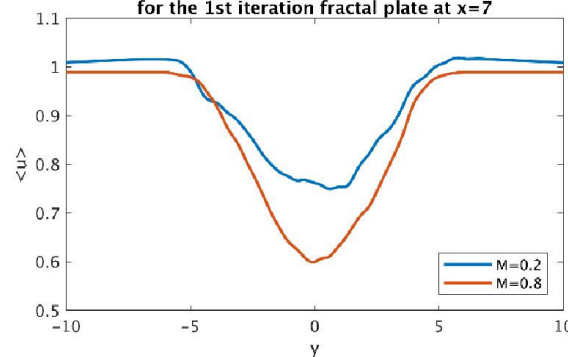
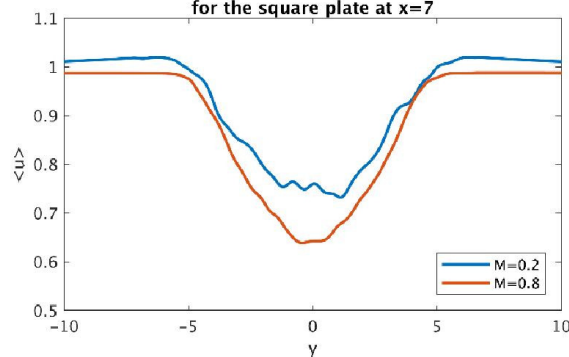


U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the square plate at  $x=7$

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the 1st iteration fractal plate at  $x=7$

U mean velocity comparison between  $M=0.2$  and  $M=0.8$  for the 2nd iteration fractal plate at  $x=7$

$X=7$

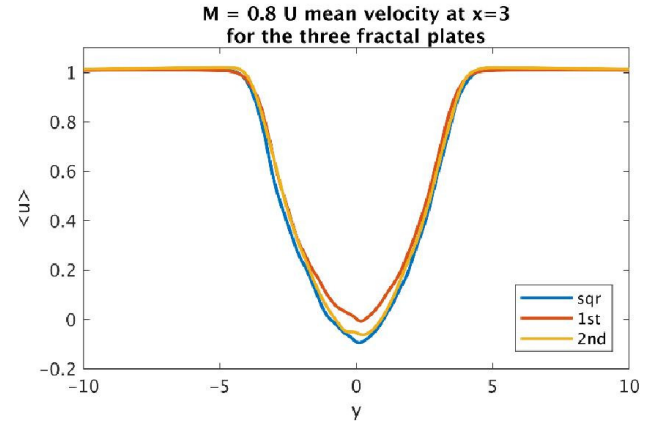
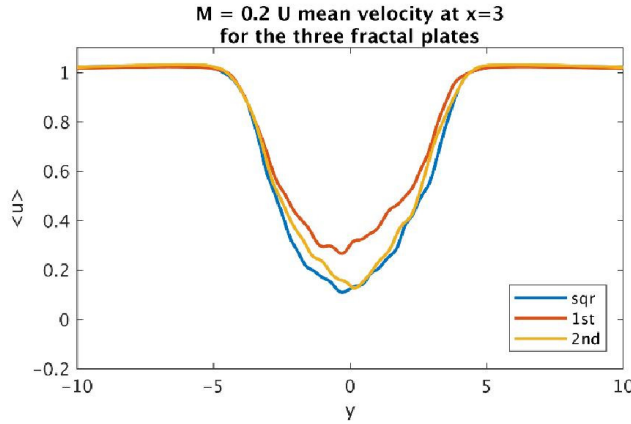


# U mean velocity

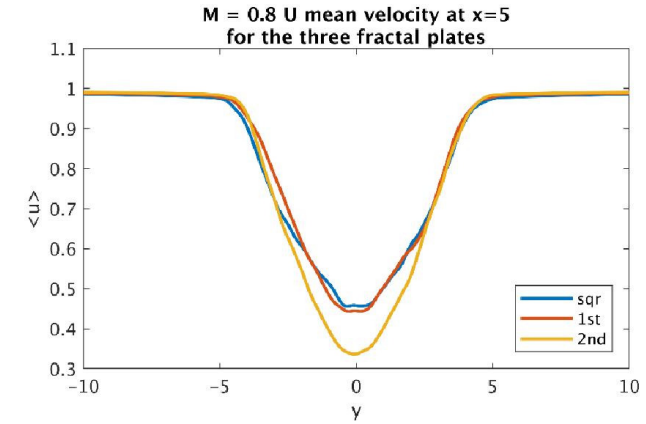
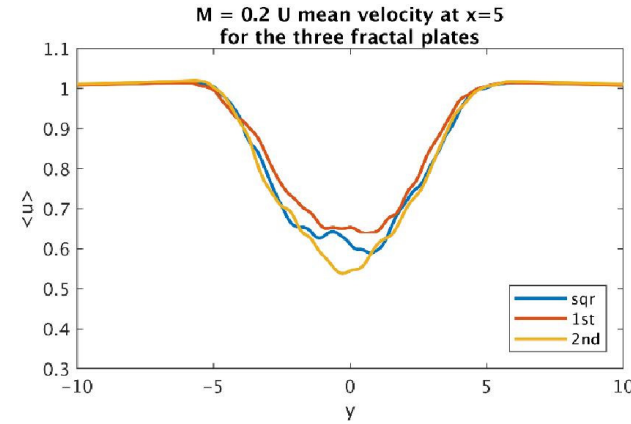
**M = 0.2**

**M = 0.8**

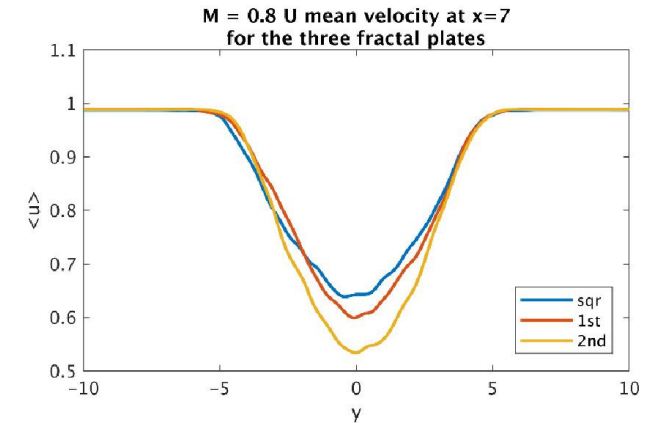
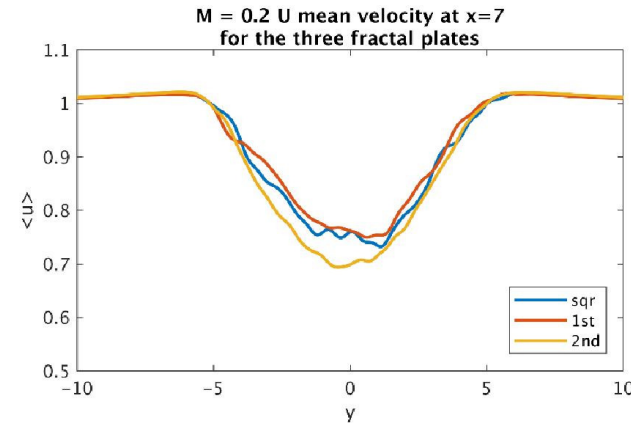
**X=3**



**X=5**



**X=7**

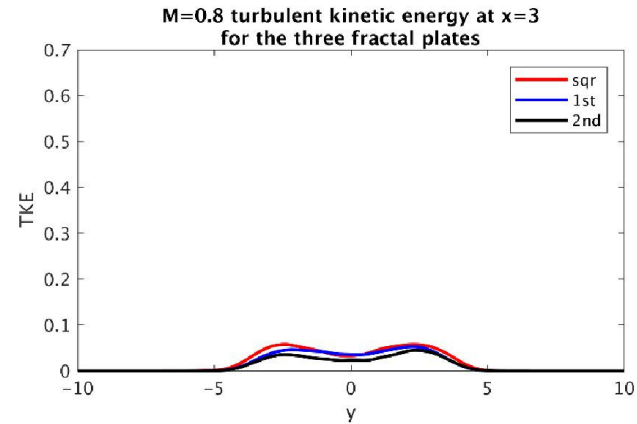
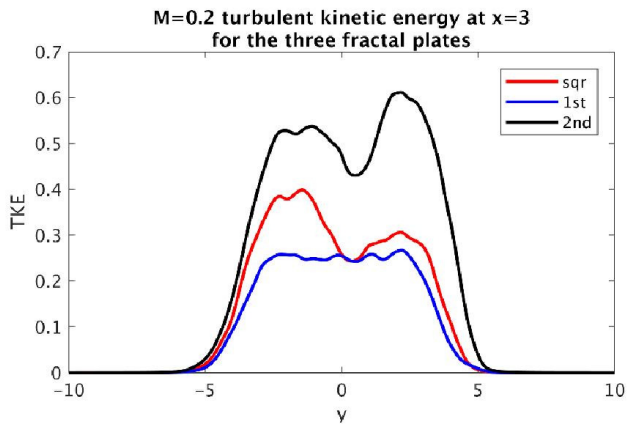


# Turbulent kinetic energy

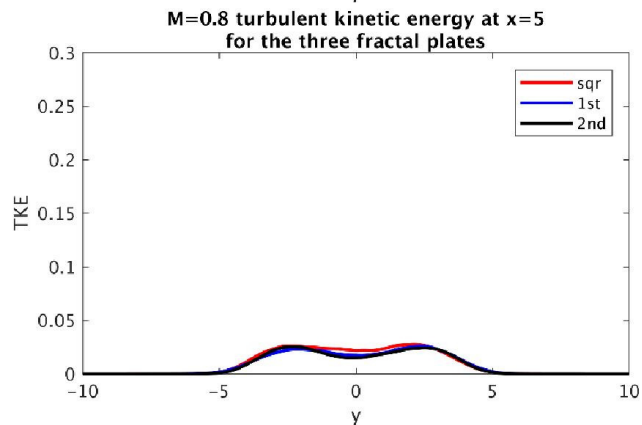
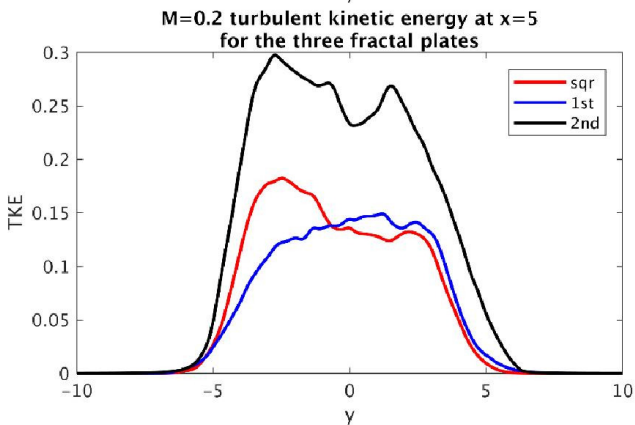
$M = 0.2$

$M = 0.8$

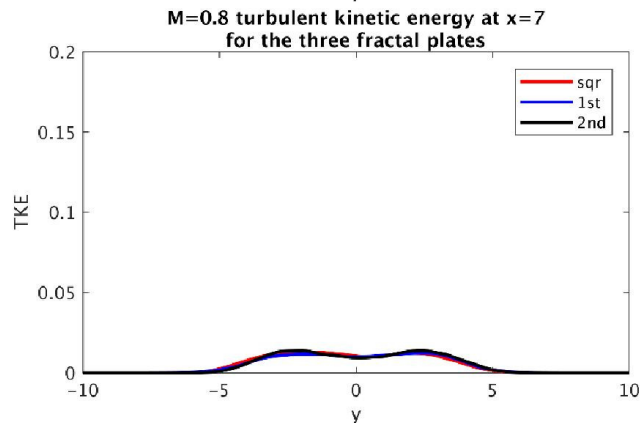
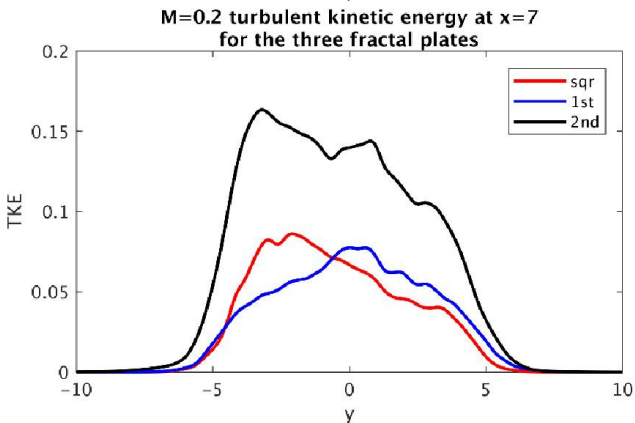
$X=3$



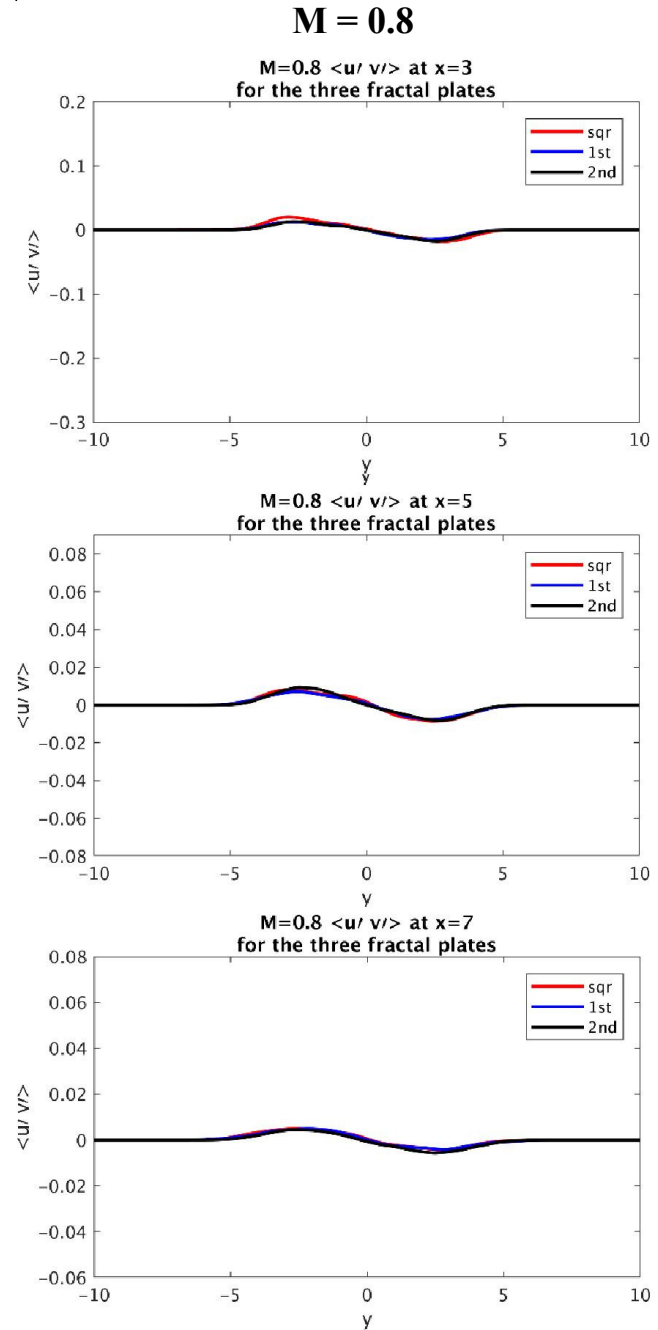
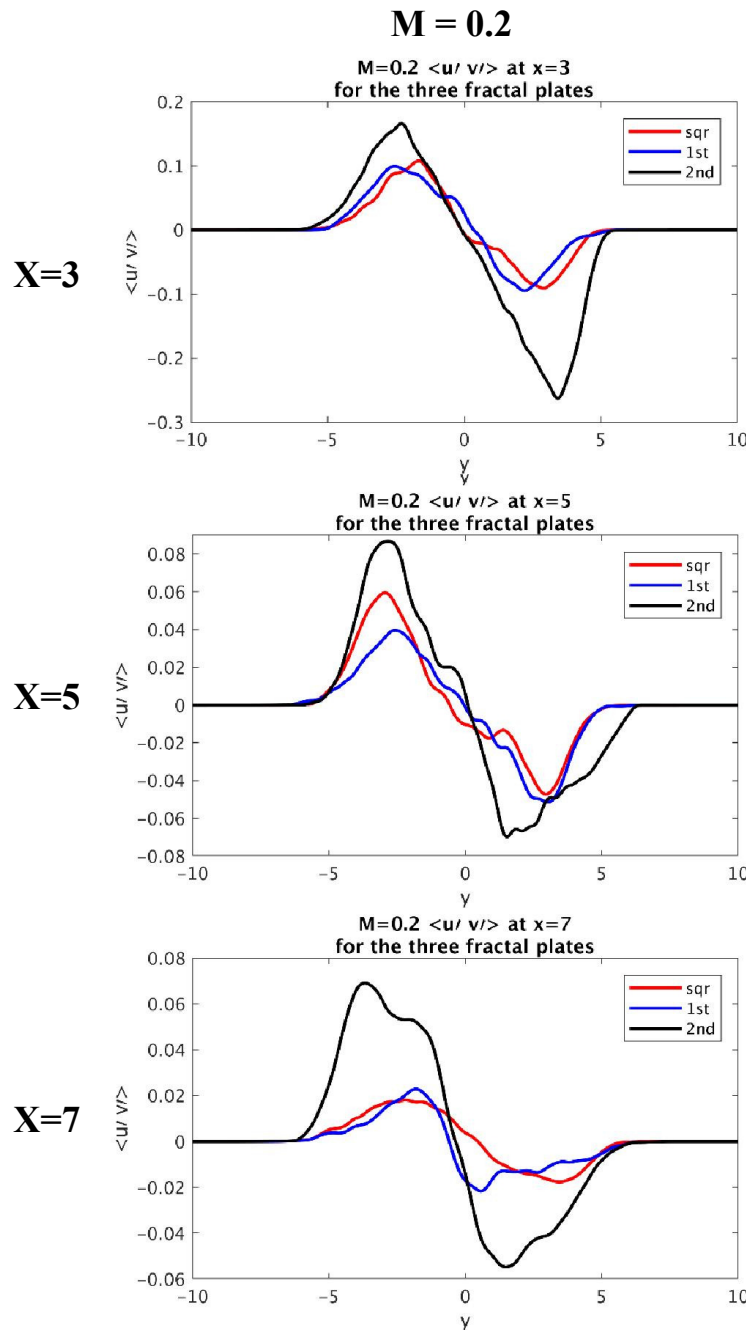
$X=5$



$X=7$



$u'v'$

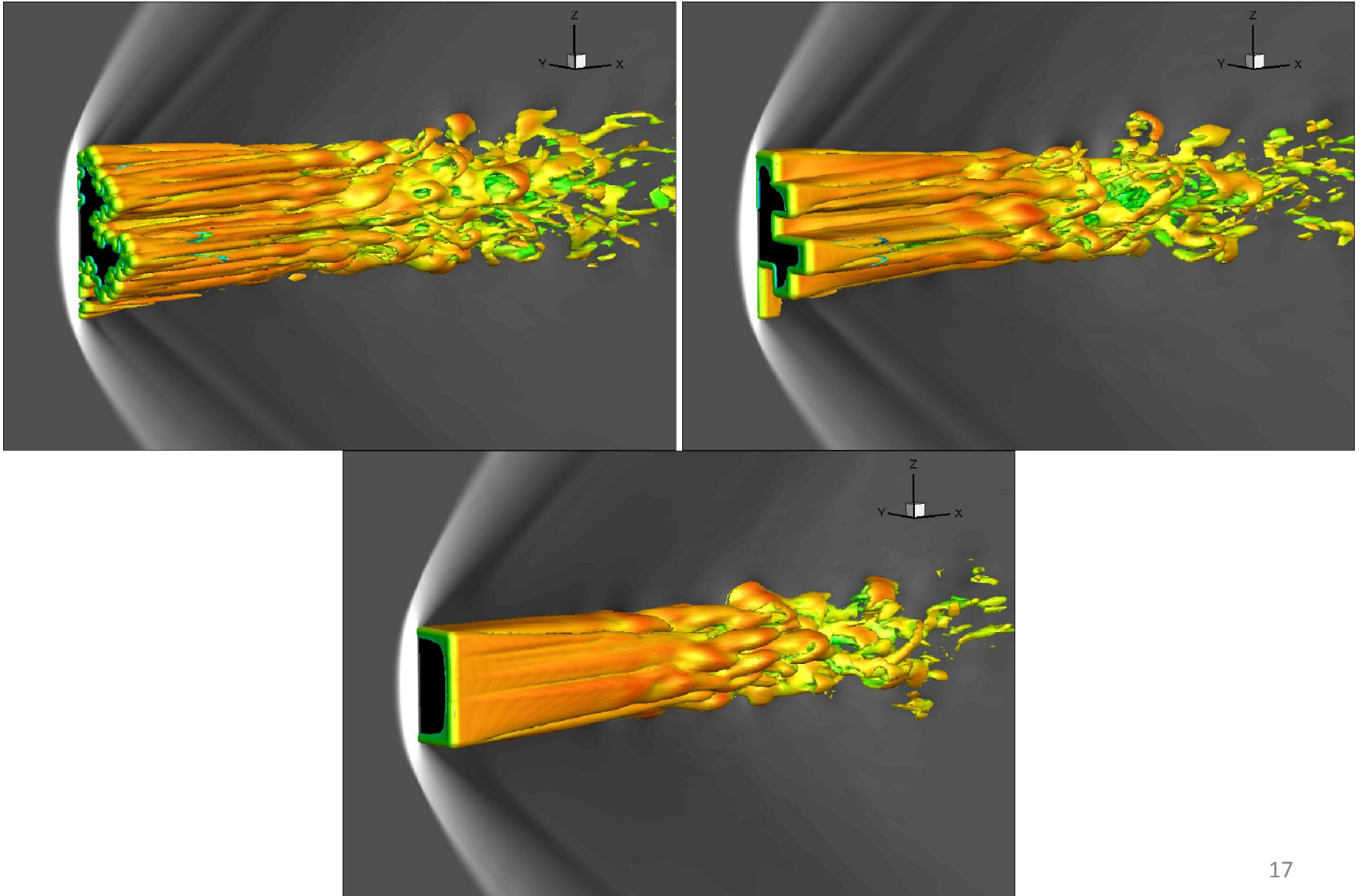




# □ Conclusion

- Ongoing work
  - Higher Mach numbers
- Future work
  - Jets

**$M = 1.5$  Iso-surfaces of the vorticity magnitude colored by the velocity magnitude**



Thank you.  
Questions?