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Denise Bolognani, University of Trento, Department of Civil, Environmental and Mechanical Engineering, via Mesiano 77, 38123 Trento, Italy
Email: denise.bolognani@unitn.it

Quantifying the benefit of SHM: what if the manager is not the owner?

Denise Bolognani[1]. Andrea Verzobio[2]. Daniel Tonelli[1,3]. Carlo Cappello[1]. Branko Glisic[3]. Daniele Zonta[1,2]. John Quigley[4]

[1]University of Trento, Department of Civil, Environmental and Mechanical Engineering, via Mesiano 77, 38123 Trento, Italy.
[2]University of Strathclyde, Department of Civil & Environmental Engineering, 75 Montrose Street, Glasgow, G1 1XJ, UK.
[3]Princeton University, Department of Civil & Environmental Engineering, Princeton, NJ 08544 USA.
[4]University of Strathclyde, Department of Management Science, 199 Cathedral Street, Glasgow, G4 OQU, UK.

Abstract

Only very recently our community has acknowledged that the benefit of Structural Health Monitoring (SHM) can be properly quantified using the concept of Value of Information (*VoI*). The *VoI* is the difference between the utilities of operating the structure with and without the monitoring system. Typically, it is assumed that there is one decision maker for all decisions, i.e. deciding on both the investment on the monitoring system as well as the operation of the structure. The aim of this work is to formalize a rational method for quantifying the Value of Information when two different actors are involved in the decision chain: the *manager*, who makes decisions regarding the structure, based on monitoring data; and the *owner*, who chooses whether to install the monitoring system or not, before having access to these data. The two decision makers, even if both rational and exposed to the same background information, may still act differently because of their different appetites for risk. To illustrate how this framework works, we evaluate a hypothetical *VoI* for the Streicker Bridge, a pedestrian bridge in Princeton University campus equipped with a fiber optic sensing system, assuming that two fictional characters, Malcolm and Ophelia, are involved: Malcolm is the *manager* who decides whether to keep the bridge open or close it following to an incident; Ophelia is the *owner* who decides whether to invest on a monitoring system to help Malcolm making the right decision. We demonstrate that when manager and owner are two different individual, the benefit of monitoring could be greater or smaller than when all the decisions are made by the same individual. Under appropriate conditions, the monitoring *VoI* could even be negative, meaning that the *owner* is willing to pay to prevent the *manager* to use the monitoring system.

Keywords

Value of Information; Bayesian Inference; Expected Utility Theory; Decision-making; Bridge Management; Fiber Optic Sensors

Introduction

Although the utility of structural health monitoring (SHM) has rarely been questioned in our community, very recently a few published papers [1] [2] have clarified the way that the benefit of monitoring can be properly quantified. Indeed, seen from a mere structural engineering perspective, the utility of monitoring may not be immediately evident. Wear for a minute the hat of the manager of a Department of Transportation (DoT), responsible for the safety of a bridge: would you invest your limited budget on a reinforcing work or on a monitoring system? A retrofit work will increase the bridge load-carrying capacity and therefore its safety. On the contrary, sensors don't change the bridge capacity, nor reduce the external loads. So how can monitoring affect the safety of the bridge? The answer to this legitimate question goes roughly along these lines: monitoring does not provide structural capacity, rather better information on the state of a structure; based on this information, the manager can make better decisions on the management of the structure, minimizing the chances of wrong choices, and eventually increasing the safety of the bridge over its lifespan. Therefore, to appreciate the benefit of SHM, we need to account for how the structure is expected to be operated and eventually recast the monitoring problem into a formal economic decision framework.

The basis of the rational decision making is encoded in axiomatic Expected Utility Theory (EUT), first introduced by Von Neumann and Morgenstern [3] in 1944, and later developed in the form that we currently know by Raiffa and Schlaifer [4] in 1961. EUT is largely covered by a number of modern textbooks (among the many, we recommend Parmigiani [5] to the Reader of SHM who is approaching the topic for the first time). Within the framework of EUT, the benefit of information, such as that coming from a monitoring system, is formally quantified by the so-called Value of Information (*VoI*). The concept of *VoI* is anything but new: it was first introduced by Lindley [6] in 1956, as a measure of the information provided by an experiment, and later formalized by Raiffa and Schlaifer [4] and DeGroot [7]. Since its introduction, it has been continuously applied in manifold fields, including statistics, reliability and operational research [8] [9] [10] [11]. Its first appearance in the SHM community, however, is much more recent and dates back, in our best knowledge, to a paper published in the proceedings of SPIE by Bernal et al. in 2009 [12], followed by Pozzi et al. [13], Pozzi and Der Kiureghian [14], Thöns & Faber [1], Zonta et al. [2], - a recent state of the art can be find in Thöns [15]. In the last few years, quantifying the value of SHM has known a renewed popularity thanks to the activity of the EU-funded COST action TU1402 [16].

Broadly speaking, the value of a SHM system can be simply defined as the difference between the benefit, or expected utility u^*, of operating the structure *with* the monitoring system and the benefit, or expect utility u, of operating the structure *without* the system. Both u^* and u are expected utilities calculated *a priori*, i.e. *before* actually receiving any information from the monitoring system. While in u we assume the knowledge of the manager is his a priori knowledge, u^* is calculated assuming the decision maker has access to the monitoring information and is sometimes referred as to *preposterior utility*. The difference between these values measures the value of the information to the decision maker. Clearly, if the monitoring does not provide any useful information, the preposterior u* is equal to the prior u, and the value of monitoring information is zero.

Typically, it is assumed that there is one decision maker for all decisions, i.e. deciding on both investments as well as operations. This individual could be for example an idealized manager of a DoT, as the fictitious character 'Tom' who appears in [2]. We must recognize that in the real world the process whereby a DoT makes decision over its stock is typically more complex, with more individuals involved in the decision chain. Even oversimplifying, we always have at least two different decision stages. First a decision is made on whether or not to buy and install the monitoring system on the structure; this is a problem of long-term planning and investment of financial resources. This decision is typically carried out by a high-level manager, that in this paper we will conventionally refer to as *owner*, whose key performance measure is return on investment. The second stage concerns the day-to-day operation of the structure which includes for example maintenance, repair, retrofit or enforcing traffic limitations, once the monitoring system is installed; if installed these decisions may be informed by the monitoring system. Most of the time, the manager and the owner of the structure are different individuals. Both decision makers are motivated to maintain a high level of long term availability for the structure, which is challenging as the state of the structure is never known precisely while in operation. Operators balance two types of errors, either removing a structure from operation prematurely for maintenance or operating too long resulting in a failure; both of which are based on imperfect information concerning the state of the structure. Decision makers will differ in their choices under uncertainty even when they have access to the same information if they have different appetites for risk. As such, the owner needs to consider the operators appetite for risk when deciding whether to install a monitoring system, as this will indicate how the system will inference the operators decision making and as such the value of this information.

The aim of this work is to formalize a rational method for quantifying the Value of Information when two different actors are involved in the decision chain: the *manager*, who makes decisions regarding the structure, based on monitoring data; and the *owner*, who chooses whether to install the monitoring system or not, before having access to these data. We start explaining why and how two different individuals, both rational and provided with the same background information, can end up with different decisions. Next, we review the basis of the *VoI*, which illustrates a method for evaluating the *VoI* in SHM-based decision-problems, and revise the framework of Zonta et al. [2], to include the difference between the manager and the owner. To illustrate how this framework works we apply it to the same decision problem reported in [2]: the Streicker Bridge case study. This is a pedestrian bridge at Princeton University campus, which is equipped with a continuous monitoring system. Some concluding remarks are reported at the end of the paper.

SHM-based decision

In this section, we review the concepts of Bayesian judgment, expected utility and value of information, as applied to SHM problems, following a similar path as in Cappello et al. [17] and Zonta et al. [2]. The Reader can find further examples of SHM-based decision problems in Flynn and Todd [18] [19], Flynn et al. [20] and Tonelli et al [21]. As observed in [17], SHM-based decision making (i.e., deciding based on the information from a SHM system) is properly a two-step process, which includes a judgement and a decision, as depicted in Figure 1: first, based on the information from the sensors **y**, we infer the state S of the structure; next, based on our knowledge of the state S we choose the optimal action a_{opt} to take.

Before proceeding with the mathematical formalization of this process let us confine the complexity of our problem through the following assumptions:

- the monitoring system provides a dataset that can be represented by a vector **y**;
- the structure (e.g.: one bridge) can be in a one out of N mutually exclusive and exhaustive states S_1, S_2, …, S_N (e.g.: S_1 = 'severely damaged', S_2 = 'moderately damaged', S_3 = 'not damaged', …);
- the state of the structure is generally *not* deterministically known, and can be only described in probabilistic terms;
- the decision maker can choose between a set of M alternative actions a_1, a_2, …, a_M (for example, a_1 = 'do nothing', a_2 = 'limit traffic', a_3 = 'close the bridge to traffic', …);
- taking an action produces measurable consequences (e.g.: a monetary gain or loss, a temporary downtime of the structure, in some case causalities); the consequences of an action can be mathematically described by several parameters (e.g.: the amount of money lost, the number of day of downtime, the number of casualties), encoded in an outcome vector **z**;
- the outcome **z** of an action depends on the state of the structure, thus it is a function of both action a and state S: $\mathbf{z}(a,S)$; when the state is certain the consequence of an action is also deterministically known; therefore, the only uncertainty in the decision process is the state of the structure S;
- for simplicity and clarity, we refer here to the case of 'single shot' interrogation, which is the case when the interrogation occurs only following an event which has a single chance to happen during the lifespan; an extension to the case of multiple interrogations is also found in [2].

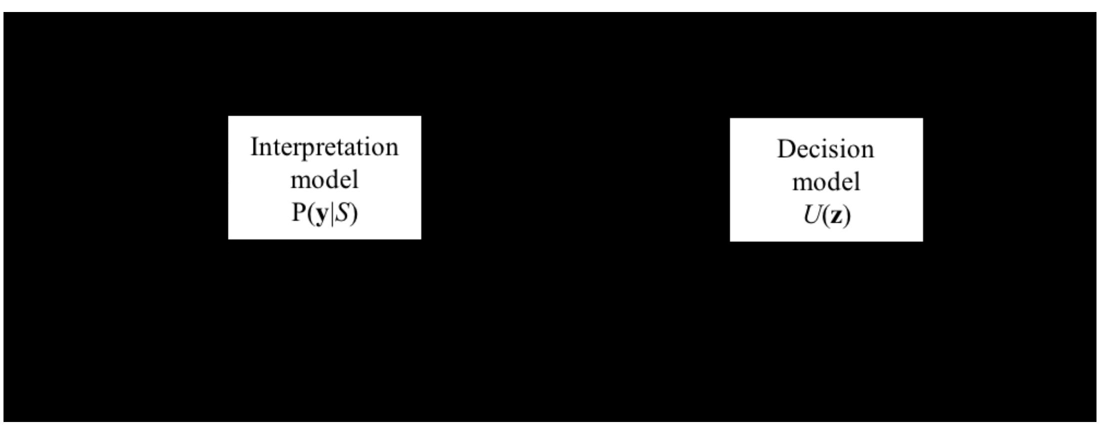

Figure 1. The process of SHM-based decision making.

Judgment is about understanding the state of the structure based on the observation, which is exactly what SHM is about from a logical standpoint. In the presence of uncertainty, the state of the structure after observing the sensors data $\mathbf{y}$ is probabilistically described by the posterior information $\mathrm{P}(S|\mathbf{y})$, and the logical inference process followed by a rational agent is mathematically encoded in Bayes' rule [22], which reads:

$$\mathrm{P}(S_i|\mathbf{y}) = \frac{\mathrm{p}(\mathbf{y}|S_i)\mathrm{P}(S_i)}{\mathrm{p}(\mathbf{y})}, \tag{1}$$

where $\mathrm{P}(\cdot)$ indicates a probability and $\mathrm{p}(\cdot)$ a probability density function. Equation (1) basically says that the *posterior* knowledge of the *i*th structural state $\mathrm{P}(S_i|\mathbf{y})$ depends on the *prior* knowledge $\mathrm{P}(S_i)$ (i.e., what I expect the state of the structure to be before reading any monitoring data) [23] and the *likelihood* $\mathrm{p}(\mathbf{y}|S_i)$ (i.e., the probability of observing the data given the state of the structure). Distribution $\mathrm{p}(\mathbf{y})$ is simply a normalization constant, referred to as *evidence*, calculated as:

$$\mathrm{p}(\mathbf{y}) = \sum_{i=1}^{N} \mathrm{p}(\mathbf{y}|S_i)\mathrm{P}(S_i). \tag{2}$$

Decision is about choosing the 'best' action based on the knowledge of the state. When the state of the system S_i is deterministically known, the rational decision-maker ranks an action based on the consequences $\mathbf{z}$ through a utility function $U(\mathbf{z})$. Mathematically, the utility function is a transformation that converts the vector $\mathbf{z}$, which describes the outcome of an action in its entire complexity, into a scalar U, which indicates the agent's order of subjective preference for any possible outcome.

When the state of the system is uncertain, and therefore the consequences of an action are only probabilistically known, the axioms of expected utility theory (EUT) state that the decision maker ranks their preferences based on the *expected* utility u, defined as:

$$u(a) = \mathrm{E}_S\left[U\big(\mathbf{z}(a, S)\big)\right], \tag{3}$$

where E_S is the expected value operator of random variable S, which we have assumed be the only uncertainty into the problem. To prevent confusion, note that in this paper capital U indicates the utility function, while lowercase u denotes an expected utility.

To better clarify the practical meaning of Equation (3), let's start from the case of a structure not equipped with a monitoring system, where the manager decides without accessing any SHM data. In this case the manager's prior expected utility $u(a_j)$ of a particular action a_j, depends on their *prior* probabilistic knowledge $\mathrm{P}(S_i)$ of each possible state S_i:

$$u(a_j) = \sum_{i=1}^{N} U\left(\mathbf{z}(a_j, S_i)\right)\mathrm{P}(S_i), \tag{4}$$

and consistently with EUT, the rational manager will choose that actions a_{opt} which carries the maximum expected utility payoff u:

$$u = \max_j \; u(a_j), \quad a_{\text{opt}} = \arg\max_j \; u(a_j). \tag{5a,b}$$

In contrast, if a monitoring system is installed, and data are accessible by the agent, the monitoring observation $\mathbf{y}$ affects the state knowledge, and therefore indirectly their decision. This time, the posterior expected utility $u(a_j,\mathbf{y})$ of actions a_j depends on the posterior probabilities $\mathrm{P}(S_i|\mathbf{y})$, which are now functions of the observation $\mathbf{y}$:

$$u(a_j, \mathbf{y}) = \sum_{i=1}^{N} U\left(\mathbf{z}(a_j, S_i)\right) \mathrm{P}(S_i|\mathbf{y}). \tag{6}$$

Because the posterior probability depends on the particular observation $\mathbf{y}$, in the posterior situation the expected utility is a function of $\mathbf{y}$ as well, and so are the maximum expected utility and the optimal choice:

$$u(\mathbf{y}) = \max_j \; u(a_j, \mathbf{y}), \quad a_{\text{opt}} = \arg\max_j \; u(a_j, \mathbf{y}). \tag{7a,b}$$

Equation (5a) and (7a) are the utilities calculates before and after a monitoring system is interrogated. Note that, in order to evaluate the posterior utility of an action $u(a_j,\mathbf{y})$, we need to know the particular realization of observation $\mathbf{y}$, so we cannot evaluate the posterior utility until the monitoring system is installed and its readings are available.

How does the utility change if we have decided to install a monitoring system, but we have still to observe the sensors' readings? Technically, what we should do is to evaluate a priori (i.e., now that the system is not installed yet) the expected value of the utility a posteriori (i.e., at the time when the system will be installed and operating). We denote this quantity *preposterior utility*, u^*, to separate it both from the prior and posterior utilities introduced above. The preposterior utility u^* is independent on the particular realization and can be derived from the posterior expected utility $u(\mathbf{y})$ by marginalizing out the variable $\mathbf{y}$, [2] [17]:

$$u^* = \mathrm{E}_{\mathbf{y}}\left[\max_j \; u(a_j,\mathbf{y})\right] = \int_{D\mathbf{y}} \max_j \; u(a_j,\mathbf{y}) \cdot \mathrm{p}(\mathbf{y}) \; d\mathbf{y}, \tag{8}$$

where distribution $\mathrm{p}(\mathbf{y})$ is the same *evidence* defined by Equation (2). The preposterior expected utility encodes the total expected utility of a decision process, based on the information provided by the monitoring system, but evaluated before the monitoring system is actually installed.

Finally, the Value of Information of the monitoring system is simply the difference between the expected utility with the monitoring system (the preposterior utility u^*) and the corresponding utility without the monitoring system (the prior utility u):

$$VoI = u^* - u = \int_{D\mathbf{y}} \max_j u(a_j, \mathbf{y}) \cdot \mathrm{p}(\mathbf{y}) \, d\mathbf{y} - \max_j u(a_j). \tag{9}$$

In other words, the *VoI* is the difference between the *expected maximum* utility and the *maximum expected* utility. It is easily mathematically verified that u^* is always greater or equal than u, and therefore the *VoI* as formulated above can only be positive. This is to say that under the assumption above SHM is always useful, consistently with the principle that "information can't hurt" [24] as reported in Pozzi [25]. It is worth reminding that these assumptions are performed before acquiring the data. That means that the value of those data is anticipated by the decision maker, even if the realized value, once the decision is made, may be quite different. As well, it may be that the cost of data exceeds its value, but this would be reflected in the calculation as we assess the utility associated with the cost of obtaining the data.

The process of deciding on the monitoring system installation can be graphically represented as a two-stage decision tree, as shown in Figure 2. At the first stage the agent decides on whether to go or not with the SHM system, while at the second stage he decides on the action $a_1, \ldots, a_j$ to undertake on the structure. The realization of the state occurs at the following chance node and the outcome z depends on the action and the state. On the 'without SHM' branch of the tree, the state is determined by the prior information and the expected utility corresponds to u in Equation (9). On the 'with SHM' branch of the tree, the second stage action is decided based on the information $\mathbf{y}$ from the monitoring system and the final outcome includes the cost $\mathbf{z}_{SHM}$ of the monitoring system. The best choice of stage one is the one that provides maximum utility, and this can be calculated by solving the two-stage tree by backward induction [5].

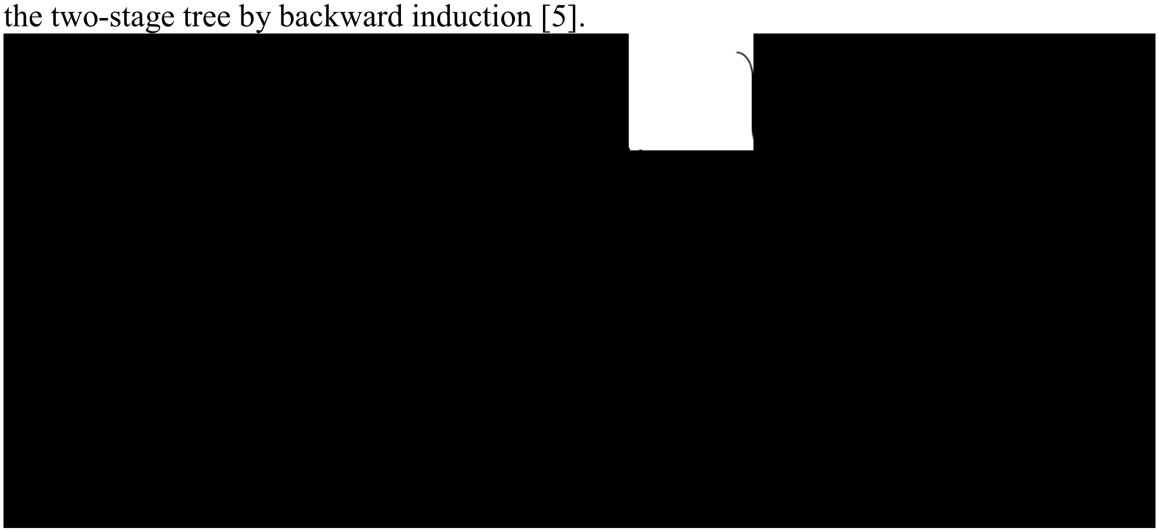

Figure 2. Graphical representation of the decision problem of whether or not to install a monitoring system (SHM).

Two individuals, two decisions

In the *classical* formulation of the *VoI* stated above, we have implicitly assumed that the decision is taken at any stage by the same rational individual, characterized by a defined background information and utility function. We address now the problem of quantifying the *VoI* when two separate individuals are involved in the decision chain. We conventionally denote *manager* (M) the one who makes decisions on the day-to-day operation of the structure, and *owner* (O), the one who is in charge of the strategic investments on the asset and decide on whether to install the monitoring system or not. Referring to Figure 2, the manager is the one who takes decisions at stage two, while the owner decides at stage one. We will refer to the *classical* formulation of *VoI*, as stated in the previous section, as to *unconditional* - in contrast with the *conditional VoI* which we are about to introduce.

A common misunderstanding, not only in our community, is that two individuals, if both rational and exposed to the same observation, should always end up with the same decision. In the real world, there are a number of components in the SHM-based decision process that are inherently subjective, so different decisions by different individuals should not be necessarily be seen as an inconsistency. This concept needs a deeper explanation: with reference to Figure 1, the reasons whereby two individuals, both rational, can take a different decision based on the same observation include:

a) the two have a different prior knowledge of the problem – i.e. they use different priors $P(S)$;
b) they interpret differently the observation – i.e. they use different interpretation models, which are encoded in the likelihood function $P(\mathbf{y}|S)$;
c) they have a different expectation or knowledge of the possible outcome of an action – i.e. they assume different outcome vectors $\mathbf{z}$;
d) they weight differently the importance of an outcome - i.e. they use different utility functions $U(\mathbf{z})$.

Differences in (a) (b) and (c) are merely about background knowledge and may actually occur in the real world; however, we expect that two individuals with similar experience and education should generally agree on any of that. For example, two structural engineers with common background will probably agree on the limited importance of a bending crack visible on an unprestressed reinforced concrete beam, while a non-expert could be over-concerned. In this paper, we will assume that the two agents fully agree on (a), (b) and (c), while they only differ in the way how they weight outcomes (d), through their utility function. The utility function is not a matter of background knowledge, rather it reflects the value of the individual as to the consequence of an action. Therefore, there is no logical argument to judge one utility function better than another one, as long as it does not violate the axioms of the expected utility theory.

Even limiting our discussion to the case where the outcome z is just a monetary loss or gain, the utility function adopted by different people can be very different based on their particular individual risk aversion [26] [27]. For instance, an agent is risk neutral if his or her utility function U is linear with the loss or gain z, as shown in Figure 3. Since the expected utility is proportional to the probability of realization, as shown in Equation (4), risk neutrality implies indifference to a gamble with an expected value of zero. So, for example, to a risk neutral agent a 1% probability of losing \$100 is equivalent to a certain loss of \$1.

In practice it is commonly observed that individuals tend to reject gambles with a neutral expected payoff: in the example above individuals often prefer to pay \$1 off the pocket rather than taking the risk of losing $100. This condition is referred to as risk aversion and can be graphically represented with a utility function with a concave (i.e., with negative second derivative) utility function, as shown in Figure 3. The condition of risk aversion is consistent with the observation that the marginal utility of most goods, including money, diminish with the amount of goods, or the wealth of the decision maker, as observed since Bernoulli [26].

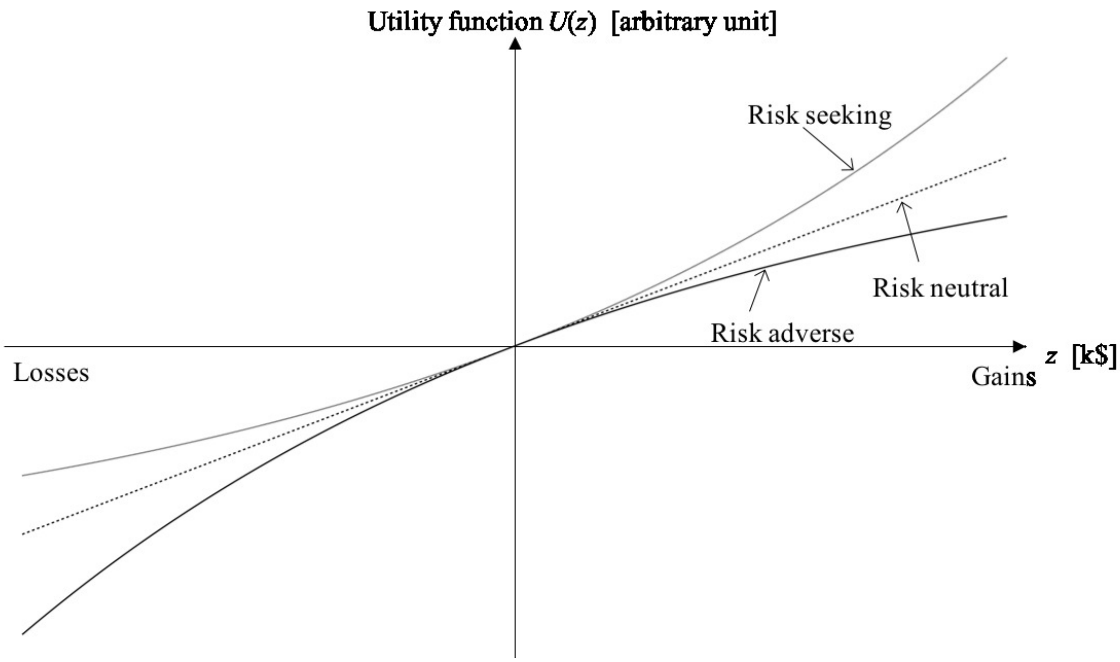

Figure 3. Utility function for risk seeking, risk neutral and risk adverse agents.

Dealing with losses, risk aversion respect to a loss depends on the amount of the loss with respect to the decision maker's own wealth or the extent of his or her own asset: when the loss is much smaller than the whole value of the asset, the agent tends to be risk neutral, while they became risk averse when the loss is a significant fraction of their asset. In our situation, the owner, who is in charge of the strategic development of the agency, typically manages a large stock of structures, and the loss corresponding to an individual structure is a much smaller than the overall asset value. In this case, it is likely that the owner is risk neutral with respect to the loss compared to the value of a single structure. In contrast the manager is responsible for the safety of a single structure: in this case the value of the structure corresponds to the value of the asset, and their behaviour is likely to be risk adverse respect to the loss of that particular structure.

To proceed with the mathematical formulation, we have to acknowledge that the two agents involved in the decision chain, the owner and the manager, may have different utility functions. We're going to use indices $^{(M)}$ or $^{(O)}$ to indicate that a quantity is intended from one of the other perspective. The expected utility of the manager is calculated as:

$$ {}^{(M)}u(a_j) = \sum_{i=1}^{N} {}^{(M)}U\left(\mathbf{z}(a_j, S_i)\right) \mathrm{P}(S_i)\,, \tag{10} $$

and we may calculate the optimal action and the maximum utility from the manager perspective as in the following:

$$ {}^{(\mathrm{M})}u = \max_j {}^{(\mathrm{M})}u\left(a_j\right) \text{ , } {}^{(\mathrm{M})}a_{\mathrm{opt}} = \arg\max_j {}^{(\mathrm{M})}u\left(a_j\right) . \tag{11a,b} $$

If the owner was in charge of the entire decision chain, we would end up with analogous expressions of optimal action ${}^{(\mathrm{O})}a_{\mathrm{opt}}$ and maximum expected utility ${}^{(\mathrm{O})}u_{\mathrm{max}}$, this time from the owner perspective. Observe that the optimal choice of the owner does not necessarily coincide with that of the manager, meaning that if the owner was in charge of the full decision chain, they would behave differently from the manager. Continuing on this rationale, we can reformulate the expression of posterior utilities, preposterior utilities and *VoI* from the owner or the manager perspective.

However, the situation we are discussing is different: the owner is the one who decides on the monitoring system installation, but the manager is the one who decides which is the optimal action at the second stage. Therefore, all utilities are from the owner perspective, but should be evaluated accounting for the action that the manager, not the owner, is expected to choose. In other words, the utility of the owner is *conditional* to the action chosen by the manager ${}^{(\mathrm{M})}a_{\mathrm{opt}}$. For example, the prior utility of the owner conditional to the decision expected by the manager reads:

$$ {}^{(\mathrm{O|M})}u = {}^{(\mathrm{O})}u\left({}^{(\mathrm{M})}a_{\mathrm{opt}}\right) = {}^{(\mathrm{O})}u\left\{\arg\max_j {}^{(\mathrm{M})}u\left(a_j\right)\right\} , \tag{12} $$

where the index ${}^{(\mathrm{O|M})}$ on the utility ${}^{(\mathrm{O|M})}u$ indicates that this utility is *conditional* to the manager's choice, in opposition to the *unconditional* utility ${}^{(\mathrm{O})}u$ calculated assuming the owner in charge of the full decision chain. We can proceed accordingly to formulate the *posterior conditional utility* (the utility of the owner after the manager has observed the monitoring response):

$$ {}^{(\mathrm{O|M})}u = {}^{(\mathrm{O})}u\left({}^{(\mathrm{M})}a_{\mathrm{opt}}(\mathbf{y})\right) = {}^{(\mathrm{O})}u\left\{\arg\max_j {}^{(\mathrm{M})}u\left(a_j,\mathbf{y}\right)\right\} , \tag{13} $$

and similarly the *preposterior conditional utility* (the utility of the owner in the expectation of what the manager would decide if a monitoring system was installed):

$$ {}^{(\mathrm{O|M})}u^* = \int_{D_y} {}^{(\mathrm{O})}u\left\{\arg\max_j {}^{(\mathrm{M})}u\left(a_j,\mathbf{y}\right)\right\} \cdot \mathrm{p}(\mathbf{y})\, dy . \tag{14} $$

Eventually the *conditional VoI* is the difference between the preposterior and the prior *conditional* utilities:

$$VoI = {}^{(O|M)}u* - {}^{(O|M)}u =$$

$$= \int_{D_y} {}^{(O)}u\left\{\arg\max_j {}^{(M)}u(a_j,\mathbf{y})\right\} \cdot \mathrm{p}(\mathbf{y})\, dy - {}^{(O)}u\left\{\arg\max_j {}^{(M)}u(a_j)\right\}. \qquad (15)$$

The unconditional and conditional formulations are summarized and compared in Table 1. At this point, it's interesting to compare the unconditional and the conditional utilities, and also the value of information. The unconditional utility, prior or preposterior, is basically the owner's utility of their favourite choice, while the conditional utility is the owner's utility of the choice of someone else. If the two choices coincide, the conditional utility is equal to the unconditional prior utility. If they do not coincide, the manager's choice can only be suboptimal from the owner's perspective, and therefore the conditional utility must be equal or lower than the unconditional. Therefore, the following relationships must hold:

$${}^{(O|M)}u \le {}^{(O)}u\,, \qquad {}^{(O|M)}u^* \le {}^{(O)}u^*. \qquad (16)$$

In a situation with one decision maker the *VoI* cannot be negative; if the decision maker anticipated misleading data it would be optimal to discard it resulting in a *VoI* of 0. However, we consider a situation of two decision makers and demonstrate that the *VoI* can be negative to one, i.e. the owner, as the new information is resulting in the other, i.e. the manager, making decisions that are less preferred by the owner than with no information.

Table 1. Value of Information of a monitoring system in the unconditional and conditional formulation.

Unconditional formulation Manager (M) = Owner (O)	Conditional formulation Manager (M) ≠ Owner (O)
Prior utility without monitoring	
${}^{(O)}u = \max_j {}^{(O)}u(a_j)$ ${}^{(O)}a_{opt} = \arg\max_j {}^{(O)}u(a_j)$	${}^{(O\|M)}u = {}^{(O)}u\left({}^{(M)}a_{opt}\right) = {}^{(O)}u\left\{\arg\max_j {}^{(M)}u(a_j)\right\}$
Posterior utility with monitoring	
${}^{(O)}u(\mathbf{y}) = max_j {}^{(O)}u(a_j,\mathbf{y})$ ${}^{(O)}a_{opt}(\mathbf{y}) = arg\ max_j {}^{(O)}u(a_j,\mathbf{y})$	${}^{(O\|M)}u(\mathbf{y}) = {}^{(O)}u\left({}^{(M)}a_{opt}(\mathbf{y})\right)$ ${}^{(O\|M)}u(\mathbf{y}) = {}^{(O)}u\left\{arg\ max_j {}^{(M)}u(a_j,\mathbf{y})\right\}$
Preposterior utility with monitoring	
${}^{(O)}u* = \int_{D_y} \max_j {}^{(O)}u(a_j,\mathbf{y}) \cdot \mathrm{p}(\mathbf{y})\, dy$	${}^{(O\|M)}u* = \int_{D_y} {}^{(O)}u\left\{\arg\max_j {}^{(M)}u(a_j,\mathbf{y})\right\} \cdot \mathrm{p}(\mathbf{y})\, dy$
Value of information of the monitoring system	
$VoI = {}^{(O)}\mathrm{u}* - {}^{(O)}u$	$VoI = {}^{(O\|M)}u* - {}^{(O\|M)}u$

The Streicker Bridge case study

To illustrate how the presence of two different decision makers in the decision chain affect the way how the *VoI* is evaluated, we consider the case of Malcolm, the fictitious manager of an imaginary Office of Design and Construction at Princeton University, protagonist in [2] and [17]. Malcolm is responsible for the Streicker Bridge, a pedestrian bridge located on Princeton University campus. The bridge and its monitoring system are illustrated in much detail in a number of past publications [28] [29] [30], we summarise the main structural features, for clarity. The deck of the bridge is a continuous thin concrete posttensioned deck featuring a characteristic X-shape connecting four different sectors of Princeton Campus. From the structural point of view, it consists of a thin post-tensioned supported by a high resistance steel lattice. The main span of the bridge overpasses Washington road, a busy public road the campus (see Figure 4(a) and Figure 4(b)).

The SHM-*lab* of Princeton University instrumented the bridge with two SHM systems: (i) global structural monitoring using discrete long-gauge strain Fiber Optic Sensors (FOS), based on fiber Bragg-grating (FBG) [31], and (ii) integrity monitoring, using truly distributed FOS based on Brillouin Optical Time Domain Analysis (BOTDA) [32]. These two approaches are complementary: discrete sensors monitor an average strain at discrete points, while the distributed sensors monitor one-dimensional strain field. Discrete FOS embedded in the bridge deck have gauge length 60 cm and feature excellent measurement properties with error limits of ±4 με. Thus, they are excellent for assessment of global structural behavior and for structural identification. Instead, distributed FOS have accuracy an order of magnitude lower than discrete sensors and so cannot be used for accurate structural identification; they are used for damage detection and localization. Figure (4c) shows the sensors map in the main span, while Figure (4d) its cross section.

Agents

To make the case study easier to understand, we imagine the bridge managed by two agents with distinct roles:

- Ophelia (O) is the *owner* responsible for Princeton's estate; she is Malcolm's supervisor and decides on whether to install the monitoring system or not.
- Malcolm (M) is the *manager* responsible for the bridge operation and maintenance, graduated in civil engineering and registered as a professional engineer, who has to take decisions on the state of the bridge based on monitoring data, exactly as in Zonta et al. [2].

We assume that Ophelia and Malcolm are both rational individuals and that have the same knowledge background as for possible damage scenarios *S* of the bridge, prior information, and they have the same knowledge of the consequence of a bridge failure. They only differ in the way how to weight the seriousness of the consequences of a failure. It is probably unnecessary to remind that, while the Streicker Bridge is a real structure, the two characters, Ophelia and Malcolm, are merely fictitious and do not reflect in any instance the way how asset maintenance and operation is performed at Princeton University.

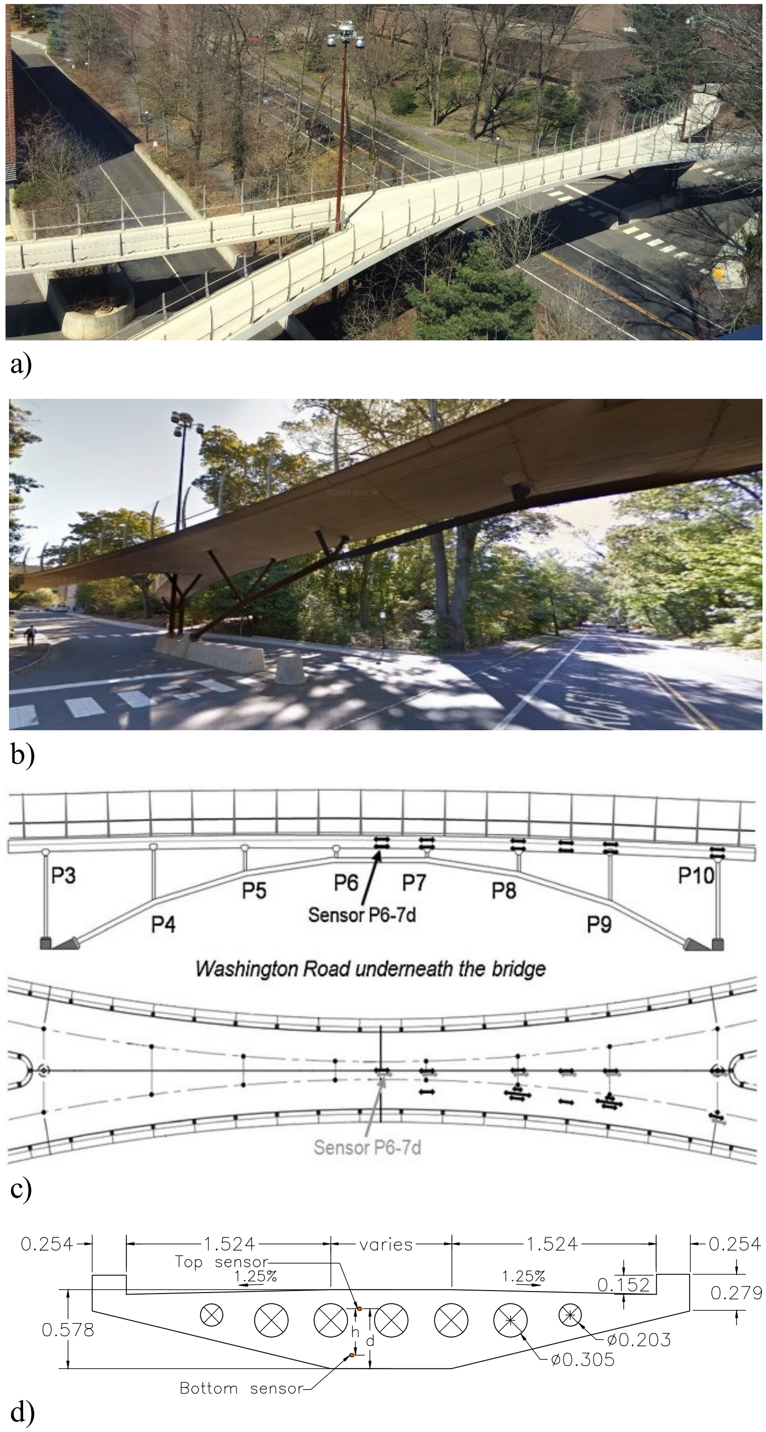

Figure 4. The Streicker bridge: view of the bridge (a)(b), location of sensors in the main span (c), main cross section (d).

States and likelihoods

As part of this fictitious story, we suppose that both Ophelia and Malcolm are concerned by a single specific scenario: a truck, maneuvering or driving along Washington road, could collide with the steel arch supporting the concrete deck of the bridge. In this oversimplified example, we will assume that after an incident the bridge will be in one of the following two states:

- *No Damage* (U): the structure has either no damage or some minor damage, with negligible loss of structural capacity.
- *Damage* (D): the bridge is still standing but has suffered major damage; consequently, Malcolm estimates that there is a chance of collapse of the entire bridge.

Similar to the assumptions in [1], we assume Malcolm (and similarly Ophelia) focuses on the sensor installed at the bottom of the middle cross-section between P6 and P7 (called Sensor P6-7d, see Figure 4(c)).

We understand that for both Ophelia and Malcolm the two states represent a set of mutually exclusive and exhaustive possibilities, which is to say that P(D) + P(U) = 1. On the basis of their experience, they both agree that scenario U is more likely than scenario D, with prior probabilities P(D) = 30% and P(U) = 70%, respectively.

We can also assume that both use the same interpretation model, i.e. they interpret identically the data from the monitoring system. As Malcolm will pay attention only to the changes at the midspan sensor (labelled P6-7d in Figure 4(c)), we presume that he expects the bridge to be undamaged if the change in strain will be close to zero. However, he is also aware of the natural fluctuation of the strain, due to thermal effects, and to a certain extent due to creep and shrinkage: he estimates this fluctuation to be in the order of ±300 με. We can represent this quantity with a probability density function pdf(ε|U), with zero mean value and standard deviation $\sigma = 300$ με, which describes Malcolm's expectation of the system response in the undamaged (U) state, i.e. this is the likelihood of no damage. On the other hand, if the bridge is heavily damaged (D) but still standing. Malcolm expects a significant change in strain; we can model the likelihood of damage pdf(ε|D) as a distribution with mean value 1000 με and standard deviation of $\sigma = 600$ με, which reflects Malcolm's uncertainty of expectation. Before the data are available, he can also predict the distribution of ε, which is practically the so-called evidence in classical Bayesian theory, through the following formula:

$$\mathrm{pdf}(\varepsilon) = \mathrm{pdf}(\varepsilon|\mathrm{D})\cdot \mathrm{P(D)} + \mathrm{pdf}(\varepsilon|\mathrm{U})\cdot \mathrm{P(U)}\ . \tag{17}$$

When the measurement ε is available, both update their estimation of the probability of damage consistently with Bayes' theorem:

$$\mathrm{pdf}(\mathrm{D}|\varepsilon) = \frac{\mathrm{pdf}(\varepsilon|\mathrm{D})\cdot \mathrm{P(D)}}{\mathrm{pdf}(\varepsilon)}\ , \tag{18}$$

where pdf(D|ε) is the posterior probability of damage. Figure 6(a) shows the two unnormalized posterior distributions along with the evidence. Note that the posterior probability of damage starts exceeding the posterior of no-damage when the measurement ε exceeds the threshold $\bar{\varepsilon}_p$ = 540 με.

Decision model

After he assesses the state of the bridge, we assume that Malcolm can decide between the two following actions:

- *Do nothing* (DN): no special restriction is applied to the pedestrian traffic over the bridge or to road traffic under the bridge.
- *Close Bridge* (CB): both Streicker Bridge and Washington Road are closed to pedestrians and road traffic, respectively; access to the nearby area is restricted for the time needed for a thorough inspection, which both Ophelia and Malcolm estimates to be 1 month.

Ophelia and Malcolm agree that the costs related to each action, for each scenario, are the same as estimated in Glisic and Adriaenssens [28], and reported in Table 2.

Table 2. Costs per action and state.

	Scenario U (no damage)	**Scenario D (bridge fails)**
Action DN (do nothing)	nothing happens you pay nothing	failure cost z_F = \$881,600
Action CB (close bridge)	1-month downtime cost z_{DT} =\$139,800	1-month downtime cost z_{DT} = \$139,800

However, Ophelia and Malcolm differ in their utility functions, which is the weight they apply to the possible economic losses. Ophelia is risk neutral, meaning that according to her a negative utility is linear with the incurred loss, as illustrated in Figure 5. Strictly speaking, a utility function is defined except for a multiplicative factor, therefore it should be expressed in an arbitrary unit sometime referred to as *util* [33]. Since Ophelia's utility is linear with loss, for the sake of clarity we will deliberately confuse negative utility with loss, and therefore we will measure Ophelia's utility in k\$.

Unlike Ophelia, Malcolm is likely to behave risk adversely, i.e. his negative utility increases more than proportionally with the loss. We can describe mathematically the risk aversion classically defined in Arrow-Pratt theory [34] [35], where the level of risk aversion of an agent is encoded in the coefficient of Absolute Risk Aversion (ARA), defined as the rate of the second derivative (curvature) to the fist derivative (slope):

$$A(z) = \frac{U''(z)}{U'(z)} \; . \tag{19}$$

To state Malcolm's utility function, we can make the following assumptions:

- Malcolm's and Ophelia's reaction are virtually identical for a small amount of loss, while their way of weighting the losses departs for bigger losses.
- For small losses, therefore, the two-utility function may be confused, and we will adopt for Malcolm's the same conventional unit (call it equivalent k\$) for measuring utility. Malcolm's utility function derivative for zero loss is equal to 1.

- We assume that Malcolm's utility has constant ARA; it is easily demonstrated that a function with constant ARA and unitary derivative at zero [36] takes the form of an exponential:

$$^{(\mathrm{M})}U(z) = \frac{1 - e^{-z \cdot \theta}}{\theta}, \tag{20}$$

where θ is the constant ARA coefficient: $A(z) = \theta$.
- To calibrate θ, we assume that for a loss equal to the failure cost, Malcolm's negative utility is twice that of Ophelia's. This results in a constant ARA coefficient $\theta = -1.425$ M\$$^{-1}$.

Using these assumptions, the resulting Malcolm's utility function is plotted in Figure 5.

We wish now to verify how the different utility functions affect the decision of the two a priori and a posteriori.

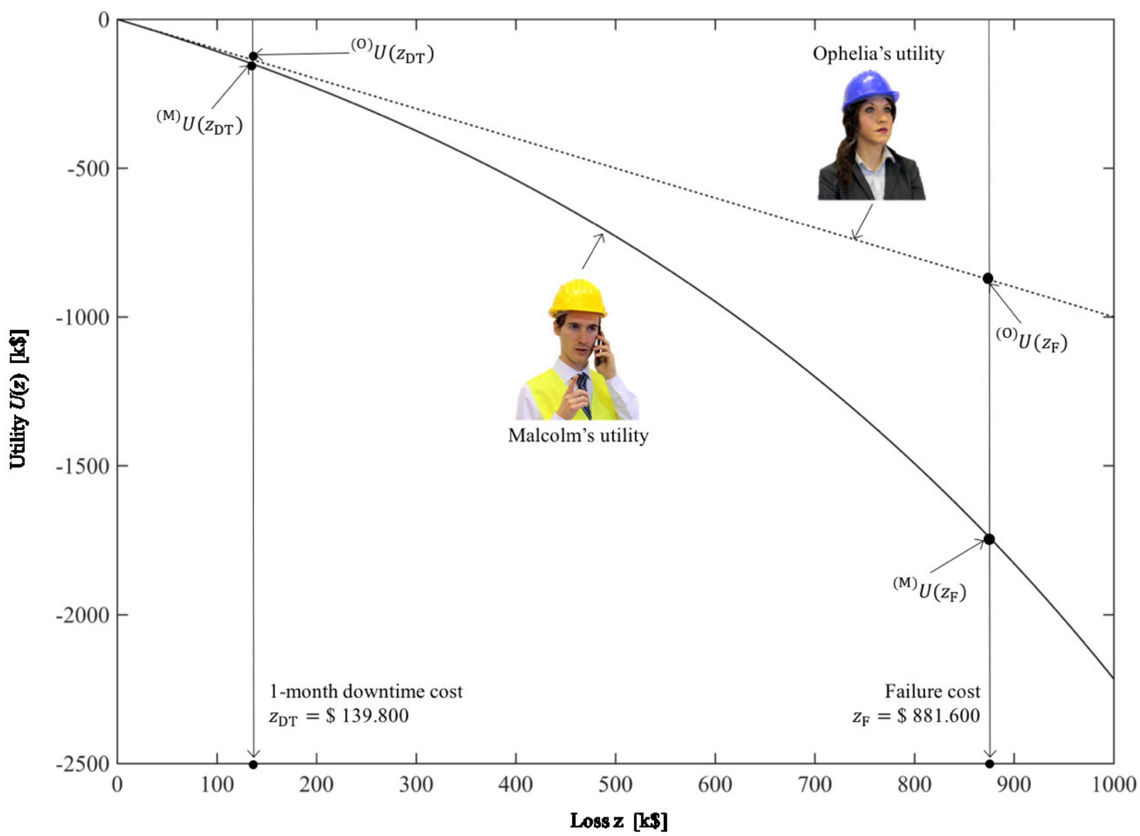

Figure 5. Representation of Ophelia's and Malcolm's utility functions.

Prior utility

Consider the case where Malcolm has no monitoring information. Based on his utility, Malcolm estimates the utilities involved in each action. Action CB depends only on the downtime cost z_{DT}, while action DN depends also on his estimate of the state of the bridge:

$$ {}^{(M)}u_{DN} = {}^{(M)}U(z_F) \cdot P(D) = -528.883 \text{ k\$}, \quad {}^{(M)}u_{CB} = {}^{(M)}U(z_{DT}) = -154.940 \text{ k\$}. \quad (21) $$

Since the utility of action CB is clearly less negative than the utility of action DN, Malcolm would always choose to close the bridge after an incident if he has no better information from the monitoring system. Therefore, Malcolm's maximum expected utility without the monitoring system is ${}^{(M)}u = {}^{(M)}u_{CB}$ = -154.940 k$.

Now imagine Ophelia in charge of the decision: her prior utilities are different from Malcolm's and their values are somewhat closer:

$$ {}^{(O)}u_{DN} = {}^{(O)}U(z_F) \cdot P(D) = -264.480 \text{ k\$}, \quad {}^{(O)}u_{CB} = {}^{(O)}U(z_{DT}) = -139.800 \text{ k\$}, \quad (22) $$

but in the end, in this particular case, her optimal action would be again 'close the bridge'.

Posterior utility

Now imagine that the monitoring system is installed and let's go back to Malcolm. Since now Malcolm can rely on the monitoring reading, in this case the expected utility of an action is calculated using the posterior probability of damage pdf(D|ε) rather than the prior:

$$ {}^{(M)}u_{CB|\varepsilon} = u(z_{DT}), \quad {}^{(M)}u_{DN|\varepsilon} = u(z_F) \cdot \text{pdf}(D|\varepsilon). \quad (23a,b) $$

Note that since the cost of closing the bridge is independent on the bridge state, the monitoring observation ε does not affect the posterior utility of closing the bridge (CB), which is always equal to -154.940 k$ as in the prior case. On the contrary, the expected utility of doing nothing (DN) does depend on the probability of having the bridge damaged, and this probability, in turn, depends on the monitoring observation through Equation (23b). Malcolm's posterior expected utilities (i.e. after observing data from the monitoring system) for actions DN and CB are plotted in the graph of Figure 6(b) as functions of the observation ε. As a rational agent, Malcolm will always take the decision that maximizes his utility. For very small values of ε, suggesting a small probability of collapse, Malcolm's utility of DN is bigger than the utility of CB, and therefore Malcolm will keep the bridge open. Malcolm's utility of closing the bridge starts exceeding the utility of doing nothing above a threshold of strain of ${}^{(M)}\bar{\varepsilon}_u$ = 170 με, and therefore Malcolm will always close the bridge above this threshold.

Note that this threshold is much smaller than the threshold $\bar{\varepsilon}_p$ whereby Malcolm would judge the damage more likely, so there is a range of values whereby Malcolm, in consideration of the possible consequences, will still prefer to close the bridge even if it is more likely the bridge is not damaged. Malcolm's maximum expected utility is plotted in bold in the graph of Figure 6(b).

Assume now that Ophelia is in charge of the decision. Since she weights the losses differently, her utility curves as functions of ε are different from Malcolm's, and are plotted in the graph of Figure 6(c). For the same reason, the threshold above which she would close the bridge, ${}^{(O)}\bar{\varepsilon}_u$ = 310 με, is different and much higher than Malcolm's, reflecting Ophelia's risk neutrality in contrast to Malcolm's risk aversion. Therefore, there is a range of values of measurements, from 170 με to 310 με, where the two decision makers, both rational, behave differently under the same information, simply because of their different level of risk aversion.

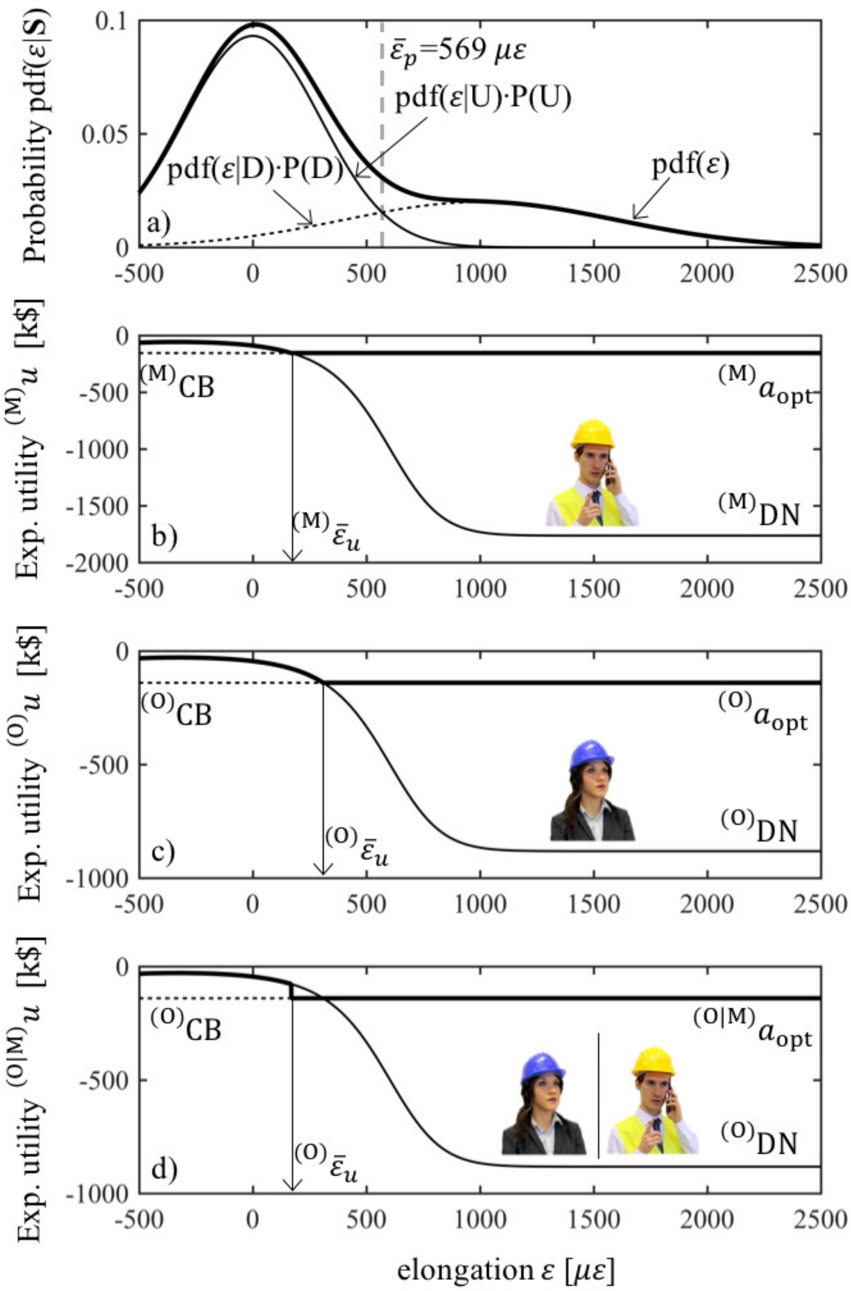

Figure 6. Representation of Malcolm's estimation of the state of the bridge a priori (a), Malcolm's decision model with monitoring data (b), Ophelia's decision model with monitoring data (c), Ophelia's decision model based on Malcolm's own (d).

Preposterior utility and Value of Information

In this scenario Ophelia and Malcolm are both involved in the decision chain. Malcolm is the operational *manager* who decides whether or not to close the bridge in the occurrence of an incident. Ophelia is the *owner* who decides on the purchase of the monitoring system. This is illustrated as a decision tree in Figure 7. We seek the VOI as anticipated by Ophelia (she has to decide), which explicitly accounts from Malcolm reacting to the signals from the monitoring system.

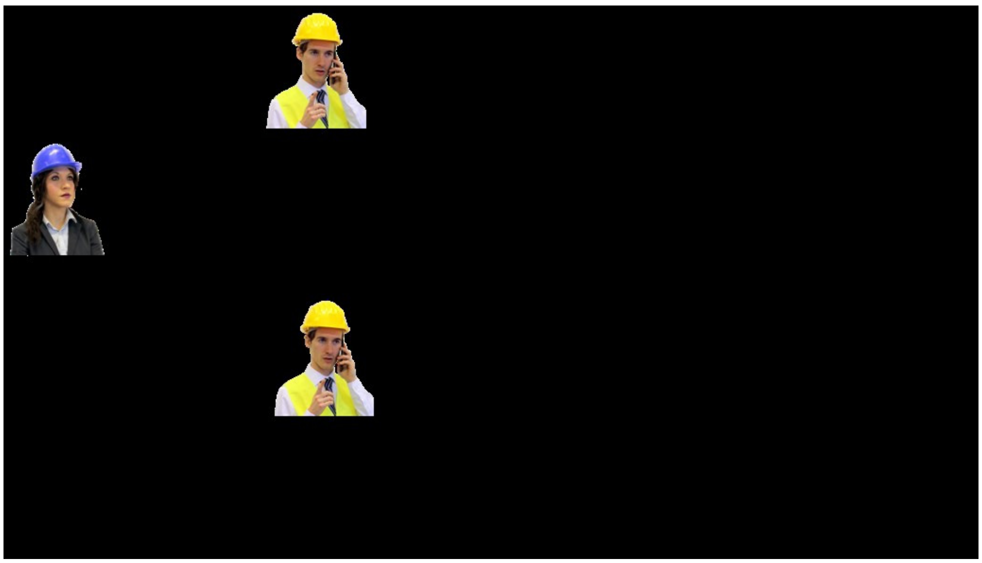

Figure 7. Decision tree for the Streicker Bridge case study.

Before attacking this problem, let's first see what happens if the decision chain was in the hands of a single individual. Let us start, for example, with Malcolm. His preposterior utility (i.e., the prior utility of operating the bridge with the monitoring system) can be calculated with the equation:

$$ {}^{(M)}u^{*} = \int_{D_\varepsilon} {}^{(M)}u\left\{\underset{j}{argmax}\, {}^{(M)}u(a_j,\varepsilon)\right\} \cdot p(\varepsilon)\, d\varepsilon = -88.504 \text{ k\$}, \tag{24} $$

where the index (M) indicates that all the utilities are calculated from Malcolm's perspective. Malcolm's *VoI* is simply the difference between the preposterior utility (i.e. the prior utility of operating the bridge with the monitoring system) and the prior utility (i.e. the utility of operating the bridge without the monitoring system):

$$ VoI = {}^{(M)}u^{*} - {}^{(M)}u = -88.505 \text{ k\$} + 154.940 \text{ k\$} = 66.435 \text{ k\$}. \tag{25} $$

Note that the *VoI* is a utility, not an actual amount of money, and is measured in Malcolm's utility unit, which in our case is Malcolm's dollar-equivalent as defined above.

Now we can calculate the *VoI* from Ophelia's perspective, assuming that she takes decisions at any stage of the decision chain. In this case being Ophelia less risk adverse than Malcolm, her utilities will be ${}^{(O)}u^{*}$ = -84.600 k\$ and ${}^{(O)}u$ = -139.800 k\$, so eventually Ophelia's *VoI* would be:

$$VoI = {}^{(O)}u^{*} - {}^{(O)}u = -84{,}600 \text{ k\$} + 139{,}800 \text{ k\$} = 55.200 \text{ k\$}\,. \qquad (26)$$

This practically means that, if Ophelia was in charge of all the decisions, she would be willing to spend up to 55.200 k\$ for the information from the monitoring system.

In reality, Ophelia is only in charge of the purchase of the monitoring system, while the one who is going to use it is her colleague Malcolm. So, in taking her decision, Ophelia has to figure out how Malcolm is going to behave both with and without the monitoring system. In other words, we have to calculate the prior and preposterior utility from Ophelia's perspective, but conditional to the action that Malcolm will undertake.

For example, to calculate the prior (i.e. the utility of Ophelia of operating the bridge without the monitoring system, conditioned to Malcolm's actions) conditional utility, Ophelia thinks: *what will Malcolm do after an accident if no monitoring system is installed? I know Malcolm, and I know he will close the bridge right away (I would do the same, but that's irrelevant). My utility, if he closes the bridge, is:*

$${}^{(O|M)}u = {}^{(O)}u\left\{\underset{j}{argmax}\, {}^{(M)}u(a_j)\right\} = {}^{(O)}u_{CB} = -139.800 \text{ k\$}\,, \qquad (27)$$

which in this case is the same as the unconditional. *And what* – Ophelia continues to think – *would Malcolm do if a monitoring system was installed. I know that he would look at the strain ε and he would close the bridge if ε* >170 με *and keep the bridge open otherwise. I personally would NOT do the same, but that's it, I have to live with Malcolm's decision!* The way Ophelia evaluates the utility on Malcolm's decisions is explained in Figure 6(d): her utilities for each possible Malcolm's choice are calculated using *her* utility function, hence all individual curves are identical to those of Figure 6(c), However, the threshold whereby she expects the bridge is closed is Malcolm's threshold, i.e. the same as in Figure 6(d). Ophelia's utility of Malcolm's choice is, for any value of *ε*:

$${}^{(O)}a_{opt} = {}^{(O)}u\left\{\underset{j}{argmax}\, {}^{(M)}u(a_j,\varepsilon)\right\} = -154.94 \text{ k\$}\,, \qquad (28)$$

and therefore the preposterior utility conditional to Malcolm is:

$${}^{(O|M)}u^{*} = \int_{D_\varepsilon} {}^{(O)}u\left\{\underset{j}{argmax}\, {}^{(M)}u(a_j,\varepsilon)\right\} \cdot p(\varepsilon)\, d\varepsilon = -88.504 \text{ k\$} \qquad (29)$$

Eventually, Ophelia's *VoI*, *conditional* on Malcolm's decision, is:

$${}^{(O|M)}VoI = {}^{(O|M)}u^{*} - {}^{(O|M)}u = -88.505 \text{ k\$} + 139.800 \text{ k\$} = 51.295 \text{ k\$}\,. \qquad (30)$$

Again, this quantity is the money Ophelia believe is worth spending on a monitoring system, having accepted that Malcolm, not her, is going to use it. The conditional ${}^{(O|M)}VoI = 51.495$ k\$ is slightly lower than the unconditional ${}^{(O)}VoI = 55.200$ k\$. Generally, it is clear from Ophelia perspective, that when Malcolm's decision is different from hers it is always suboptimal. Therefore, the conditional prior and pre-posteriors are always smaller than the corresponding unconditional: ${}^{(O|M)}u \leq {}^{(O)}u$, ${}^{(O|M)}u^* \leq {}^{(O)}u^*$. In the present example, Ophelia and Malcolm agree on what to do a priori ${}^{(O|M)}u = {}^{(O)}u$, the conditional ${}^{(O|M)}VoI$ is necessarily smaller than the conditional ${}^{(O)}VoI$. In simple words, Ophelia's rationale goes along these lines: *I can exploit the monitoring system better than Malcolm, therefore the benefit of the monitoring system would be greater if I was using the monitoring system rather than Malcolm*.

However, this is not the most general case. Assume for example the prior probability of damage P(D) is 10%: Ophelia's prior utility of action DN ${}^{(O)}u_{DN}$ = -88.160 k\$, small enough for Ophelia to keep the bridge open; on the contrary Malcolm's prior utility ${}^{(M)}u_{DN}$ = -176.294 k\$, is still big enough for Malcolm to close it. In this case the unconditional prior is much bigger than the conditional one, since Ophelia doesn't agree with Malcolm's choice, and the conditional ${}^{(O|M)}VoI$ = 103.670 k\$ is much bigger than the unconditional ${}^{(O)}VoI$ = 53.217 k\$, meaning that monitoring is much more useful in this case. We can almost hear Ophelia commenting: *This Malcolm can't make the right decision alone, hopefully some monitoring will help him! For sure a monitoring system is more useful to him rather than me!*

Negative Value of Information?

We noted above that in the unconditional case (i.e. when Ophelia is both *owner* and *manager*), the preposterior utility u^* is always greater or equal than the prior u, hence the *VoI* cannot be negative. In simpler words, if a monitoring system if offered to Ophelia at no cost, she has no reason not to accept it. Of course, if at any time Ophelia realizes that the monitoring system yields junk data, she can always decide to disregard this information, but she has no economic reason to refuse a priori to see the data ('*Take each man's censure, but reserve thy judgment*').

We also noted that in the unconditional case (i.e. when Ophelia is the *owner* but someone else, Malcolm, is the *manager* who decide based on the SHM data) there is no logical necessity whereby Ophelia's preposterior utility must be greater than her prior. So in principle we can always find a combination of prior probabilities and utility functions which ultimately yield a negative conditional *VoI*. We illustrate this concept with an example.

Imagine that Malcolm, instead of being risk adverse, is risk seeking. This is to say that his utility function is convex (i.e., with positive second derivative), as shown in Figure 8: for this exercise we can again assume an Arrow-Pratt's utility model, as in Equation (20), but this time with a positive ARA coefficient $\theta = 5.234$ M\$$^{-1}$. Also, assume, both for Ophelia and Malcolm, a high prior probability of damage, say P(D) = 55%.

Using these assumptions, Ophelia's prior utilities for doing nothing (DN) and closing the bridge (CB) are ${}^{(O)}u_{DN}$ = -484.88 k\$ and ${}^{(O)}u_{CB}$ = -139.800 k\$ respectively, while Malcolm's are ${}^{(M)}u_{DN}$ = -108.660 k\$ and ${}^{(M)}u_{CB}$ = -100.680 k\$. For both, closing the bridge (CB) is the action that yields the maximum expected utility a priori: so they both agree that, without a monitoring system, the best thing to do is to close the bridge.

Their decisions start departing after receiving data from the monitoring system. Figure 9 shows how Ophelia's and Malcolm's decision models change based on the new assumptions.

We note that:

- because of the high prior risk of collapse, risk-neutral Ophelia is very conservative and thinks it is a good idea to close the bridge as soon as the elongation recorded is greater than ${}^{(O)}\bar{\varepsilon}_u = 70\ \mu\varepsilon$;
- risk-seeking Malcolm doesn't take a collapse so seriously and he would rather keep the bridge open unless the sensor reads an elongation greater than ${}^{(M)}\bar{\varepsilon}_u = 423\ \mu\varepsilon$.

So there is a very wide range of values, from 70 με to 423 με, whereby Malcolm would keep the bridge open in disagreement with Ophelia, who believes this is a dangerous practice which can potentially result in a big loss. Based on these premises, Ophelia's conditional preposterior (i.e. Ophelia expected utility conditional to Malcolm's decision) is calculated, using equation (29), in ${}^{(O|M)}u^* = -150.362$ k$, and eventually her conditional value of information is:

$$ {}^{(O|M)}VoI = {}^{(O|M)}u^* - {}^{(O|M)}u = -150.362\text{ k\$} + 139.800\text{ k\$} = -10.562\text{ k\$}. \qquad (31) $$

Contrary to the example above, now the conditional value of information is negative, meaning that Ophelia's perceives the monitoring information as damaging. Ophelia thinks that, in observing the monitoring data, Malcolm may wrongly decide to keep the bridge open even when, in her opinion, it should be closed. She concludes that, after all, it is better not to install the monitoring system at all. In Ophelia's own words: *Malcolm is an irresponsible and should not use the monitoring system! I would rather pay money than letting him use the system!* Indeed, the negative value of information is exactly the amount of money Ophelia is willing to pay to prevent Malcolm using the monitoring system.

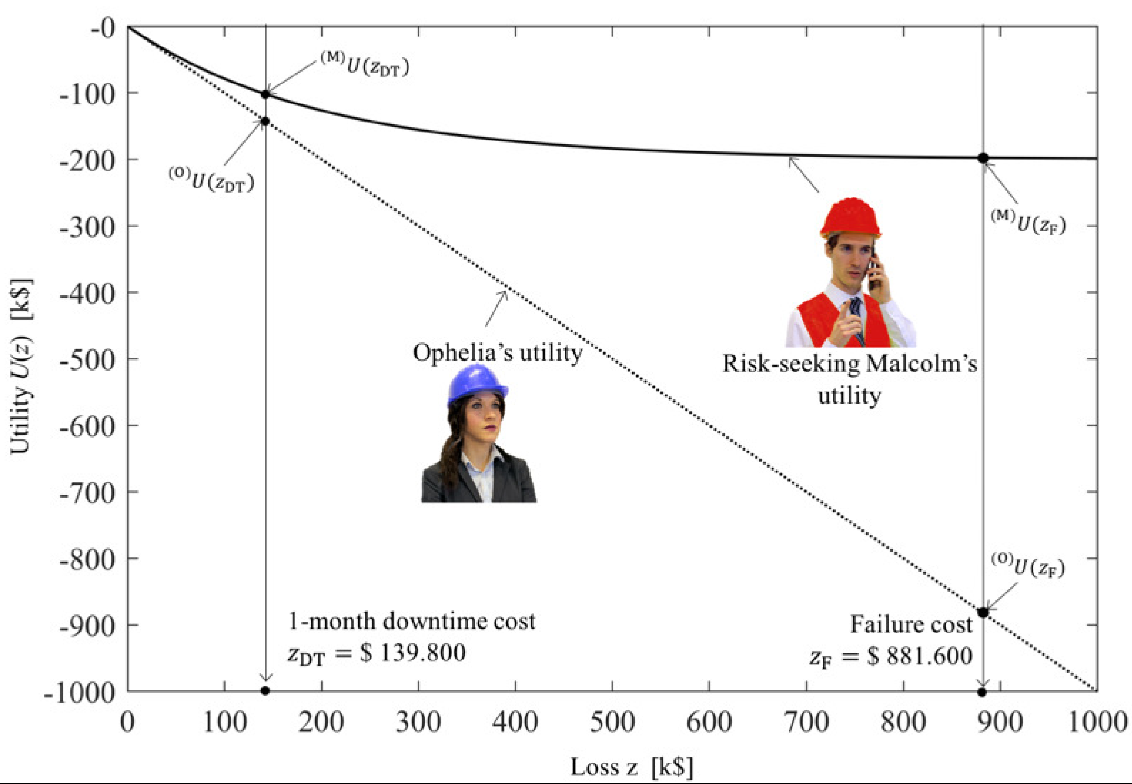

Figure 8. Representation of Ophelia's and risk-seeking Malcolm's utility functions.

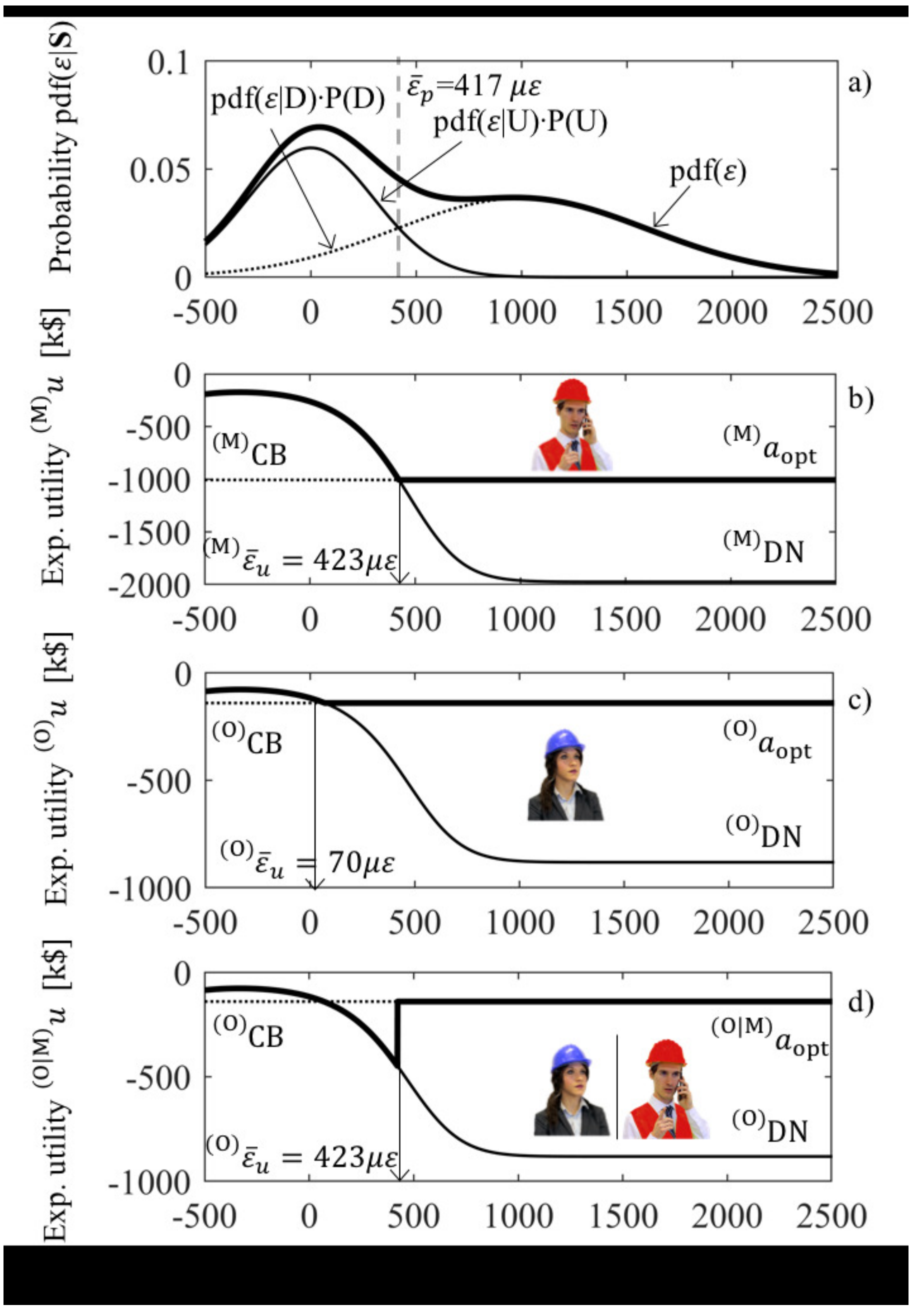

Figure 9. Representation of risk-seeking Malcolm's estimation of the state of the bridge a priori (a), risk-seeking Malcolm's decision model with monitoring data (b), Ophelia's decision model with monitoring data (c), Ophelia's decision model based on risk-seeking Malcolm's own (d).

Concluding remarks

The benefit of SHM can be quantified using the concept of Value of Information. This is the difference between the anticipated utilities of operating the structure with the monitoring system (the *preposterior* utility) and without the monitoring system (the *prior* utility). *Preposterior* utility, *Prior* utility and *Value of Information* are all subjective quantities: they depend on the particular background information and risk appetite of the individual in charge of the decision. In calculating the *VoI*, a commonly understood assumption is that the individual who decide on the installation of the monitoring system is the same rational agent who will later use it.

In the real world, these could be two separate subjects. We labelled conventionally *owner* the individual who decides on buying a monitoring system and *manager* the one who is going to use it, once the system has been installed. The two decision makers, even if both rational and exposed to the same background information, may still act differently because of their different appetites for risk.

We developed a formulation to properly evaluate the *VoI* from the owner perspective, when the manager is a different individual. The rationale of the formulation is that the owner, in evaluating the benefit of the monitoring system, must anticipate the way how the manager will actually react to the monitoring information. The calculation requires the definition of the owner's prior and preposterior utilities *conditional* to the manager anticipated behavior. For convenience, we defined the *VoI conditional* in the case when the manager is not the owner, and *unconditional* when manager and owner coincide.

To illustrate how this framework works, we have evaluated a hypothetical *VoI* for the Streicker Bridge, a pedestrian bridge in Princeton University campus equipped with a fiber optic sensing system, assuming that two fictional characters, Ophelia the *owner* and Malcolm the *manager*, are involved in the decision chain. In the example, Malcolm is the manager who decide whether to keep the bridge open or close it, following to an incident that could potentially jeopardize its safety. Ophelia is the owner who decide whether to purchase a monitoring system to help Malcolm making the right decision in that event. We noted that:

- Seen from the owner's perspective, the choices of the manager are always suboptimal: Malcolm's decisions don't necessarily coincide with what Ophelia would have made in the same situation.
- In the prior situation (i.e. without SHM), the conditional utility (i.e. when the manager is not the owner) is always equal or lower than the unconditional one (i.e. when manager and owner coincide).
- The conditional (i.e. manager is not owner) *VoI* could be bigger or smaller than the unconditional (i.e. manager is owner); if Ophelia agree on how Malcolm makes decision without the monitoring system, the conditional value of monitoring is always lower than the unconditional.
- If Ophelia doesn't agree with Malcolm, the conditional value of information may be bigger than the unconditional: Ophelia would strongly support the purchase of the monitoring system in the hope it will help Malcolm to make the right decision.

While the unconditional *VoI* is never negative, we demonstrate that under appropriate combination of prior information and utility functions, the conditional value of information could be negative. This can happen when Ophelia believe than the monitoring system can seriously mislead Malcolm's decision. The negative value of information is exactly the amount of money Ophelia is willing to pay to prevent Malcolm using the monitoring system.

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Declaration of conflicting interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

[1] Thons S and Faber MH, “Assessing the Value of Structural Health Monitoring,” in *11th International Conference on Structural Safety & Reliability (ICOSSAR 2013)*, New York, USA, 2013.

[2] Zonta D, Glisic B and Adriaenssens S, "Value of information: impact of monitoring on decision-making," *Structural Control and Health Monitoring,* no. 21, pp. 1043-1056, 2014.

[3] Von Neumann J and Morgenstern O, Theory of Games and Economic Behavior, Princeton, NJ, USA: Princeton University Press, 1944.

[4] Raiffa H and Schlaifer R, Applied Statistical Decision Theory, Boston: Clinton Press, 1961.

[5] Parmigiani G and Inoue L, Decision Theory: Principles and Approaches, Baltimore: John Wiley & Sons, Ltd, 2009.

[6] Lindley DV, "On a measure of the information provided by an experiment," vol. 27, no. 4, pp. 986-1005, 1956.

[7] DeGroot MH, "Changes in utility as information," *Theory and Decision,* vol. 17, no. 3, pp. 287-303, 1984.

[8] Wagner HM, Principles of operations research, NJ.: Prentice-Hall Inc., 1969.

[9] Sahin F and Robinson EP, "Flow coordination and information sharing in supply chains: Review, implications, and directions for future research," *Decision Sciences,* vol. 334, no. 4, pp. 505-536, 2002.

[10] Ketzenberg ME, Rosenzweig ED, Marucheck AE and Matters RD, "A framework for the value of information in inventory replenishment," *European Journal of Operational Research,* pp. 1230-1250, 2007.

[11] Quigley J, Walls L, Demirel G, MaccCarthy BL and Parsa M, "Supplier quality improvement: The value of information under uncertainty," *European Journal of Operational Research,* 2017.

[12] Bernal D, Zonta D and Pozzi M, "An examination of the ARX as a residual generator for damage detection," in *Proceedings of SPIE - The International Society for Optical Engineering*, 2009.

[13] Pozzi M, Zonta D, Wang W and Chen G, "A framework for evaluating the impact of structural health monitoring on bridge management," in *Proc. of 5th International Conf. on Bridge Maintenance, Safety and Management (IABMAS2010)*, Philadelphia, 2010.

[14] Pozzi M and Der Kiureghian A, "Assessing the value of information for long-term structural health monitoring," in *Proc. of SPIE 7984*, San Diego, CA, USA, 2011.

[15] Thons S, "On the Value of Monitoring Information for the Structural Integrity and Risk Management," *Computer-Aided Civil and Infrastructure Engineering,* 2017.

[16] Thons S, Limongelli MP, Ivankovic AM, Val D, Chryssanthopoulos M, Lombaert G, Dohler M, Straub D, Chatzi E, Kohler J, Wenzel H and Sorensen JD, "Progress of the COST Action TU1402 on the Quantification of the Value of Structural Health Monitoring," in *Proceedings of the 11th International Workshop on Structural Health Monitoring (IWSHM 2017)*, Stanford, USA, 2017.

[17] Cappello C, Zonta D and Glisic B, "Expected utility theory for monitoring-based decision making," in *Proceedings of the IEEE*, Trento, 2016.

[18] Flynn E and Todd M, "A Bayesian approach to optimal sensor placement for structural health monitoring with application to active sensing," *Mech. Syst. Signal. Pr.,* vol. 24, no. 4, pp. 891-903, 2010.

[19] Flynn E and Todd M, "Optimal Placement of piezoelectric actuators and sensors for detecting damage in plate sensors," *J. Intel. Mat. Syst. Str.,* vol. 21, no. 3, pp. 265-274, 2010.

[20] Flynn E, Todd M, Croxford A, Drinkwater B and Wilcox P, "Enhanced detection through low-order stochastic modeling for guided-wave structural health monitoring," *Struct. Health Monit.,* vol. 11, no. 2, pp. 149-160, 2011.

[21] Tonelli D, Verzobio A, Cappello C, Bolognani D, Zonta D, Bursi OS and Costa C, "Expected utility theory for monitoring-based decision support system," in *Proceedings of the 11th International Workshop on Structural Health Monitoring*, Stanford, USA, 2017.

[22] Sivia D and Skilling J, Data analysis: A Bayesian Tutorial, Oxford: Oxford University Press, 2006.

[23] Cappello C, Zonta D, Pozzi M, Glisic B and Zandonini R, "Impact of prior perception on bridge health diagnosis," *Journal of Civil Structural Health Monitoring,* vol. 5, no. 4, pp. 509-525, 2015.

[24] Cover TM and Thomas JA, Elements of information theory, John Wiley & Sons, 2012.

[25] Pozzi M, Malings C and Minca AC, "Negative value of information in systems' maintenance," in *12th Int. Conf. on Structural Safety and Reliability*, Vienna, Austria, 2017.

[26] Bernoulli D, "Exposition of a new theory on the measurement of risk," *Econometria,* vol. 22, no. 1, pp. 23-36, 1954.

[27] Kahneman D and Twersky A, "Choices, values and frames," *American Psychologist,* vol. 39, no. 4, pp. 341-350, 1984.

[28] Glisic B and Adriaenssens S, "Streicker Bridge: initial evaluation of life-cycle cost benefits of various structural health monitoring approaches," in *Proceedings of the 5th International Conference on Bridge Maintenance, Safety and Management*, 2010.

[29] Glisic B and Inaudi D, "Development of method for in-service crack detection based on distributed fiber optic sensors," *Structural Health Monitoring,* vol. 11, no. 2, pp. 696-711, 2012.

[30] Glisic B, Chen J and Hubbell D, "Streicker Bridge: a comparison between Bragg-gratings long-gauge strain and temperature sensors and Brillouin scattering-based distributed strain and temperature sensors," in *Proceedings of SPIE - The International Society for Optical Engineering*, 2011.

[31] Kang DH, Park SO, Hong CS and Kim CG, "Mechanical strength characteristics of fiber bragg gratings considering fabrication process and reflectivity," in *Journal of Intelligent Material Systems and Structures*, 2007.

[32] Nikles M, Thevenaz L and Robert P, "Simple distributed fiber sensor based on Brillouin gain spectrum analysis," in *Optics Letters*, 1996.

[33] McConnell CR, Economics: Principles, Problems, and Policies - 3rd Edition, New York: McGraw-Hill Book Company, 1966, p. 792.

[34] Pratt JW, "Risk Aversion in the Small and in the Large," *Econometrica,* vol. 32, no. 1-2, pp. 122-136, 1964.

[35] Arrow KJ, Aspects of the Theory of Risk Bearing, Helsinki: Yrjo Jahnssonin Saatio, 1965.

[36] Wakker PP, Prospect Theory for Risk and Ambiguity, Cambridge University Press, 2008, p. 104.