Rebuttal to "Comments on The Quaternion LMS Algorithm for Adaptive Filtering of Hypercomplex Processes"

Danilo Mandic, *Fellow, IEEE*, and Clive Cheong Took, *Senior Member, IEEE*

***Abstract*—This is the authors' rebuttal on the correspondence article "Comments on The Quaternion LMS Algorithm for Adaptive Filtering of Hypercomplex Processes" by Gang Wang and Rui Xue.**

***Index Terms*—adaptive filters, quaternions, quaternion gradient, HR & GHR calculus.**

I. Introduction

The comment correspondence by Wang and Xue [1] presents a valid and technically correct argument. The authors are correct that the original QLMS in [2] was derived based on a pragmatic rather than a mathematical assumption, like many established algorithms in engineering practice (including the LMS). However, we would like to point out that the argument presented by the authors is not novel, as the same argument was already published by Barthelemy *et al.* in [3].

It is clear that the authors Wang and Xue were not aware of the above work, which also cannot be found in the references of their correspondence in [1]. We would like to mention that in several conference and journal papers by Mandic *et al.* it was clearly highlighted that the original QLMS from 2009 indeed provides an approximate gradient calculation, which in the context of the already approximate stochastic gradient type LMS training does converge and represents a valid QLMS algorithm. The convergent and adequate operation of the original QLMS can be verified using the Matlab code for the original QLMS from the link in [4].

Indeed, the only difference from the rigorous QLMS derived using the $\mathbb{HR}$ and $\mathbb{GHR}$ calculus [5], [6] (by the same authors) is that the gradient in the real part of the quaternion variable within the original QLMS attains half the value of the correct gradient. This, therefore, does not affect 3D processes which use a pure quaternion representation.

It is important to note that the $\mathbb{GHR}$ calculus (see below) resolves the 180-year old issue of the product and chain rule in quaternion calculus, and represents a rigorous and unifying basis for the advanced in gradient-based quaternion learning. The same argument about the original QLMS in [2] was reiterated and rectified (including the QLMS convergence) in the references [5], [6], [7], [8].

We again congratulate the authors for pointing out to this issue with the original QLMS [2] and would like to encourage them to continue working in this exciting area. Unfortunately, their argument, albeit correct, was already addressed in the open literature.

D. Mandic is with the Department of Electrical and Electronic Engineering, Imperial College London, e-mail: d.mandic@ic.ac.uk.

C. Cheong Took is with the Department of Electronic Engineering, Royal Holloway, University of London, e-mail: clive.cheongtook@rhul.ac.uk.

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