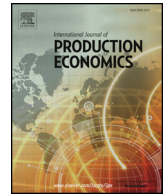**Syed Ali, Sharifah Aishah Binti and Doostmohammadi, Mahdi and Akartunali, Kerem and van Der Meer, Robert (2018) A theoretical and computational analysis of lot-sizing in remanufacturing with separate setups. International Journal of Production Economics, 203. pp. 276-285. ISSN 0925-5273 , http://dx.doi.org/10.1016/j.ijpe.2018.07.002**

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A theoretical and computational analysis of lot-sizing in remanufacturing with separate setups

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ABSTRACT

Due to the stricter government regulations on end-of-life product treatment and the increasing public awareness towards environmental issues, remanufacturing has been a significantly growing industry over the last decades, offering many potential business opportunities. In this paper, we investigate a crucial problem apparent in this industry, the remanufacturing lot-sizing problem with separate setups. We first discuss two reformulations of this problem, and remark an important property with regards to their equivalence. Then, we present a theoretical investigation of a related subproblem, where our analysis indicates that a number of flow cover inequalities are strong for this subproblem under some general conditions. We then investigate the computational effectiveness of the alternative methods discussed for the original problem. Detailed numerical results are insightful for the practitioner, indicating that in particular when the return variability increases or when the remanufacturing setup costs decrease relevant to manufacturing setup costs, the flow covers can be very effective.

1. Introduction

The increasing scarcity of earth's natural resources and disposal capacities present significant global environmental problems. The worryingly increasing amount of waste in many sectors is driven by technological development of new products that lead to the excessive consumption of raw materials as well as energy. Therefore, Original Equipment Manufacturers (OEMs) in various industries are increasingly in a transition towards applying circular economy principles. As a crucial component of circular economy, remanufacturing is an industrial process that brings used products to "at least an OEM functioning order with a warranty to match" (Ijomah, 2009), that is, rebuilding of a product using a combination of reused, repaired and new parts so that the remanufactured product is at least at the same quality level as the original manufactured product. Remanufacturing is the most advanced product recovery option that offers value-added recovery, extends product life cycle, reduces landfills and energy consumption (Shi et al., 2011), and takes place in industries ranging from aviation equipment and medical instruments to copiers, computers and automotive parts (Matsumoto and Umeda, 2011).

Remanufacturing also offers a big potential for employment and profitable business opportunities. As reported by the Centre for Remanufacturing and Reuse, UK remanufacturing industry approximately contributes £5 billion per year to the economy, creates jobs for more than 500,000 people, and saves 270,000 tonnes of materials (mostly metals) from recycling or scrapping (Chapman et al., 2009). Parker et al. (2015) also note similar benefits in the European context, where an annual contribution of €30 billion is estimated to the EU economies from nine key remanufacturing sectors.

Remanufacturing can be operated either under a dedicated model (i.e., remanufacturing only) or a hybrid model (remanufacturing combined with its forward production). As noted by Li et al. (2009), remanufacturing in North America generally adopts a dedicated model, in contrast to most remanufacturing operations in European countries employing a hybrid model. OEMs who opt out their operations to third-party remanufacturers are owing to the fact that remanufacturing is much more reactive and less visible compared to manufacturing. Remanufacturing involves inherently complex manufacturing process that requires specific tools, high technology machinery and multi-skilled labours. Furthermore, the three main sub-processes of disassembly, reprocessing and reassembly incorporate higher degree of uncertainty associated with end-of-life products, which further complicate
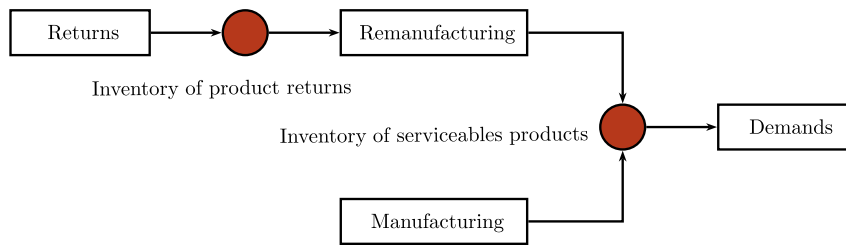
Fig. 1. Material flow in a hybrid remanufacturing environment.

production planning and control (Guide, 2000).

Complications become even more crucial in hybrid systems. Therefore, in this paper, we investigate production planning of the hybrid systems, where the remanufacturing and manufacturing have separate setups and the demands can be fulfilled by both remanufactured and new products, as illustrated in Fig. 1. Since our aim is to gain a better analytical understanding of the inherent structure of these problems, we study in this paper a single-item uncapacitated version of the problem. In order to remain consistent with the notation used in the literature, we denote this problem as ELSRs (Economic Lot Sizing Problem with Remanufacturing and Separate Setups). Next, we present a brief literature review.

1.1. Literature review

Studying basic problems that have obvious limitations compared to a real-world problem has a number of benefits. First of all, as often is the case, only simplified problems allow a fully analytical approach, such as polyhedral analysis, and any extra levels of complexity can easily hinder such analysis. Moreover, a good understanding of a basic problem can often be extended to more complicated problems in terms of solution methods, so that the effectiveness of such methods can be substantially improved. Therefore, a number of researchers presented such analysis in the lot-sizing literature. Milestone contributions that are also effectively extended to more complicated problem include the (ℓ, S) inequalities of Barany et al. (1984) providing the full description for single-item uncapacitated problems, and the facility location (Krarup and Bilde, 1977) and shortest path (Eppen and Martin, 1987) reformulations that are compact for single-item uncapacitated problems. More recent noteworthy theoretical achievements include Akbalik and Pochet (2009) and Küçükyavuz and Pochet (2009), and a thorough review of single-item problems is presented by Brahimi et al. (2017). We also refer the interested reader to Akartunalı et al. (2016) and Doostmohammadi and Akartunalı (2018) for recent overviews on complex lot-sizing problems.

Lot-sizing in remanufacturing has been an increasingly more active research area over the last decade. The earlier works of Richter and Weber (2001) and Golany et al. (2001) investigated theoretical properties under special conditions such as linear or time-invariant costs, and proposed polynomial or heuristic algorithms for such cases. The study of van den Heuvel (2004) presented the first complexity analysis of the problem, which was recently extended by Akartunalı and Arulselvan (2016), and Yang et al. (2005) established complexity results for the case of concave costs. Teunter et al. (2006) proposed a polynomial dynamic programming algorithm for the case with joint setups and time-invariant costs, and Pineyro and Viera (2010, 2014) provided complexity analysis in case of a disposal option. Pan et al. (2009) showed some important properties for capacitated variants such as constant capacities and proposed a pseudo-polynomial algorithm, and Baki et al. (2014) proposed a constructive heuristic following an alternative reformulation. As proven by Retel Helmrich et al. (2014), the problem with separate setups is $\mathscr{NP}-$ hard if even all costs are time invariant, which provides further motivation for the problem we study. The recent papers of Cunha et al. (2017); Sifaleras and Konstantaras

(2017) propose innovative solution methods for remanufacturing problems with multiple items, and the study of Sahling (2013) proposes a column generation approach for the case with capacities. Finally, it is noteworthy to remark the recent studies of Agra et al. (2016) and Attila et al. (2017) proposing robust optimization approaches in order to address some inherent uncertainties in these problems.

1.2. Contributions and outline of the paper

The main contributions of our paper are as follows: i) Following the recent work of Retel Helmrich et al. (2014) presenting a shortest path (SP) reformulation and of Cunha and Melo (2016) presenting a facility location (FL) reformulation for ELSRs, we note an important theoretical property with regards to the equivalence of the LP relaxations of these two reformulations; ii) We study a related subproblem defined as a mixed integer set, and theoretically prove the strength for some crucial flow cover inequalities for this set; and iii) We present a detailed computational analysis for both theoretical contributions, and in particular highlight the strengths and weaknesses of the inequalities, in order to provide valuable insights to the practitioners.

In the next section, we present a simple formulation for ELSRs. Then, in Section 3, we discuss shortest path and facility location reformulations, and remark their theoretical equivalence. Section 4 is a thorough theoretical analysis for a related subproblem, indicating that most of the flow cover inequalities are strong under reasonable general conditions. We present the results of thorough computational experiments in Section 5, which is in particular focused on the effectiveness of flow cover cuts. Finally, we conclude with some key remarks and potential future directions in Section 6.

2. Problem definition and formulation

Following the nomenclature of Teunter et al. (2006), we briefly describe ELSRs, and present a mathematical formulation in this section. The problem considers remanufacturing and manufacturing operations on separate production lines, each with its own setup and production costs, where the quality of remanufactured products is assumed to be as good as manufactured products. Given returns and customer demands over a finite planning horizon with n periods, the problem seeks to find a feasible production plan such that the total costs (production, inventory and setup costs) are minimized while demands are satisfied on time (and hence no backlogging allowed). A network representation of ELSRs is illustrated in Fig. 2, where the higher level shows the flow of returns and the lower level shows the flow of serviceable products (i.e., those remanufactured/manufactured). Next, we define the notation used in the remainder of the paper.

Decision Variables:
x_t^r: number of remanufactured products produced in period t;
x_t^m: number of manufactured products produced in period t;
y_t^r: = 1 if remanufacturing takes place in period t, = 0 otherwise;
y_t^m: = 1 if manufacturing takes place in period t, = 0 otherwise;
I_t^r: inventory of returns held at the end of period t;
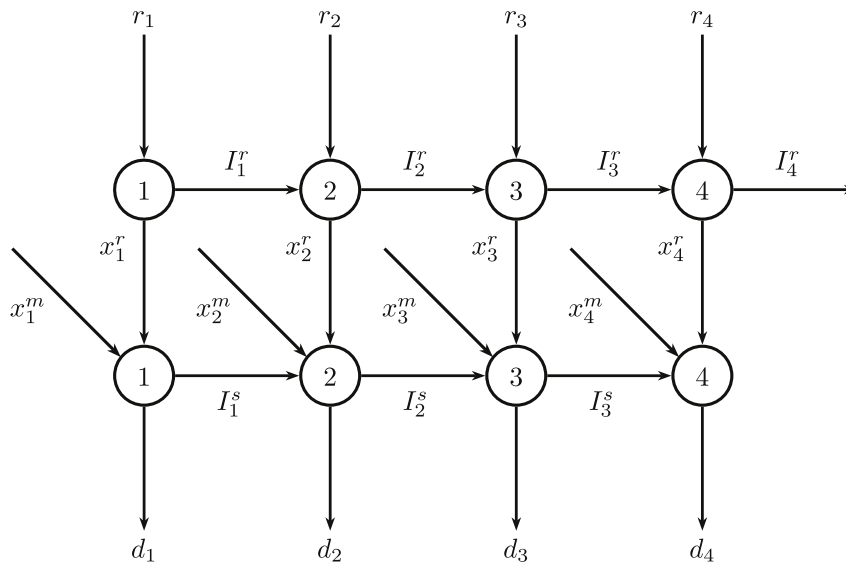I_t^s: inventory of serviceable products held at the end of period t.

Fig. 2. Network representation of ELSRs problem with period, $n = 4$ (adapted from Retel Helmrich et al. (2014)).

Parameters

p_t^r: unit production cost of remanufacturing in period t;

p_t^m: unit production cost of manufacturing in period t;

h_t^r: unit holding cost for inventory of returns in period t;

h_t^s: unit holding cost for inventory of serviceable products in period t;

K_t^r: setup cost for remanufacturing in period t;

K_t^m: setup cost for manufacturing in period t;

d_t: amount of demand in period t, where $d_{t,t'} = \sum_{i=t}^{t'} d_i$;

r_t: amount of returns in period t, where $r_{t,t'} = \sum_{i=t}^{t'} r_i$.

Let $N = \{1, ..., n\}$. The mathematical formulation of ELSRs is as follows:

$$Z = \min \sum_{t=1}^{n} (K_t^r y_t^r + K_t^m y_t^m + p_t^r x_t^r + p_t^m x_t^m + h_t^r I_t^r + h_t^s I_t^s) \tag{1}$$

$$\text{s.t. } I_t^r = I_{t-1}^r + r_t - x_t^r \quad \forall\, t \in N \tag{2}$$

$$I_t^s = I_{t-1}^s + x_t^r + x_t^m - d_t \quad \forall\, t \in N \tag{3}$$

$$x_t^r \leq \min\{r_{1,t}, d_{t,n}\} y_t^r \quad \forall\, t \in N \tag{4}$$

$$x_t^m \leq d_{t,n} y_t^m \quad \forall\, t \in N \tag{5}$$

$$y_t^r, y_t^m \in \{0,1\}^n \quad \forall\, t \in N \tag{6}$$

$$x_t^r, x_t^m, I_t^r, I_t^s \geq 0 \quad \forall\, t \in N \tag{7}$$

$$I_0^r = I_0^s = 0 \tag{8}$$

The objective function (1) minimizes the sum of setup and production costs for remanufacturing and manufacturing, and holding costs for returns and serviceable products. The constraints (2) and (3) represent flow conversation for returns and serviceable products, respectively, so that demands are satisfied on time. The constraints (4) and (5) ensure setups take place when remanufacturing and manufacturing is strictly positive, respectively. Finally, (6) and (7) state the integrality and nonnegativity conditions. Without loss of generality and in line with previous work such as Retel Helmrich et al. (2014), we assume no initial inventory for returns and serviceable products, as stated in (8). Then, the feasible region X of the basic formulation for ELSRs is defined as $X = \{(x^r, x^m, y^r, y^m, I^r, I^s) \mid (2) - (8)\}$, and the problem ELSRs as $Z = \min\{(1) \mid (x^r, x^m, y^r, y^m, I^r, I^s) \in X\}$.

3. Extended reformulations for ELSRs

As proven by Retel Helmrich et al. (2014), the ELSRs problem is $\mathcal{NP}$-hard for time-invariant cost parameters, and therefore, any analytical insights are essential for developing effective solution methods for problems in practice. In this section, similar to the analysis presented in Akartunalı and Miller (2012) for multi-item lot-sizing problems, we aim to establish theoretical strengths of extended reformulations, which have proven to be very insightful for many integer programming problems in the past. We consider two extended reformulations for ELSRs, which we also evaluate computationally later in the paper.

First of all, we consider the shortest path (SP) reformulation, as originally proposed by Eppen and Martin (1987) for the uncapacitated single-item lot-sizing problem, for which it is proven to obtain integer solutions. We note that the study of Retel Helmrich et al. (2014) has already presented an SP reformulation for ELSRs, where they also developed a partial SP reformulation for the sake of computational efficiency, and Syed Ali (2016) has also investigated this reformulation further. Therefore, we omit to present a full formulation of this reformulation in this paper, and refer the interested reader to these references. In the remainder of the paper, we denote the optimization problem using the SP reformulation by Z_{SP}, and use the superscript LP to indicate the LP relaxation value of a problem, e.g., Z_{SP}^{LP} indicating the LP relaxation value of the SP reformulation.

The second reformulation we consider is a facility location (FL) reformulation (Krarup and Bilde, 1977). We note that to the best of our knowledge, Cunha and Melo (2016) is the first study presenting a FL reformulation for ELSRs problem. This reformulation disaggregates the production variables x_t^r and x_t^m by defining new decision variables with respect to the destinations of products, as follows:

$w_{t,t'}^{sr}$: amount of remanufactured products in period t to satisfy the demand in period $t' \geq t$;

$w_{t,t'}^{sm}$: amount of manufactured products in period t to satisfy the demand in period $t' \geq t$.

In addition, we introduce new decision variables to identify the origin of returns, which can be also linked to the variables $w_{t,t'}^{sr}$:

$w_{t,t'}^r$: amount of returns in period t, which are remanufactured in period $t' \geq t$.

Then, the following constraints are added into the original formulation.

$$w_{t,t'}^{sr} \leq d_{t'} y_t^r \ \forall \ t \in [1, n], \quad \forall \ t' \in [t, n] \tag{9}$$

$$w_{t,t'}^{sm} \leq d_{t'} y_t^m \ \forall \ t \in [1, n], \quad \forall \ t' \in [t, n] \tag{10}$$

$$\sum_{t=1}^{t'} \left(w_{t,t'}^{sr} + w_{t,t'}^{sm} \right) = d_{t'} \quad \forall \ t' \in [1, n] \tag{11}$$

$$w_{t,t'}^r \leq r_t y_t^r \ \forall \ t' \in [1, n], \quad \forall \ t \in [1, t'] \tag{12}$$

$$\sum_{t'=t}^{n} w_{t,t'}^r \leq r_t \quad \forall \ t \in [1, n] \tag{13}$$

$$x_t^r = \sum_{t'=1}^{t} w_{t',t}^r = \sum_{t'=t}^{n} w_{t,t'}^{sr} \quad \forall \ t \in [1, n] \tag{14}$$

$$x_t^m = \sum_{t'=t}^{n} w_{t,t'}^{sm} \quad \forall \ t \in [1, n] \tag{15}$$

$$w^{sr}, w^{sm}, w^r \geq 0 \tag{16}$$

Constraints (9), (10) and (12) enforce setup variables to take correct values. Constraint (11) ensures the satisfaction of demand, and constraint (13) limits the production of remanufactured products by the number of returns. Constraints (14) and (15) details relationships between the old and new variables, and (16) denotes the nonnegativity requirements. We note that this reformulation has only very minor differences (such as equations for demand satisfaction and upper bounds on $w_{t,t'}^{sr}$ variables) from the multicommodity formulation of Cunha and Melo (2016). The feasible region and objective function associated with this reformulation can be defined as $X_{FL} = \{(x^r, x^m y^r, y^m, I^r, I^s, w^r, w^{sr}, w^{sm}) | (2) - (8), (9) - (16)\}$, and $Z_{FL} = \min\{(1) | (x^r, x^m y^r, y^m, I^r, I^s, w^r, w^{sr}, w^{sm}) \in X_{FL}\}$, respectively. Moreover, following the previously introduced notation, Z_{FL}^{LP} denotes the LP relaxation value of the FL reformulation. We conclude this section with the following important result.

Proposition 1. $Z^{LP} \leq Z_{FL}^{LP} = Z_{SP}^{LP}$.

In words, the lower bounds provided by SP and FL reformulations are equal to each other, and they do provide at least as strong a lower bound as the basic formulation of the problem. This equivalence is very important in practice, as it allows us two options to choose from when computationally evaluating a problem. The proof is similar to a result presented in Akartunalı and Miller (2012), and it is also presented in detail in Syed Ali (2016), hence we omit it. We also note that this equivalence is remarked in the computational results of Cunha and Melo (2016).

4. Polyhedral analysis for ELSRs

Polyhedral analysis is an analytical tool set, allowing us to evaluate the strength of linear inequalities by checking whether they define the "facets" of a feasible region or not, and hence it is valuable to understand the complications inherent in challenging integer programming problems. Therefore, in this section, we investigate the polyhedral structure of a mixed integer set arising from the feasible set of ELSRs, which considers two knapsack sets simultaneously based on the well-known single node fixed-charge network (SNFCN). Before explaining this further, we first define this mixed integer set formally:

$$X^s = \left\{ (x^r, x^m, y^r, y^m) \in \mathbb{R}_+^n \times \mathbb{R}_+^n \times B^n \times B^n \left| \sum_{t \in N} x_t^r \right. \right.$$

$$\left. \leq R, \sum_{t \in N} (x_t^r + x_t^m) \geq D, x_t^r \leq m_t^r y_t^r, x_t^m \leq m_t^m y_t^m, \forall \ t \in N \right\}.$$

Here, $R = \sum_{t=1}^n r_t$ denotes the total amount of returns, and

$D = \sum_{t=1}^n d_t$ is the total amount of demands. Note that the big-M constraints can be structured based on the initial formulation, using $m_t^r = \min\{r_{1,t}, d_{t,n}\}$ and $m_t^m = d_{t,n}$ for any $t \in N$. In order to investigate the polyhedral set $conv(X^s)$, we first refer to Padberg et al. (1985) and the SNFCN set defined as follows:

$$X_{\nabla} = \left\{ (x, y) \in \mathbb{R}_+^n \times B^n \left| \sum_{t \in N} x_t \nabla d, x_t \leq m_t y_t, \forall \ t \in N \right. \right\},$$

where $\nabla \in \{\leq, =, \geq\}$ and $conv(X_{\nabla})$ is denoted by P_{∇}. Using "surrogate knapsack" problem and the associated knapsack polytope $K = conv\{y \in \mathbb{R}^n | \sum_{t \in N} m_t y_t \geq d, y_t \in \{0,1\}, \forall \ t \in N\}$, which is a relaxation of $P_{\geq}$ and $P_{=}$, the authors show that almost all facets of K are facets for $P_{\geq}$. Moreover, a class of "flow cover" facets for $P_{=}$ can be described from a large class of valid inequalities for $P_{\leq}$. These insights will be beneficial to our polyhedral study of $conv(X^s)$. Next, we state our assumptions:

(i) $D > R > 0$,
(ii) $\sum_{t \in N \setminus \{k\}} m_t^m \geq D$ for each $k \in N$,
(iii) $D = m_1^m > m_2^m > \cdots > m_n^m > 0$,
(iv) $\sum_{t \in N} m_t^r > R$.

Note that the second assumption allows manufacturing to satisfy all demands even when it is set to zero in any chosen period, the third assumption simply uses the structure of ELSRs used to define big-M parameters, and the last assumption ensures that all returned products can indeed be remanufactured.

Proposition 2. $dim(conv(X^s)) = 4n$.

We note that we provide the proofs of this and any following propositions in the Online Supplement, in order not to disrupt the flow of the paper while ensuring that the technically interested reader can find these details. We also note that the proofs of Corollaries are straightforward by following the results presented in Padberg et al. (1985).

Proposition 3. The trivial facet-defining inequalities for $conv(X^s)$ (and their facet-defining conditions if applicable) are:

(i) $x_i^r \geq 0, \forall \ i \in N$.
(ii) $x_i^r \leq m_i^r y_i^r, \forall \ i \in N$.
(iii) $x_i^m \leq m_i^m y_i^m, \forall \ i \in N$.
(iv) $y_i^m \leq 1, \forall \ i \in N$.
(v) $y_i^r \leq 1, \forall \ i \in N$.
(vi) $\sum_{t \in N} x_t^r \leq R$, when $\sum_{t \in N \setminus \{k\}} m_t^r > R$ for each $k \in N$ holds.
(vii) $\sum_{t \in N} x_t^r + \sum_{t \in N} x_t^m \geq D$.
(viii) $x_i^m \geq 0, \forall \ i \in N$, when $\forall \ k \in N \setminus \{i\}, \sum_{t \in N \setminus \{i,k\}} m_t^m + \sum_{i \in N} m_t^r \geq D$ holds.

The following definitions will be used throughout the paper.

Definition 1. A cover set can be defined as follows:

- A set $S^r \subseteq N$ is a cover for R if $\lambda_1 = \sum_{t \in S^r} m_t^r - R > 0$.
- A set $S^m \subseteq N$ is a cover for $D - R$ if $\lambda_2 = \sum_{t \in S^m} m_t^m - (D - R) > 0$.
- For $S^r, S^m \subseteq N$ such that $S^r \cap S^m = \varnothing$, pair (S^r, S^m) is a cover for D if $\lambda_3 = \sum_{t \in S^r} m_t^r + \sum_{t \in S^m} m_t^m - D > 0$.

We also define $(x)^+ = \max\{x, 0\}$.

It can be readily seen that set $X_{\leq}$ is a relaxation of set X^s. Thus, any valid inequality for $X_{\leq}$ is also valid for X^s. Next, we present the well-known valid inequalities for $X_{\leq}$, and refer the interested reader to (Padberg et al., 1985) for validity proofs of these inequalities. Our theoretical contribution comes from the fact that, under certain and general conditions, these inequalities are facet-defining for $conv(X^s)$.

Corollary 1. Let $S^r \subseteq N$ be a cover for R, with $\overline{m^r} = \max_{t \in S^r} m_t^r$. Then, the following inequality (called returns cover inequality) is valid for X^s.

$$\sum_{t \in S^r} x_t^r - \sum_{t \in S^r} (m_t^r - \lambda_1)^+ y_t^r \leq R - \sum_{t \in S^r} (m_t^r - \lambda_1)^+. \tag{17}$$

Proposition 4. *Let $S^{r+} = \{t \in S^r | m_t^r - \lambda_1 > 0\}$. If $|S^{r+}| \geq 2$, then* (17) *defines a facet of $conv(X^s)$.*

It is natural to extend inequality (17) as follows.

Corollary 2. *Let $S^r \subseteq N$ be a cover for R with $\overline{m^r} = \max_{t \in S^r} m_t^r$ and $L^r \subseteq N \setminus S^r$. Assume $\overline{m_t^r} = \max(\overline{m^r}, m_t^r)$ for all $t \in L^r$. Then the following inequality (called returns-extended cover inequality) is valid for X_s.*

$$\sum_{t \in S^r \cup L^r} x_t^r - \sum_{t \in S^r} (m_t^r - \lambda_1)^+ y_t^r - \sum_{t \in L^r} (\overline{m_t^r} - \lambda_1) y_t^r \leq R - \sum_{t \in S^r} (m_t^r - \lambda_1)^+. \tag{18}$$

Proposition 5. *The inequality* (18) *is facet-defining for $conv(X^s)$ if both $0 < \overline{m^r} - \lambda_1 < m_t^r \leq \overline{m_t^r}$ for any $t \in L^r$ and the condition of Proposition 4 hold.*

Next, we investigate some well-known inequalities originally proposed for $X_{\geq}$, which is again an obvious relaxation of set X^s.

Corollary 3. *Let $S^m \subseteq N$ be a cover for $D - R$. Then, the following inequality (called demands cover inequality) is valid for X^s.*

$$\sum_{t \in N \setminus S^m} x_t^m \geq \sum_{t \in S^m} (m_t^m - \lambda_2)^+ (1 - y_t^m). \tag{19}$$

We note that the validity of these inequalities follows simply the fact that $\{(x^m, y^m) | \sum_{t \in N} x_t^m \geq D - R, 0 \leq x_t^m \leq m_t^m y_t^m\}$ is a relaxation of X^s.

Proposition 6. *Let $S^{m+} = \{t \in S^m | m_t^m - \lambda_2 > 0\}$. If $|S^{m+}| \geq 1$, $\sum_{t \in N \setminus S^m} m_t^m > \max_{t \in S^m} m_t^m - \lambda_2$, and $\sum_{t \in N} m_t^r > R + \max_{t \in N} m_t^r$, then the inequality* (19) *defines a facet for $conv(X^s)$.*

Corollary 4. *Let $S^m \subseteq N$ be a cover for $D - R$, and $L^m \subseteq N \setminus S^m$ such that $\overline{m^m} = \max_{t \in S^m} m_t^m > \lambda_2$ and $\overline{m_t^m} = \max\{m_t^m, \overline{m^m}\}$, $\forall t \in L^m$. Then, the following inequality (called demands-extended cover inequality)*

$$\sum_{t \in N \setminus (S^m \cup L^m)} x_t^m + \sum_{t \in L^m} (\overline{m_t^m} - \lambda_2) y_t^m \geq \sum_{t \in S^m} (m_t^m - \lambda_2)^+ (1 - y_t^m), \tag{20}$$

is valid for X^s.

Proposition 7. *Let $S^{m+} = \{t \in S^m | m_t^m - \lambda_2 > 0\}$. If $0 < \overline{m^m} - \lambda_2 < m_t^m \leq \overline{m_t^m}$ for any $t \in L^m$, $\sum_{t \in N \setminus (S^m \cup L^m)} m_t^m > \max_{t \in S^m} m_t^m - \lambda_2$, and $\sum_{t \in N} m_t^r > R + \max_{t \in N} m_t^r$, then the inequality* (20) *defines a facet for $conv(X^s)$.*

Corollary 5. *For $S^r, S^m \subseteq N$, let (S^r, S^m) be a pair cover for D. Then, the inequality (called returns-and-demands cover inequality)*

$$\sum_{t \in N \setminus S^r} x_t^r + \sum_{t \in N \setminus S^m} x_t^m \geq \sum_{t \in S^r} (m_t^r - \lambda_3)^+ (1 - y_t^r)$$
$$+ \sum_{t \in S^m} (m_t^m - \lambda_3)^+ (1 - y_t^m), \tag{21}$$

is valid for X^s.

Proposition 8. *Let $S^{r+} = \{t \in S^r | m_t^r - \lambda_3 > 0\}$ and $S^{m+} = \{t \in S^m | m_t^m - \lambda_3 > 0\}$. If $\lambda_3 > 0$, $\sum_{t \in N \setminus S^r} m_t^r < R$, $\exists i_1 \in S^{r+}$ such that $m_{i_1}^r - \lambda_3 \leq m_t^r$ holds $\forall t \in N \setminus S^r$ and $m_{i_1}^r - \lambda_3 \leq m_t^m$ holds $\forall t \in N \setminus S^m$, $|S^{r+}| + |S^{m+}| \geq 2$ and $N \setminus S^r \neq \emptyset \neq N \setminus S^m$, then the inequality* (21) *is facet-defining for $conv(X^s)$.*

We note that we omit a detailed discussion of exact separation algorithms for these inequalities, as they are straightforward extensions of the minimization problem proposed by Padberg et al. (1985) (or alternatively the maximization problem proposed by Agra and Doostmohammadi (2014)) for flow cover inequalities. To conclude this section, we note that the feasible region of the basic formulation for ELSRs is now updated with the additional flow cover inequalities and hence can be written as

$X^{ext} = \{(x^r, x^m, y^r, y^m, I^r, I^s) | (2), (3), (5) - (8), (17) - (21)\},$

with the objective function $Z^s = \min\{(1) | (x^r, x^m, y^r, y^m, I^r, I^s) \in X^{ext}\}$.

5. Numerical results

In this section, we investigate two aspects of the theory presented from a computational perspective. First of all, we provide a brief evaluation of the two different reformulations presented in Section 3 and compare it to a method presented in the literature. Then, in line with the primary theoretical contribution of our paper, we provide a thorough discussion with detailed numerical results concerning the facet-defining inequalities of Section 4 and their strength for a range of problems. All computations are run on a PC with Intel Core® i7-4500U CPU 2.40 GHz processor and 8 GB RAM, and all formulations and separation algorithms are implemented in FICO® Xpress-MP Optimization Suite (Mosel version 7.7) without any solver cuts.

5.1. Brief analysis of extended reformulations

For the first set of tests, we experiment with 360 test instances presented in Retel Helmrich et al. (2014). These problems have planning horizons of 25, 50 and 75 periods, and demand is generated using a normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 50$. On the other hand, the returns parameters are generated using a normal distribution with three different parameter settings: i) low return ($\mu = 10$, $\sigma = 5$), ii) medium return ($\mu = 50$, $\sigma = 25$), and iii) high return ($\mu = 90$, $\sigma = 45$). The setup cost for both remanufacturing and manufacturing is set to 125, 250, 500 and 1000, and holding costs for both product returns and manufactured products are set to 1 for all test instances. We note that equal holding costs are used primarily for the sake of simplicity, as varying these values make only negligible difference based on our preliminary experimentation, but also holding costs in these settings may depend on other factors than the pure cost of products, e.g., manufactured items may be neatly packaged and stacked up while returned items need specific shelf space, and hence we did not want to put specific cost emphasis on either one. Following the model presented in Teunter et al. (2006) and the argument that involving variable production costs may distort the balance between setups and inventory holding significantly, we set the variable costs of manufacturing and remanufacturing to zero for the sake of computational simplicity. The default time is set to 600 s for each test instance.

As noted theoretically before, facility location (FL) reformulation and shortest path (SP) reformulation provide identical lower bounds at the root node. From our computational experience, we found out that FL reformulation performs better in the given limited run time, and therefore, we present here only the duality gaps achieved by this reformulation. As a benchmark from the recent literature, we report duality gaps achieved by (ℓ, S, WW) inequalities of Retel Helmrich et al. (2014) using the same time limit. Here, we note that our aim is not to evaluate different methods from the perspective of computational times but rather understand the strengths of different approaches. These results are presented in Table 1, where the FL reformulation often provides a much stronger gap compared to (ℓ, S, WW) inequalities, where each entry shows the average of 10 instances.

The results in Table 1 indicate that the advantage of using FL reformulation varies significantly between different settings of the problem. We first observe that the performance of FL increases when the setup costs increase, which indicates that FL is more capable of dealing with fixed costs. On the other hand, a more obvious performance strength for FL can be seen when low levels of returns are present in the system. This makes intuitive sense, since at low return, the manufacturing will dominate the production more than remanufacturing, and therefore, a valid inequality involving returns naturally becomes less effective. We also note that in case of low returns, and in particular when number of periods is limited to 25, many of the instances could be

Table 1
Comparisons of average duality gaps after 600 s.

	Low Return				Medium Return				High Return			
Setup Cost =	125	250	500	1000	125	250	500	1000	125	250	500	1000
n = 25, lsWW	21.77	19.54	18.57	17.36	19.59	17.77	16.72	16.03	12.73	12.83	12.88	13.34
n = 25, FL	0.99	0.88	0.84	0.14	5.92	5.48	4.2	3.63	9.58	9.03	7.71	6.12
n = 50, lsWW	22.37	20	19.41	17.53	21.3	19.19	16.7	15.35	11.35	11.66	12.19	11.34
n = 50, FL	1.67	1.02	1.06	0.67	6.98	6.31	4.7	3.79	7.35	7.86	7.84	6.25
n = 75, lsWW	22.55	20.64	19.99	19.83	22.76	20.02	19	17.63	12.65	12.33	13.31	11.53
n = 75, FL	1.34	1.19	0.97	0.74	7.93	6.72	5.35	4.41	8.53	8.46	8.72	6.89

Table 2
[Low] Numerical results for facet-defining inequalities of the ELSRs problem.

n	K_t^r	Root	% root gap closed		# cuts added (FC)					$(\ell, S) + FC$
		Gap (%)	FL	$(\ell, S) + FC$	R	RE	D	DE	RD	vs. FL
2	10	28.8541	31.4328	99.8529	0	0	2	0	1	68.5072
	30	26.5228	40.6872	100	0	0	2	0	1	59.3128
	50	22.7890	50.0475	100	0	0	2	0	2	49.9525
	90	14.1078	66.7907	100	0	0	2	0	2	33.2093
	200	0.9273	100	100	0	0	0	0	0	0
4	10	23.9931	39.0948	89.6031	0	0	3	0	1	56.5679
	30	23.5822	59.2905	98.6087	0	0	3	0	1	39.7904
	50	21.8994	75.8580	99.5070	0	0	3	0	1	23.8318
	90	19.3709	94.4144	99.8686	0	0	2	0	0	5.4741
	200	12.2554	100	100	0	0	0	0	0	0
	500	6.3232	100	100	0	0	0	0	0	0
6	10	35.0957	70.9881	76.6853	0	0	3	4	0	7.5911
	30	34.9737	79.5150	83.9535	0	1	3	4	0	5.4238
	50	34.7577	85.1482	88.3908	1	1	2	2	0	3.7090
	90	33.7086	90.1658	93.2182	2	3	0	2	0	3.3214
	200	28.8887	98.8462	99.1722	0	1	0	0	0	0.3401
	500	21.5066	100	100	0	0	0	0	0	0
8	10	39.8131	70.8516	72.5591	0	0	3	6	0	2.8802
	30	38.6172	77.0986	78.2567	0	0	3	6	0	1.7400
	50	37.0794	82.2884	83.0450	1	2	5	5	0	1.0082
	90	35.3551	86.9766	87.5638	2	3	1	4	0	0.7226
	200	32.8643	90.4093	90.9856	1	6	1	4	0	0.6035
	500	24.7705	97.0644	97.0519	0	0	0	0	0	−0.0131
12	10	53.6100	69.2926	69.4444	0	0	1	2	0	0.2653
	30	52.3521	75.3886	75.1378	0	0	0	2	0	−0.3476
	50	50.9684	79.5795	78.9228	0	0	0	2	0	−0.9031
	90	48.4052	84.4135	83.1792	0	0	0	0	0	−1.6194
	200	42.8525	90.6798	89.1232	0	1	0	0	0	−1.8462
	500	34.2976	94.4163	93.2686	0	0	0	0	0	−1.2693

solved to optimality with FL reformulation, which can be observed from particularly low average percentage gaps.

5.2. Computational strength of flow cover inequalities

The primary aim of this section is to gain an empirical understanding with respect to the strength of the flow covers that are theoretically proven to be facet-defining under general conditions for the mixed integer set related to the actual problem. This necessitates us to computationally evaluate lower bounds generated by these inequalities, rather than aiming to design sophisticated frameworks for computational efficiency. Therefore, we implemented exact separation algorithms as noted in Section 4, which are embedded within a Branch-and-Cut framework. In order to facilitate this separation procedure to be computationally more effective, we added a version of (ℓ, S) inequalities (Barany et al., 1984) a priori into the original formulation presented in Section 2 before separating flow covers. The reason of using the original formulation with (ℓ, S) inequalities include: i) the problem size is kept small and in the original space of the problem, ii) (ℓ, S)

inequalities have been computationally effective in many lot-sizing problem settings, and iii) although the lower bound obtained by adding (ℓ, S) is theoretically weaker than the reformulations presented in the paper, our preliminary experiments indicated that in practice, it can often achieve a lower bound close to the bound obtained by these reformulations. Next, we briefly discuss our experimental design.

Since the exact separation algorithms are excessively time-consuming when the problem size gets bigger, we generated for this numerical experimentation problems with small planning horizons containing $n = 2, 4, 6, 8$ and 12 periods, which allows us to use the previous time limit of 600 s per problem with the guarantee of successful completion of the exact separation process. We set the setup costs for remanufacturing not higher than manufacturing, in order to motivate remanufacturing taking place. Therefore, we vary setup costs for remanufacturing between 10, 30, 50, 90, 200 and 500, whereas the setup costs for manufacturing are kept fixed at 500. Then, similar to previous section, we consider low, medium and high return variabilities, where return parameters, r_t, are generated randomly from uniform distributions with intervals [5, 15], [5, 35] and [5, 50], respectively.

Table 3
[Medium] Numerical results for facet-defining inequalities of the ELSRs problem.

n	K_t^r	Root	% root gap closed		# cuts added (FC)					$(\ell, S) + FC$
		Gap (%)	FL	$(\ell, S) + FC$	R	RE	D	DE	RD	vs. FL
2	10	45.0950	26.9075	94.4611	0	0	2	0	0	72.5786
	30	42.8864	31.1835	95.2767	0	0	2	0	0	68.3412
	50	40.2386	34.7207	96.4270	0	0	2	0	0	64.7389
	90	32.8733	40.3537	98.5051	0	0	2	0	0	59.2821
	200	15.9223	78.7215	100	0	0	1	0	0	21.2786
4	10	49.7403	35.3968	68.0173	0	0	2	1	0	52.1367
	30	47.6494	46.0011	74.8182	0	0	2	1	0	40.7644
	50	45.2408	53.6317	80.5717	0	0	2	1	0	34.9492
	90	40.3614	58.9151	90.0060	1	1	2	1	0	26.3695
	200	29.4189	91.5988	96.9788	0	0	1	0	0	5.8726
	500	3.2088	100	100	0	0	0	0	0	0
6	10	49.3599	49.9758	60.1883	0	0	2	5	0	18.4488
	30	47.9154	59.4096	67.6241	0	0	2	6	0	13.3061
	50	46.4717	66.3107	72.9706	0	0	2	6	0	9.9277
	90	43.8035	77.6025	82.3302	0	0	2	5	0	6.1446
	200	36.0323	89.7151	91.4156	0	1	1	0	0	2.1343
	500	24.3197	98.7498	100	1	1	0	0	0	1.2502
8	10	54.8327	56.2942	67.5875	0	0	5	6	0	19.2394
	30	52.2178	64.0220	72.3168	0	0	4	5	0	13.2511
	50	49.9530	69.0474	76.2635	0	0	5	5	0	10.3687
	90	46.1356	76.6449	77.4577	1	2	3	3	0	0.7057
	200	38.4081	83.8108	83.2445	1	1	1	2	0	−1.1520
	500	26.3794	91.7442	92.2815	1	0	0	1	0	0.4788
12	10	59.2999	52.0590	51.8342	0	0	2	6	0	1.2009
	30	57.5021	59.1354	60.3081	0	0	2	8	0	2.6055
	50	56.0083	64.7985	64.9259	0	0	2	8	0	0.5619
	90	53.6279	71.6369	69.5996	0	0	1	7	0	−3.0779
	200	46.4377	83.4929	79.4322	0	0	0	5	0	−5.6437
	500	34.2895	90.1208	87.6771	0	1	0	2	0	−2.8061

Moreover, the demand parameters, d_t, are generated from the interval [10,60], the holding costs for both product returns, h_t^r, and manufactured products, h_t^s, are generated randomly from [0.5, 2], and similar to previous setting, production costs for both remanufacturing and manufacturing are set to zero for the sake of simplicity. The variation between periods, remanufacturing setup costs, and return variabilities provide us a set of 90 combinations, and for each combination, we generate 5 test problems, therefore resulting in a total of 450 test instances.

We present detailed computational results in Tables 2–4 for low, medium and high return variabilities, respectively. The first column in these tables, n, indicates the number of periods, and the second column, K_t^r, lists remanufacturing setup costs. Next column presents the root gaps (in %) using the original formulation itself, followed by two columns indicating what percentage of this root gap was closed by using simply the facility location (FL) reformulation and by our separation algorithms that add flow cover cuts on top of (ℓ, S) inequalities ($(\ell, S) + FC$). In the following five columns, we provide details for number of cuts generated on average for each set of 5 instances, where we followed the same order of the inequalities as presented earlier: Returns cover (17), Returns-Extended cover (18), Demands cover (19), Demands-Extended cover (20), and Returns-and-Demands cover (21). The last column denotes the pairwise comparison of the average percentage of gap closed between (FL) and ($(\ell, S) + FC$), where a higher value indicates the superiority of flow covers and a negative value indicates the superiority of FL. We calculate this quantity using the equation

$$\frac{((\ell, S) + FC)\text{gap closed} - FL\text{ gap closed}}{((\ell, S) + FC)\text{gap closed}} \times 100$$

We note that we do not report computational times in these tables,

as our focus is to evaluate the strength of lower bounds with flow covers (rather than computational efficiency) and the impact of different inequalities. Although we noted earlier that the time limit of 600 s was sufficient to complete the exact separation process for all 450 test instances, it would be valuable to briefly comment on the computational effort needed for this process. For the smallest problems with $n = 2$, the average computational time for all instances (i.e., all parameter settings) is only 3.1 s, whereas this average increases to 97.6 s for instances with 12 periods. Although there is at times significant variation in computational times of individual instances due to randomization, we have not observed any significant differences in computational times due to different parameters settings.

As Tables 2–4 indicate, the average percentage of gap closed for both approaches slowly deteriorates from low return to high return. On the other hand, both FL and $(\ell, S)+$ FC close bigger percentages of the initial gaps as the setup cost for remanufacturing approaches the setup cost for manufacturing, while the root gaps decreases for these increasing remanufacturing setup costs. An important point to note is that an increase in the number of periods does not seem to affect the capability of either FL or $(\ell, S)+$ FC approaches with respect to closing the root gaps.

With regards to the numbers of flow cover cuts added, it seems that R and RE cuts become less often violated when the return variability is increased. It is noticeable from the results that the D and DE cuts are the most violated inequalities in most settings, which make them perfect candidates for a more computationally effective framework. Finally, we note that the contributions made by the RD cuts are only limited to instances with small number of periods, which is not helpful in case of practical settings.

When we compare relative performances of $(\ell, S)+$ FC and FL, we observe that the advantage of $(\ell, S)+$ FC deteriorates in general with

Table 4
[High] Numerical results for facet-defining inequalities of the ELSRs problem.

N	K_t^r	Root	% root gap closed		# cuts added (FC)					$(\ell, S) + FC$
		Gap (%)	FL	$(\ell, S) + FC$	R	RE	D	DE	RD	vs. FL
2	10	57.5876	24.5649	79.4025	0	0	1	0	0	71.2885
	30	54.3917	27.2093	80.5020	0	0	1	0	0	69.1753
	50	51.1973	29.2590	80.5231	0	0	1	0	0	66.8088
	90	44.7598	31.1094	80.7856	0	0	1	0	0	64.2208
	200	22.2754	37.5051	89.1584	0	0	1	0	0	59.0290
4	10	55.3992	32.8398	55.2681	0	0	2	1	0	47.3823
	30	52.6569	41.0820	62.1798	0	0	2	1	0	37.9533
	50	50.3678	46.6797	67.0364	0	0	2	1	0	32.8480
	90	45.3242	51.0317	73.3504	0	0	1	1	0	23.8992
	200	31.5619	81.5030	90.3511	0	0	1	0	0	10.4204
	500	13.7853	100	100	0	0	0	0	0	0
6	10	55.6137	45.5415	56.9709	0	0	2	3	0	20.1430
	30	52.9243	53.6717	61.9214	0	0	3	3	0	12.9995
	50	50.7867	59.5644	67.9446	0	0	3	3	0	11.4530
	90	47.5418	67.9697	73.8230	0	0	3	4	0	6.9820
	200	37.4267	82.9882	86.8705	1	2	2	2	0	4.4484
	500	23.4955	97.7132	97.9943	0	0	0	0	0	0.2914
8	10	56.9692	58.4371	62.8906	0	0	3	3	0	6.7093
	30	54.5931	64.2582	67.4552	0	0	3	3	0	4.6992
	50	51.6575	68.7684	71.4358	0	0	3	1	0	3.5303
	90	47.5161	75.2219	76.6774	0	0	2	1	0	1.8891
	200	39.3639	87.0346	86.4874	0	0	2	0	0	−0.7299
	500	25.7692	93.9996	94.9315	0	1	0	0	0	0.9840
12	10	63.0503	48.3811	53.9802	0	0	2	6	0	13.6969
	30	60.8534	57.1965	60.7815	0	0	2	5	0	7.1225
	50	59.0818	62.7979	65.0870	0	0	1	6	0	4.3014
	90	55.7265	70.2096	70.7575	0	0	1	6	0	1.1469
	200	48.3331	82.5790	81.1046	0	0	1	5	0	−1.7651
	500	34.6776	88.4368	87.5205	0	0	1	0	0	−1.0873

Table 5
Numerical results for larger [High] instances.

n	K_t^r	Root	% root gap closed		# cuts added (FC)					$(\ell, S) + FC$
		Gap (%)	FL	$(\ell, S) + FC$	R	RE	D	DE	RD	vs. FL
24	10	63.3546	59.5230	60.9084	0	0	1	8	0	2.3387
	30	63.9191	67.1785	67.5560	0	0	1	8	0	0.5511
	50	64.2062	71.4966	71.1044	0	0	1	8	0	−0.5887
	90	63.9053	77.2983	75.8265	0	0	1	5	0	−2.0085
	200	61.8837	83.5224	81.3234	0	0	0	6	0	−2.7801
	500	54.0915	91.1676	88.9633	0	0	0	4	0	−2.4821
48	10	73.1216	62.1041	61.8368	0	0	0	4	0	−0.4401
	30	74.0453	69.3531	68.1933	0	0	0	4	0	−1.7068
	50	74.3979	74.2511	72.3792	0	0	0	6	0	−2.5911
	90	74.6121	79.4440	76.5107	0	0	0	1	0	−3.8559
	200	73.1370	86.8438	83.8011	0	0	0	6	0	−3.6592
	500	68.1152	92.0747	90.2282	0	0	0	1	0	−2.0533

the increasing number of periods. On the other hand, we observe that $(\ell, S)+$ FC consistently has a much stronger performance when return variability increases and when setup cost for remanufacturing decreases. This results in superior performance even in the case of the biggest problems considered here with 12 periods, where the high return variability and lowest levels of remanufacturing setup costs show significant improvements over the FL, in contrast to poor performance seen in low variability case with 12 periods. This observation makes intuitive sense, since in both cases of increasing return variability and decreasing remanufacturing setup costs, remanufacturing will be more attractive and hence take place more often, resulting in more effective flow cover cuts.

Finally, we present further computational results with some bigger

size instances that have 24 and 48 periods in order to provide further insights. First of all, we run some preliminary tests in order to gain insight into running times necessary to complete exact separation, and we determined to use a 1 h time limit. For this test, we focus on the instances with high return variability, for two reasons: 1) gaps closed by the two methods were weakest in this set, and 2) flow covers performed better in this set, relative to the facility location reformulation. The same experimental design principles and parameter settings were used to generate these instances, and we present the results in Table 5. We first note that all instances with 24 periods could be completed with this time limit (hence providing us an exact as in earlier tests), and although the running times were more volatile for instances with 48 periods and a third of these 30 instances were not completed within this time limit,

it was still valuable to obtain an intermediate lower bound in such cases to observe general trends. As in previous tests, both FL and (ℓ, S)+ FC close bigger percentages of the initial gaps as the setup cost for re-manufacturing approaches the setup cost for manufacturing, while the root gaps decreases for these increasing remanufacturing setup costs. It is also encouraging to see that the capability of both FL and (ℓ, S)+ FC approaches remain stable with respect to closing the root gaps. In line with the previous observations, the D and DE cuts are again the most violated inequalities, and (ℓ, S)+ FC has a stronger performance when the setup cost for remanufacturing decreases. Due to the incomplete computational runs (also observed from the lower number of cuts generated), the performance of (ℓ, S)+ FC seems to be poorer than FL for instances with 48 periods, though the methods do not differ sig-nificantly with respect to % root gaps closed.

6. Conclusions and future research

In this paper, we discussed a lot-sizing problem with re-manufacturing, where setups for manufacturing and remanufacturing are separate. Following some recent work in this area, we presented two reformulations of this problem and presented an important theoretical property with regards to their strength, indicating that they do indeed provide equivalent lower bounds. Then, we presented a detailed poly-hedral analysis for a mixed integer set that is not only a relaxation of the original problem but also is an intersection of two knapsack sets based on the well-known single node fixed-charge network (SNFCN). This polyhedral analysis indicated that a number of flow cover cuts are facet-defining for this mixed integer set under some general conditions. In order to computationally evaluate the theory developed, we pre-sented detailed numerical results, which in particular indicated which types of inequalities are more advantageous and in which settings. These results indicated that these cuts are in particular strong and useful when the return variability increases and/or when the re-manufacturing setup costs decrease relevant to manufacturing setup costs. Such knowledge is crucial for building effective computational frameworks for real-world problems, when the problem in question becomes large-scale and contain further complicating elements.

There are some immediate research directions following this study. First of all, the rich structure of the mixed integer set presented in Section 4 deserves further analysis, not only for the particular set stu-died but also to extend this analysis to more sophisticated problems, such as when capacities are present on the remanufacturing, which is more often the case in practice than capacities for manufacturing, as the latter can often be satisfied with orders arriving from outside the system. One particularly interesting direction is the case when multiple items to be remanufactured share a resource with a constant capacity (i.e., remanufacturing with big bucket capacity) and no setup times. This case is also interesting from a practical point of view, as many SMEs with limited and often inflexible resources remanufacture for a range of manufacturers. From a theoretical perspective, the recent study of Doostmohammadi and Akartunalı (2018) on a manufacturing problem with similar properties hints at some strategies for further analysis, e.g., extending valid inequalities with a single period nature to valid inequalities covering consecutive periods. Though such theore-tical analysis is challenging due to its complex nature, it is very pro-mising to identify further strong valid inequalities, and therefore, we are currently investigating this.

Moreover, such theoretical understanding would help us to develop appropriate solution methods that are computationally capable to solve sophisticated real world problems. As we have already noted in the computational results, exact separation of valid inequalities is often time consuming, and there is a need to develop customized methods for computational efficiency. One immediate direction is to develop heur-istic separation algorithms in order to generate only a small number of effective inequalities, and then integrate these with an overall suc-cessful heuristic method, such as the variable descent algorithm of

Sifaleras and Konstantaras (2017). Such integrated methods often benefit from the strengths of both exact and heuristic methods, e.g., valid inequalities directing the heuristic search in more promising re-gions while computational times not being hindered thanks to heuristic components.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.ijpe.2018.07.002.

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