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Abstract: The paper proposes a Trust Relationship-based Conflict Detection and Elimination decision making (TR-CDE) model, applicable for Large-Scale Group Decision Making (LSGDM) problems in social network contexts. The TR-CDE model comprises three processes: a trust propagation process; a conflict detection and elimination process; and a selection process. In the first process, we propose a new relationship strength-based trust propagation operator, which allows to construct a complete social network by considering the impact of relationship strength on propagation efficiency. In the second process, we define the concept of conflict degree and quantify the collective conflict degree by combining the assessment information and trust relationships among decision makers in the large group. We use social network analysis and a nonlinear optimization model to detect and eliminate conflicts among decision makers. By finding the optimal solution to the proposed nonlinear optimization model, we promote the modification of the assessments from the DM who exhibits the highest degree of conflict in the process, as well as guaranteeing that a sufficient reduction of the group conflict degree is achieved. In the third and last process, we propose a new selection method for LSGDM that determines decision makers' weights based on their conflict degree. A numerical example and a practical scenario are implemented to show the feasibility of the proposed TR-CDE model.

<u>Authors' response to the reviewers' comments on the manuscript No. EJOR-D-18-00858</u>

Large-scale group decision making model based on social network analysis: trust relationship-based conflict detection and elimination

(Bingsheng Liu, Qi Zhou, Ru-Xi Ding, Iván Palomares and Francisco Herrera)

We are very grateful to the editor and the reviewer for their constructive comments and valuable suggestions on the manuscript. We have carefully read the comments and revised the manuscript according to the reviewer' comments. These modifications are marked in the updated manuscript with the blue color. In the new version, there are also some sentences marked in blue color, which are used to give further explanations in this reply letter with little revision.

For Reviewer 1

Comment 1:   Table 1 in the response to reviewers and the discussion following that table in the two paragraphs beginning with "In the two" and "On the other hand" should be included in the paper in 6.1.5 (or at least the two paragraphs).

Response: Thanks for your suggestion. We added the Table with the two paragraphs in the new version. It is presented on Page 31 Line 46 to Page 32 Line 24 with blue color. The detail is: *In order to better explain the meaning and the setting influence of the conflict threshold Φ on LSGDM events, we list the values of group conflict degree ρ in each iteration of the numerical example and the practical scenarios in Table 1.*

Table 1. The group conflict degree ρ in each iteration.

iteration(T)	The numerical example	The practical scenario
	ρ	ρ
$T = 1$	0.51	0.423
$T = 2$	0.50	0.410
$T = 3$	0.41	0.376
$T = 4$	0.33	0.350
$T = 5$	0.32	0.326
$T = 6$	0.29	0.310
$T = 7$	-	0.279

* Table 1 is Table 18 in the manuscript.

In the two experiments, the conflict threshold is set the same as $\Phi = 0.3$. It means that within the limited iterations T_{max}, when the calculated group conflict degree is less that the threshold, the acceptable consensus is reached and the final decision can be made. As the results recorded in Table 1, if we set

the value of Φ higher than 0.3, we can make the final decision within a few iterations. Such as, if we set $\Phi = 0.42$ in the two experiments, we can chose the final decision with the two group conflict degrees are 0.41 after three iterations and two iterations, respectively. In other words, the higher the value of conflict threshold is, the less iterations are needed and the higher the group conflict degree is allowed before making the final decision in the LSGDM event. A smaller number of iterations implies a reduction in the temporal cost. However, the higher value of group conflict degree means the higher dissatisfaction among DMs, which may lead to some other serious group events.

As there are the results of the practical scenario experiment in Table 1, we removed the section for the behaviors of parameters to Section 6.3 to make it more readable.

On the other hand, if we set $\Phi = 0.28$ in the two experiments, the numerical example can not reach the consensus in the limited 6 iterations. That is, the lower the conflict threshold is, the more iterations are needed for the LSGDM event.

Comment 2: This is a wording suggestion to consider: in definition 6.1 consider using urgency level/level of urgency vs. emergency level. A decision could be very urgent but in no way an emergency.

Response: Thanks for Reviewer's suggestion. We changed the "emergency level" to "urgency level" in Definition 6.1 and other related context in the manuscript, as shown On Page 20 Line 29 and Line 36, Page 32 Line 57 and Page 33 Line 2.

We really appreciate for Reviewer #1's so constructive comments to make the paper improved to a great extent. We also cited recent and relevant publications in EJOR and other OR journals.

Highlights

An SNA based conflict detection and elimination decision making process is presented.

The impact of relationship strength on trust propagation efficiency is considered.

Multi-path trust propagation operator is presented to complete the social network.

Nonlinear optimization model guarantees a sufficient reduction of group conflict.

We promote the modification of the assessments by finding the optimal solution.

Large-scale group decision making model based on social network analysis: trust relationship-based conflict detection and elimination

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Abstract

The paper proposes a Trust Relationship-based Conflict Detection and Elimination decision making (TR-CDE) model, applicable for Large-scale Group Decision Making (LSGDM) problems in social network contexts. The TR-CDE model comprises three processes: a trust propagation process; a conflict detection and elimination process; and a selection process. In the first process, we propose a new relationship strength-based trust propagation operator, which allows to construct a complete social network by considering the impact of relationship strength on propagation efficiency. In the second process, we define the concept of conflict degree and quantify the collective conflict degree by combining the assessment information and trust relationships among decision makers in the large group. We use social network analysis and a nonlinear optimization model to detect and eliminate conflicts among decision makers. By finding the optimal solution to the proposed nonlinear optimization model, we promote the modification of the assessments from the DM who exhibits the highest degree of conflict in the process, as well as guaranteeing that a sufficient reduction of the group conflict degree is achieved. In the third and last process, we propose a new selection method for LSGDM that determines decision makers' weights based on their conflict degree. A numerical example and a practical scenario are implemented to show the feasibility of the proposed TR-CDE model.

Keywords: Decision processes, Large-scale group decision making, Conflict detection and elimination, Trust propagation operator, Social network analysis

1. Introduction

Large-Scale Group Decision Making (LSGDM) refers to the selection of the best option from a set of feasible alternatives, predicated on the preferences of a large number of decision makers (DMs) [1]. It is a common form of decision making problems that has recently attracted widespread research attention

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[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. In LSGDM problems, DMs often represent different interest groups and are diverse in their status, education, expertise, and understanding of the problem at hand. Hence, they tend to have strongly different preferences on the available decision alternatives. Such discrepancies in opinions of DMs could cause intra-group conflicts, which are a kind of disharmony relationship among DMs, and can potentially lead to resignations, unmotivated DMs and non-cooperation [11]. Moreover, intra-group conflict is not conducive to the overall decision process. It could lead to social unrest situations that may, for example, greatly impact the reputation of a country or society on behalf of whom the large group acts. Consequently, the decisions made by the group could ultimately become influential at a considerable scale [12].

To eliminate the conflict among DMs by reducing it under an acceptable degree for the decision making events, a frequently used approach consists in undertaking a Consensus Reaching Processes (CRP), aimed at obtaining a collective solution as close as possible to unanimous agreement [6, 13, 14, 15, 16, 17, 18]. It is widely believed that conflicts among DMs derive from non-conformity in their individual preferences towards alternatives. For this reason, many decision making models focus primarily on obtaining a reasonable degree of consensus. Consensus measures can be defined either (i) based on the distance between each DM's preference and the collective preference [19, 20], or (ii) predicated on the distance between DMs' preferences [5, 21, 22]. These models clearly adopt the principle that eliminating differences between DMs' opinions leads to higher-quality group decisions.

Interestingly, most consensus-based approaches regard DMs as independent individuals who have no relationships with each other. In practice, however, most DMs in an LSGDM problem are stakeholders or have some type of relationships and expertise in the problem being addressed. This implies, in turn, that they may have already developed some social relationships with some other DMs who participate in the same LSGDM problem, thereby influencing their opinions on the reliability of other DMs' judgments [23].

There is wide support among decision-making researchers for distinguishing two categories of conflict: *task conflict* and *relationship conflict* [24, 25, 26, 27, 28, 29]. Where DMs involved in LSGDM have relationships with one another, both dissension between preferences (task conflict) and disharmonious relationships among DMs (relationship conflict) are cornerstone factors of intra-group conflict. Existing works show correlations between the two kinds of conflict [24, 30, 31, 32]: that is, if A trusts B, then A is more likely to accept and recognize B's opinion, even if it differs from A's. This implies that conflict may not necessarily occur between DMs with rather distinct opinions if they trust each other. Conversely, highly similar opinions cannot ensure the absence of conflict between two DMs if they distrust each other. Therefore, when DMs' relationships are arranged in a social network structure characterized by diverse social relationships, traditional LSGDM models − which focus exclusively on analyzing preference information for consensus reaching − are no longer feasible and sufficient in some practical situations and

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real-world domains. In other words, social relationship information should be properly integrated (along with preferential information) in the process of measuring the group's overall level of harmony (i.e., the absence of intra-group conflict) to develop models focused on conflict elimination.

To detect and eliminate conflict defined with both social relationships and preferential information, it is necessary to obtain complete information on the relationships among DMs. Social Network Analysis (SNA) is a theoretical tool to study relationships between individuals, groups, organizations and societies [33]. Recent LSGDM literature has introduced approaches to studying how social relationships influence collective decision making processes [34, 35, 36]. These methods treat information among DMs as non-transitive. Such social networks are incomplete because some relationship information cannot be inferred for DMs who do not know one another. To obtain complete information for a social network, an increasingly large body of research is devoted to investigating how to propagate relation information among members in a social organization [37, 38, 39]. These studies have endeavored to obtain a complete social network by introducing a trust propagation operator, which is used to generate a trust relation between two indirect nodes/individuals via some mediators, founded on the hypothesis that trust information can be fully propagated by mediators. Indeed, relationship strength, which is defined as the intimacy/closeness degree between two nodes [40] or the contact time and frequency of two nodes [41] in a social network, is an important indicator of information spreading and propagation. If the relationship between two individuals is strong, they are more likely to share information and influence each other's opinions. On the contrary, if the relationship is weak, the efficiency of information propagation is likely to be lower [42, 43, 44]. Therefore, in developing SNA-based decision models, the impact of relationship strength on trust propagation efficiency should be also considered in the process of calculating "indirect" trust degrees via a mediator.

In summary, despite the extensive prior research proposing numerous LSGDM models, they still suffer from a number of limitations. Concretely, our interest lies in investigating the following three challenges for LSGDM problems in a social network-driven setting:

(1) Only a small number of these models consider the characteristics and inherent diversity of social relationships among DMs.

(2) Traditional consensus reaching LSGDM models which only consider the impact of preference disagreement on conflict [6, 16, 45, 46], are not realistically applicable for the LSGDM problems where DMs have well-defined relationships with each other.

(3) In the existing SNA-based models, the relationship propagation operators do not consider the impact of relationship strength on the efficiency of the propagation.

Based on the motivations outlined above, we draw on SNA to propose a novel Trust Relationship-based Conflict Detection and Elimination decision making (TR-CDE) model for LSGDM scenarios. An important innovation in TR-CDE model is considering both the preference inconformity and the relationship disharmony as the causal factors of conflict. Since it is infeasible for DMs to evaluate a priori their

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relationships with other DMs they do not know, a trust propagation process is necessary to derive the complete social network from the known relationship information. Therefore, our cornerstone contribution of a TR-CDE model comprises the following three processes:

(1) *Trust propagation process.* In this process, by introducing the Einstein product operator, and based on the trust propagation operator proposed by Wu [39], we develop a new relationship strength-based trust propagation operator which considers the relationship strength between pairs of DMs. Based on this, a novel multi-path relationship strength-based trust propagation operator is proposed. We show that this propagation operator surpasses previous methods by considering the influence of relationship strength on propagation efficiency and synthesizing all the propagation paths to obtain comprehensive relationship information. By using the trust propagation operator, information on the complete social network structure is obtained for the subsequent processes. An algorithm termed "relationship strength-based trust propagation algorithm" (Algorithm 1) is drawn to describe the details of the process.

(2) *Conflict detection and elimination process.* As intuitionistic fuzzy sets (IFSs) can suitably represent the uncertainty and hesitation of decision makers' assessment information [6, 47, 48, 49], we consider the use of IFSs to represent DMs' assessment information on alternatives in the LSGDM process. By integrating social network information with assessment consensus information, a conflict network is developed, which facilitates visualizing conflicts among DMs. The "key" DMs who hamper group harmony (i.e. the absence of group conflict) are detected, and an opinion modification plan is computed for them accordingly, by building a nonlinear optimization model with the aim of guaranteeing as much reduction in the group conflict degree as possible throughout an iterative process. In this way, the conflicts among DMs are detected and eliminated from the decision making process. The conflict detection and elimination process is dynamic, hence DMs can modify their assessment to reach higher collective agreement stepwise. The process also constitutes an innovative and feasible tool for handling practical LSGDM situations, as it tries to reach a high level of harmony based on both social relationships and preference consistency, rather than simply improving the degree of consensus among preferences. To summarize this process, we present an algorithm termed "conflict detection and elimination algorithm" (Algorithm 2).

(3) *Selection process.* It is proposed that conflict information should be introduced into the selection process. By using this information to determine DMs' importance weights, the influence of DMs with a high conflict degree will be further reduced so as to ensure fair and less biased collective decisions.

A numerical example is likewise implemented to show the feasibility of the proposed TR-CDE model. Moreover, to demonstrate the effectiveness of the conflict detection and elimination process, we apply the TR-CDE model in a practical scenario to compare against several other LSGDM models. The comparison experiment shows that a greater effectiveness is achieved by the proposed TR-CDE model in eliminating conflict in LSGDM.

The remainder of this paper is organized as follows. In Section 2, we introduce IFSs and SNA as two

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paramount concepts in the remainder of the paper. The three parts of the proposed TR-CDE model are then introduced in the following three sections. Beginning with the trust propagation process in Section 3. First, we propose a new single-path trust propagation operator considering the impact of relationship strength on propagation efficiency. To this aim, we introduce a multi-path trust propagation operator. In this way, we can obtain the completed social network. In Section 4, we introduce the conflict detection and elimination process that integrates the complete social network with assessment information to form the conflict network; by identifying the key DMs who are causing the presence of group conflict, thereby eliminating such conflict accordingly. Subsequently, section 5 is devoted to introducing the selection process, in which DMs' weights are derived from the conflict network and the optimal selection for the LSGDM problem is calculated. In Section 6, we present a numerical example and practical scenario to illustrate and validate the feasibility of the proposed TR-CDE approach through its application to an LSGDM problem. In Section 7, we draw some conclusions and summarize the paper's innovations.

2. Preliminaries

In this section, we first introduce the basic concepts of IFSs and an information aggregation operator for intuitionistic fuzzy values. Then, SNA is briefly introduced, along with several SNA-related concepts.

2.1. Intuitionistic fuzzy sets

Due to their appropriateness to represent the hesitancy exhibited by DMs, IFSs are considered a suitable means to represent assessment information; hence, they have been widely adopted in recent research [50, 51, 52]. Therefore, this study utilizes IFSs to represent DMs' assessment information on alternatives.

As a generalization of a fuzzy set, the concept of IFS was introduced by Atanassov [53] as follows:

Definition 2.1. *Let X be a non-empty set, then we term:*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

as an IFS, where $\mu_A(x)$ and $\nu_A(x)$ represent the membership degree and the non-membership degree of the element x in X to A, respectively.

The above statement implies $\mu_A : (x) \to [0,1]$, $x \in X \to \mu_A(x) \in [0,1]$, $\nu_A : (x) \to [0,1]$, $x \in X \to \nu_A(x) \in [0,1]$. For each IFS A, if $\pi_A(x) = 1 - \mu_{(A)} - \nu_A(x)$, then $\pi_A(x)$ is called the hesitation degree (or intuitionistic index) of x to A. Obviously, $\pi_A(x) \in [0,1]$. If $\pi_A(x) = 0$ for all $x \in X$, then A is reduced to a fuzzy set; if $\mu_A(x) = \nu_A(x) = 0$ for all $x \in X$, then the IFS A is completely intuitionistic.

To simplify the notation of IFS, $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ is defined as an intuitionistic fuzzy value. It is used to represent the element x in an IFS, here $\mu_\alpha \in [0,1]$, $\nu_\alpha \in [0,1]$, $0 \le \mu_\alpha + \nu_\alpha \le 1$. For instance, supposing

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an IFS $A = \langle x_1, 0.4, 0.2 \rangle, \langle x_2, 0.2, 0.7 \rangle, \langle x_3, 0.4, 0.4 \rangle$, then the elements $\alpha_{x_1} = \langle 0.4, 0.2 \rangle$, $\alpha_{x_2} = \langle 0.2, 0.7 \rangle$ and $\alpha_{x_3} = \langle 0.4, 0.4 \rangle$ are the intuitionistic fuzzy values of that IFS.

The intuitionistic fuzzy weighted average (IFWA) operator proposed by Xu [54], is one of the extant aggregation operators for IFSs. IFWA is defined as follows.

Definition 2.2. *Let $\alpha_j = \langle \mu_{\alpha_j}, \nu_{\alpha_j} \rangle$ $(j = 1, 2, \cdots, n)$ be a set of intuitionistic fuzzy values, and consider the mapping IFWA: $\widetilde{\Theta}^n \to \widetilde{\Theta}$, then*

$$IFWA_\omega(\alpha_1, \cdots, \alpha_n) = \left(1 - \prod_{j=1}^{n}(1 - \mu_{\alpha_j})^{\omega_j}, \prod_{j=1}^{n} \nu_{\alpha_j}^{\omega_j} \right) \tag{1}$$

is called the IFWA operator, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\alpha_j (j = 1, 2, \cdots, n)$, meeting the condition $\omega_j \in [0, 1](j = 1, 2, \cdots, n)$ and $\sum \omega_j = 1$. In particular, if $\omega = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})$, then the IFWA operator is reduced to the intuitionistic fuzzy averaging (IFA) operator.

2.2. Social network analysis

SNA studies the relationships between social entities, such as members of a group, corporations, or nations [55]. It provides a framework that allows, for instance, to examine the structural and location-based properties of the social group, including centrality, prestige, and structural balance [23]. Consequently, SNA can be utilized to model the relationships among a group of people. The three main elements in an SNA are: the set of actors, the relationships among them, and the actor attributes. We refer to important network concepts in a unified manner, using the following three representation schemes (see Table 1):

- **Sociometric**: relational data are presented as a two-dimensional matrix, called a sociometric or adjacency matrix.
- **Algebraic**: this notation distinguishes distinct relations and presents combinations of relations.
- **Graph theoretical**: the network is viewed as a graph, consisting of nodes joined by edges.

Table 1. Different representation schemes in social network analysis.

Due to the transitivity of information, if a node A trusts B, and B trusts C, then A will most likely trust C even if they do not know each other. That is, there exists a potential trust relationship from A to C. Some related literature divides the relationships into three types, as illustrated in Fig.1.

- **Direct Relationship** As the top path in Fig.1 shows, if there is an edge from A to B in the social network, then A has a direct relationship with B. In this case, A may interact with B in real life, enabling him/her to exactly evaluate B.

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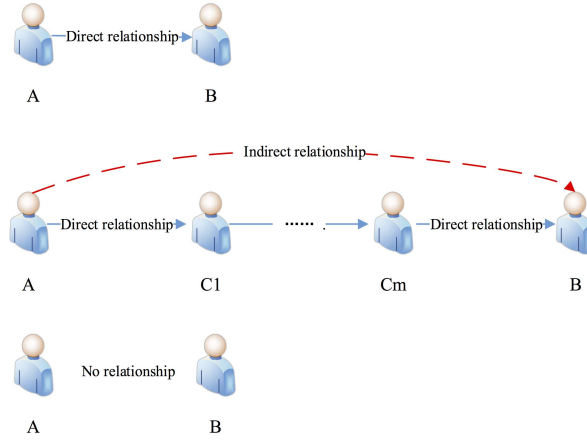
Fig. 1. An example of social network to illustrate types of relationships.

- **Indirect Relationship** As the middle path in Fig.1 shows, if there is no edge from A to B in the social network, but A can build an edge to B through several mediators $(C_1, \cdots, C_m)$, then A has an indirect relationship with B. In this case, A does not typically know B, but we can obtain reliable relationship information from A to B by the direct relationship information between their mediators.

- **Irrelevant Relationship** As the bottom path in Fig.1 shows, if there is neither a direct nor indirect relationship between A and B, then A has an irrelevant relationship with B.

Any social network has at least one of the the three types of relationship mentioned above. The nodes and the edges between them form the structure of the social network. For a social network with m nodes, we introduce several important concepts in SNA:

- **Density of social network**: this represents the ratio of the number of direct pairwise relationships with respect to the maximum possible number of direct relationships. If there are n directed edges in a social network with m nodes, then the density is $\rho = \frac{n}{m(m-1)}$.

- **In $-$ degree and out $-$ degree** A node's in-degree is the number of edges that start from other nodes and end in that node. Similarly, a node's out-degree is the number of edges that start from that node and end in the others.

3. Trust propagation process

In this paper, we introduce social network information in LSGDM problems and study the impact of social relationships on the intra-group conflicts. To study the conflict among DMs, first, it is necessary to obtain their complete social network. The Einstein product operator has been found to be suitable for trust propagation [39]. Based on the Einstein product operator, we first propose a single-path relationship strength-based trust propagation operator with one mediator. Then, we extend the operator and propose a trust propagation operator applicable to paths with multiple mediators. Finally, the algorithm for multi-path trust propagation operator is devised.

3.1. Basic concepts of the relationship strength-based trust propagation in LSGDM problems

In the LSGDM problem, there are m DMs that are indexed by $E = \{e_1, e_2, \cdots, e_m\}$. This subsection presents the trust propagation process applied to obtain the complete social network of DMs.

Direct relationship information can be gathered, for instance, through a questionnaire in which DMs evaluate other DMs whom they know. In this paper, we consider relationship strength as an important factor of propagation efficiency. Thus, a DM e_i will evaluate other DMs e_j ($j = \{1, 2, \cdots, m\}$, $j \neq i$) that he/she knows with a tuple of the form:

$$\lambda_{i,j} = (t_{i,j}, s_{i,j}).$$

Where $t_{i,j}$ represents the trust degree of e_i on e_j, satisfying the condition $0 \leq t_{i,j} \leq 1$. If $\lambda_{i,j} = 0$, it implies that e_i fully distrusts e_j; conversely, if $\lambda_{i,j} = 1$, we have e_i fully trusts e_j. $s_{i,j}$ represents the degree of strength of the relationship from e_i to e_j, which satisfies the condition $0 \leq s_{i,j} \leq 1$. Relationship strength is the quantitative expression of the contact frequency among DMs: the bigger its value, the closer the relationship of the two DMs, and the more relevant and meaningful the trust information (given by t) becomes. We assume that a DM's evaluation of him-/herself is $\lambda_{i,i} = (1, 0)$, which implies a DM cannot propagate his information to him-/herself.

(a) Incomplete social network. (b) Complete social network.
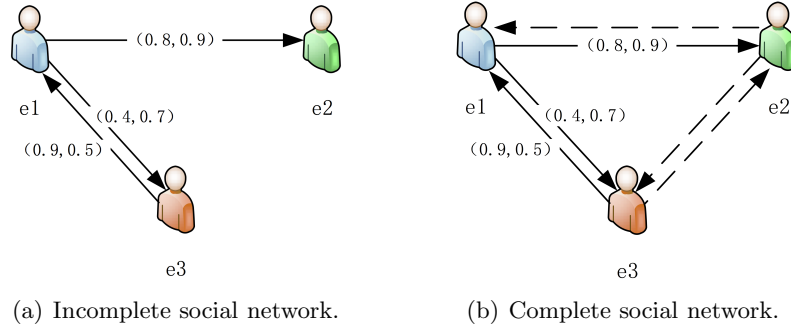
Fig. 2. The two types of the social network.

Every DM gives evaluation information about his/her acquainted DMs. However, in LSGDM problems, the number of DMs is large, so it is common for some DMs to be unable to give an exact evaluation of other DMs whom they do not know. For the sake of illustration, considering the small group example shown in Fig.2(a), we can obtain the direct relationships of $\lambda_{1,2}$, $\lambda_{1,3}$, and $\lambda_{3,1}$, but the other relationships $\lambda_{2,1}$, $\lambda_{2,3}$, and $\lambda_{3,2}$ remain unknown. We can use an initial matrix (or incomplete) matrix (A) to represent the incomplete social network as follows:

$$A = \begin{pmatrix} (1,0) & (0.8, 0.9) & (0.4, 0.7) \\ (-, 0) & (1,0) & (-, 0) \\ (0.9, 0.5) & (-, 0) & (1,0) \end{pmatrix}.$$

To obtain the full relationship information, we should build a relationship propagation mechanism and compute the indirect relationship information via the known direct relationships, as Fig.2(b) shows.

It should be noted that trust can only be propagated by the relationship where $s > 0$, and we have $s = 0$ for all the indirect relationships.

3.2. Single-path relationship strength-based trust propagation

In a social network, information is transitive, therefore trust can be propagated by one or more mediators. We define the set of nodes and connections across which information is transmitted, thereby forming a propagation path. Taking Fig.3 as an example: $e_1 \to e_2 \to \cdots \to e_k$ is a trust propagation path from e_1 to e_k with $(k-2)$ mediators. The propagation paths do not contain any loops, in other words, a propagation path does not involve any repeated nodes in it.
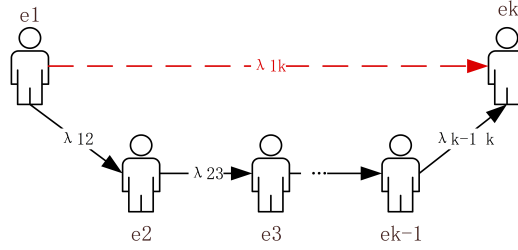
Fig. 3. An example of social network to illustrate types of relationships.

Related studies [56] show that the information strength wanes on the propagation process. Since the relationship strength s represents the closeness between two DMs, a DM will be more likely to share his/her information with the DMs with whom they have closer relationships. Put another way, propagation efficiency must be deemed as a function of relationship strength. In this paper, we define the propagation efficiency p as follows.

Definition 3.1. *Suppose that the relationship information from e_i to e_j is $\lambda_{i,j} = (t_{i,j}, s_{i,j})$, then the propagation efficiency from e_i to e_j is defined as:*

$$p(s) = 1 - cos\frac{\pi s_{i,j}}{2}.$$

Theorem 3.1. *The value of p is in the interval of $[0, 1]$.*

Proof. For $s_{i,j}$ being the relationship strength and in the interval of $[0,1]$, which is the domain of definition for function $p(s)$. Then, we can have $0 \leq \frac{\pi s}{2} \leq \frac{\pi}{2}$. For the function $y = cos(x)$, y is monotone decreasing when x is in the interval $[0, \frac{\pi}{2}]$. Thus, we can easy obtain that the fuction $p(s)$ is strictly monotonic increasing function in the domain. As $cos(0) = 1$, $cos(\frac{\pi}{2}) = 0$, therefore, we can conclude that the value of $p(s)$ is in the interval of $[0, 1]$. $\square$

A plot representing the values taken by $p(s)$ across the domain $[0,1]$, is shown in Fig.4. In addition, p is a convex function of s, which implies that the increment of p accentuates as s increases.

Suppose that the trust information of e_1 to e_3 is unknown in the initial social network. Let us consider the simplest situation of only one propagation path with one mediator e_2 between the two DMs e_1 and e_3. In this case, the relationship between e_1 and e_2 and that between e_2 and e_3 are direct relationships, the values of which are known as: $\lambda_{1,2} = (t_{1,2}, s_{1,2})$, $\lambda_{2,3} = (t_{2,3}, s_{2,3})$.

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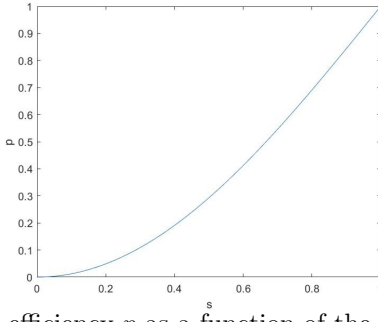
Fig. 4. Propagation efficiency p as a function of the relationship strength s.

There is a single propagation path from e_1 to e_3, which can be denoted as $e_1 \to e_2 \to e_3$. Referring to the Einstein product operator $E_\otimes$, we can obtain the trust value of e_1 towards e_3 under the assumption that the information can be fully propagated:

$$E_\otimes(t_{1,2}, t_{2,3}) = t_{1,3}^f = \frac{t_{1,3}t_{2,3}}{1 + (1 - t_{1,3})(1 - t_{2,3})}. \tag{2}$$

In Eq.(2), $t_{1,3}^f$ the fully propagated trust value from e_1 to e_3.

Theorem 3.2. *The propagation operator exhibits the following properties:*

- **Commutativity**: $E_\otimes(t_{1,2}, t_{2,3}) = t_{1,3}^f = \frac{t_{1,2}t_{2,3}}{1+(1-t_{1,2})(1-t_{2,3})} = \frac{t_{2,3}t_{1,2}}{1+(1-t_{2,3})(1-t_{1,2})} = E_\otimes(t_{2,3}, t_{1,2})$.

- **Boundary Conditions**:

  Full trust propagation: if $t_{1,2} = 1$, we have $t_{1,3}^f = E_\otimes(1, t_{2,3}) = \frac{1 \times t_{2,3}}{1+(1-1) \times (1-t_{2,3})} = t_{2,3}$;
  Similarly, if $t_{2,3} = 1$, we have $t_{1,3}^f = t_{1,2}$.

  Full distrust propagation: if $t_{1,2} = 0$, we have $t_{1,3}^f = E_\otimes(0, t_{2,3}) = \frac{0 \times t_{2,3}}{1+(1-0) \times (1-t_{2,3})} = 0$;
  Similarly, if $t_{2,3} = 0$, we have $t_{1,3}^f = 0$.

Considering the propagation efficiency, we can use a tree map (Fig.5(a)) to show the trust propagation.

(a) With one mediator. (b) With k mediators.
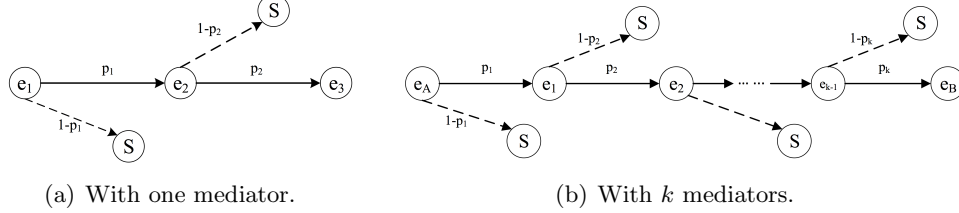
Fig. 5. The single-path trust propagation path with different numbers of mediators.

In the tree, only the path $e_1 \to e_2 \to e_3$ can propagate trust information from e_1 to e_3. In this case, we use Eq.(2) to calculate the trust value t_{13}. In the tree map, the node S represents a stranger individual, such that those paths ending in S imply that trust cannot be propagated to e_3. Using mathematical expectation, we can calculate the trust value from e_1 to e_3 as follows.

$$t_{1,3} = p_{1,2} \times p_{2,3} \times t_{1,3}^f + (1 - p_{1,2}) \times t_S + p_{1,2} \times (1 - p_{2,3}) \times t_S.$$

The trust value towards a stranger (not known user by her/him) is $t_S = 0$; therefore, we simplify the above equation as:

$$t_{1,3} = p_{1,2} \times p_{2,3} \times t_{1,3}^f = (1 - cos\frac{\pi S_1}{2}) \times (1 - cos\frac{\pi S_2}{2}) \times \frac{t_{1,2}t_{2,3}}{1 + (1 - t_{1,2})(1 - t_{2,3})}. \tag{3}$$

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Consider now the case when the propagation path can involve more than one mediator, as depicted in Fig.5(b). We can now extend the single-path propagation operator by Eq.(2) to deal with $k-1$ mediators, under the assumption that the information can be fully propagated.

$$t_{A,B}^f = \frac{2 \prod_{i,j=1}^k t_{i,j}}{\prod_{i,j=1}^k (2 - t_{i,j}) + \prod_{i,j=1}^k t_{i,j}}. \tag{4}$$

Considering the propagation efficiency, we have:

$$p_{A,B} = \prod_{i,j=1}^k p_{i,j}. \tag{5}$$

In Eq.(5), p_{AB} is the overall propagation efficiency for the path between A and B. Taking Fig.5(b) as an example, we deduce the trust propagation operator as Eq.(6):

$$t_{A,B} = p_{A,B} \times t_{A,B}^f = \prod_{i,j=1}^k (1 - cos\frac{\pi s_{ij}}{2}) \times \frac{2 \prod_{i,j=1}^k t_{i,j}}{\prod_{i,j=1}^k (2 - t_{i,j}) + \prod_{i,j=1}^k t_{i,j}}. \tag{6}$$

According to the discussion introduced formerly, the value of t_{AB} lies strictly in the interval of $[0, 1]$. In addition, when trust is propagated from one node to another, the indirect relationship strength decreases. That is, the more mediators in the trust propagation path, the closer the value of relationship strength s approaches to 0. Therefore, according to Eq. (6), when the propagation path is long enough, the value of t_{AB} closes to 0. In other words, its influence on the trust degree can eventually become negligible [57], when there are a certain number of mediators involved in the propagation path.

3.3. Multi-path trust propagation

In some social networks, there may exist more than one possible path to obtain an indirect social relationship between two nodes e_A and e_B. Trust propagation under this situation is termed multi-path trust propagation. Consider the example shown as the following incomplete social network matrix:

$$A = \begin{pmatrix} (1, 0) & (0.8, 0.9) & (0.4, 0.7) & (0.9, 0.7) \\ (-, 0) & (1, 0) & (0.4, 0.6) & (0.3, 0.4) \\ (0.9, 0.5) & (-, 0) & (1, 0) & (0.1, 0.1) \\ (0.7, 0.4) & (-, 0) & (-, 0) & (1, 0) \end{pmatrix},$$

to obtain the trust value of t_{21}, we notice that there are three paths to obtain his/her trust value on e_B, they are: Path 1: $e_2 \rightarrow e_3 \rightarrow e_1$; Path 2: $e_2 \rightarrow e_4 \rightarrow e_1$; Path 3: $e_2 \rightarrow e_3 \rightarrow e_4 \rightarrow e_1$. We use a tree diagram to show the propagation paths, as shown in Fig.6:
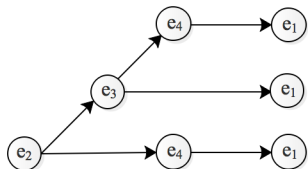

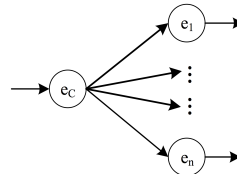**Fig. 6.** Multi-path trust propagation.

Fig. 7. Furcate node.

When there are multiple paths for an indirect relationship, we define the overall trust value for the indirect relationship as the mathematical expectation of trust values for all propagation paths, that is:

$$t_{AB} = \sum_{i=1}^{k} p_{AB}^{path_i} \times t_{AB}^{f,path_i}. \tag{7}$$

Where $path_i$ represents the overall propagation efficiency for path i, and $t_{AB}^{f,path_i}$ is the fully-propagated trust value from e_A to e_B.

In the cases where some edges connecting nodes appear in several different paths, such as e_2 in Fig.6, we refer to those (pairs of) nodes as furcate nodes. Taking e_2 in Fig.6 as an example, let us suppose that the propagation efficiency to the following mediators e_3 and e_4 is 0.9 and 0.7, respectively. That indicates $p_{23} + p_{24} = (1 - cos\frac{\pi \cdot s_{23}}{2}) + (1 - cos\frac{\pi \cdot s_{24}}{2}) > 1$, which may result in $s_{AB} > 1$. To avoid this situation, we should normalize the propagation efficiency within the unit interval.

Suppose that there is a furcate node e_C, as depicted in Fig.7, that is followed by n branches: we make the following judgement to obtain the propagation efficiency for each branch.

- If $\sum_{i=1}^{n}(1 - cos\frac{\pi \cdot s_{Ci}}{2}) \leq 1$, let $p_{Ci} = 1 - cos\frac{\pi \cdot s_{Ci}}{2}$ for $\forall i = \{1, 2, \cdots, n\}$.

- Otherwise, if $\sum_{i=1}^{n}(1 - cos\frac{\pi \cdot s_{Ci}}{2}) > 1$, let $p_{Ci} = \frac{1-cos\frac{\pi \cdot s_{Ci}}{2}}{\sum_{i=1}^{n}(1-cos\frac{\pi \cdot s_{Ci}}{2})}$ for $\forall i = \{1, 2, \cdots, n\}$.

We execute this judgment to normalize the propagation efficiency whenever there is a furcate node. Based on this, we propose a relationship strength-based trust propagation algorithm (Algorithm 1) to obtain the indirect or irrelevant relationship information from e_A to e_B.

Algorithm 1 Relationship strength-based trust propagation algorithm

---

Step 1 Determine whether there exists at least one path from e_A to e_B.
      Yes, turn to **Step 2**.
      No, turn to **Step 7**.

Step 2 Calculate the fully-propagated trust value $(t_{AB}^{f,path_1}, \cdots, t_{AB}^{f,path_k})$ for each path, by Eq.(4);

Step 3 Identify whether there are furcate nodes in the paths.
      Yes, turn to **Step 4**.
      No, turn to **Step 5**.

Step 4 For every furcate node e_C, do the following.
      If $\sum_{i=1}^{n}(1 - cos\frac{\pi \cdot s_{Ci}}{2}) \leq 1$, let $p_{Ci} = 1 - cos\frac{\pi \cdot s_{Ci}}{2}$ for $\forall i = 1, 2, \cdots, n$.
      Otherwise, if $\sum_{i=1}^{n}(1 - cos\frac{\pi \cdot s_{Ci}}{2}) > 1$, let $p_{Ci} = \frac{1-cos\frac{\pi \cdot s_{Ci}}{2}}{\sum_{i=1}^{n}(1-cos\frac{\pi \cdot s_{Ci}}{2})}$ for $\forall i = 1, 2, \cdots, n$.

Step 5 Calculate the propagation efficiency for each path by Eq.(5) to obtain $p_{AB}^{path_1}, \cdots, p_{AB}^{path_k}$.

Step 6 Calculate t_{AB} by Eq.(7), Output $\lambda_{m,n} = (t_{ab}, 0)$.

Step 7 Output $\lambda_{AB} = (0, 0)$. End.

By applying the trust propagation process, as summarized in Algorithm 1, we can obtain the relationship information for the indirect relationships and irrelevant relationships. Thus, the missing data in the initial social network is completed. We use the following matrix, termed the complete social network matrix B, to represent the social relationships among DMs:

$$B = \begin{pmatrix} \lambda_{1,1} & \cdots & \lambda_{1,m} \\ \vdots & \ddots & \vdots \\ \lambda_{m,1} & \cdots & \lambda_{m,m} \end{pmatrix} = \begin{pmatrix} (t_{11}, s_{11}) & \cdots & (t_{1m}, s_{1m}) \\ \vdots & \ddots & \vdots \\ (t_{m1}, s_{m1}) & \cdots & (t_{mm}, s_{mm}) \end{pmatrix}.$$

4. Conflict detection and elimination in LSGDM problems

In the previous section, we introduced a relationship strength-based trust propagation operator to complete DMs' pairwise relationship information across a social network. In this section, we propose a method that, by combining the complete social network information with DMs' assessments on alternatives, allows for detecting the conflicts between DMs. The conflict network mapped thereby shows the distribution of existing conflicts among the DMs. By analyzing the structure of the conflict network, we can detect the key DMs in the decision making process, i.e. DMs whose importance in their roles can potentially help to reduce group conflict. Accordingly, some effective suggestions for modifying preferences and eliminating conflict, are provided to to the identified DMs for improving harmony by reducing the conflict level. Together with the conflict detection and elimination algorithm (Algorithm 2), the conflict detection and elimination process is presented in Section 4.3.

4.1. Problem and data formulation

Let us suppose an LSGDM problem in which there are m DMs and we index them by $E = \{e_1, e_2, \ldots, e_m\}$. Those DMs give their assessments on a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, one of which shall be chosen as the solution for the LSGDM problem. The assessment information of an arbitrary DM $e_i \in E$ can be denoted as a vector $D_i = \{d_1^i, d_2^i, \ldots, d_n^i\}$. As noted above, IFSs can well represent DMs' assessment information. Therefore, without loss of generality in this paper we consider that the assessment information of an arbitrary DM e_i is in the form of an IFS comprising n intuitionistic fuzzy values:

$$D_i = \{\langle \mu_1^i, \nu_1^i \rangle, \langle \mu_2^i, \nu_2^i \rangle, \ldots, \langle \mu_n^i, \nu_n^i \rangle\}, \ (1 \leq i \leq m).$$

In D_i, a DM e_i can express his/her satisfaction with alternative x_j by assigning a larger value to the membership degree μ_j^i than to the non-membership degree ν_j^i. Conversely, if he/she is not satisfied with the alternative, the value of ν_{ji} should be larger than μ_j^i. Moreover, if a DM e_i has no obvious preference on x_j, he/she can express their hesitation by assigning a small number on both μ_j^i and ν_j^i, so that the hesitancy degree π becomes larger. Accordingly, a $n \times m$ group decision matrix ($\mathbf{D}$) with the following structure is obtained by combining the individual IFSs.

$$\mathbf{D} = (D_1^T, D_2^T, \ldots, D_m^T) = \begin{pmatrix} \langle \mu_1^1, \nu_1^1 \rangle & \cdots & \langle \mu_1^m, \nu_1^m \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_n^1, \nu_n^1 \rangle & \cdots & \langle \mu_n^m, \nu_n^m \rangle \end{pmatrix}.$$

In addition, by applying the previously defined trust propagation operator, we obtain a complete social network matrix B with the following structure:

$$B = \begin{pmatrix} (t_{11}, s_{11}) & \cdots & (t_{1m}, s_{1m}) \\ \vdots & \ddots & \vdots \\ (t_{m1}, s_{m1}) & \cdots & (t_{mm}, s_{mm}) \end{pmatrix}.$$

As mentioned earlier, when the DMs involved in LSGDM problems have relationships with one another, two important factors of intra-group conflict are preference nonconformity (task conflict) and relationship disharmony (relationship conflict). Therefore, in the next subsections, we describe the process of combining both the group decision matrix $\mathbf{D}$ and the complete social network matrix B to derive a conflict network for DMs, which is subsequently analyzed to detect and eliminate conflicts among DMs.

4.2. Formation of conflict network

Due to the differences in their educational background, status and interests, DMs' assessments on alternatives in are usually diverse in many LSGDM situations. We define the similarity degree δ of assessment information for a pair of DMs, i.e. the degree to which the preferences of e_i and e_j is similar.

Definition 4.1. *The similarity degree δ_{ij} for each pair of DMs (e_i and e_j) is defined as:*

$$\delta_{ij} = 1 - \frac{1}{2n} \sum_{k=1}^{n} \left(\mid \mu_k^i - \mu_k^j \mid + \mid \nu_k^i - \nu_k^j \mid \right). \tag{8}$$

Where μ_k^i and ν_k^i represent e_i's intuitionistic fuzzy value assessment on alternative x_k.

The similarity degree of all DM pairs can be integrated into a similarity matrix (S) as $S = (\delta_{ij})_{m \times m}$. As Eq.(8) shows, $\delta_{ij} \in [0, 1]$, and the higher the value of δ, the lower the disagreement or divergence level among their individual preferences. Clearly, S is a symmetric matrix in which the elements on the main diagonal are 1.

In addition, when DMs are in a social network, relationships between them would either aggravate or alleviate the previously computed divergence that stems from assessment information. That is, when there is a certain assessment divergence between DMs e_i and e_j, e_i is more liable to have conflict with e_j, if e_i distrusts e_j. Conversely, if e_i trusts e_j, then e_i would be more willing to accept e_j's opinion. Thus, we define the disharmony degree c_{ij} between two DMs e_i and e_j as follows:

Definition 4.2. *The disharmony degree between two DMs e_i and e_j is*

$$c_{ij} = \delta_{ij} \times (1 - t_{ij}). \tag{9}$$

After computing disharmony degrees for all DMs, the conflict information among DMs can be integrated into a matrix called Conflict Information Matrix (CIM). Thus, we have $CIM = (c_{ij})_{m \times m}$. Since

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both a DM's trust value for him-/herself (t_{ii}) and the similarity degree equal 1, the elements on the leading diagonal of $CIM(c_{ij})$ are 0, which naturally complies with the situation that a DM would not conflict with him-/herself.

A reasonable threshold $\theta(0 < \theta < 1)$ is set to distinguish whether conflict exists between two DMs. For two DMs e_i and e_j, if their disharmony degree c_{ij} meets the condition of $c_{ij} > \theta$, then they are assumed to have conflict. Conversely, if $c_{ij} \leq \theta$, we consider that the assessment divergence and disharmony between e_i and e_j is not significant enough to cause conflict.

Definition 4.3. *We use the cut matrix of CIM C_θ to show the distribution of conflicts between DMs. The cut matrix C_θ is defined as follows.*

$$C_\theta = (c_{ij}(\theta)_{m \times m}), where \ c_{ij}(\theta) = \begin{cases} 1, & for \ c_{ij} > \theta, \\ 0, & for \ c_{ij} \leq \theta. \end{cases} \tag{10}$$

According to C_θ, we can construct a conflict network, in which there exists a directed edge from e_i to e_j, if $C_{ij}(\theta) = 1$. Similarly as with ordinary social networks, the nodes in the conflict network have in-degree and out-degree, which can be easily obtained from C_θ.

- **In $-$ degree** of a node e_i in the conflict network C_θ is $I_i = \sum_{j=1}^{m} c_{ji}(\theta)$.
- **Out $-$ degree** of a node e_i in the conflict network C_θ is $O_i = \sum_{j=1}^{m} c_{ij}(\theta)$.

4.3. Conflicts detection and elimination process

After obtaining the conflict network, we can easily visualize the distribution of conflicts between DMs. The following steps are undertaken to analyze the characteristics of the network and further decrease the occurrence of conflicts. Based on the in-degree and out-degree of a conflict network, we define the conflict degree from the perspective of individuals and the entirety of the group, respectively.

Definition 4.4. *We define the conflict degree Ω of a DM e_i as:*

$$\Omega_i = \frac{I_i + O_i}{2m}.$$

For those DMs with high Ω_i, further measures should be taken to decrease their negative influence in the LSGDM process.

Similar to the notion of (preference-based) consensus degree in the decision making process [5], we utilize a parameter ρ to describe the (conflict-based) consensus degree of the entire large group organized under a social network structure.

Definition 4.5. *Given its resemblance to the concept of density of a social network, ρ is defined as the density of conflict in a group, as follows:*

$$\rho = \frac{\sum_{i=1}^{m} \Omega_i}{m}.$$

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Obviously, we should control ρ under a reasonable threshold Φ $(0 < \Phi < 0.5)$ before making the final selection, both to decrease the occurrence of conflict and to ensure an optimal alternative selection process is made. On the other hand, those DMs with low similarity degree to each other contribute more to ρ. In practice, this means that modifying their assessment or evaluation information would be more effective in decreasing ρ than modifying the other DMs' assessments. Therefore, we identify the DM e_k with the highest conflict degree, of which $\Omega_k = max\{\Omega_1, \Omega_2, \dots, \Omega_m\}$, as the one who should modify their assessment information on alternatives under the guidance of a moderator [5, 58, 59, 60].

Whereas the modified assessment information of e_k can be represented by $D'_k = (\langle \mu_1^{k'}, \nu_1^{k'} \rangle, \langle \mu_2^{k'}, \nu_2^{k'} \rangle,$ $\dots, \langle \mu_n^{k'}, \nu_n^{k'} \rangle)$, we provide some reasonable guidelines about the nature of the modification process. The following guidelines are adopted:

- Maximally eradicate in-degrees and out-degrees of DMs to decrease the overall conflict degree.
- Because the elements in D'_k are intuitionistic fuzzy values, they should meet the conditions: $0 \leq \mu_j^{k'} \leq 1$, $0 \leq \nu_j^{k'} \leq 1$, and $0 \leq \mu_j^{k'} + \nu_j^{k'} \leq 1$.
- For convenience in expression, the accuracy of assessment is limited to one decimal place.

Based on these guidelines, and aiming to guarantee that the group conflict degree ρ decreases as much as possible at each iteration, we build a nonlinear optimization model following the guidelines described above to compute the modification orientation of e_k's assessment matrix:

$$max \; H_k = \rho - \rho',$$

$$s.t. \begin{cases} 0 \leq \mu_j^{k'} \leq 1, \; 0 \leq \nu_j^{k'} \leq 1, and \\ 0 \leq \mu_j^{k'} + \nu_j^{k'} \leq 1, \\ \mu_j^{k'}, \nu_j^{k'} \in \{0, 0.1, 0.2, \dots, 0.9, 1\}, \\ j \in \{1, 2, \dots, n\}. \end{cases} \tag{11}$$

In the above optimization model, ρ' is the overall conflict degree after the DM e_k modifies his/her assessment information on alternatives.

Theorem 4.1. *An optimal solution is existed for the nonlinear optimization problem (11).*

Proof. As mentioned in the constraint condition that $\mu_j^{k'}, \nu_j^{k'} \in \{0, 0.1, 0.2, \dots, 0.9, 1\}$, thus, for any $j \in \{1, 2, \dots, n\}$, the intuitionistic fuzzy value $\langle \mu_j^{k'} \nu_j^{k'} \rangle$ has 10! possible values. Therefore, for the optimization problem (11), the number of feasible points are $(10!)^n$. That is, there might be no more than $(10!)^n$ values of H_k, which represents the reduction of group conflict degree between two continuous iterations. In other words, H_k has a maximum value. Therefore, an optimal solution exists for the nonlinear optimization problem (11). $\square$

As the analyses presented in Theorem 4.1 and its proof show, the proposed nonlinear optimization model (11) constitutes an efficient tool to decrease the group conflict degree ρas well as fostering a certain preference modification for the DM who presents a great conflict relation in the LSGDM problem.

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Using MATLAB to solve the problem defined above, we can obtain the recommended assessment information for e_k, which can be represented by $D'_k = (\langle \mu_1^{k'}, \nu_1^{k'} \rangle, \langle \mu_2^{k'}, \nu_2^{k'} \rangle, \ldots, \langle \mu_n^{k'}, \nu_n^{k'} \rangle)$. A moderator is introduced in the LSGDM process to invite the DM to accept the modification scheme. Accordingly,

- If e_k agrees to modify his/her assessment from D_k to D'_k following the advice from the moderator, we calculate the new conflict degree ρ of the entire group, to determine whether $\rho \leq \Phi$. If ρ meets this condition, we proceed to determine the weights of DMs and make the final selection. Otherwise, calculate Ω for all DMs and determine the DM (besides e_k) with the highest Ω. By computing the nonlinear optimization model (11), we can determine how the DM should modify his/her assessment.

- If e_k disagrees to modify his/her assessment, identify the DM (besides e_k) with the highest Ω. By using the optimization model (11), determine how the DM should modify his/her assessment.

To clearly present the steps of the overall conflict detection and elimination process, we provide the following algorithm (Algorithm 2) to illustrate this process.

Algorithm 2 The proposed conflict detection and elimination algorithm

---

Step 1   Let the iteration round be $T = 1$. Input group decision matrix ($\mathbf{D}_{(\mathbf{T=1})}$), complete social network matrix B, and the maximum cycle time T_{max}.

Step 2   If $T \leq T_{max}$, use Eq.(8) to obtain similarity matrix S.
Else, turn to **Step 10**.

Step 3   Use Eq.(9) to obtain conflict information matrix (CIM).

Step 4   Use Eq.(10) to obtain conflict network, and let $i = 1$.

Step 5   Calculate conflict degree $\Omega_{(T)}$ for all DMs, and calculate group conflict degree ρ.

Step 6   Let $\sigma : \{1, 2, \ldots, n\} \rightarrow \{1, 2, \ldots, n\}$ be a permutation such that $\Omega_{\sigma_i} \geq \Omega_{\sigma_{i+1}}$ for $i = \{1, 2, \ldots, m\}$.

Step 7   If $\rho \leq \Phi$, turn to **Step 11**;
Else, if $\sigma_i \leq m$, turn to **Step 8**; otherwise, turn to **Step 10**.

Step 8   Use Eq.(11) to compute the modification scheme for the key DM e_i.

Step 9   If the key DM agrees with the modification plan, let $T = T + 1$, and update the group decision matrix $\mathbf{D}_{(T)}$ by replacing the modification scheme D'_i in it. Then, turn to **Step 2**.
Else, let $i = i + 1$, and turn to **Step 7**.

Step 10   Decision making failed, end .

Step 11   Output D, end.

---

5. Selection process

After eliminating the conflicts among DMs, the value of ρ is intuitively decreased. If ρ meets the condition of $\rho \leq \Phi$, we consider that a sufficient level of harmony has been reached among DMs. In this situation, a high-quality (i.e., low-conflict) large group decision can be made via the selection process, which involves determining DMs weights and ranking the alternatives. Accordingly, in this section we propose a selection method based on conflict to determine DM weights and subsequently rank alternatives predicated on the collective preference information.

The prior modification process normally results in some changes to DMs' assessment information. We

assume that the final group decision matrix is represented by:

$$\mathbf{D} = \begin{pmatrix} d_1^{1'} & \cdots & d_1^{m'} \\ \vdots & \ddots & \vdots \\ d_n^{1'} & \cdots & d_n^{m'} \end{pmatrix} = \begin{pmatrix} <\mu_1^{1'},\nu_1^{1'}> & \cdots & <\mu_1^{m'},\nu_1^{m'}> \\ \vdots & \ddots & \vdots \\ <\mu_n^{1'},\nu_n^{1'}> & \cdots & <\mu_n^{m'},\nu_n^{m'}> \end{pmatrix}.$$

It should be noted that some DMs' assessment information may remain unchanged. Meanwhile, we can compute the conflict degree of each DM, referring to Section 3.3 as $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$.

To minimize the risk of making a wrong decision, many LSGDM models assign low weights to DMs with a low trust degree or consensus degree[5, 16, 38, 39]. As the conflict degree combines trust and assessment information, it is reasonable to use it as the basis for calculating DMs' weights. Therefore, in order to decrease the influence of the conflict among DMs, we define the weights of DMs as below.

Definition 5.1. *Combining the conflict degree Ω_i for the DM e_i, we define the weight of DM e_i as:*

$$\omega_i = \frac{\Omega_{max} - \Omega_i}{\sum_{j=1}^m (\Omega_{max} - \Omega_i)}. \tag{12}$$

Remark 5.1. *It can be easily seen that for DM e_i, the lower his/her conflict degree Ω_i is, the higher the corresponding weight ω_i is. Moreover, if DM e_i exhibits the maximum conflict degree Ω_{max}, his/her resulting weight calculated by Eq. (12) is zero. In essence, the definition of DMs' weights can well decrease the negative effect of their conflicts. DMs' weights are utilized in the following IFWA operator, which is an exponential operation with ω_i. Thus, even a DM e_i shares the weight as $\omega_i = 0$, the contribution of his/her opinion utilized by IFWA operator is not zero.*

Thus, we obtain the weight vector $\omega = (\omega_1, \ \omega_2, \ \ldots, \ \omega_m)$ for all DMs. The weight vector of DMs should meet the conditions: $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$. Moreover, there is no less than one element with the zero value in the weight vector ω.

Based on the above calculated DM weights, we aggregate the assessment information on alternatives x_i into a group assessment using the IFWA operator (see Eq.(1)):

$$\dot{d}_i = IFWA_\omega(d_i^{1'}, \ldots, d_i^{m'}) = \left(1 - \prod_{j=1}^m (1 - \mu_i^{j'})^{\omega_j}, \prod_{j=1}^m \nu_i^{j'\omega_j}\right). \tag{13}$$

Where $\mu_i^{j'}$ and $\nu_i^{j'}$ are the membership and non-membership degrees of e_j with respect to alternative x_i.

In Eq.(13), the DMs' assessment information is aggregated into group assessment information:

$$\dot{d} = \{\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_n\}.$$

Where $\dot{d}_i = \langle \mu_i, \nu_i \rangle$ $(\dot{d}_i \in \dot{d})$ represents the group assessment on alternative x_i. Using the score function

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[54] of IFSs as follows, we can obtain the scores of alternatives:

$$s_i = \mu_i - \nu_i.$$

Finally, we rank the alternatives in decreasing score order, such that the optimal solution to the LSGDM problem is given by the alternative x_j satisfying $s_j = max\{s_1, s_2, \ldots, s_n\}$.

6. Illustrative examples

In this section, we first apply the proposed TR-CDE decision making model to a numerical example (Section 6.1) to show how the model works and demonstrate its feasibility. We then apply our model in a practical LSGDM scenario mainly focused on the conflict detection and elimination process, with the aim of demonstrating that our model is practicable in solving LSGDM problems (Section 6.2). In addition, to show its effectiveness and added value, we compare our model against several other LSGDM models. The behaviors of parameters of TR-CDE model are presented in Section 6.3.

6.1. Numerical example

The main purpose of this subsection is to show the TR-CDE model's feasibility in solving LSGDM problems in a social network context, characterized by existing social relationships among participants. Thus, we first implement the TR-CDE model in a simple decision making problem, containing three alternatives $x_i(i = 1, 2, 3)$ and 10 DMs $e_k(k = 1, 2, \ldots, 10)$, to clearly illustrate how the TR-CDE model works. All DMs adopt IFSs to evaluate the alternative x_i, and they also assess those DMs with whom they have social relationships, in the form of pairs (t, s). In the trust propagation process, we consider that information wanes across this process.

Before showing the numerical example in detail, below we draw some explanations and assumptions for the parameter settings.

i It is assumed that any propagation path with strictly more than two mediators is irrelevant. That is, to obtain the complete trust network, there are no more than two mediators utilized in any trust propagation path in this experiment. The main reason is that, as the value of relationship strength s belongs to [0,1], according to Definition 3.1 and Eq. (5), the more mediators involved in, the lower the propagation efficiency $p_{A,B}$ becomes. Thus, in this experiment, there are no more than two mediators involved in the relationship strength-based trust propagation operator.

ii In the conflict detection and elimination process, the threshold to distinguish whether conflict exists between two DMs is set as $\theta = 0.09$. As shown in Eq. (10), if the disharmony degree c_{ij} meets the condition that $c_{ij} > \theta$ $(0 < \theta < 1)$, the DMs e_i and e_j are regarded as having conflict, and $c_{ij}(\theta) = 1$. Thus, we can conclude that the higher the value of θ is, the less conflict relationships are reflected in the conflict information matrix C_θ. The value of θ can be set according to the values of

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obtained disharmony degree $c_{ij}(i \neq j)$. In this paper, we set $\theta = 0.09$ to dismiss sufficiently weak conflicts (with a low disharmony degree). The detail is shown in Section 6.1.3.

iii The DMs are asked to decrease their conflict level towards a minimum harmony level of $\Phi = 0.30$ before selection. As mentioned in Section 4.3, $0 < \Phi < 0.5$ and if $\rho < \Phi$ the conflict detection and elimination process ends. The upper bound for Φ is 0.5, which means that if no more than half of DMs in the decision making session present conflicts, a sufficient consensus level has been reached. The lower the value of Φ is, the less conflict relationships are allowed for achieving the consensus. Therefore, we set Φ as 0.3, which lies within a reasonable range.

vi The maximum number of iteration rounds (T) for the conflict elimination process is set to be $T_{max} = 6$. It is necessary to set the maximum iteration round T_{max}, since time-efficiency is a key factor in iterative decision processes. If consensus is reached within T_{max} rounds, the process is deemed effective. The setting for T_{max} depends on the temporal cost allowed in the decision making session, which is shown in the following definition.

Definition 6.1. *We suggest that*

$$T_{max} = \ulcorner (1 - \beta) \cdot m \urcorner,$$

where $\beta \in (0,1)$ is the urgency level, fixed by a representative of the group depending on the nature of the problem. The more urgent the LSGDM problem is, the higher the value of β can be set. Thus, the lower the value of T_{max} is, which can guarantee the decision should be made in less iterations. The value of β should be set higher. Based on this rule, as the decision making event is not an urgent event, we set $\beta = 0.4$, thus, T_{max} as 6 in this numerical example.

6.1.1. Initial data

Every DM evaluates the other DMs they know with a pair of the form (t, s), where t is the trust degree and s is the relationship strength. We collect the relationship information and obtain the initial social network matrix B, as shown in Table 2.

Table 2. The initial social network matrix containing social relationship pairs (t, s).

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	$(1,0)$	$(0.8,0.1)$	$(0.6,0.4)$	$(0.2,0.7)$	$(0.6,0.2)$	$(0,1)$	$(-,0)$	$(0.2,0.1)$	$(-,0)$	$(0.7,0.2)$
e_2	$(-,0)$	$(1,0)$	$(-,0)$	$(0.9,0.1)$	$(0.1,0.8)$	$(0.4,0.5)$	$(0.1,0.8)$	$(0.7,0.2)$	$(0.2,0.8)$	$(-,0)$
e_3	$(-,0)$	$(-,0)$	$(1,0)$	$(0.3,0.7)$	$(-,0)$	$(0.3,0.5)$	$(0.7,0.2)$	$(-,0)$	$(0.7,0.2)$	$(0.3,0.5)$
e_4	$(0.1,0.9)$	$(0.2,0.8)$	$(0.4,0.5)$	$(1,0)$	$(0.8,0.1)$	$(-,0)$	$(0.1,0.8)$	$(0.6,0.2)$	$(0.4,0.2)$	$(-,0)$
e_5	$(0.7,0.2)$	$(0.7,0.2)$	$(0.8,0.5)$	$(0.2,0.1)$	$(1,0)$	$(0.4,0.6)$	$(1,0)$	$(0.7,0.2)$	$(-,0)$	$(0.1,0.8)$
e_6	$(0.4,0.1)$	$(-,0)$	$(0.6,0.3)$	$(0.7,0.2)$	$(0.8,0.1)$	$(1,0)$	$(0.8,0.1)$	$(0.5,0.3)$	$(1,1)$	$(0.3,0.7)$
e_7	$(1,1)$	$(0.3,0.7)$	$(-,0)$	$(0.2,0.7)$	$(1,1)$	$0.4,0.6$	$(1,0)$	$(1,1)$	$(-,0)$	$(-,0)$
e_8	$(-,0)$	$(0.7,0.2)$	$(0.4,0.2)$	$(0.4,0.3)$	$(-,0)$	$(0.7,0.1)$	$(0.4,0.5)$	$(1,0)$	$(-,0)$	$(-,0)$
e_9	$(0.9,0.1)$	$(0.3,0.7)$	$(0.6,0.4)$	$(0.3,0.4)$	$(0.1,0.7)$	$(0.7,0.2)$	$(0.9,0.1)$	$(-,0)$	$(1,0)$	$(0.2,0.8)$
e_{10}	$(0.7,0.1)$	$(1,1)$	$(0.1,0.9)$	$(-,0)$	$(0.7,0.1)$	$(-,0)$	$(0.9,0.8)$	$(0.8,0.8)$	$(0.4,0.8)$	$(1,0)$

Meanwhile, DMs provide their assessments with intuitionistic fuzzy values. We collect the assessment information as described in Section 4.1 and obtainthe group decision matrix $\mathbf{D}_{(T=1)}$, as shown in Table 3.

Table 3. The initial group decision matrix $\mathbf{D}_{(T=1)}$.

DMs	x_1	x_2	x_3
e_1	$\langle 1, 0 \rangle$	$\langle 0.4, 0.6 \rangle$	$\langle 0.1, 0.9 \rangle$
e_2	$\langle 0.4, 0.5 \rangle$	$\langle 0.1, 0.8 \rangle$	$\langle 0, 0.8 \rangle$
e_3	$\langle 0.8, 0.1 \rangle$	$\langle 0.7, 0.2 \rangle$	$\langle 0.3, 0.5 \rangle$
e_4	$\langle 0.1, 0.6 \rangle$	$\langle 0.9, 0 \rangle$	$\langle 0.5, 0.5 \rangle$
e_5	$\langle 0.9, 0.1 \rangle$	$\langle 0.3, 0.3 \rangle$	$\langle 0.1, 0.8 \rangle$
e_6	$\langle 0.5, 0.4 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.9, 0.1 \rangle$
e_7	$\langle 0.5, 0.5 \rangle$	$\langle 0.3, 0.2 \rangle$	$\langle 0.4, 0.6 \rangle$
e_8	$\langle 0.2, 0.7 \rangle$	$\langle 0.6, 0.4 \rangle$	$\langle 0.8, 0.1 \rangle$
e_9	$\langle 0.5, 0.3 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.2, 0.6 \rangle$
e_{10}	$\langle 0.6, 0.4 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0.5, 0.5 \rangle$

6.1.2. Trust propagation process

As Table 2 shows, the trust values $t_{1,7}$, $t_{1,9}$, $t_{2,1}$, $t_{2,3}$, $t_{3,1}$, $t_{3,2}$, $t_{3,5}$, $t_{3,8}$, $t_{4,6}$, $t_{4,10}$, $t_{5,9}$, $t_{6,2}$, $t_{7,3}$, $t_{7,9}$, $t_{7,10}$, $t_{8,1}$, $t_{8,5}$, $t_{8,9}$, $t_{8,10}$, $t_{9,8}$, $t_{10,4}$, and $t_{10,6}$ are missing. We take the computation of $t_{2,1}$ (i.e. the level of trust from e_2 to e_1) as an example to show how the proposed trust propagating operator works.

Referring to Table 2, we can draw a tree diagram (see Fig.8) comprising all the paths from e_2 to e_1 with at most two mediators between them.
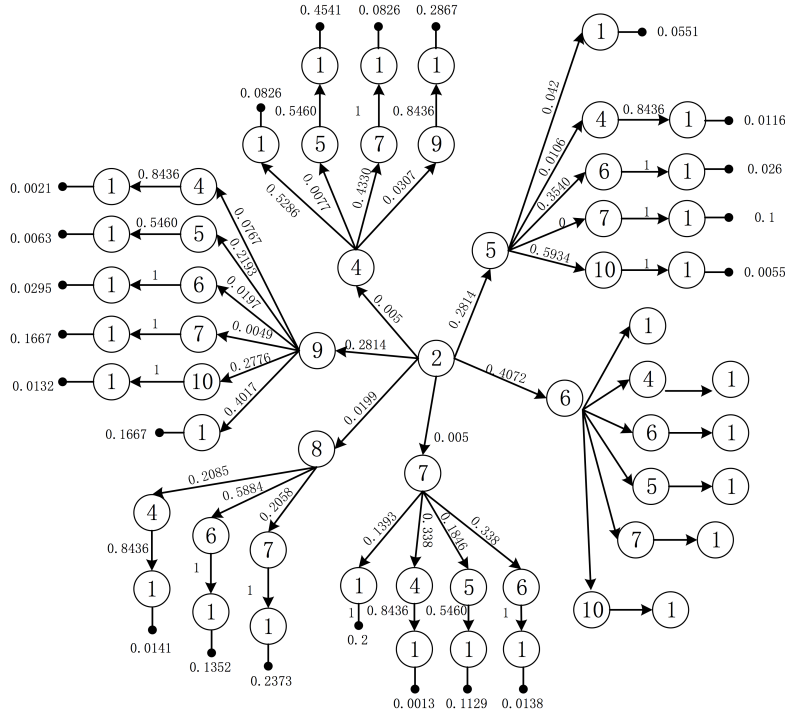
Fig. 8. Tree diagram showing propagation paths from e_2 to e_1.

As Fig.8 shows, there are six branches describing participants about whom e_2 supplied direct trust information ($e_2 \to e_4$, $e_2 \to e_5$, $e_2 \to e_6$, $e_2 \to e_7$, $e_2 \to e_8$, and $e_2 \to e_9$), constituting the trust propagation paths from e_2 to e_1. We next calculate the propagation efficiency p from one DM to another. Then, according to relationship strength-based trust propagation operator (see Algorithm 1), we normalize the propagation efficiency for the furcate DMs. The values of p (including normalized p) are presented on the edges. Using Eq.(2) and Eq.(6), we calculate the full propagated trust values for all the paths, shown alongside the terminal node of each path. It should be noted that the relationship information for

e_2 with respect to e_6 is 1, according to the full distrust propagation, the trust value for all the paths on this branch is 0. Therefore, for convenience, we can omit the calculation.

Following this, we calculate the trust value for each branch. Taking the branch $e_2 \rightarrow e_8$ as an example, the trust value for this branch is: $0.0199 \times (0.2085 \times 0.8436 \times 0.0141 + 0.5884 \times 1 \times 0.1352 + 0.2058 \times 1 \times 0.2373) = 2.617 \times 10^{-3}$.

By summing all the trust values for each branch together, we obtain the trust value from e_2 to e_1 of $t_{2,1} = 0.02029$. Similarly, the trust values are computed for other indirect relationships: $t_{1,7} = 0.2178$, $t_{1,9} = 0.1223$, $t_{2,1} = 0.0203$, $t_{2,3} = 0.4032$, $t_{3,1} = 0.0592$, $t_{3,2} = 0.2687$, $t_{3,5} = 0.4572$, $t_{3,8} = 0.3111$, $t_{4,6} = 0.1782$, $t_{4,10} = 0.2004$, $t_{5,9} = 0.1673$, $t_{6,2} = 0.1792$, $t_{7,3} = 0.2243$, $t_{7,9} = 0.1578$, $t_{7,10} = 0.2433$, $t_{8,1} = 0.4552$, $t_{8,5} = 0.0034$, $t_{8,9} = 0.5224$, $t_{8,1} = 0.1231$, $t_{9,8} = 0.4722$, $t_{10,4} = 0.1930$, and $t_{10,6} = 0.0853$. We thereby obtain the complete social network, which is the basis for the following conflict detection and elimination process.

6.1.3. Conflict detection and elimination

In this application example, we use MATLAB to simulate the conflict detection and elimination process, and to compute the result over the course of the decision process. We first set the iteration $T=1$, and compute the similarity matrix by Eq.(8) based on $\mathbf{D}_{(T=1)}$, as shown in Table 4.

Table 4. The similarity matrix SM in the iteration round $T = 1$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	1.00	0.63	0.80	0.50	0.87	0.55	0.65	0.50	0.68	0.78
e_2	0.63	1.00	0.57	0.53	0.70	0.52	0.75	0.53	0.68	0.82
e_3	0.80	0.57	1.00	0.70	0.77	0.72	0.78	0.60	0.82	0.68
e_4	0.50	0.53	0.70	1.00	0.47	0.68	0.75	0.73	0.82	0.55
e_5	0.87	0.70	0.77	0.47	1.00	0.52	0.72	0.43	0.65	0.85
e_6	0.55	0.52	0.72	0.68	0.52	1.00	0.77	0.85	0.73	0.60
e_7	0.65	0.75	0.78	0.75	0.72	0.77	1.00	0.68	0.87	0.77
e_8	0.50	0.53	0.60	0.73	0.43	0.85	0.68	1.00	0.65	0.55
e_9	0.68	0.68	0.82	0.82	0.65	0.73	0.87	0.65	1.00	0.73
e_{10}	0.78	0.82	0.68	0.55	0.85	0.60	0.77	0.55	0.73	1.00

By Eq.(9), we compute the conflict information matrix CIM (Table 5).

Table 5. The conflict information matrix (CIM) in the iteration round $T = 1$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0.00	0.04	0.04	0.20	0.02	0.23	0.14	0.20	0.12	0.05
e_2	0.18	0.00	0.13	0.02	0.13	0.15	0.11	0.07	0.14	0.13
e_3	0.09	0.16	0.00	0.11	0.05	0.10	0.03	0.14	0.02	0.08
e_4	0.23	0.19	0.09	0.00	0.05	0.13	0.11	0.05	0.03	0.11
e_5	0.02	0.04	0.02	0.19	0.00	0.13	0.00	0.08	0.11	0.12
e_6	0.14	0.20	0.06	0.05	0.04	0.00	0.02	0.04	0.13	0.08
e_7	0.00	0.09	0.08	0.10	0.00	0.07	0.00	0.00	0.07	0.05
e_8	0.14	0.07	0.12	0.08	0.26	0.02	0.10	0.00	0.09	0.12
e_9	0.01	0.12	0.03	0.08	0.12	0.04	0.01	0.10	0.00	0.09
e_{10}	0.05	0.00	0.10	0.11	0.04	0.11	0.06	0.11	0.09	0.00

We can obtain a statistic result from Table 5, which is drawn in Fig.9. As there are 10 DMs involved in the decision making event, there exist 90 valid values of disharmony degrees, c_{ij} $(i \neq j)$, for the 90

DM pairs in CIM. The value of $c_{ij}(i \neq j)$ ranges from 0 to 0.26. In order to dismiss some DM pairs with low disharmony degrees, which are regarded as having weak influence, we consider keeping half of the conflict information among the 90 DM pairs (that is 45) in this example, which follows the absolute majority principle in terms of amount of non-conflicting participants versus conflicting participants. It can be easily seen in Fig.9, there are 47 DMs, which is the closest number with 45, sharing a disharmony degrees not lower than 0.09. Thus, we choose 0.09 to be the value of the threshold θ in the numerical example.
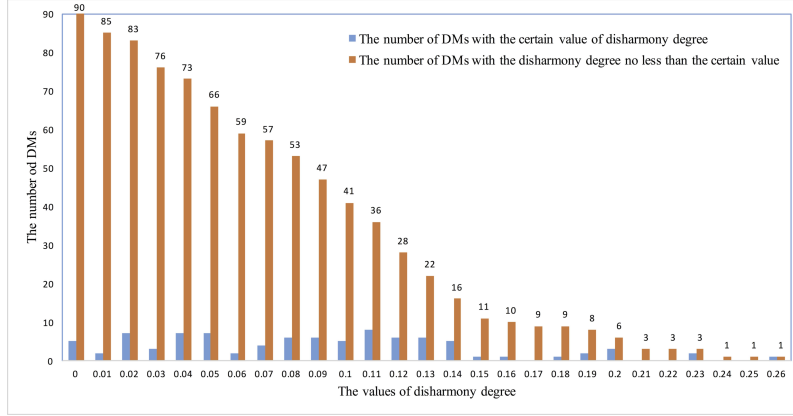
Fig. 9. Number of DMs with different level of disharmony degree $c_{ij}(i \neq j)$.

According to Table 5, the conflict network can be detected with Eq.(10), as shown in Table 6. Thus, we obtain the conflict degree Ω for DMs: $\Omega_{(T=1)}$={0.56,0.61,0.50,0.61,0.39,0.50,0.28,0.56,0.56,0.56}. The group conflict degree is $\rho = 0.51$, which exceeds the threshold $\Phi = 0.30$.

Table 6. The cut matrix (C_θ) in the iteration round $T = 1$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0	0	0	1	0	1	1	1	1	0
e_2	1	0	1	0	1	1	1	0	1	1
e_3	1	1	0	1	0	1	1	0	0	1
e_4	1	1	0	0	0	1	1	0	0	1
e_5	0	0	0	1	0	1	0	0	1	1
e_6	1	1	0	0	0	0	0	0	1	0
e_7	0	0	0	1	0	0	0	0	0	0
e_8	1	0	1	0	1	0	1	0	0	1
e_9	0	1	0	0	1	0	0	1	0	1
e_{10}	0	0	1	1	0	1	0	1	1	0

Therefore, the intra-group conflict should be reduced before proceeding to make a group decision. e_2 has the highest conflict degree, thus being the target DM in this round. Using Eq.(11) to compute the modification plan, we can get the suggested assessment information vector $D_2' = \{\langle 0.1, 0.1 \rangle, \langle 0.4, 0.5 \rangle, \langle 0.2, 0.7 \rangle\}$. The DM e_2, guided by the moderator, agrees with the modification plan, then we replace the D_2 with the modified assessment vector D_2' and update the group decision matrix $\mathbf{D}$.

In the second round, $T=2$, we calculate the SM based on the updated $\mathbf{D}_{(\mathbf{T=2})}$, and output CIM, shown in Table 7. We calculate the cut matrix of CIM, and further obtain the conflict degree Ω for

Table 7. The conflict information matrix (CIM) in the iteration round $T = 2$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0.00	0.03	0.04	0.20	0.02	0.23	0.14	0.20	0.12	0.05
e_2	0.13	0.00	0.08	0.02	0.10	0.13	0.11	0.06	0.10	0.10
e_3	0.09	0.10	0.00	0.11	0.05	0.10	0.03	0.14	0.02	0.08
e_4	0.23	0.13	0.09	0.00	0.05	0.13	0.11	0.05	0.03	0.11
e_5	0.02	0.03	0.02	0.05	0.00	0.13	0.00	0.08	0.11	0.12
e_6	0.14	0.17	0.06	0.06	0.04	0.00	0.02	0.04	0.13	0.08
e_7	0.00	0.09	0.08	0.10	0.00	0.07	0.00	0.00	0.07	0.05
e_8	0.14	0.06	0.12	0.13	0.26	0.02	0.10	0.00	0.09	0.12
e_9	0.01	0.09	0.03	0.08	0.12	0.04	0.01	0.10	0.00	0.09
e_{10}	0.05	0.00	0.10	0.09	0.04	0.11	0.06	0.11	0.09	0.00

DMs: $\Omega_{(T=2)}$={0.56, 0.50, 0.4, 0.61, 0.39, 0.50, 0.28, 0.56, 0.50, 0.56}. The group conflict degree ρ is 0.50, which still exceeds the threshold $\Phi = 0.30$. e_4 shows the highest conflict degree and becomes the key DM. Computing the modification plan, we obtain $D_4' = \{\langle 0.7, 0.2\rangle, \langle 0.5, 0.4\rangle, \langle 0, 0.8\rangle\}$. The DM e_4 agrees to modify his/her assessment information, so the group decision matrix is updated to $\mathbf{D}_{(T=3)}$ and the third iteration begins, T=3. The CIM for T=3 is shown in Table 8.

Table 8. The conflict information matrix (CIM) in the iteration round $T = 3$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0.00	0.03	0.04	0.05	0.02	0.23	0.14	0.20	0.12	0.05
e_2	0.13	0.00	0.08	0.01	0.10	0.13	0.11	0.06	0.10	0.10
e_3	0.09	0.10	0.00	0.07	0.05	0.10	0.03	0.14	0.02	0.08
e_4	0.06	0.08	0.06	0.00	0.01	0.16	0.11	0.09	0.03	0.09
e_5	0.02	0.03	0.02	0.19	0.00	0.13	0.00	0.08	0.11	0.12
e_6	0.14	0.17	0.06	0.05	0.04	0.00	0.02	0.04	0.13	0.08
e_7	0.00	0.09	0.08	0.10	0.00	0.07	0.00	0.00	0.07	0.05
e_8	0.14	0.06	0.12	0.08	0.26	0.02	0.10	0.00	0.09	0.12
e_9	0.01	0.09	0.03	0.08	0.12	0.04	0.01	0.10	0.00	0.09
e_{10}	0.05	0.00	0.10	0.11	0.04	0.11	0.06	0.11	0.09	0.00

By calculating the cut matrix of CIM, we can obtain the conflict degree Ω for DMs as $\Omega_{(T=3)} = $ {0.44, 0.44, 0.33, 0.22, 0.33, 0.50, 0.28, 0.61, 0.50, 0.44}. Similarly, we calculate the group conflict degree and obtain $\rho = 0.41$, which is less than the value with T=2, but it is still higher than the preset threshold $\Phi = 0.30$. Thus, the process continues. e_8 is the key DM. We compute the modification plan, which is $D_8' = \{\langle 0.6, 0.3\rangle, \langle 0.9, 0\rangle, \langle 0.3, 0.6\rangle\}$. e_8 agrees to modify his assessment information.

Therefore, the group decision matrix is updated as $\mathbf{D}_{(T=4)}$, after which T=4. Following this, the CIM for T=4 is achieved and is showed in Table 9.

Table 9. The conflict information matrix (CIM) in the iteration round $T = 4$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0.00	0.03	0.04	0.05	0.02	0.23	0.14	0.13	0.12	0.05
e_2	0.13	0.00	0.08	0.01	0.10	0.13	0.11	0.05	0.10	0.10
e_3	0.09	0.10	0.00	0.07	0.05	0.10	0.03	0.05	0.02	0.08
e_4	0.06	0.08	0.06	0.00	0.01	0.16	0.11	0.05	0.03	0.09
e_5	0.02	0.03	0.02	0.05	0.00	0.13	0.00	0.05	0.11	0.12
e_6	0.14	0.17	0.06	0.06	0.04	0.00	0.02	0.08	0.13	0.08
e_7	0.00	0.09	0.08	0.10	0.00	0.07	0.00	0.00	0.07	0.05
e_8	0.09	0.05	0.05	0.08	0.15	0.05	0.06	0.00	0.02	0.10
e_9	0.01	0.09	0.03	0.08	0.12	0.04	0.01	0.02	0.00	0.09
e_{10}	0.05	0.00	0.10	0.09	0.04	0.11	0.06	0.09	0.09	0.00

For $T=4$, we have Ω for DMs as $\Omega_{(T=4)} = \{0.39, 0.44, 0.22, 0.17, 0.33, 0.50, 0.22, 0.22, 0.39, 0.44\}$, the group conflict degree is $\rho = 0.33$, which exceeds the threshold $\Phi = 0.30$. Thus, we obtain that e_6 is the key DM, and compute the modification plan, which is $D_6^{'} = \{\langle 0.2, 0.7 \rangle, \langle 0.3, 0.6 \rangle, \langle 0.2, 0.3 \rangle\}$. The DM e_6 agrees to modify his assessment information. Therefore, let $T=5$ and we update $\mathbf{D}_{(T=4)}$ to $\mathbf{D}_{(T=5)}$. Following this, the CIM for $T=5$ is showed in Table 10.

Table 10. The conflict information matrix (CIM) in the iteration round $T = 5$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0.00	0.03	0.04	0.05	0.02	0.23	0.14	0.13	0.12	0.05
e_2	0.13	0.00	0.08	0.01	0.10	0.07	0.11	0.05	0.10	0.10
e_3	0.09	0.10	0.00	0.07	0.05	0.13	0.03	0.05	0.02	0.08
e_4	0.06	0.08	0.06	0.00	0.01	0.14	0.11	0.05	0.03	0.09
e_5	0.02	0.03	0.02	0.05	0.00	0.12	0.00	0.05	0.11	0.12
e_6	0.14	0.09	0.08	0.05	0.04	0.00	0.02	0.19	0.17	0.08
e_7	0.00	0.09	0.08	0.10	0.00	0.07	0.00	0.00	0.07	0.05
e_8	0.09	0.05	0.05	0.08	0.15	0.06	0.06	0.00	0.02	0.10
e_9	0.01	0.09	0.03	0.08	0.12	0.05	0.01	0.02	0.00	0.09
e_{10}	0.05	0.00	0.10	0.09	0.04	0.11	0.06	0.09	0.09	0.00

For $T=5$, we have Ω for DMs as $\Omega_{(T=5)} = \{0.39, 0.33, 0.22, 0.17, 0.33, 0.44, 0.22, 0.28, 0.39, 0.44\}$, the group conflict degree is $\rho = 0.32$, which exceeds the threshold $\Phi = 0.30$. Thus, we again set e_6 as the key DM, and compute the modification plan, which is $D_6^{'} = \{\langle 0.1, 0.6 \rangle, \langle 0.3, 0.2 \rangle, \langle 0.7, 0.2 \rangle\}$. However, the DM e_6 refuses to make compromises this time. Therefore, we turn to e_{10}, and compute the modification plan for him, which is $D_{10}^{'} = \{\langle 0, 0 \rangle, \langle 0.3, 0.6 \rangle, \langle 0.6, 0.3 \rangle\}$. The DM e_{10} also refuses to make compromise. Thus, we turn to the DM with the third largest conflict degree, that is e_1, and compute the modification plan as follows: $D_1^{'} = \{\langle 0.6, 0.4 \rangle, \langle 0.4, 0.5 \rangle, \langle 0.5, 0.5 \rangle\}$. Through persuasion by the moderator, e_1 finally changes his/her assessment information, and the group decision matrix $\mathbf{D}$ is updated. Let the iteration round be $T=6$, the CIM for this round is calculated as shown in Table 11.

Table 11. The conflict information matrix (CIM) in the iteration round $T = 6$.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
e_1	0.00	0.02	0.06	0.06	0.05	0.10	0.07	0.12	0.10	0.03
e_2	0.07	0.00	0.08	0.01	0.10	0.07	0.11	0.05	0.10	0.10
e_3	0.015	0.10	0.00	0.07	0.05	0.13	0.03	0.05	0.02	0.08
e_4	0.07	0.08	0.06	0.00	0.01	0.14	0.11	0.05	0.03	0.09
e_5	0.04	0.03	0.02	0.05	0.00	0.11	0.00	0.05	0.11	0.12
e_6	0.06	0.09	0.08	0.05	0.04	0.00	0.02	0.10	0.17	0.08
e_7	0.00	0.09	0.08	0.10	0.00	0.07	0.00	0.00	0.07	0.05
e_8	0.08	0.05	0.05	0.08	0.15	0.06	0.06	0.00	0.02	0.10
e_9	0.01	0.09	0.03	0.08	0.12	0.05	0.01	0.02	0.00	0.09
e_{10}	0.03	0.00	0.10	0.09	0.04	0.11	0.06	0.09	0.09	0.00

We calculate the conflict degree for individuals and the group. The individual conflict degree is $\Omega_{(T=6)} = \{ 0.22, 0.28, 0.22, 0.17, 0.33, 0.39, 0.17, 0.28, 0.39, 0.44 \}$ and the group conflict degree is $\rho = 0.29$. As the group conflict degree is less than the preset threshold $\Phi = 0.30$, the ending condition is met. We output the final group decision matrix $\mathbf{D}_{(T=6)}$ as shown in Table 12.

Table 12. The final group decision matrix $\mathbf{D}_{(T=6)}$.

DMs	x_1	x_2	x_3
e_1	$\langle 0.6, 0.4 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0.5, 0.5 \rangle$
e_2	$\langle 0.1, 0.1 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0.2, 0.7 \rangle$
e_3	$\langle 0.8, 0.1 \rangle$	$\langle 0.7, 0.2 \rangle$	$\langle 0.3, 0.5 \rangle$
e_4	$\langle 0.7, 0.2 \rangle$	$\langle 0.5, 0.4 \rangle$	$\langle 0, 0.8 \rangle$
e_5	$\langle 0.9, 0.1 \rangle$	$\langle 0.3, 0.3 \rangle$	$\langle 0.1, 0.8 \rangle$
e_6	$\langle 0.2, 0.7 \rangle$	$\langle 0.3, 0.6 \rangle$	$\langle 0.2, 0.3 \rangle$
e_7	$\langle 0.5, 0.5 \rangle$	$\langle 0.3, 0.2 \rangle$	$\langle 0.4, 0.6 \rangle$
e_8	$\langle 0.6, 0.3 \rangle$	$\langle 0.9, 0 \rangle$	$\langle 0.3, 0.6 \rangle$
e_9	$\langle 0.5, 0.3 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.2, 0.6 \rangle$
e_{10}	$\langle 0.6, 0.4 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0.5, 0.5 \rangle$

6.1.4. Selection process

Through the conflict detection and elimination process, the group conflict degree has been reduced below a reasonable threshold. This implies that the DMs have reached sufficient mutual acceptance of other DMs' assessments. Therefore, the decision group moves on to the selection process.

According to Eq. (12), we calculate DMs' weights obtain the weight vector, as follows:

$$\omega = \{0.1429, 0.1071, 0.1429, 0.1786, 0.0714, 0.0357, 0.1786, 0.1071, 0.0357, 0\}.$$

As DM e_{10} shares the maximum conflict degree $\Omega_{10} = 0.44$, thus we have $\Omega_{max} = \Omega_{10} = 0.44$. According to Eq. (12), the weight for e_{10} is zero.

Using Eq.(13), we obtain the group assessment on each alternative, as $\dot{d}_1 = \langle 0.63, 0.23 \rangle$, $\dot{d}_2 = \langle 0.56, 0.35 \rangle$, $\dot{d}_3 = \langle 0.28, 0.61 \rangle$. The scores of the alternatives are: $s_1 = 0.40, s_2 = 0.21$, and $s_3 = -0.33$. We can get $s_1 \succ s_2 \succ s_3$. Therefore, s_1 is the final choice for the LSGDM problem.

6.1.5. Discussion

In the trust propagation process illustrated in this example, we consider the number of mediators for each path as strictly no more than two for propagation to occur. The indirect relationship information is calculated stepwise with the proposed trust propagation operator. Finally, we obtain the complete social network, constituting the necessary data foundation for the subsequent conflict removal process.

In the conflict detection and elimination process, we utilize MATLAB to simulate the optimization problem. By obtaining the cut matrix of CIM, we gain clear insight into the distribution of conflicts across the group. Calculating the conflict degree of participants enables the DMs who significantly hamper the group harmony to be identified and targeted for assessment modification. By solving the nonlinear optimization model proposed in Eq.(11), we derive the optimal modification plan for the key DM. Under the guidance of a moderator, the group conflict is decreased stepwise. The variation trends in the conflict degree for individuals (Ω) and the overall group (ρ) per round are presented in Fig.10.

As described in the objective function of the nonlinear optimization problem, which is to maximize the decrease of group conflict degree ρ, the group conflict degree ρ is decreased stepwise, it is implied
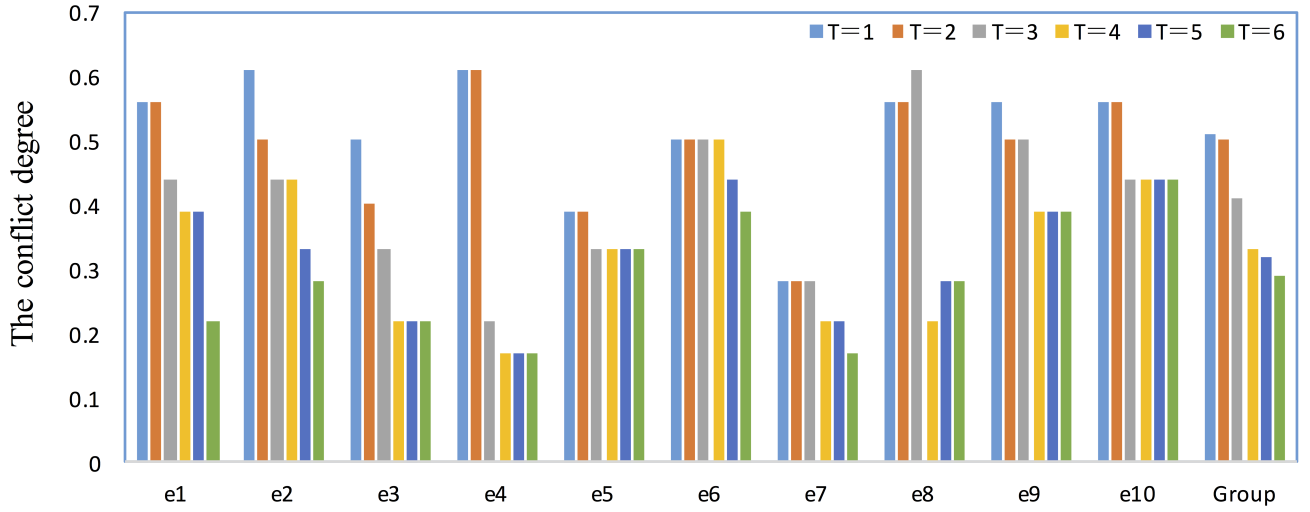
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Fig. 10. The variation trend of conflict degree for individuals (Ω)/group (ρ).

in Fig.10. Meanwhile, for the majority of DMs, their individual conflict degree Ω also decreases in the process, except for DM e_8 in the iteration round $T = 3$. It is an acceptable result, as the objective function guarantees a decrease of group conflict ρ. After six iterations, the aggregated group conflict degree ρ had dropped from 0.51 to 0.29, which demonstrates that the proposed TR-CDE decision making model can efficiently eliminate conflict.

6.2. Practical scenario

As mentioned in [4, 61], the number of DMs in the LSGDM problem is typically assumed to be no less than 20. In this subsection, we apply the TR-CDE model to a practical LSGDM scenario with 20 DMs and make some comparisons with some other representative LSGDM models.

6.2.1. Calculation process for the TR-CDE model

The LSGDM scenario is described as follows. A department that is responsible for a certain river basin in China plans to build a large hydropower station. A village comprising 20 families, is located nearby the selected site and will need to be relocated. The government proposes four alternative relocation compensation schemes for these families: 1) money-dominant scheme; 2) housing-dominant scheme; 3) job-dominant scheme; and 4) lowland-dominant scheme. To ensure scientific decision making, each family nominates a representative, and the 20 representatives must collectively decide which scheme is the most acceptable one for the villagers. We use $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, \ldots, e_{20}\}$ to represent the alternative set and DM set, respectively. The thresholds for the case are: each path comprises no more than one mediator, $\theta = 0.09$, $\beta = 0.5$ and conflict threshold $\Phi = 0.30$. Then we have the maximum iteration $T_{max} = 10$.

The 20 DMs' evaluation is shown in the group decision matrix in Table 13.

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Table 13. The initial group decision matrix $\mathbf{D}_{(T=1)}$.

DMs	x_1	x_2	x_3	x_4
e_1	$\langle 0.1, 0.7 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.1, 0.3 \rangle$	$\langle 0.8, 0.1 \rangle$
e_2	$\langle 0.9, 0.1 \rangle$	$\langle 0.3, 0.5 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0, 0.4 \rangle$
e_3	$\langle 0.5, 0.5 \rangle$	$\langle 0.2, 0.2 \rangle$	$\langle 0.3, 0.7 \rangle$	$\langle 0.3, 0.7 \rangle$
e_4	$\langle 0.8, 0.2 \rangle$	$\langle 0.1, 0.9 \rangle$	$\langle 0.3, 0.5 \rangle$	$\langle 0.3, 0.4 \rangle$
e_5	$\langle 0.4, 0.5 \rangle$	$\langle 0.7, 0.3 \rangle$	$\langle 0, 0.4 \rangle$	$\langle 0.1, 0.9 \rangle$
$\cdots$				
e_{16}	$\langle 0.6, 0.3 \rangle$	$\langle 0.4, 0 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.2, 0.8 \rangle$
e_{17}	$\langle 0.3, 0 \rangle$	$\langle 0.1, 0.5 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.1, 0.6 \rangle$
e_{18}	$\langle 0.4, 0.4 \rangle$	$\langle 0.1, 0.6 \rangle$	$\langle 0.1, 0.3 \rangle$	$\langle 0, 0.8 \rangle$
e_{19}	$\langle 0.4, 0.1 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0, 0.1 \rangle$	$\langle 0.1, 0.4 \rangle$
e_{20}	$\langle 0.6, 0.2 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0.3, 0.3 \rangle$	$\langle 0.3, 0.5 \rangle$

Those DMs evaluate the other DMs they know, forming the following initial social network matrix (Table 14).

Table 14. The initial social network matrix containing social relationship pairs (t, s).

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$	e_{12}	e_{13}	e_{14}	e_{15}	$\cdots$	e_{20}
e_1	(0,1)	(0.5,1)	(0.1,0.1)	(−,**0**)	(0.3,0.6)	(0.1,0.9)	(0.7,0.2)	$\cdots$	(0.8,0.9)	(0.5,0.6)	(−,**0**)	(0.6,1)	$\cdots$	(0.6,0.2)
e_2	(0.3,0.9)	(0,1)	(0.7,0.3)	(0.4,0.6)	(0.1,0.7)	(−,**0**)	(−,**0**)	$\cdots$	(0.1,0.3)	(0.5,0.4)	(0.8,0.5)	(0.1,1)	$\cdots$	(0.7,0.6)
e_3	(0.5,0.1)	(0.7,0)	(0,1)	(0.1,0.9)	(0.6,0.6)	(0.9,0.9)	(0.4,0.1)	$\cdots$	(0.9,0.3)	(0.1,0.1)	(0.2,0.4)	(−,**0**)	$\cdots$	(0.4,0.4)
e_4	(−,**0**)	(0.2,0.8)	(−,**0**)	(0,1)	(0.3,0.9)	(0.9,0.7)	(0.1,0.5)	$\cdots$	(0.1,0.9)	(−,**0**)	(0.1,0.2)	(0.1,0.5)	$\cdots$	(0.9,0.1)
e_5	(0.9,0.6)	(0.7,0.8)	(0.5,09)	(0.2,0.2)	(0,1)	(0.3,0.6)	(0.8,0.3)	$\cdots$	(0.4,0.3)	(0.9,0.5)	(0.9,0.5)	(0.5,0.9)	$\cdots$	(0.9,0.4)
e_6	(−,**0**)	(0.2,0.9)	(0.1,0.7)	(0.3,0.4)	(0.8,0.7)	(0,1)	(0.3,0.3)	$\cdots$	(0.6,0.1)	(0.1,0.3)	(0.9,0.3)	(0.1,0.2)	$\cdots$	(0.8,0.3)
$\cdots$								$\cdots$					$\cdots$	
e_{19}	(0.4,0.7)	(0.6,0.2)	(0.7,0.1)	(0.6,0.6)	(0.4,0.5)	(0.2,0.9)	(0.7,0.1)	$\cdots$	(−,**0**)	(0.5,0.3)	(1,0.7)	(0.5,0.8)	$\cdots$	(1,0.6)
e_{20}	(0.6,0.2)	(0.6,0.2)	(0.6,0.7)	(0.4,0.1)	(0.2,0.6)	(0.4,1)	(0.6,0.3)	$\cdots$	(0.3,0.1)	(0.1,1)	(0.5,0.9)	(0.4,0.7)	$\cdots$	(0,1)

For the sake of space efficiency, we calculate the complete social network directly by the proposed trust propagation operator, showed in Table 15.

Table 15. The completed social network matrix.

DMs	e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$	e_{12}	e_{13}	e_{14}	e_{15}	$\cdots$	e_{20}
e_1	(0,1)	(0.5,1)	(0.1,0.1)	(0.2,0)	(0.3,0.6)	(0.1,0.9)	(0.7,0.2)	$\cdots$	(0.8,0.9)	(0.5,0.6)	(0.2,0)	(0.6,1)	$\cdots$	(0.6,0.2)
e_2	(0.3,0.9)	(0,1)	(0.7,0.3)	(0.4,0.6)	(0.1,0.7)	(0.3,0)	(0.2,0)	$\cdots$	(0.1,0.3)	(0.5,0.4)	(0.8,0.5)	(0.1,1)	$\cdots$	(0.7,0.6)
e_3	(0.5,0.1)	(0.7,0)	(0,1)	(0.1,0.9)	(0.6,0.6)	(0.9,0.9)	(0.4,0.1)	$\cdots$	(0.9,0.3)	(0.1,0.1)	(0.2,0.4)	(0.2,0)	$\cdots$	(0.4,0.4)
e_4	(0.4,0)	(0.2,0.8)	(0.2,0)	(0,1)	(0.3,0.9)	(0.9,0.7)	(0.1,0.5)	$\cdots$	(0.1,0.9)	(0.2,0)	(0.1,0.2)	(0.1,0.5)	$\cdots$	(0.9,0.1)
e_5	(0.9,0.6)	(0.7,0.8)	(0.5,09)	(0.2,0.2)	(0,1)	(0.3,0.6)	(0.8,0.3)	$\cdots$	(0.4,0.3)	(0.9,0.5)	(0.9,0.5)	(0.5,0.9)	$\cdots$	(0.9,0.4)
e_6	(0.2,0)	(0.2,0.9)	(0.1,0.7)	(0.3,0.4)	(0.8,0.7)	(0,1)	(0.3,0.3)	$\cdots$	(0.6,0.1)	(0.1,0.3)	(0.9,0.3)	(0.1,0.2)	$\cdots$	(0.8,0.3)
$\cdots$								$\cdots$					$\cdots$	
e_{19}	(0.4,0.7)	(0.6,0.2)	(0.7,0.1)	(0.6,0.6)	(0.4,0.5)	(0.2,0.9)	(0.7,0.1)	$\cdots$	(0.2,0)	(0.5,0.3)	(1,0.7)	(0.5,0.8)	$\cdots$	(1,0.6)
e_{20}	(0.6,0.2)	(0.6,0.2)	(0.6,0.7)	(0.4,0.1)	(0.2,0.6)	(0.4,1)	(0.6,0.3)	$\cdots$	(0.3,0.1)	(0.1,1)	(0.5,0.9)	(0.4,0.7)	$\cdots$	(0,1)

To compare with the traditional CRP-based method, we calculate the consensus degree (c) and detect the key DM based on DMs' consensus degree for each iteration. The simulation results for the conflict detection and elimination process are shown in Table 16.

In Table 16, we use "Y" to represent that the key DM chooses to accept the modification, while "N" means that he/she rejects the change. In addition, the TR-CDE model treats the DM with highest conflict degree as the key DM, similarly as extant CRP models that designate the DM with the lowest consensus degree.

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Table 16. Comparison of the TR-CDE model and the traditional CRP.

Iteration	TR-CDE model Key DM/Choice	ρ	Traditional CRP model Key DM	c
1	e_{12}/N	0.423	e_7	0.691
	e_4/Y			
2	e_7/Y	0.410	e_7	0.693
3	e_{12}/N	0.376	e_{12}	0.712
	e_9/Y			
4	e_{19}/Y	0.350	e_{12}	0.722
5	e_{15}/Y	0.326	e_{12}	0.731
6	e_6/Y	0.310	e_{12}	0.737
7		0.279	e_{12}	0.751

After the conflict detection and elimination process, we can obtain the conflict degree for each DM:

$\Omega_{(T=7)} = \{0.53, 0.21, 0.26, 0.26, 0.39, 0.16, 0.13, 0.34, 0.21, 0.39, 0.45, 0.50, 0.18, 0.29, 0.34, 0.18, 0.16, 0.24, 0.24, 0.10\}$.

Meanwhile, the updated group decision matrix is obtained as shown in Table 17.

Table 17. The updated group decision matrix $\mathbf{D}_{(T=7)}$.

DMs	x_1	x_2	x_3	x_4
e_1	$\langle 0.1, 0.7 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.1, 0.3 \rangle$	$\langle 0.8, 0.1 \rangle$
e_2	$\langle 0.9, 0.1 \rangle$	$\langle 0.3, 0.5 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0, 0.4 \rangle$
e_3	$\langle 0.5, 0.5 \rangle$	$\langle 0.2, 0.2 \rangle$	$\langle 0.3, 0.7 \rangle$	$\langle 0.3, 0.7 \rangle$
e_4	$\langle 0.3, 0.6 \rangle$	$\langle 0.3, 0.5 \rangle$	$\langle 0.5, 0.4 \rangle$	$\langle 0.3, 0.6 \rangle$
e_5	$\langle 0.4, 0.5 \rangle$	$\langle 0.7, 0.3 \rangle$	$\langle 0, 0.4 \rangle$	$\langle 0.1, 0.9 \rangle$
$\dots$				
e_{16}	$\langle 0.6, 0.3 \rangle$	$\langle 0.4, 0 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.2, 0.8 \rangle$
e_{17}	$\langle 0.3, 0 \rangle$	$\langle 0.1, 0.5 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.1, 0.6 \rangle$
e_{18}	$\langle 0.4, 0.4 \rangle$	$\langle 0.1, 0.6 \rangle$	$\langle 0.1, 0.3 \rangle$	$\langle 0, 0.8 \rangle$
e_{19}	$\langle 0.5, 0.3 \rangle$	$\langle 0.6, 0.3 \rangle$	$\langle 0.5, 0.4 \rangle$	$\langle 0.1, 0.6 \rangle$
e_{20}	$\langle 0.6, 0.2 \rangle$	$\langle 0.2, 0.7 \rangle$	$\langle 0.3, 0.3 \rangle$	$\langle 0.3, 0.5 \rangle$

Following this, in the selection process, the DMs' weights are calculated based on their conflict degree:

$$\omega = \{0, 0.0635, 0.0536, 0.0536, 0.0278, 0.0734, 0.0794, 0.0377, 0.0635, 0.0278,$$
$$0.0159, 0.0060, 0.0694, 0.0476, 0.0377, 0.0694, 0.0734, 0.0575, 0.0575, 0.0853\}.$$

As mentioned in Remark 5.1, the DM e_1 shares the maximum conflict degree 0.53, thus, the weight for e_1 is zero calculated by Eq (12).

Using the IFWA operator, the following group assessment on alternatives is obtained:

$$\dot{d}_1 = \langle 0.83, 0.05 \rangle; \ \ \dot{d}_2 = \langle 0.75, 0.06 \rangle; \ \ \dot{d}_3 = \langle 0.41, 0.15 \rangle; \ \ \dot{d}_4 = \langle 0.38, 0.45 \rangle.$$

The scores of the alternatives are: $s_1 = 0.78$, $s_2 = 0.69$, $s_3 = 0.26$, and $s_4 = -0.07$. Thus, we can obtain that $s_1 \succ s_2 \succ s_3 \succ s_4$. Therefore, s_1 is the final selected decision for the LSGDM problem.

6.2.2. Some comparisons with other models

In this subsection, to validate the feasibility of the proposed TR-CDE model, we compare the results derived therefrom with those produced by other representative group decision making models or methods. The analysis is based on the same practical scenario. The assessment information on alternatives and the relationship information are denoted by $\langle \mu, \nu \rangle$ and (t, s), respectively. However, some of the following

methods use other forms of information. Therefore, we make some minor changes in each aggregation method to suit each case.

(1) When the influence of relationships among DMs on the decision making process is not considered, we can utilize the the intuitionistic fuzzy average operator [54] on the obtained final group decision matrix D, shown on Table 17, to calculate the group assessment on alternatives. The difference in this comparison is in the selection process. The obtained group assessment on alternatives are $\dot{d}_1 = \langle 0.75, 0.07 \rangle$; $\dot{d}_2 = \langle 0.67, 0.08 \rangle$; $\dot{d}_3 = \langle 0.35, 0.21 \rangle$; $\dot{d}_4 = \langle 0.37, 0.46 \rangle$. So, the ranking result is $s_1 \succ s_2 \succ s_3 \succ s_4$.

(2) When the DMs are in a social network context, trust information is considered. The decision making model proposed by Wu [35], uses the incomplete trust network to calculate the trust degree for each DMs and determine their weights. Its main difference with our model lies in the utilization of trust degree solely in the social network, without taking the relationship strength into consideration. Trust is a type of relationship among DMs, which is regarded as transitive in the proposed TR-CDE model. Combining the relationship strength s, the provided incomplete trust network can be extended to a complete trust social network. The completed trust network is used to calculate the disharmony degree c_{ij} between two DMs (as shown in Eq.(9)) in TR-CDE model. Thus, the weights are calculated by the individual conflict degree Ω, which is indirect with the trust relationship. Nevertheless, in the decision making model in [35], the transitive of DMs' trust relationships and the relationship strength are ignored: the DMs' weights are defined based on both the trust degree and consensus degree.

We set β (which is the parameter in the decision making model proposed by Wu [35]), and the parameter to control the degree of consensus and trust is 0.5 in the decision making model of [35]. The decision result derived from this method is: $\dot{d}_1 = \langle 0.68, 0.10 \rangle$; $\dot{d}_2 = \langle 0.81, 0.12 \rangle$; $\dot{d}_3 = \langle 0.38, 0.19 \rangle$; $\dot{d}_4 = \langle 0.36, 0.48 \rangle$. Thus, the ranking result can be obtained: $s_2 \succ s_1 \succ s_3 \succ s_4$.

(3) The model proposed by Wu [39] is applicable to the decision making problem in which the DMs are in a social network context and the trust information is transitive. The concepts of trust degree and distrust degree are utilized in the trust propagation operator, although it does not consider the relationship strength s, to obtain the complete trust social network.

By implementing the method in [39] on this practical scenario, we can have $\dot{d}_1 = \langle 0.69, 0.10 \rangle$, $\dot{d}_2 = \langle 0.84, 0.12 \rangle$, $\dot{d}_3 = \langle 0.38, 0.17 \rangle$, $\dot{d}_4 = \langle 0.37, 0.46 \rangle$ and the alternative ranking result $s_2 \succ s_1 \succ s_3 \succ s_4$.

6.2.3. Discussion

In this subsection, we applied the proposed TR-CDE model in a practical LSGDM scenario. The relationship information is completed through the relationship strength-based trust propagation operator. In the conflict detection and elimination process, the group conflict degree is decreased from 0.42 to 0.28.

To identify the relation between the group conflict degree and consensus degree, we calculate the consensus degree for each iteration. Table 16 clearly shows that as the group conflict degree falls, the consensus degree rises. This indicates that the proposed TR-CDE decision making model can improve the consensus degree by conflict elimination.

We also apply some other group decision models to the practical scenario, and find a largely identical group assessment. The main difference in ranking alternatives concerns x_1 and x_2. Our model and the method (1) both find that x_1 is more optimal than x_2, whereas the methods (2) and (3) leads to the opposite case. The difference between the proposed TR-CDE model and the compared method (1) is the aggregation operator in selection process. In addition, the main differences among TR-CDE model and the compared methods (2) and (3) are utilizations of trust degree and the way to obtain the complete trust network. Both two methods fail to consider the relationship strength in the trust propagation, which can not guarantee the propagation efficiency. This is arguably the reason why the methods (2) and (3) select alternative x_2 as the solution. As mentioned before, x_1 represents the money-dominant scheme and x_2 represents housing-dominant scheme. In the reality, people show a strong preference on the money-dominant scheme or money and housing-dominant scheme compared with housing-dominant only scheme.

Thus, these comparison results can mainly be attributed to the unique features in the TR-CDE model: 1) the relationship strength-based trust propagation operator; 2) the mechanism for conflict detection and elimination process; and 3) the conflict-driven weight determination method. Overall, the proposed model introduces a reasonable trust propagation operator, which considers about the influence of relationship strength on propagation efficiency. The TR-CDE model's feasibility in detecting and eliminating conflicts among DMs in LSGDM has been demonstrated and validated.

6.3. The behaviors of parameters

In this subsection, we introduce some discussion and practical guidelines for decision groups on setting such parameters depending on the specific problem being tackled.

In order to better explain the meaning and the setting influence of the conflict threshold Φ on LSGDM events, we list the values of group conflict degree ρ in each iteration of the numerical example and the practical scenarios in Table 18.

In the two experiments, the conflict threshold is set the same as $\Phi = 0.3$. It means that within the limited iterations T_{max}, when the calculated group conflict degree is less that the threshold, the acceptable consensus is reached and the final decision can be made. As the results recorded in Table 18, if we set the value of Φ higher than 0.3, we can make the final decision within a few iterations. Such as, if we set $\Phi = 0.42$ in the two experiments, we can chose the final decision with the two group conflict degrees are 0.41 after three iterations and two iterations, respectively. In other words, the higher the value of conflict threshold is, the less iterations are needed and the higher the group conflict degree is allowed

Table 18. The group conflict degree ρ in each iteration.

iteration(T)	The numerical example ρ	The practical scenario ρ
$T = 1$	0.51	0.423
$T = 2$	0.50	0.410
$T = 3$	0.41	0.376
$T = 4$	0.33	0.350
$T = 5$	0.32	0.326
$T = 6$	0.29	0.310
$T = 7$	-	0.279

before making the final decision in the LSGDM event. A smaller number of iterations implies a reduction in the temporal cost. However, the higher value of group conflict degree means the higher dissatisfaction among DMs, which may lead to some other serious group events.

On the other hand, if we set $\Phi = 0.28$ in the two experiments, the numerical example can not reach the consensus in the limited 6 iterations. That is, the lower the conflict threshold is, the more iterations are needed for the LSGDM event.

Furthermore, we implement the TR-CDE model on the numerical example with different $\theta = \{0.05, 0.06, \ldots, 0.08, 0.10, \ldots, 0.14\}$ within 6 iterations. All the detected key DMs accept the suggested modifications. Combining the results with $\theta = 0.09$, the group conflict degrees are recorded in Table 19.

Table 19. The group conflict degree ρ in each iteration with different threshold θ.

iteration(T)	$\theta = 0.05$	$\theta = 0.06$	$\theta = 0.07$	$\theta = 0.08$	$\theta = 0.09$	$\theta = 0.10$	$\theta = 0.11$	$\theta = 0.12$	$\theta = 0.13$	$\theta = 0.14$
$T = 1$	0.69	0.63	0.60	0.54	0.51	0.42	0.39	**0.29**	**0.22**	**0.16**
$T = 2$	0.68	0.62	0.56	0.49	0.50	0.33	0.32			
$T = 3$	0.63	0.61	0.54	0.47	0.41	**0.28**	**0.27**			
$T = 4$	0.59	0.59	0.44	0.44	0.33					
$T = 5$	0.58	0.57	0.38	0.40	0.32					
$T = 6$	0.57	0.51	0.31	0.36	**0.29**					

From Table 19, comparing the group conflict degree ρ in the iteration $T = 1$, we can conclude that the lower the threshold θ is, the more conflicts present in the LSGDM event, which represent the initial conflict level. When θ is set less than 0.10, the value of ρ is more than 0.5. Within 6 iterations, we can learn that the lower the value of θ is, the more difficult to reach the preset conflict threshold Φ within limited iterations. On the contrary, the higher the θ is, the less conflicts are detected in the consensus. With the same assessment matrix, when the value of θ is set more than 0.11, the initial group conflict is less than $\Phi = 0.30$, and the consensus is reached directly. This means the more conflicts are ignored. It will increase the risk for the occurrence of DMs' dissatisfied events. Thus, we suggest to set θ at the half level of the initial CIM, which means to remain half of the disharmonies as the detected conflicts in the decision making events.

Besides, T_{max} (which is determined by the urgency level β), the threshold θ to cut the disharmony level, and the acceptable conflict degree level Φ are the three important parameters in the TR-CDE model. From Table 19, we can easily summary that these three parameters can mutually restrict and influence each other, which is reflected in the following aspects.

- To reach the same Φ, with different θ, the TR-CDE model needs different numbers of iterations T. By setting different urgency levels β, we can obtain different values of T_{max}. The more urgency the LSGDM is, the higher the value of β should be set and the lower the value of T_{max} is. If $T \leq T_{max}$, the consensus is reached; if $T > T_{max}$, the TR-CDE model fail. As T represents the cost of the decision making process, we can set T_{max} much higher, which means setting β much lower, within an acceptable range.

- With the same T_{max}, the initial group conflict present differently with different values of θ. It implies that the initial conflict level is different in the LSGDM event. The higher the value of θ is, the lower initial conflict level is for the LSGDM event. In this situation, the consensus can be achieved or not is depended on level of Φ. Within limited iterations, if the initial conflict level is high, the possible achieved conflict level may be high. It implies that, in that situation, the higher value of Φ is set, the more possibility the consensus can be reached. If the initial conflict level is low, even though a lower value of Φ is set, it is possible to reach the consensus with the limited T_{max} iteration.

- Once the value of θ is fixed, the initial conflict performance of the LSGDM event is certain. Except for the DMs' actions, the higher T_{max} is, the lower group conflict degree can be reached, which means the lower Φ can be set. On the other hand, the lower Φ is, the more iterations are required to reach the consensus, which means to make the TR-CDE model succeed, a large value of T_{max} is needed to be set.

7. Conclusion

We proposed a TR-CDE model for resolving LSGDM problems where some intra-group social relationships exist among participating decision makers. Since both the relationship and assessment information are causative factors of conflict, the TR-CDE model focuses on conflict detection and elimination, in contrast to conventional LSGDM models incorporating consensus reaching approaches. The model comprises three processes:

(1) **Trust propagating process**. It is proposed to obtain the complete social network where some indirect relationship information cannot be directly obtained. For this purposes, a new relationship strength-based trust propagation operator is proposed. The proposed operator is an effective and reasonable solution with the following characteristics: 1) it considers the impact of relationship strength on the propagating efficiency, since the information cannot be fully propagated in practice; and 2) it can suitably handle the situation of multiple propagation paths to obtain an indirect relationship. Based on the relationship strength-based trust propagation operator, we can obtain the completed social network.

(2) **Conflict detection and elimination process**. The assessments are represented by IFSs, which can faithfully express the uncertainty and hesitation of DMs. Combining the obtained completed

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social network and the assessment information, the corresponding conflict network can be well established. Based on the conflict network, an SNA-based conflict detection and elimination process is implemented. This can effectively identify the conflict relationships among DMs and serves to decrease group conflict degree by reducing the conflict degree of key DMs identified towards an ideal level. By building a nonlinear optimization model, which guarantees the group conflict degree can present the maximum reduction in every iteration, modification plans are obtained for the key DMs, thereby efficiently and quickly reducing the conflict below the desired threshold.

(3) **Selection process**. In this process, the conflict degree is used to determine each DM's importance weight, taking both trust and consensus on assessment into consideration, thereby making the weight determination more reasonable.

In conclusion, we have successfully demonstrated the proposed TR-CDE model's efficiency and feasibility in solving LSGDM problems in a social network context.

Aside from the shown advantages of TR-CDE model, there are some limitations can be further considered in the future works for LSGDM scenarios. From the numerical example we find that the trust values calculated by the proposed method are generally low, we contribute this to over-counting the information loss as well as overlooking the influence of trust value on propagation efficiency during the propagation. The option for the DM exhibiting the highest conflict level in the decision making process is to accept the modification which is calculated by solving the nonlinear optimization model (11) or to reject the modification without the partial acceptance. Therefore, in the future studies, we can focus on developing a more efficient trust propagation operator, and develop the conflict elimination optimization model which can eliminate the group conflict degree, as well as providing more options for the DMs who present conflict in LSGDM events and favoring those DMs who behave more cooperatively during the conflict reduction process.

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