On the Impact of the Physical Topology on the Optical Network Performance

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***Abstract*—Network performance optimization is a poly-parametric task where usually the physical network topology is already provided and given. The latter is mainly defined by the geographical needs and the business models developed in a region that reflect the particular data traffic requirements. However, during the design of a new network, the physical topology can be considered as an additional parameter, and this work aims to study the potential impact of a network topology in the performance of the network. Two relevant metrics of optical WDM networks are evaluated, throughput and blocking ratio. Through the simulation of 350k implementations of more than 500 network topologies, we demonstrate a performance variation on networks with the same number of nodes and edges up to 73.9% on the total network load under same levels of requests blocking ratio, and up to 17.3% on the network throughput.**

***Index Terms*—Network Design, Network Topology, Optical Networks.**

I. Introduction

Optical networks are still the dominant backbone networking technology to offer connectivity in the Metro and Core areas providing long range and high capacity communication infrastructure. DWDM technology is an essential enabler that apart from the high capacity offers increased survivability and network adaptability. New network components such as the ROADM (Reconfigurable Optical Add-Drop Multiplexer) and the OXC (optical cross-connectors) have turned the old-fashion static optical networks into dynamic environments [1]. In this context, the planning has become a very complex activity since the bandwidth allocation on a virtual topology offered through the different wavelengths has to be mapped to a physical optical fiber topology.

Current optical networks exploit the ROADM technology to allow the deployment of more versatile network topologies that go beyond traditional point-to-point, star and ring implementations, enabling more interesting hybrid mesh topologies. However, this extra flexibility adds a new degree of freedom in the network design process that needs to be considered together with the other critical parameters on which a network is evaluated, e.g., network capacity, reliability, cost, scalability, and operational simplicity. To this end, the physical topology can be part of the design of optical networks, that is inherently a multi-objective optimization problem with at least two conflicting goals: to minimize costs and to maximize performance. Choosing the correct physical topology may help in relaxing the unavoidable trade-offs that network designers often face while pursuing an optimal balance between performance and reduced costs.

While the effects of the network topology have been studied with respect to the survivability of a network [2], [3], its impact on the overall network performance has not yet been adequately assessed, mainly because in a typical network design process the topology is considered as an input of the problem [4], [5], [6]. Previous works have revealed how the topology affects critical aspects of the network design, such as wavelength requirements [7], [8] (i.e., the minimum resources needed to support a given traffic demand) and network reliability [2] (e.g., its fault tolerance). However, the impact of physical topology versus network throughput has yet to be evaluated. Although [9] have measured and compared the throughput of several popular High-Performance Computers and Data Center topologies, to the best of our knowledge, no similar studies have been proposed for optical metro/core networks.

In this paper we aim to introduce a novel perspective on the network optimization problem and to suggest considering the physical fiber topology in the set of network optimization parameters as a way to achieve a better performance/cost trade-off. With the use of graph theory and simulations, we demonstrate the significant influence of the topology on the performance of optical networks. Extensive simulation diagrams of throughput and blocking ratio analyze all possible 2-connected graphs for a 7-node network and reveal a performance variation of up to 73% on the total network load under same levels of requests blocked ratio, and up to 17.3% on the network throughput.

II. Concept Description

Graph theory states that there are $2^{n(n-1)/2}$ [10] ways to interconnect n nodes to form a graph. For example, there are 64 ways to interconnect four nodes to form a graph, and more than 2×10^6 graphs on seven nodes. However, many of these possible graphs are not connected, i.e., they are composed by at least two parts incommunicable with each other (e.g., Fig. 1 (a) and Fig. 1 (b)). Such graphs are not suitable to the topological design of an optical network.

Even after disregarding the not connected graphs, there are many possible ways to interconnect n nodes. For instance, there are 38 connected graphs on four nodes and $1,866,256$ connected graphs on seven nodes [11], e.g., Fig. 1 (c) and Fig. 1 (d). However, many of these graphs are topologically equivalent. In graph theory literature, topologically equivalent graphs are called isomorphic graphs. More precisely, two graphs G and H are isomorphic if there is a one-to-one correspondence between the node sets of G and H, $f : V(G) \rightarrow V(H)$, such that any two nodes u and v are linked in G if, and only if, $f(u)$ and $f(v)$ are linked in H. For example, Fig. 1 (d) and Fig. 1 (h) are two isomorphic graphs with four nodes and five edges. Notice that, besides the labeling of the nodes of these graphs, they have the same topological structure: a 4-node cycle (or ring) plus an edge between two non-adjacent nodes. From the 38 connected graphs on four nodes, only six are topologically distinct (i.e., Fig. 1 (e) to (j)). Furthermore, from the $1,866,256$ connected graphs on seven nodes, only 853 are topologically distinct.

Among the six distinct connected graphs on four nodes shown in Fig. 1, there are three graphs for which a single node or link removal is enough to disconnect, or "split" it into two or more parts incommunicable with each other. These graphs are not suitable to the topological design of an optical WDM network since they do not satisfy fault-tolerance constraints.

Graphs that remain connected after any single node or link removal are called 2-connected graphs. In a 2-connected graph, each pair of nodes is interconnected by at least two disjoint paths. All network topologies tested in this work are topologically distinct 2-connected graphs, a reasonable assumption in the context of optical network design [8]. Hence, this study is interested in only three from all 64 graphs on four nodes, Fig. 1 (h) to (j).

The number of topologically distinct connected, and 2-connected graphs for networks up to 11 nodes is shown in Table I, whose data was obtained by enumeration [12], [13] and is available at [11][1]. At this point we would like to note that the number of possible graphs scales to the number of nodes, leading to very high numbers of topologies even for a relatively moderate number of nodes. For example, although our filters return only three topologies on four node topologies, this number explodes when considering 11 nodes, thus making it impractical to conduct exhaustive studies even for relatively small networks. For instance, the set of distinct 2-connected networks of order $n = 7$ has 468 possibilities. By increasing by one or two nodes, the number of possible topologies grows from 468 to over $7,000$ and over $190,000$, respectively.

In the set of three distinct 2-connected graphs on four nodes shown in Fig. 1, there is only one graph with four edges (the ring), one with five edges, and one with six edges (the full-mesh). For each n larger than four, there is still only one ring, for which the number of edges m equals to the number of nodes, and only one full-mesh, for which $m = \frac{n(n-1)}{2}$. Between the unique ring topology and full-mesh topology,

[1]Data up to 26 nodes is available on *The On-Line Encyclopedia of Integer Sequences*, sequences A001349 ("Number of connected graphs with n nodes"), and A002218 ("Number of unlabeled nonseparable (or 2-connected) graphs (or blocks) with n nodes").

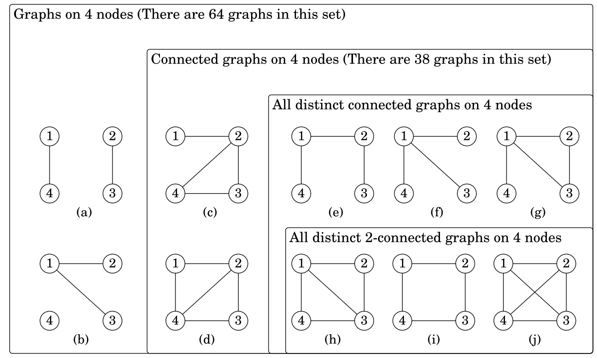

Fig. 1. Possible network topologies on 4 nodes.

TABLE I
TOTAL NUMBER OF DISTINCT CONNECTED AND 2-CONNECTED TOPOLOGIES UP TO 11 NODES [11].

Topology order (number of nodes)	All possible (distinct) connected topologies	All possible (distinct) 2-connected topologies
4	6	3
5	21	10
6	112	56
7	853	468
8	11,117	7,123
9	261,080	194,066
10	11,716,571	9,743,542
11	1,006,700,565	900,969,091

many hybrid graphs vary on the number of edges. Fig. 2 shows the distribution of the distinct 2-connected topologies on seven nodes, according to the number of edges. The most topologies have 11 to 13 edges whereas there are 94 12-edges topologies.

From the above discussion it becomes evident that despite the restriction for topologically distinct 2-connected topologies, there is still a large number of topologies to choose. Such topologies differ only in the way the n nodes are interconnected through the m edges. Thus, one could expect that networks with the same number of nodes and similar numbers of edges would perform identically, but the paper will evaluate and quantify the network performance variations for all the possible cases in order to reveal the patterns of dependence between the number of nodes, number of edges and actual networking topology.

This research considers the performance of the network without taking into account the physical constraints and parameters related to network deployment during the design process, such as fiber lengths, and costs. Although this is not the way networks are currently designed, it allows us to analyze how the network performance depends on its topology.

In order to investigate the influence of the topology on the network's performance, evaluated by the throughput and blocking ratio metrics, we devised a test scenario consisting in exhaustive studies of all distinct 2-connected topologies with $4 \leq n \leq 7$, following the steps below:

1) For each n, generate the set S_n of all distinct 2-connected network topologies on n nodes;
2) Simulate the topologies in S_n under the same conditions;
3) Create the throughput and blocking ratio curves for each topology in S_n.

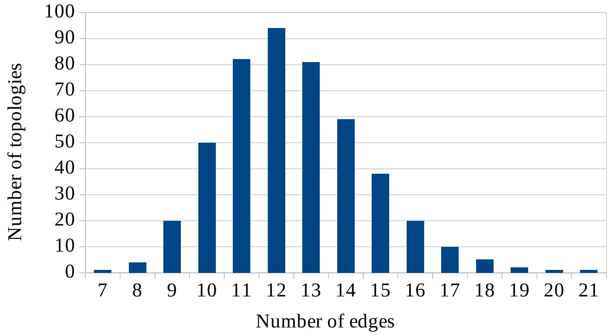

Fig. 2. Distribution of the number of distinct 2-connected topologies on 7 nodes, according to the number of edges.

The first step consists in generating the set of topologies to compare. For each n, the set S_n of all topologically distinct 2-connected network topologies on n nodes was generated using the nauty library [13][2].

The second step of our process is running simulations for all the topologies in S_n. Each topology was simulated using the ElasticO++ [14] simulation tool that has been previously developed by the authors. ElasticO++ is a simulation framework for WDM and EON networks, capable of running and extracting data from a significant volume of simulations with a reduced workload for the users.

All topologies in S_n are simulated and compared under the same conditions. For that, the ElasticO++ was set to run simulations under a dynamic environment, measuring the throughput and request blocked rates obtained by each network under various traffic loads. Each simulation follows the Erlang model with new requests arriving at a Poisson rate and holding time exponentially distributed, with a normalized mean of $1/\mu = 1$. Network load is given by $\rho = \lambda/\mu = \lambda$ (Erlang). In each simulation run, 10^5 requests are generated, and each point in the charts represents the average of 30 simulations, with different random seeds. Each new request demands 100 Gbps between a random uniformly distributed source-destination pair, following a uniform traffic pattern. Although using a uniform traffic demand is an unrealistic assumption, it is largely used on works comparing algorithms performance.

Since aggregation/grooming is not considered, each request is allotted in one of the 80 optical channels available in each bidirectional link. The combination of the Shortest Path and the First-Fit heuristics were used as the Routing and Wavelength Assignment (RWA) algorithm.

To evaluate the influence of the topology on the performance, we isolated all other influencing factors on our tests and kept fixed every configurable parameter in our simulations.

III. Results

The tests evaluated the distinct 2-connected topologies on n nodes, with $4 \leq n \leq 7$. Due to space constraints, we present only the most representative results obtained for S_7, i.e., the seven nodes topologies. Fig. 3 shows the results obtained after running over 350k simulations in total.

[2]Available at < http://pallini.di.uniroma1.it >.

Fig. 3 (a) presents the request blocked ratio curves, highlighting the difference between the full mesh (darkest blue) and the ring (darkest red) topologies. Considering only the number of resources available in each case (i.e., $80 \times m$), one would expect a variation of 300%. However, the simulation results reveal a difference of $\approx 580\%$ on the requests blocked ratio evaluation metric.

The remaining 466 topologies in Fig. 3 (a) were plotted in distinct colors as follows: i) each group of networks with the same number of links/edges receives the same color, and ii) the color scale is organized from fewer edges in dark red ($m = 7$ for the ring topology) to more edges in dark blue ($m = 21$ for the full mesh topology). The visualization of the number of edges versus blocking ratio and throughput reveals that networks with the same number of nodes but with more edges exhibit smaller blocking ratios and better throughput performance. Under heavy loads and unrealistic high blocking ratios, all networks with the same number of resources tend to saturate on the same "blocking level."

Fig. 3 (b) shows in detail a zoom in of Fig. 3 (a) on the load varying from 200 to 400 Erlang. For low network loads, we observe a mix of colors revealing that in this case, it is possible for topologies with a varied number of resources to exhibit similar performance. Fig. 3 (d) shows the throughput results for all the tested topologies and uses the same color scheme applied on Fig. 3 (a). Under lower loads (up to ≈ 250 Erlang), all topologies achieve the same throughput since even the ring topology has enough resources to provide connections for all the traffic demanded. For higher loads, the topologies start to cluster based on the total number of edges, likewise to Fig. 3 (a). Similarly to Fig. 3 (b), Fig. 3 (e) shows a zoom in on the throughput results, revealing the mix of colors, further reinforcing topologies' performance discrepancy.

Next, Fig. 3 (c) and (f) presents a more comprehensive visualization of the topologies blocking ratios and throughput overlapping. The colored shades represent the differences between the lowest and the highest request blocked ratio results for all topologies with 10 and 14 edges. The overlapping on the range from ≈ 180 to ≈ 500 Erlang is noticeable, meaning that there are networks with 10 edges performing similarly as networks with 14 edges. Since in this work the number of edges of a topology is directly proportional to the network number of resources, the overlapping indicates that for low to moderate traffic we can achieve the same network performance with fewer resources. Moreover, in Fig. 3 (c) it is noticeable the variation on the requests blocked ratio, up to $\approx 73.9\%$ and $\approx 50.0\%$ for the 10 and 14 edges topologies, respectively.

Finally, Fig. 3 (f) presents the throughput variation of the 50 and 59 topologies with 10 and 14 edges, respectively. As it is highlighted in Fig. 3 (f), the throughput varies considerably between topologies with the same number of resources, up to $\approx 17.3\%$ and $\approx 10.4\%$ for the 10 edges and 14 edges topologies, respectively.

IV. Conclusion

With the use of graph theory, we have demonstrated the significant influence of the topology on the performance of

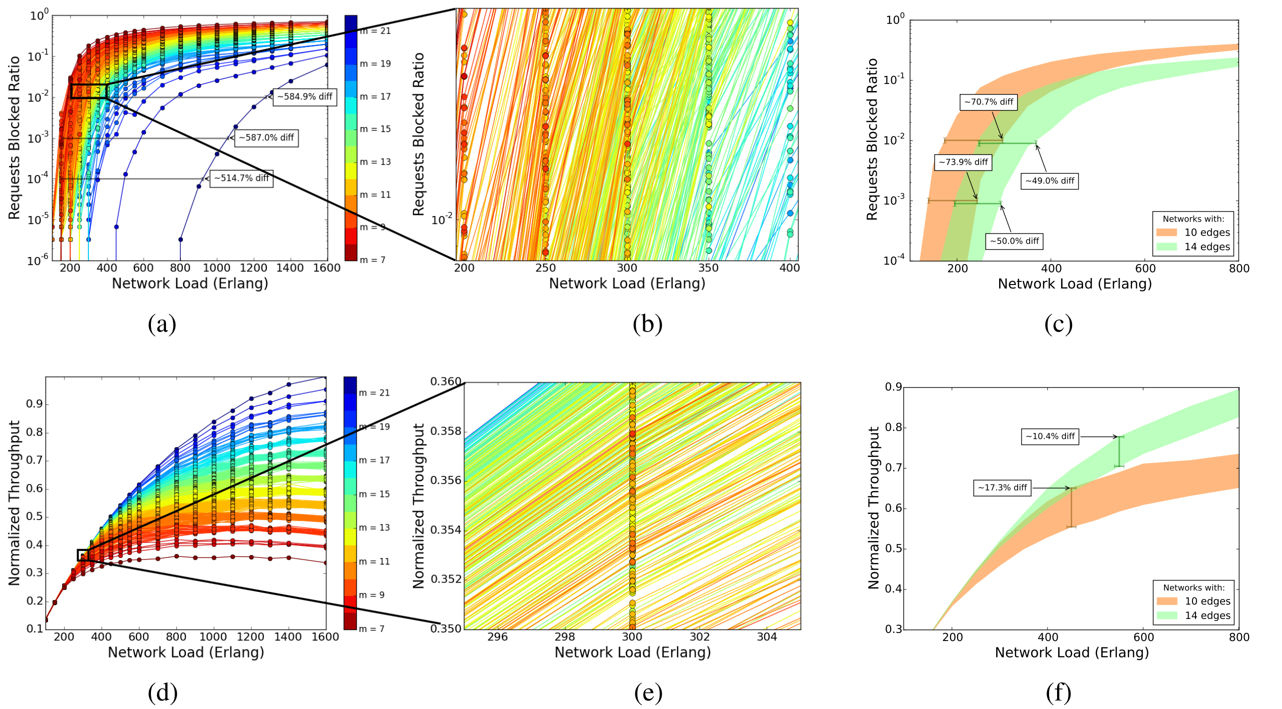

Fig. 3. Performance results of the 468 2-connected 7-nodes networks simulated: (a) overview of the request blocked ratio results. The topologies with the same number of edges share the same color. (b) Zoom in on the load range of 200 to 400 Erlang. The mix of colors represent topologies with a varied number of resources but similar performance. (c) The shaded areas represent the differences between the lowest and highest request blocked ratio results for networks with 10 and 14 edges. It is noticeable an intersection between the shades, representing similar performance even with 40% variation in total resources. (d) Throughput results. (e) Zoom in on the 300 Erlang load. (f) The throughput variation of the 50 and 59 topologies on 10 and 14 edges, respectively.

optical networks, expressed through diagrams of throughput and blocking ratio. Considering all possible distinct 2-connected graphs for a seven nodes network, we demonstrated a performance variation on networks with the same number of resources up to 73.9% on the total network load under same levels of requests blocked ratio, and up to 17.3% on the network throughput. The simulation did not consider physical parameters constraints, such as fiber lengths, losses, optical power etc, however their validity is guaranteed by the use of Elastico++ simulator that was explicitly designed for DWDM and EON networks.

To better explore these results for designing optical network topologies, it is necessary to investigate which topological characteristics lead to the better performance observed. We also plan to extend the presented tests for bigger topologies, to other technologies, e.g., Elastic Optical Networks, and for different allocation algorithms.

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References

[1] S. Gringeri, B. Basch, V. Shukla, R. Egorov, and T. J. Xia, "Flexible architectures for optical transport nodes and networks," *IEEE Communications Magazine*, vol. 48, no. 7, 2010.

[2] C. Pavan, L. S. de Lima, M. H. M. Paiva, and M. E. V. Segatto, "How reliable are the real-world optical transport networks?" *IEEE/OSA Journal of Optical Communications and Networking*, vol. 7, no. 6, pp. 578–585, June 2015.

[3] S. Orlowski, R. Wessäly, M. Pióro, and A. Tomaszewski, "Sndlib 1.0-survivable network design library," *Networks: An International Journal*, vol. 55, no. 3, pp. 276–286, 2010.

[4] C. S. R. Murthy and M. Gurusamy, *WDM optical networks: concepts, design, and algorithms*. Prentice Hall PTR Englewood Cliffs, 2002.

[5] R. Ramaswami, K. Sivarajan, and G. Sasaki, *Optical networks: a practical perspective*. Morgan Kaufmann, 2009.

[6] J. M. Simmons, *Optical network design and planning*. Springer, 2014.

[7] S. Baroni and P. Bayvel, "Wavelength requirements in arbitrarily connected wavelength-routed optical networks," *Journal of Lightwave Technology*, vol. 15, no. 2, pp. 242–251, Feb 1997.

[8] C. Pavan, R. Morais, R. F. Rocha, and A. N. Pinto, "Generating realistic optical transport network topologies," *IEEE/OSA Journal of Optical Communications and Networking*, vol. 2, no. 1, pp. 80–90, 2010.

[9] S. A. Jyothi, A. Singla, P. Godfrey, and A. Kolla, "Measuring and understanding throughput of network topologies," in *Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis*. IEEE Press, 2016, p. 65.

[10] F. Harary and E. Palmer, *Graphical Enumeration*. Elsevier Science, 2014. [Online]. Available: https://books.google.com.br/books?id=ZrvSBQAAQBAJ

[11] "OEIS, The On-Line Encyclopedia of Integer Sequences," 2010, available at <http://oeis.org>.

[12] P. Butler and R. Robinson, "On the computer calculation of the number of nonseparable graphs," in *Proceedings, 2nd Caribbean Conf. on Combinatorics and Computing*. Letchworth Press Barbados, 1977, pp. 191–208.

[13] B. D. McKay and A. Piperno, "Practical graph isomorphism, {II}," *Journal of Symbolic Computation*, vol. 60, no. 0, pp. 94 – 112, 2014.

[14] R. S. Tessinari, B. Puype, D. Colle, and A. S. Garcia, "Elastico++: An elastic optical network simulation framework for omnet++," *Optical Switching and Networking*, vol. 22, pp. 95–104, 2016.