



Maroni, Andrea and Tubaldi, Enrico and Douglas, John and Ferguson, Neil and Zonta, Daniele and McDonald, Hazel and Greenoak, Euan and Walker, Douglas and Green, Christopher (2018) A Bayesian network approach to assess underwater scour around bridge foundations. In: 9th European Workshop on Structural Health Monitoring Series (EWSHM), 2018-07-10 - 2018-07-13, Hilton Manchester Deansgate. ,

This version is available at <https://strathprints.strath.ac.uk/66014/>

Strathprints is designed to allow users to access the research output of the University of Strathclyde. Unless otherwise explicitly stated on the manuscript, Copyright © and Moral Rights for the papers on this site are retained by the individual authors and/or other copyright owners. Please check the manuscript for details of any other licences that may have been applied. You may not engage in further distribution of the material for any profitmaking activities or any commercial gain. You may freely distribute both the url (<https://strathprints.strath.ac.uk/>) and the content of this paper for research or private study, educational, or not-for-profit purposes without prior permission or charge.

Any correspondence concerning this service should be sent to the Strathprints administrator: strathprints@strath.ac.uk

A Bayesian Network approach to assess underwater scour around bridge foundations

Andrea Maroni¹, Enrico Tubaldi¹, John Douglas¹, Neil Ferguson¹, Daniele Zonta¹,
Hazel McDonald², Euan Greenoak³, Douglas Walker², Christopher Green³

1 University of Strathclyde, Department of Civil and Environmental Engineering,
Glasgow, UK, andrea.maroni@strath.ac.uk

2 Transport Scotland, Trunk Road and Bus Operations, Glasgow, UK

3 Network Rail Scotland Route, Glasgow, UK

Abstract

Flood-induced scour is by far the leading cause of bridge failures, resulting in fatalities, traffic disruption and significant economic losses. In Scotland, there are around 2,000 structures, considering both road and railway bridges, susceptible to scour. Scour assessments are currently based on visual inspections, which are expensive and time-consuming. The two main transport agencies in Scotland, Transport Scotland (TS) and Network Rail (NR), spend £2m and £0.4m per annum, respectively, in routine inspections. Nowadays, sensor and communication technologies offer the possibility to assess in real-time the scour depth at critical bridge locations; yet monitoring an entire infrastructure network is not economically sustainable. This paper proposes a methodology overcoming this limitation, based on the installation of monitoring systems at critical locations, and the use a probabilistic approach to extend this information to the entire population of assets. The state of the bridge stock is represented through a set of random variables, and ad-hoc Bayesian networks (BNs) are used to describe their conditional dependencies. The BN can estimate, and continuously update, the present and future scour depth at bridge foundations using real-time information provided by the monitored scour depth and river flow characteristics. In the occurrence of a flood, monitoring observations are used to infer probabilistically the posterior distribution of the state variables, giving the real-time best estimate of the total scour depth. Bias, systematic and model uncertainties are modelled as nodes of the BN in such a way that the accuracy of predictions can be updated when information from scour monitoring systems is incorporated into the BN. The functioning and capabilities of the BN is illustrated by considering a small network of bridges managed by TS in south-west Scotland. They cross the same river (River Nith) and only one of them is instrumented with a scour monitoring system.

1. Introduction & Background

1.1 Scour hazard

Flood-induced scour is the principal cause of failure of bridges, resulting in significant loss of life, traffic disruption and economic losses (1). Scour can be defined as the excavation of material around bridge foundations as a result of the erosive action of flowing water. Scour processes are classified according to the circumstances and structures that have caused it. The different types of scour are general scour, constriction scour, and local scour (2). While the first type is associated to the natural evolution of the river bed, the two other types are associated with the presence of a bridge. Constriction scour is the result of confining the width of the river channel, for instance between bridge abutments and piers, while local scour is caused by the interference of individual structural elements, such as piers or abutments, with the flow. In particular, local scour is

characterised by the formation of scour holes only in the immediate vicinity of those elements (3).

Scour processes occur naturally and are expected to occur at most bridges during their service life (4), since every hydraulic structure founded on river bed is prone to scour around its foundations. The scour mechanisms listed above work additively to give total scour (Figure 1), and a bridge may fail due to a combination of different scour types; however, one mechanism is often the major cause to bridge failure. When the depth of scour becomes significant, the capacity of abutment or pier foundations may be severely compromised, leading to structural instability and ultimately catastrophic failure.

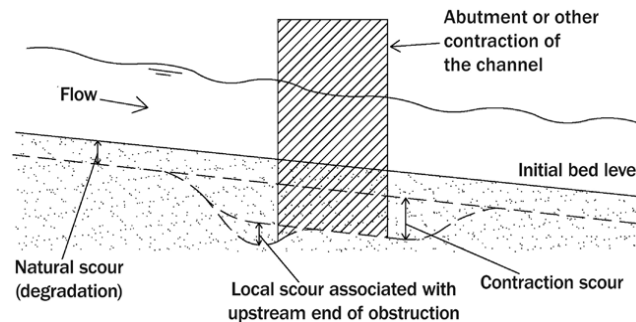


Figure 1. Schematic illustrating total scour (2)

In the UK, there are more than 9,000 major bridges over waterways. According to (5), abutment and pier scour was identified as the most common cause of 138 rail bridge failures during the period 1846-2013. Almost 95,000 bridge spans and culverts are susceptible to scour processes. Reviews of 1,502 river crossing failures that occurred in the United States in the period 1966 - 2005 revealed flooding and scour were the cause of 58% of the recorded failures (6). Following record daily rainfalls for the UK in November 2009, 20 road bridges across Cumbria were damaged or destroyed and the town of Workington was severed (7). Furthermore, the Winter storms of 2015 resulted in serious damage/destruction to bridges across Scotland and the north of England (8). This included the Lamington viaduct, which resulted in the closure of the West Coast mainline between Glasgow and London for nearly three months (9).

NR owns and operates around 19,000 underline bridges nationally: 8,700 of these structures are held within a National Scour Database and the projected spend on scour protective works from 2014-2019 is in the region of £27m. For the Scotland Route only, 1,750 structures are routinely inspected for scour and 58 are considered to be at high risk. TS is responsible for the Scottish trunk road network including 1,567 bridges or culverts over water. Of these, around 8% are currently classified as needing detailed consideration, including possible monitoring and scour protection measures. TS is currently aware of about £3.5m of known scour repairs and scour resilience works to carry out.

1.2 Scour risk assessment

The current practice for bridge scour inspection depends on visual checks at regular intervals. TS and NR assess the risks associated with scour on highway and railway structures during floods using the Procedures BD 97/12 (10) and EX2502 (11), respectively. They both provide a scour vulnerability index (SVI), based on value of total scour depth D_T and foundation depth D_F , in order to obtain a rating for the prioritisation of bridge intervention after a hazardous event or in advance of a predicted extreme flood event. The former procedure is based on a two-level assessment. The first level consists

of visual checks to identify structures that are not at risk from scour. When this condition is not met, a Level 2 Assessment is performed; it includes a framework for estimating scour depth at bridge locations, which eventually provides the SVI. In the latter procedure, visual inspections are instead used to assess existing conditions at bridges and river history before computing the SVI. The total NR Scotland Route spend on scour assessments in 2017 was approximately £440,000. Similarly, TS spends £2m per annum on routine inspections of bridges and one-third of its total assets are inspected each year. In addition, all bridges over water are visually inspected for scour effects following periods of heavy rainfall and underwater visual inspections are even more expensive.

In general, the evaluation of the risk of a structure should combine information on the hazard, the vulnerability, and the consequences of failure. The first examples of structural risk assessment frameworks have been developed in the context of seismic engineering (12), but in the recent years probabilistic frameworks have been proposed also for flood and coastal engineering (13) as well as hurricane engineering (14). The applications to the problem of scour are rather limited (15, 16).

Vulnerability (or fragility) analysis is an important component of any structural risk assessment because it allows to define how a structure is likely to fail given the occurrence of a hazardous event (15). In general, the vulnerability of a structural system such as a building or a bridge can be expressed by means of fragility functions or hazard indexes (17). Few studies have analysed the vulnerability of bridges to scour, and in the literature, it is possible to find three different approaches:

- (i) *Numerical approaches* involving finite element analyses of the soil-foundation-bridge components (18, 19, 20);
- (ii) *Analytical approaches* considering the reduction of bearing capacity of bridge foundations due to scour (21);
- (iii) *Empirical approaches* based on an SVI, typically defined as the ratio between total scour depth at the pier and foundation depth (22).

In this paper, a scour hazard model is developed by building a BN able to estimate the depth of scour in the surrounding of bridge foundations. In particular, the BN can estimate, and update, the present and future scour depth using information from the continuously monitored scour depth and river flow characteristics. Once a new observation is available, it is spread across the network, thus appraising and updating scour at unmonitored bridges. Then, the bridge vulnerability analysis based on an SVI is performed. This work is the first application of BNs to bridge scour risk management, and the first implemented case where updating of the network is based on real-time information from a monitoring system.

In section 2, we describe the developed BN for scour depth prediction and the two numerical algorithms employed to update the variables involved. Section 3 presents the small bridge network consisting of bridges managed by TS in south-west Scotland. It was built by choosing bridges over the same river (River Nith) in order to demonstrate the functioning of the BN. Only one bridge is instrumented with a scour monitoring system. In section 4, the results obtained with the two algorithms are reported.

2. Methodology

Sensor and communication technologies offer nowadays the possibility to monitor in real-time every change in characteristics of a bridge; yet monitoring an entire infrastructure

network is economically unsustainable. A way to overcome this limitation is to install monitoring systems at critical locations and use a probabilistic approach to extend this information to the entire asset. The idea is to represent the state of the bridge stock through a set of random variables and to use a BN to describe their conditional dependencies.

A BN, depicted in Figure 2, is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph comprised of nodes and links (30). The presence of a link between two nodes means that the node that appears earlier in the chain has a direct influence on the other connected node. Each node represents a random variable in the Bayesian sense, i.e., the relation between the variables is always given by the Bayes' rule:

$$\text{pdf}(\theta | \bar{y}) = \frac{\text{pdf}(\theta) \times \text{pdf}(\bar{y} | \theta)}{\text{pdf}(\bar{y})} \quad (1)$$

where $\text{pdf}(\bar{y} | \theta)$ is the probability distribution function (pdf) known as the likelihood of the observed data \bar{y} given the parameter θ , $\text{pdf}(\theta)$ is the prior pdf of parameter θ , $\text{pdf}(\theta | \bar{y})$ is called the posterior probability of θ , and the dominator $\text{pdf}(\bar{y})$ is a normalising factor called evidence. Bayes' rule describes how the probability of parameter θ changes given information gained from measured data \bar{y} . In Bayesian network terminology, a node is a parent of a child if there is a link from the former to the latter.

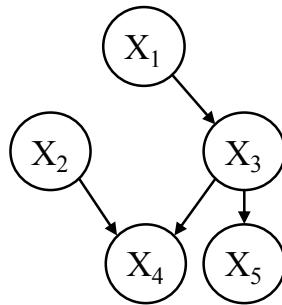


Figure 2. An example of a Bayesian Network

Probabilistic inference in BNs takes two forms: forward (predictive) analysis and backward (diagnostic) analysis. The former type of analysis for the node X_i is based on evidence nodes connected to X_i through its parent nodes. Instead, the diagnostic analysis for the node X_i is based on evidence nodes connected to X_i through its child nodes (24). This backward analysis is called Bayesian learning as well.

The true power in using BNs comes from the ease with which they facilitate information updating when a new observation becomes available (23). When evidence (e.g., information that a node is in a particular state) on one or more variables is entered into the BN, the information propagates through the network to yield updated probabilities in light of the new observations.

For these reasons, BN frameworks can be merged with monitoring systems to update the risk map of infrastructure systems. This capability of updating is particularly advantageous when information is evolving, as in the case of a real-time monitoring system. If we consider the bridge scour problem, in the occurrence of a flood, monitoring observations are used to probabilistically infer the posterior distribution of all the parent nodes of the network by exploiting features of Bayesian Learning, and to give in real-time the best estimate of scour depth, even in unmonitored bridges.

2.1 Bayesian Network for scour depth estimation

The BN employed in the scour hazard model is developed according to the Procedure BD 97/12, called just BD 97/12 for brevity. Figure 3a depicts the probabilistic correlation among variables involved in the appraisal of total scour depth D_T . Starting from the river flow characteristics (such as assessment flow Q_A and river level y_B) it is possible to estimate the depth of the two components of scour, constriction (D_C) and local scour (D_L), whose sum is equal to the total scour depth. The appraisal of the former type involves variables like mean threshold velocity $v_{B,C}$ below which scour does not occur and type of bed material. Constriction scour leads to an increase ΔA in cross-section area of flow that allows estimating an average value, $D_{C,ave}$. The variable D_C refers to the depth at the location of interest. Local scour principally depends on the shape and width of the pier. The factor f_y , called depth factor, takes into account the relative depth of the approach flow to the pier width and, for this reason, it depends on the depth of constriction scour D_C itself.

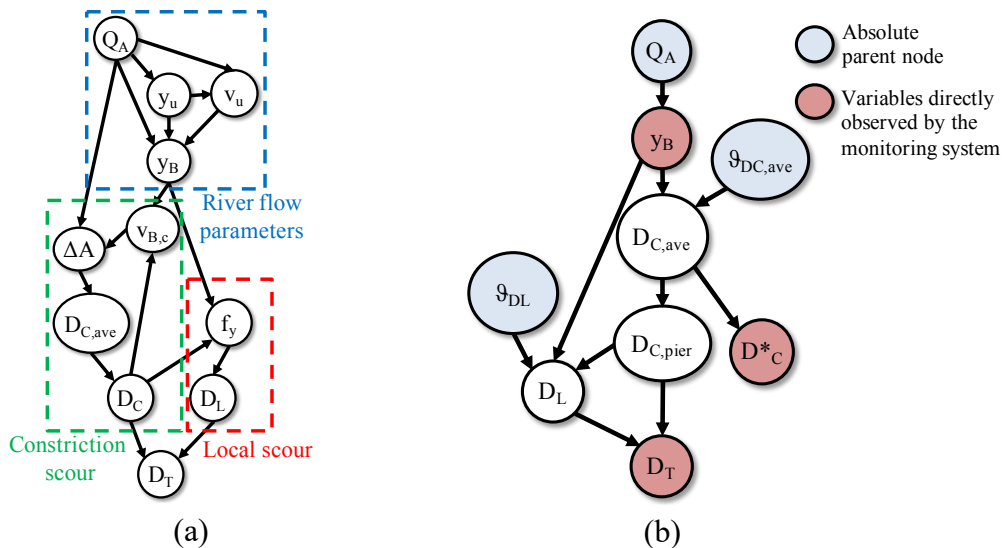


Figure 3. BN for scour depth prediction based on BD 97/12 (a), and the simplified version (b)

The models employed in the BN can utilise two types of variables relationships: deterministic and probabilistic. The former correlations consist of well-established models. The latter ones, for their probabilistic nature, must always deal with uncertainties and errors. Models are nothing more than a simplification of the reality, but the “perfect” model does not exist. Therefore, a modified version of the BN is shown in Figure 3b.

Let us focus on the quantities that can be monitored, that is, river level and depths of scour, and the utilised models. The water level y_B is measured by gauging stations; an observation of y_B updates the water flow Q_A . The model employed is assumed to be deterministic using the well-known Manning’s equation. A scour monitoring system can provide data about scour depth, for instance, in the middle of the channel (constriction scour, D^*_C) and at the pier (total scour, D_T). Observations of these variables cannot update Q_A because the path is blocked by observations of y_B . In order to exploit these scour observations within the BN, two new variables, $\theta_{DC,ave}$ and θ_{DL} , were included; they are model uncertainties added to the mathematical models used to estimate the variables $D_{C,ave}$ and D_L , respectively. These new absolute parent nodes are named not-fixed model uncertainty because they are updated every time new observation of D^*_C and D_T enter the network. Through their employment, the value of scour depths obtained with the

empirical formulas provided by BD 97/12 is corrected thanks to observations from scour monitoring system.

Let us summarise the three steps for solving the network and updating the posterior pdfs of the nodes once observation about some variables become available:

- (i) the BN starts with the prior pdfs of the parent nodes: flow Q_A and the not-fixed model uncertainties $\theta_{DC,ave}$ and θ_{DL} . Observations of river level y_B , constriction scour in the middle of the river D^*_C and total scour D_T enter into the network (Figure 4a);
- (ii) the BN is figuratively split into three sub-networks because there are three different updates: the observation of y_B updates Q_A ; the observation of D^*_C and the updated pdf of y_B update $\theta_{DC,ave}$; and the observation of D_T , the updated pdf of y_B and $D_{C,pier}$ update θ_{DL} (Figure 4b);
- (iii) descendant nodes are updated through the models provided by BD 97/12 exploiting updated information given by evidence on the parent nodes (Figure 4c).

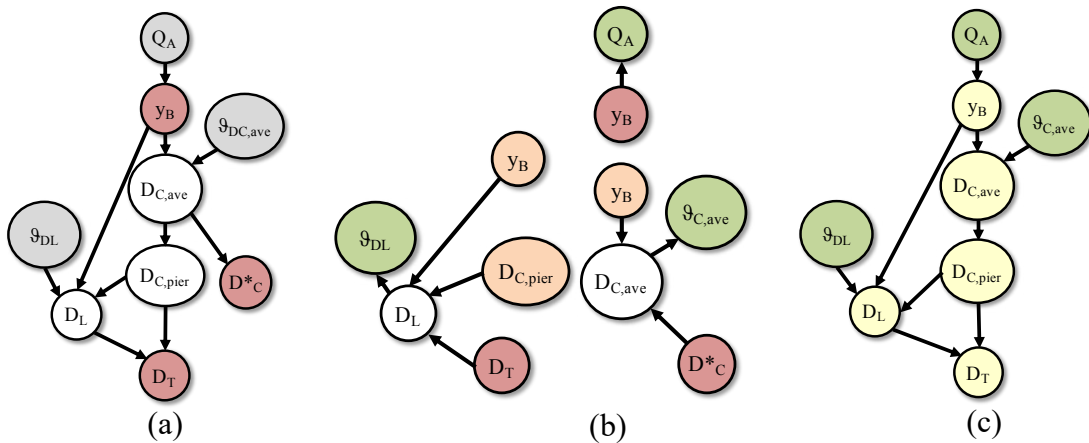


Figure 4. Starting with prior pdfs (a), updating of parent (b) and descendant nodes (c)

By following the same stages described above in the construction of the BN, a network on a bigger scale can be developed. For instance, Figure 5 shows a BN for correlating the total scour depth prediction at two different bridges, each of them with N piers. The estimation of the scour at the second bridge is based on the models corrected by the model uncertainty variables updated by direct observations of D^*_C and D_T at the first bridge. The two not-fixed model uncertainties are parent nodes of both sub-network because the models between the variables to estimate scour depth are the same for any bridge. Consequently, uncertainties and error are correlated at all bridges.

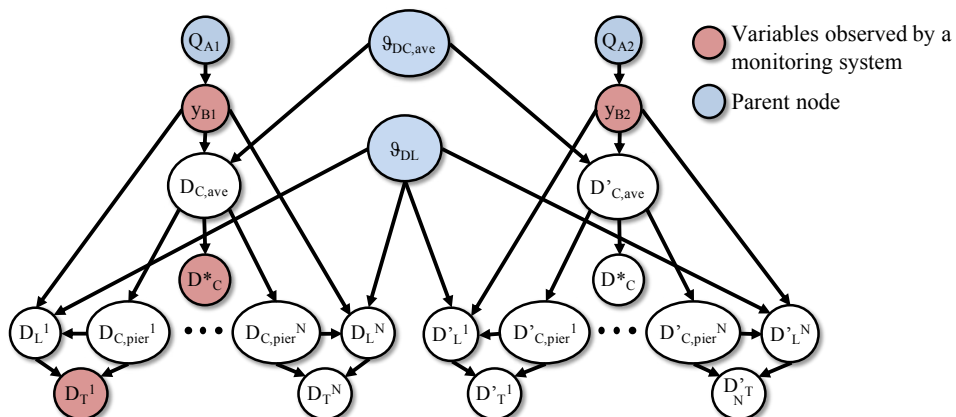


Figure 5. Bayesian Network for two different bridges, both with N piers

2.2 Numerical algorithms for model updating

Correlations present in a BN are expressed in Bayesian terms, so Eq. 1 is always the basis of Bayesian statistic inference, but a closed form to calculate it exists only in a few simple cases. To solve Eq. 1 and find the shape and estimators of posterior pdf we need a numerical algorithm for Bayesian inference. In the past few years, computer algorithms have been developed to draw a random sample from the posterior pdf, without having to completely evaluate it. Examples of sampling methods are the Markov Chain Monte Carlo (MCMC) and the Metropolis-Hasting (MH).

In this section we will present two different algorithms to solve numerically Eq. 1 into a BN. The two numerical algorithms are based on the Hessian Matrix method and the Transitional Markov Chain Monte Carlo (TMCMC) method, respectively.

2.2.1 Linear Gaussian Bayesian Networks

The first developed algorithm can solve any Linear Gaussian Bayesian Network (LGBN) by updating parent nodes' pdfs when data or observations about one of their child nodes enter into the BN. LGBN involves variables that can be described only by Normal (Gaussian) or Log-Normal pdfs and with linear relationships among them.

In mathematics, the Hessian matrix or Hessian (H) is a square matrix of second-order partial derivatives of a scalar-valued function. This algorithm is based on another definition of H : by defining the variable LH as the negative logarithm of the likelihood, in the word of statistics H is the inverse of likelihood covariance matrix. The basic equations to calculate estimators of posterior pdf($\theta | \bar{y}$) are given below:

$$LH = -\log[g(\bar{y} | \theta)] \quad (2)$$

$$H = (\sigma_{\bar{y}|\theta}^2)^{-1} = \left[\frac{\partial^2 LH}{\partial \theta^2} \right] \quad (3)$$

$$(\sigma_{\theta|\bar{y}}^2)^{-1} = (\sigma_{\theta}^2)^{-1} + (\sigma_{\bar{y}|\theta}^2)^{-1} \quad (4)$$

$$\mu_{\theta|\bar{y}} = \sigma_{\theta|\bar{y}}^2 \left[\mu_{\theta} (\sigma_{\theta}^2)^{-1} + \bar{y} (\sigma_{\bar{y}|\theta}^2)^{-1} \right] \quad (5)$$

where estimators with $\theta | \bar{y}$ as a subscript refer to the posterior pdf, with θ to the prior pdf, with $\bar{y} | \theta$ to the likelihood, and $g(\bar{y} | \theta)$ indicates the likelihood function.

2.2.2 Transitional Markov Chain Monte Carlo method

The MCMC method can simulate random samples from a target pdf that can only be evaluated up to a scaling constant. From the Bayesian point of view, the target pdf is the posterior pdf, and the scaling constant, i.e., the evidence that appears at the denominator of Bayes' Theorem. The most popular MCMC method is the MH algorithm. MH algorithm can draw samples from the target pdf without knowing the model evidence, but it cannot evaluate it (25).

In 2007, a modified version of the MCMC method was proposed, called the transitional Markov chain Monte Carlo (TMCMC) algorithm (26). The TMCMC algorithm is a marriage between the MH algorithm and the sampling-importance-resampling (SIR) method and it was motivated by the Adaptive MCMC (27). Similar to the MH algorithm, the TMCMC algorithm can draw samples from the target pdf without the knowledge of the model evidence. Nonetheless, it can estimate the model evidence, without extra

computation cost. TCMCMC algorithm is more complicated to implement than the MH algorithm, but there is no need to specify the proposal pdf, no need to determine the burn-in period, the convergence issue is minimised, and the computational time is extremely reduced (25, 26).

3. Case studies

The functioning of the developed BN is demonstrated using a small bridge network, consisting of bridges managed by TS in south-west Scotland (Figure 6). It is built by bridges over the same river (River Nith) and only the first bridge is instrumented with a scour monitoring system. The aim is to exploit observations on Bridge 1 in order to predict scour depth at other bridge locations.

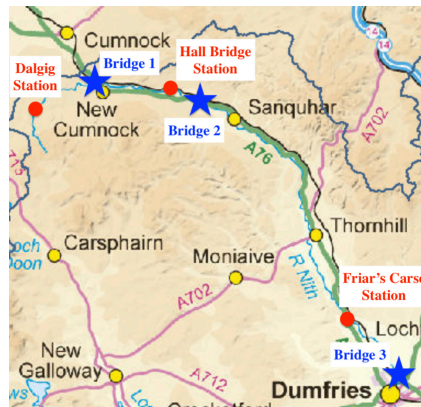


Figure 6. Network of bridges over the River Nith. Red circles represent SEPA’s gauging stations

Three bridges were chosen from the TS scour database because they all have experienced significant scour events in the past. In the following, some information and details about the three bridges are reported:

- *Bridge 1*: A76 200 Bridge in New Cumnock. It is a 3-span (9.1m, 10.7 m and 9.1 m) masonry arch bridge, with two piers in the riverbed.
- *Bridge 2*: A76 120 Guildhall bridge in Kirkconnel. It is a 3-span (8.8m, 11.3 m and 11.3 m) masonry arch bridge, with one pier in the riverbed.
- *Bridge 3*: A75 300 Dalscone bridge in Dumfries. It is a 7-span (spans of 35 m and two of 28 m) steel-concrete composite bridge, with three piers in the riverbed.



Figure 7. A76 200 Bridge (a), A76 120 Guildhall bridge (b) and A75 300 Dalscone bridge (c)

Abutments and piers of the three bridges are founded on spread footings on the natural ground except Dalscone bridge’s abutments that are founded on made up ground.

The river discharge is certainly correlated at all bridges since they cross the same river, but, given that SEPA’s gauging station precedes every bridge of the network, there is no

need to set Q_A as an absolute parent node in common for all the bridges. The whole BN for the estimation of scour depth at every pier of A76, Guildhall and Dalscone bridge is depicted in Figure 10.

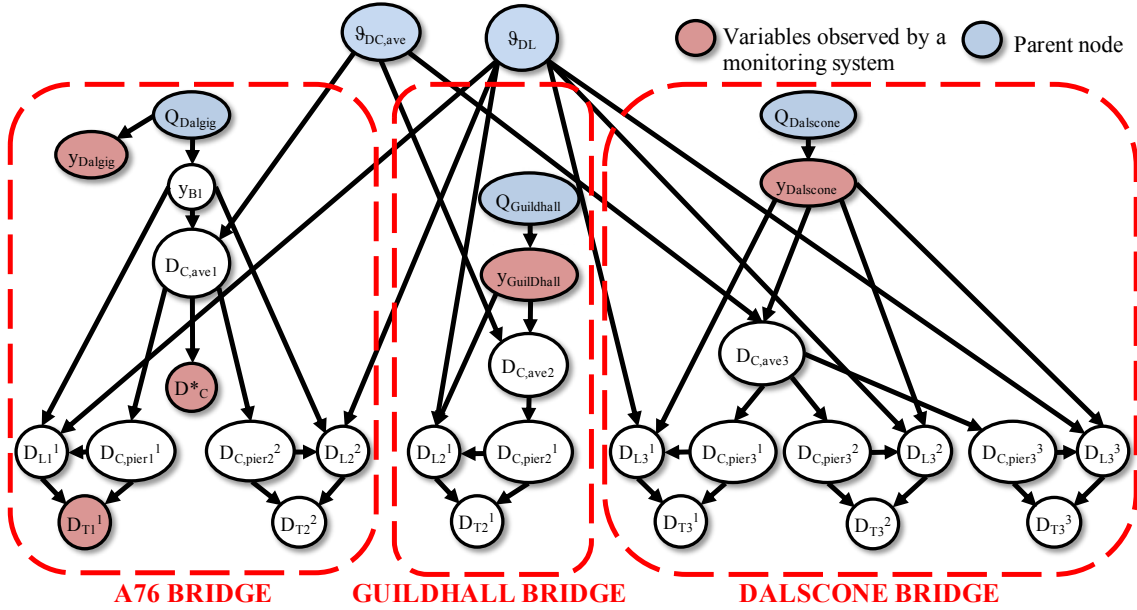


Figure 10. Final BN for depth of scour estimation at three bridges in the south-west of Scotland

4. Results

Normal distributions were employed for every variable except for the river flow; a log-normal distribution was adopted because the discharge cannot be negative. The pdfs of the not-fixed model uncertainties were set as Normal distributions with zero mean and a standard deviation of 1 m. The parameters of the Log-Normal pdf were obtained from the data recorded by SEPA's gauging station of last ten years.

Let us focus now on the observations collected from monitoring systems that are entering the BN. Scour is induced by a flood event, consequently, the peak value of river level was chosen to simulate a heavy river flood condition. Table 1 shows these peak values.

SEPA's station	Bridge	Water level [m] 30/12/2013
Dalgig	A76	1.879
Hall Bridge	Guildhall	3.015
Friar's carse	Dalscone	1.512

The scour data entering the network were hypothesised to mimic a critical situation since the monitoring system had not yet been installed at the time of this analysis. The hypothesised values are 20 cm for constriction scour D^*_C and 45 cm for total scour D_T .

4.1 Linear gaussian Bayesian network

The employed models have to be linearised in order to apply the algorithm that solves LGBN. The variable scale was changed to logarithm scale, which allows overcoming problems with exponents or products. To linearise more complicated models, such as the relationship between D_C and y_B shown in Eq. (6), a simple linear regression was performed to find the linear function that predicts the dependent variable values (D_C) as

a function of the independent variable (y_B). In Eq. (6), which is provided by BD 97/12, Manning's equation was employed to describe Q_A as a function of y_B , while the mean threshold velocity $v_{B,C}$ was calculated using the Colebrook-White equation (2).

$$D_C = \frac{\Delta A}{B_B} = \frac{Q_A}{B_B v_{b,c}} - y_B = \frac{1/n \cdot B^{5/3} \cdot y_B^{5/3} \cdot (B + 2y_B)^{-2/3} \cdot S^{1/2}}{B_B \left[-\sqrt{32} u_0 \cdot \log_{10} \left(\frac{d}{12y_B} + \frac{0.222\nu}{y_B \cdot u_0} \right) \right]} - y_B = D_C(y_B) \quad (6)$$

Table 2 depicts the results obtained by solving the LGBN. Mean values and standard deviations of D_C and D_T at piers of every bridge are reported. It is worth recalling that the BN starts from observations about D_C^* and D_T on Pier 1 of A76 200 bridge.

Table 2. Mean values and standard deviations of scour depth obtained by solving the LGBN

	A76 200		Guildhall		Dalscone	
	Pier 1 (M)	Pier 2 (E)	Pier 1 (E)	Pier 1 (E)	Pier 2 (E)	Pier 3 (E)
μ_{DC} [m]	0.20	0.191	0.654	0.478	0.464	0.471
σ_{DC} [m]	-	0.247	0.609	0.621	0.614	0.619
μ_{DT} [m]	0.45	0.438	0.953	0.791	0.807	0.794
σ_{DT} [m]	-	0.436	0.632	0.763	0.761	0.758

M: Measured, E: Estimated

4.2 TCMCMC

The prior pdfs and the hypothesised values chosen were the same used with the previous method. Table 3 shows the results obtained in the form of mean values and standard deviations of constriction and total scour depth.

Table 3. Mean values and standard deviations of scour depth obtained with TCMCMC

	A76 200		Guildhall		Dalscone	
	Pier 1 (M)	Pier 2 (E)	Pier 1 (E)	Pier 1 (E)	Pier 2 (E)	Pier 3 (E)
μ_{DC} [m]	0.20	0.199	0.607	0.421	0.420	0.432
σ_{DC} [m]	-	0.137	0.225	0.192	0.190	0.187
μ_{DT} [m]	0.45	0.452	0.932	0.802	0.798	0.805
σ_{DT} [m]	-	0.194	0.248	0.243	0.240	0.238

M: Measured, E: Estimated

Estimations of mean values of scour depth are consistent between the two algorithms whereas the TCMCMC method obtains lower values (from 45% to 65% lower than LGBN results) of standard deviations. This can be explained by TCMCMC algorithm's capacity to handle non-linear models and relationships among variables; the need to linearise robust non-linear models in order to build a LGBN has significantly increased the uncertainties and reduced the accuracy of variable estimations.

5. Conclusions

In this paper, a BN able to estimate the depth of scour at the foundations of a bridge network is presented. In particular, the BN can estimate, and continuously update, the present and future scour depth using real-time information from monitoring of scour depth and river flow characteristics. Once an observation collected from a scour monitoring system installed on a critical bridge enters into the BN, its information can be

spread across the network thus appraising and updating scour depth at unmonitored bridges. This work is the first application of BNs to bridge scour risk management, and also the first implemented case where updating of the network is based on real-time information from a monitoring system.

The resolution of the BN starts by defining the prior pdfs of parent nodes. The parent nodes consist of the uncertainty of the model for the prediction of total scour depth so that they can guarantee correlations among every bridge since the estimation models are employed for every bridge of the network. In order to make inference by updating the parent nodes, observations of river level and data of scour depths are entered into the BN.

Two different algorithms were developed to solve the Bayes' rule, the basis of Bayesian statistic inference and, in turn, of BN. The two numerical algorithms are based on, respectively, the Hessian Matrix method and the TMCMC method.

The functioning of the developed BN was demonstrated using a small bridge network, consisted of three bridges managed by TS in south-west Scotland. They cross the same river, with only the first bridge being instrumented with a scour monitoring system. A flood event was simulated using river level data from SEPA's gauging stations. Scour depth values were instead hypothesised since the monitoring system had not yet been installed at the time of this analysis.

Both methods led to same results of the first estimator (i.e., mean value) of scour depth posterior pdfs. In contrast, using the TMCMC algorithm results in lower values of standard deviations (the second estimators) for all the cases because it allows the implementation of any models and variable relationships (i.e., linear and non-linear). This decrease ranges from 45% to 65% with respect to LGBN results. A lower value of standard deviation means a higher accuracy in the estimation of the variable.

Acknowledgements

This study has been carried out within the project titled "Early warning decision support system for the management of underwater scour risk for road and railway bridges", which has received funding from the NERC ERIIP (Environmental Risk to Infrastructure Innovation Programme) under grant agreement no. NE/R009090/1. Many thanks to Simone Pavan and Andrea Giovannini from University of Trento for their help in developing the two algorithms.

References and footnotes

1. K Wardhana and FC Hadipriono, "Analysis of Recent Bridge Failures in the United States", *J. Perfor. Constr. Fac.* 17, pp 144-150, 2003.
2. AM Kirby, M Roca, A Kitchen, M Escarameia and OJ Chesterton, *Manual on scour at bridges and other hydraulic structures*, second edn, CIRIA, 2015.
3. CS Lauchlan and BW Melville, "Riprap protection at bridge piers", *J. Hydraul. Eng. ASCE* 127(5), pp 412-418, 2001.
4. EV Richardson and SR Davis, *Evaluating Scour at Bridges*, fourth edn, *Hydraulic Engineering Circular No.18*, Publication No. FHWA NHI 01-001, 2001.
5. Z van Leeuwen and R Lamb, *Flood and scour related failure incidents at railway assets between 1846 and 2013*. Railway Safety & Standards Board, 2014.
6. JL Briaud, L Brandimarte, J Wang and P D'Odorico, "Probability of scour depth exceedance owing to hydrologic uncertainty", *Georisk* 1(2), pp 77-88, 2007.

7. Cumbria Intelligence Observatory, Cumbria Floods November 2009: An Impact Assessment, 2010. Retrieved from www.cumbria.gov.uk.
8. Department for Transport, £40 million to help rebuild flood-hit roads for people in Cumbria and Lancashire, 2015. Retrieved from <https://www.gov.uk/>.
9. Network Rail Lamington Viaduct to Reopen in March following Storm Repairs, 2016. Retrieved from <https://www.networkrail.co.uk>.
10. Department for Transport, The assessment of scour and other hydraulic actions at highway structures (UK) [BD 97/12], Design manual for roads and bridges, 2012.
11. HR Wallingford, Hydraulic Aspects of Bridges - Assessment of the Risk of Scour, Report EX2502, HR Wallingford, 1993.
12. KA Porter, "An overview of PEER's performance-based earthquake engineering methodology", in ICASP9 Conf. Proc., A Der Kiureghian et al, 2, pp 973–980, 2003.
13. FEMA, HAZUS MR4 technical manual: Flood model, Technical report, Federal Emergency Management Agency, Washington D.C, 2005.
14. M Barbato, F Petrini, VU Unnikrishnan and M Ciampoli, "Performance-Based Hurricane Engineering (PBHE) framework", *Struc. Saf.* 45, pp. 24-35, 2013.
15. M Roca and R Whitehouse, "Scour risk assessment at river crossings", ICSE6 Conf. Proc., 2012.
16. E Tubaldi, L Macorini, B Izzuddin, C Manes and F Laio, "A framework for probabilistic assessment of clear-water scour around bridge piers", *Struc. Saf.* 69, pp 11-22, 2017.
17. GM Calvi, R Pinho, G Magenes, JJ Bommer and H Crowley, "Development of seismic vulnerability assessment methodologies over the past 30 years", *ISST J. Earthq. Technol.* 43(3), pp 75-104, 2006.
18. CC Hung and WG Yau, "Behavior of scoured bridge piers subjected to flood-induced loads", *Eng. Struct.* 80, pp 241-250, 2014.
19. JV Klinga and A Alipour, "Assessment of structural integrity of bridges under extreme scour conditions", *Eng. Struct.* 82, pp 55-71, 2015.
20. E Tubaldi, L Macorini, and B Izzuddin, "Three-dimensional mesoscale modelling of multi-span masonry arch bridges subjected to scour", *Eng. Struct.* 165, pp 486-500, 2018.
21. F Federico, G Silvagni, F Volpi, "Scour vulnerability of river bridge piers", *J. Geotech. Geoenviron.* 129(10) pp 890-899, 2003.
22. S Barbetta, S Camici and T Moramarco, "A reappraisal of bridge piers scour vulnerability: a case study in the upper tiber river basin (central italy)", *J. Flood Risk Manag.*, 2015.
23. FV Jensen and TD Nielsen, Bayesian networks and decision graphs, second edn, Information Science and Statistics, Springer, Berlin, Germany, 2007.
24. I Ben Gal, Bayesian Networks, in Ruggeri F, Faltin F and Kenett R, "Encyclopedia of Statistics in Quality & Reliability", John Wiley & Sons, Chichester, UK, 2007.
25. J Ching and JS Wang, "Application of the Transitional Markov Chain Monte Carlo to Probabilistic Site Characterization", *Eng. Geol.* 203, pp 151-167, 2016.
26. J Ching and YC Chen, "Transitional Markov chain Monte Carlo method for Bayesian model updating, model class selection and model averaging", *ASCE J. Eng. Mech.* 133(7), pp 816-832, 2007.
27. JL Beck and SK Au, "Bayesian updating of structural models and reliability using Markov chain Monte Carlo simulation", *ASCE J. Eng. Mech.* 128(4), pp 380–391, 2002.