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The Power of the Higgs Mechanism: Higher-Derivative BLG Theories

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Abstract: We use the novel Higgs mechanism of arXiv:0803.3218 to determine the leading higher-derivative corrections to the Euclidean $\mathcal{N} = 8$ Bagger-Lambert-Gustavsson field theory. The result matches that previously found for Lorentzian 3-algebras, pointing to a universal answer for all maximally supersymmetric 3-algebra theories. We also comment on the extension to the lower-supersymmetric case of ABJM theory.

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1. Introduction

Recent theoretical research has yielded several classes of $(2+1)$ dimensional field theories that appear to have some relevance to multiple M2-branes. The first breakthrough occurred when Bagger and Lambert proposed a Lagrangian depending on an arbitrary 3-algebra and possessing $\mathcal{N} = 8$ supersymmetry and conformal invariance [1, 2, 3]. Closure of the supersymmetry algebra was also demonstrated independently by Gustavsson [4], and we will refer to these field theories collectively as BLG theories. The $\mathcal{A}_4$-theory of Ref. [3] is a special case, characterised by an integer Chern-Simons level k, and was the first to be discussed in detail. It was conjectured in Refs. [5, 6] to describe a pair of multiple membranes at a kind of generalised orbifold. The precise definition of this orbifold is not yet known. Moreover these theories have no generalisation to $N > 2$ membranes.

Another special case of BLG theories are the Lorentzian 3-algebra theories, discussed in Refs. [7, 8, 9].[1] Being BLG theories, these too are Chern-Simons-like and have maximal $\mathcal{N} = 8$ supersymmetry, but unlike the $\mathcal{A}_4$-theory, they can be generalised to arbitrary Lie algebras and they also do not have any coupling parameter analogous to the level k in the $\mathcal{A}_4$ case. It was subsequently shown in Ref. [12] that they can be derived from maximally supersymmetric Yang-Mills theory using a non-Abelian duality. While this makes it quite compelling that they are correct, it is not yet clear that they provide a concretely useful description of membranes.[2]

A different class of theories are the ABJM theories [15] which are Chern-Simons-matter theories having manifest $\mathcal{N} = 6$ supersymmetry and an integer Chern-Simons level k. These describe N multiple membranes at a conventional geometric $\mathbb{C}^4/\mathbb{Z}_k$ orbifold. It has been shown that these theories are also described by 3-algebras, albeit with complex and non-anti-symmetric structure constants [16, 17].

All the above proposals were made to describe M-theory membranes to lowest order in ℓ_p, the M-theory Planck length. In each case these worldvolume theories are related, via the novel Higgs mechanism [18], to the Yang-Mills theory on D2-branes, which is the limit of the D-brane worldvolume theory where α' corrections are ignored. The Yang-Mills coupling depends on the vev v of a scalar in the original Chern-Simons-type theory. It is an interesting problem then to ask how to generalise the above proposals to incorporate the first nontrivial corrections (which arise at order α'^2) to the D2-brane effective theory. As we will explain below, these appear as ℓ_p^3 corrections to the corresponding multiple membrane theories.

The initial goal of the present work is to obtain the leading higher derivative corrections to the $\mathcal{A}_4$-theory. We will do this by using the Higgs mechanism of [18] and comparing the results with the α'^2 corrections to the D2-brane effective worldvolume action, which to the given order amounts to a symmetrised-trace non-Abelian DBI theory.

To check the order of the correction we are after, note that the Abelian DBI action for D2-branes is:

$$\mathcal{L} = -\frac{1}{(2\pi\alpha')^2 g_{YM}^2}\sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}\ , \tag{1.1}$$

[1] Here we are interested primarily in the variant of these theories where a constraint manifestly eliminates negative-norm states, as explained in Refs. [10, 11].

[2] However, see also Refs. [13, 14] for alternative viewpoints on Lorentzian theories.

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The factor of $(g_{YM})^{-2}$ in front of the entire action reflects the fact that it is a tree-level action in open string theory. Abelian duality is implemented by replacing the above action by the equivalent action:

$$\mathcal{L} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda} - \frac{1}{(2\pi\alpha')^2 g_{YM}^2}\sqrt{-\det(g_{\mu\nu} + (2\pi\alpha')^2 g_{YM}^4 B_\mu B_\nu)}\ . \tag{1.2}$$

This can be seen by integrating out B_μ whereupon one recovers the original action.

If instead we integrate out the gauge field A_μ, its equation of motion tells us that $\partial_\mu B_\nu - \partial_\nu B_\mu = 0$ and therefore B_μ is the gradient of a scalar, which we write as:

$$B_\mu \to -\frac{1}{g_{YM}}\partial_\mu X^8\ , \tag{1.3}$$

where the coefficient is chosen so that the eventual kinetic term for X^8 is correctly normalised.

Noting also that in static gauge $g_{\mu\nu} = \eta_{\mu\nu} + (2\pi\alpha')^2 g_{YM}^2 \partial_\mu X^i \partial_\nu X^i$,[3] and that $(\alpha')^2 g_{YM}^2 = (\alpha')^{\frac{3}{2}} g_s = \ell_p^3$, we end up with the action:

$$\mathcal{L} = -\frac{1}{(2\pi)^2\ell_p^3}\sqrt{-\det(\eta_{\mu\nu} + (2\pi)^2\ell_p^3\,\partial_\mu X^I \partial_\nu X^I)} \sim -\frac{1}{2}\partial_\mu X^I \partial^\mu X^I + (2\pi)^2\ell_p^3\,\mathcal{O}(\partial X)^4 + \cdots \tag{1.4}$$

Apparently this action depends solely on ℓ_p. However quantisation of flux in the original gauge theory imposes the periodicity condition:

$$X^8 \sim X^8 + 2\pi g_{YM}\ . \tag{1.5}$$

Therefore only in the limit $g_{YM} \to \infty$ (which is the same as the M-theory limit $g_s \to \infty$) does the dependence on g_{YM} disappear. In this limit we find an action that depends solely on ℓ_p and has SO(8) invariance. This can then be interpreted as the action for a single M2-brane. We see that the first nontrivial correction to this action is of order ℓ_p^3 and this comes multiplied by the dimension-six operator $(\partial X)^4$. This fixes the order of the corrections in which we will be interested in the non-Abelian case as well.

The way in which we will proceed is by first presenting an ansatz for the $\mathcal{A}_4$-theory action at four derivative order, in the bi-fundamental notation of Ref. [19]. This is motivated by the above considerations and the expectation that the $\mathcal{O}(\ell_p^3)$ corrections can be expressed in terms of 3-algebra quantities, notably the totally anti-symmetric triple-product, with arbitrary coefficients for each possible term. We will then match these term-by-term with the equivalent structures arising in the D-brane effective action at order α'^2, following the Higgsing procedure of [18]. To this order, the latter is given entirely by applying Tseytlin's

[3] The coefficient of the second term is again determined by requiring a standard kinetic term.

symmetrised trace prescription to the non-Abelian Dirac-Born-Infeld (DBI) action [20], including the fermionic terms, as shown *e.g.* in Refs. [21, 22].

As with several discussions of the leading-order BLG $\mathcal{A}_4$ and ABJM theories, the classical action is most meaningful for large k where the theory is weakly coupled and loop corrections can be ignored. Nevertheless, it is usually written down and studied as a function of k and one hopes it has some significance even for small k. In this spirit, we investigate higher-derivative corrections keeping in mind that they too are applicable primarily in the large k regime, but the action we will obtain can then be extrapolated to small k with due caution.

After obtaining the result for the $\mathcal{A}_4$-theory in the bi-fundamental formulation, we move on to express the answer entirely in terms of 3-algebra notation. We then revisit the results of Refs. [23, 24], where higher-derivative corrections to Lorentzian 3-algebra theories were obtained by showing that the non-Abelian duality, used in Ref. [12], extends to α'^2 corrections.[4] We finally show that both the Euclidean and Lorentzian four-derivative 3-algebra theories are obtainable from a general four-derivative BLG 3-algebra action, upon making a Euclidean or Lorentzian choice for the 3-algebra metric.

Though we work only to order ℓ_p^3, we conjecture that all subsequent corrections to BLG theories can be organised in terms of the 3-algebra triple-product. We refer to the action including all such corrections as the "3BI action". Thus:

$$S_{\rm 3BI} = S_{\ell_p^0} + S_{\ell_p^3} + \dots \ . \tag{1.6}$$

We close by discussing possible generalisations of our result to $\mathcal{N}=6$ 3-algebra constructions that include the ABJM theory.

2. Review of the novel Higgs mechanism

We begin with a careful review of the Higgsing procedure for the $\mathcal{A}_4$-theory in the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ formalism of [19], including the fermions. This will be useful to set up notation and normalisations before we proceed to the more complicated four-derivative order.

[4] In Ref. [24] there is a formal extension to all orders in α' but the starting point used there, of a DBI-type non-Abelian D2-brane action, is strictly correct only up to order α'^2. For other applications of the procedure of Ref. [12] see [25].

2.1 Higgs mechanism for the $\mathcal{A}_4$-theory

The $\mathcal{A}_4$-theory action is given by the expression:[5]

$$
\begin{aligned}
S_{\mathcal{A}_4} = \frac{k}{2\pi}\int d^3x\,\mathrm{Tr}\Big[&-(\tilde{D}^\mu X^I)^\dagger \tilde{D}_\mu X^I + i\,\bar{\Psi}^\dagger\Gamma^\mu\tilde{D}_\mu\Psi - \tfrac{8}{3}\,X^{IJK\dagger}\,X^{IJK}\\
&-i\,\bar{\Psi}^\dagger\Gamma_{IJ}[X^I,X^{J\dagger},\Psi] + i\,\bar{\Psi}\Gamma_{IJ}[X^{I\dagger},X^J,\Psi^\dagger]\\
&+\tfrac{1}{2}\,\varepsilon^{\mu\nu\lambda}\Big(A^{(1)}_\mu\partial_\nu A^{(1)}_\lambda + \tfrac{2}{3}A^{(1)}_\mu A^{(1)}_\nu A^{(1)}_\lambda - A^{(2)}_\mu\partial_\nu A^{(2)}_\lambda - \tfrac{2}{3}A^{(2)}_\mu A^{(2)}_\nu A^{(2)}_\lambda\Big)\Big]\ , \qquad (2.1)
\end{aligned}
$$

where the fermions are 32-component spinors satisfying $\Gamma^{012}\Psi = -\Psi$ and we have also defined:

$$
\begin{aligned}
X^{IJK} &= X^{[I}X^{J\dagger}X^{K]}\\
[X^I,X^{J\dagger},\Psi] &= \tfrac{1}{3}\Big(X^{[I}X^{J]\dagger}\Psi - X^{[I}\Psi^\dagger X^{J]} + \Psi X^{[I\dagger}X^{J]}\Big)\ . \qquad (2.2)
\end{aligned}
$$

Note that the explicit anti-symmetrisation is in the indices while leaving the position of the $\dagger$ fixed and that the anti-symmetrised products are defined with weight one.

In the above the indices $\mu = 0,1,2$ while $I = 1,...,8$. The $A^{(1)}_\mu$ and $A^{(2)}_\mu$ are the SU(2) gauge fields giving rise to the two Chern-Simons terms with equal but opposite levels k, which are quantised in integer units. The X's are complex scalars obeying the reality condition:

$$
X_{a\dot{b}} = \epsilon_{ab}\,\epsilon_{\dot{b}\dot{a}}\,X^{\dagger\dot{a}b} \qquad (2.3)
$$

and transforming in the bi-fundamental representation $(\mathbf{2},\bar{\mathbf{2}})$ according to:

$$
\tilde{D}_\mu X^I = \partial_\mu X^I + A^{(1)}_\mu X^I - X^I A^{(2)}_\mu\ . \qquad (2.4)
$$

The next step is to create linear combinations of the gauge fields:

$$
\begin{aligned}
A_\mu &= \tfrac{1}{2}\big(A^{(1)}_\mu + A^{(2)}_\mu\big)\\
B_\mu &= \tfrac{1}{2}\big(A^{(1)}_\mu - A^{(2)}_\mu\big)\ . \qquad (2.5)
\end{aligned}
$$

With these definitions the form of the bosonic part of the action becomes:

$$
\begin{aligned}
S^b_{\mathcal{A}_4} = \frac{k}{2\pi}\int d^3x\,\mathrm{Tr}\Big[&-(D^\mu X^I)^\dagger D_\mu X^I - \tfrac{8}{3}\,X^{IJK\dagger}\,X^{IJK}\\
&+\{B_\mu,X^I\}\{B^\mu,X^{I\dagger}\} + D_\mu X^{I\dagger}\,\{B_\mu,X^I\} - \{B^\mu,X^{I\dagger}\}\,D_\mu X^I
\end{aligned}
$$

[5] This action is related to the one in Ref. [19] by a re-scaling $X\to\sqrt{\frac{k}{2\pi}}\,X$, $\Psi\to\sqrt{\frac{k}{2\pi}}\,\Psi$, and a redefinition $A_\mu \to -iA_\mu$ so that the matrix-valued gauge fields are anti-Hermitian. Our spinor and Γ-matrix notation and conventions can be found in Appendix A.

$$+\varepsilon^{\mu\nu\lambda}\Big(B_\mu F_{\nu\lambda}+\tfrac{1}{3}B_\mu B_\nu B_\lambda\Big)\Big]\ ,$$
$$(2.6)$$

where we have substituted:

$$\tilde{D}_\mu X^I = D_\mu X^I - \{B_\mu, X^I\}\ , \tag{2.7}$$

with:

$$\begin{aligned} D_\mu X^I &= \partial_\mu X^I + [A_\mu, X^I] \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\ . \end{aligned} \tag{2.8}$$

Note that the new gauge field A_μ is in the diagonal subgroup of $\mathrm{SU}(2)\times\mathrm{SU}(2)$ and has an adjoint action on the X's.

In this form for the action, one can expand the scalars into trace and traceless parts, in a suitable basis, and also give a vev v to one of them, say X^8:

$$\begin{aligned} X^8 &= \tfrac{1}{2}(v+\tilde{x}^8)\,\mathbb{1}+\boldsymbol{x}^8 \\ X^i &= \tfrac{1}{2}\tilde{x}^i\,\mathbb{1}+\boldsymbol{x}^i\ . \end{aligned} \tag{2.9}$$

Here:

$$\boldsymbol{x}^I = i\,x^{Ia}\,\frac{\boldsymbol{\sigma}^a}{2}\ , \tag{2.10}$$

with $\boldsymbol{\sigma}^a$ the usual Pauli matrices. Recall that in the above, $i=1,...,7$ while $I=1,...,8$.

In what follows we will be interested in the limit of large vev $v\to\infty$. Having performed a decomposition of the bi-fundamental scalars into a trace and a traceless part, we substitute back into the action to get:

$$\begin{aligned} S^b_{\mathcal{A}_4} = \frac{k}{2\pi}\int d^3x\Big\{ &-\tfrac{1}{2}\partial^\mu\tilde{x}^I\partial_\mu\tilde{x}^I + \mathrm{Tr}\Big(D^\mu\boldsymbol{x}^I D_\mu\boldsymbol{x}^I + \tfrac{v^2}{2}[\boldsymbol{x}^i,\boldsymbol{x}^j][\boldsymbol{x}^i,\boldsymbol{x}^j] \\ &+ 2vB^\mu D_\mu\boldsymbol{x}^8 + v^2 B^\mu B_\mu + \varepsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda}\Big)\Big\} + \text{higher order}\ . \end{aligned} \tag{2.11}$$

The higher order terms that we omitted writing in the above are the ones that will be negligible in the final action when $v\to\infty$. We will ignore them for now and return to them later.

One can see that after giving the vev v, the gauge field B_μ has acquired a mass term by the Higgs mechanism. Moreover the corresponding Goldstone boson that is 'eaten' is $\boldsymbol{x}^8$, as is evident if we group all terms depending on $\boldsymbol{x}^8$ and B_μ as follows:

$$v^2\left(B_\mu+\tfrac{1}{v}D_\mu\boldsymbol{x}^8\right)^2+\varepsilon^{\mu\nu\lambda}\left(B_\mu+\tfrac{1}{v}D_\mu\boldsymbol{x}^8\right)F_{\nu\lambda} \tag{2.12}$$

(to obtain this form, we have added a term proportional to $\varepsilon^{\mu\nu\lambda} D_\mu \boldsymbol{x}^8\, F_{\nu\lambda}$ which vanishes by partial integration and the Bianchi identity). The shift $B_\mu \to B_\mu - \frac{1}{v} D_\mu \boldsymbol{x}^8$ now eliminates $\boldsymbol{x}^8$ from the Lagrangian.

The novel feature of this Higgs mechanism is that B_μ has no kinetic term, therefore it can be integrated out and the effect of this is to render the other gauge field A_μ dynamical. To see this, note that the equation of motion for B_μ is:

$$B_\mu = -\frac{1}{2v^2}\varepsilon^{\mu\nu\lambda} F_{\nu\lambda}\ . \tag{2.13}$$

Eliminating B_μ from the action:

$$S^b_{\mathcal{A}_4} = \frac{k}{2\pi}\int d^3x \Big\{ -\tfrac{1}{2}\partial^\mu \tilde{x}^I \partial_\mu \tilde{x}^I + \mathrm{Tr}\Big(\tfrac{1}{2v^2} F^{\mu\nu}F_{\mu\nu} + D^\mu \boldsymbol{x}^i D_\mu \boldsymbol{x}^i + \tfrac{v^2}{2}[\boldsymbol{x}^i,\boldsymbol{x}^j][\boldsymbol{x}^i,\boldsymbol{x}^j]\Big)\Big\}\ . \tag{2.14}$$

The fields in the above action are 8 singlets $\tilde{x}^I$ along with adjoint SU(2) scalars $\boldsymbol{x}^i$ and an SU(2) gauge field, all described as matrix-valued fields in the fundamental representation:

$$A_\mu = iA^a_\mu \frac{\boldsymbol{\sigma}^a}{2}, \qquad \boldsymbol{x}^i = ix^{i\,a}\frac{\boldsymbol{\sigma}^a}{2}\ . \tag{2.15}$$

Extracting a factor $\frac{1}{v^2}$ from the action, and re-scaling $\boldsymbol{x}^i \to \frac{1}{v}\boldsymbol{x}^i$ and $\tilde{x}^I \to \frac{1}{v}\tilde{x}^I$, we have:

$$S^b_{\mathcal{A}_4} = \frac{k}{2\pi v^2}\int d^3x \Big\{ -\tfrac{1}{2}\partial^\mu \tilde{x}^I \partial_\mu \tilde{x}^I + \mathrm{Tr}\Big(\tfrac{1}{2} F^{\mu\nu}F_{\mu\nu} + D^\mu \boldsymbol{x}^i D_\mu \boldsymbol{x}^i + \tfrac{1}{2}[\boldsymbol{x}^i,\boldsymbol{x}^j][\boldsymbol{x}^i,\boldsymbol{x}^j]\Big)\Big\}\ . \tag{2.16}$$

The last step is to combine the seven singlet scalars $\tilde{x}^i$ with the SU(2) adjoints $\boldsymbol{x}^i$ to make U(2) adjoints:

$$\boldsymbol{X}^i = \tfrac{i}{2}\tilde{x}^i\,\mathbb{1} + \boldsymbol{x}^i\ . \tag{2.17}$$

This only leaves the singlet scalar $\tilde{x}^8$, which can instead be dualised into an Abelian gauge field. This is done as follows:

$$-\int d^3x\ \tfrac{1}{2}\partial^\mu \tilde{x}^8 \partial_\mu \tilde{x}^8 \to \int d^3x\ \left(-\tfrac{1}{4}F^{\mu\nu}_{\mathrm{U}(1)}F^{\mathrm{U}(1)}_{\mu\nu} + \tfrac{1}{2}\varepsilon^{\mu\nu\lambda}\partial_\mu \tilde{x}^8 F^{\mathrm{U}(1)}_{\nu\lambda}\right)\ , \tag{2.18}$$

where $F^{U(1)}_{\mu\nu}$ is treated as an independent field. The equation of motion for $F^{U(1)}_{\mu\nu}$ leads us back to the LHS. Instead, integrating out $\tilde{x}^8$ on the RHS gives us the Bianchi identity for $F^{U(1)}_{\mu\nu}$, solving which we have:

$$F^{U(1)}_{\mu\nu} = \partial_\mu A^{U(1)}_\nu - \partial_\nu A^{U(1)}_\mu\ . \tag{2.19}$$

Once the Bianchi identity has been imposed, the second term on the RHS drops out and the new Abelian gauge field combines with the SU(2) part to form a U(2) gauge field:

$$\begin{aligned} \boldsymbol{A}_\mu &= \tfrac{i}{2}A^{U(1)}_\mu\,\mathbb{1} + A_\mu \\ \boldsymbol{F}_{\mu\nu} &= \tfrac{i}{2}F^{U(1)}_{\mu\nu}\,\mathbb{1} + F_{\mu\nu}\ . \end{aligned} \tag{2.20}$$

Putting these ingredients together, one ends up with the familiar-looking expression:[6]

$$S^b_{YM} = \frac{k}{2\pi v^2}\int d^3x\,\mathrm{Tr}\Big(\tfrac{1}{2}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} + D^\mu \boldsymbol{X}^i D_\mu \boldsymbol{X}^i + \tfrac{1}{2}[\boldsymbol{X}^i,\boldsymbol{X}^j][\boldsymbol{X}^i,\boldsymbol{X}^j]\Big)\ . \tag{2.21}$$

The higher-order terms that we had dropped earlier do indeed decouple in the limit $k\to\infty$, $v\to\infty$ with the ratio $\frac{k}{v^2}$ fixed. This is because they are of higher order in inverse powers of v but their k-dependence is the same as for the leading terms.

For the fermion kinetic term and the Yukawa-type interaction with two scalars and two fermions the procedure is now straightforward. Since the fermions transform and decompose like the scalars:

$$\Psi = \tfrac{1}{2}\tilde{\psi}\,\mathbb{1} + \boldsymbol{\psi} \quad \text{with} \qquad \boldsymbol{\psi} = i\psi^a\frac{\boldsymbol{\sigma}^a}{2}\ , \tag{2.22}$$

the trace part will reduce immediately to the required kinetic term, while the extra term present for the traceless kinetic part, including $\{\mathbf{A}^\mu,\Psi\}$, will be sub-leading in $\frac{1}{v}$ after the re-scaling $\Psi\to\frac{\Psi}{v}$. The interaction term also reduces straightforwardly upon Higgsing and by combining the trace and SU(2) parts into anti-Hermitian fields:

$$\boldsymbol{\Psi} = \tfrac{i}{2}\tilde{\psi}\,\mathbb{1} + \boldsymbol{\psi}\ , \tag{2.23}$$

one gets:

$$S^f_{\mathcal{A}_4} = \frac{k}{2\pi v^2}\int d^3x\ \mathrm{Tr}\Big[-i\bar{\boldsymbol{\Psi}}\Gamma^\mu D_\mu\boldsymbol{\Psi} - i\bar{\boldsymbol{\Psi}}\Gamma^i\Gamma^8[\boldsymbol{X}^i,\boldsymbol{\Psi}]\Big] + \mathcal{O}(\tfrac{1}{v})\ . \tag{2.24}$$

The last thing we need is to do away with the Γ^8 matrix appearing in the second term of Eq. (2.24). This is straightforward if we rewrite it as:

$$\begin{aligned} -i\bar{\boldsymbol{\Psi}}\Gamma^i\Gamma^8[\boldsymbol{X}^i,\boldsymbol{\Psi}] &= -i\bar{\boldsymbol{\Psi}}\Gamma^i(1+\Gamma^8)[\boldsymbol{X}^i,\boldsymbol{\Psi}] \\ &= -i\bar{\boldsymbol{\Psi}}\tfrac{1}{\sqrt{2}}(1-\Gamma^8)\Gamma^i\tfrac{1}{\sqrt{2}}(1+\Gamma^8)[\boldsymbol{X}^i,\boldsymbol{\Psi}] \\ &= -i\bar{\boldsymbol{\Psi}}'\Gamma^i[\boldsymbol{X}^i,\boldsymbol{\Psi}']\ , \end{aligned} \tag{2.25}$$

where in the first step we have used $\bar{\boldsymbol{\Psi}}\Gamma^i\boldsymbol{\Psi}=0$ and in the last step we have defined:

$$\boldsymbol{\Psi}' = \tfrac{1}{\sqrt{2}}(1+\Gamma^8)\boldsymbol{\Psi}\ . \tag{2.26}$$

Note that the above redefinition leaves the first term of Eq. (2.24) invariant:

$$-i\bar{\boldsymbol{\Psi}}\Gamma^\mu D_\mu\boldsymbol{\Psi} = -i\bar{\boldsymbol{\Psi}}'\Gamma^\mu D_\mu\boldsymbol{\Psi}' \tag{2.27}$$

and that the chirality condition becomes:

$$\Gamma^{012}\boldsymbol{\Psi} = -\boldsymbol{\Psi} \to \Gamma^8\boldsymbol{\Psi}' = \boldsymbol{\Psi}'\ . \tag{2.28}$$

[6]We will denote all fields in D2-brane actions using bold-face symbols throughout to avoid confusion with the $\mathcal{A}_4$-theory expressions.

Dropping the prime on $\mathbf{\Psi}'$ for notational economy, we have:

$$S^f_{\mathcal{A}_4} = \frac{k}{2\pi v^2}\int d^3x \ \mathrm{Tr}\Big[-i\bar{\mathbf{\Psi}}\Gamma^\mu D_\mu\mathbf{\Psi} - i\bar{\mathbf{\Psi}}\Gamma^i[\boldsymbol{X}^i,\mathbf{\Psi}]\Big] + \mathcal{O}(\tfrac{1}{v}) \ , \tag{2.29}$$

where the above is the action obtained by the dimensional reduction of the fermion kinetic term of 10d YM down to 3d involving the usual set of $\mathrm{SO}(9,1)$ Γ-matrices in a 32 dimensional representation, with Γ^8 being the $\mathrm{SO}(9,1)$ chirality matrix.

Therefore, by adding the bosonic and fermionic pieces together, what we finally recover in the limit $k\to\infty$, $v\to\infty$ with the ratio $\frac{k}{v^2}$ fixed, is the action of maximally supersymmetric U(2) Yang-Mills theory, namely the (lowest-order in α') worldvolume field theory on two D2-branes. The coupling constant is $g^2_{YM} = 2\pi v^2/k$.

If one keeps k finite while taking $v\to\infty$, the theory on the D2-branes becomes strongly coupled. Since this belongs to the moduli space of the $\mathcal{A}_4$-theory and also is, by definition, the theory on 2 M2-branes in flat space, this amounts to a proof that $\mathcal{A}_4$ describes membranes (assuming the moduli space does not receive significant quantum corrections, which is likely to be true given the maximal supersymmetry). The spacetime interpretation of the $\mathcal{A}_4$-theory is not completely understood, though some of its properties are known and it has been proposed that it corresponds to a pair of membranes on an exotic orbifold that exists only in M-theory [5, 6].

2.2 Effective Higgs rules

Let us summarise what has happened to the theory after giving a vev to one of the original bi-fundamental scalars, $\langle X^8\rangle = \frac{v}{2}\,\mathbb{1}$: The traceless part of X^8 has disappeared during the Higgsing process. The trace part $\tilde{x}^8$ has become an Abelian gauge field after using the Abelian duality Eq. (2.18). Of the two non-dynamical gauge fields, one has been integrated out while the other has become a dynamical SU(2) in the diagonal of the original $\mathrm{SU}(2)\times\mathrm{SU}(2)$, which combines with the above U(1) into a U(2). The fermions follow directly along similar lines. One also has higher order terms $\mathcal{O}(\frac{1}{v})$, which decouple in the limit where $k\to\infty$, $v\to\infty$. Finally the scalars in the $i = 1,...,7$ directions, which were originally bi-fundamentals under $\mathrm{SU}(2)\times\mathrm{SU}(2)$, were first separated into their trace and trace-free parts in Eq. (2.9) and later recombined (slightly differently) into U(2) adjoint scalars in Eq. (2.17).

We can summarise the above discussion into a set of effective rules that capture the net result of the Higgsing process at this order, up to a total derivative and $\mathcal{O}(\frac{1}{v})$ terms. For that, we start with the action Eq. (2.1) and make the following substitutions:

- For the CS terms in the gauge fields:

$$\mathcal{L}_{CS} \to -\tfrac{2}{v^2}\mathbf{f}^\mu\mathbf{f}_\mu \ , \tag{2.30}$$

where we have defined $\mathbf{f}^\mu = \frac{1}{2}\varepsilon^{\mu\nu\lambda}\boldsymbol{F}_{\nu\lambda}$ and in 'mostly-plus' notation for the metric $(-++)$, the inverse transformation is $\boldsymbol{F}_{\mu\nu} = -\varepsilon_{\mu\nu\lambda}\mathbf{f}^\lambda$.

- For the scalars:[7]

$$\begin{aligned}
&\tilde{D}^\mu X^8 \to \tfrac{1}{v}\mathbf{f}^\mu\ , \quad \tilde{D}^\mu X^i \to \tfrac{1}{v}D^\mu \boldsymbol{X}^i\ , \quad X^{ij8} \to -\tfrac{1}{4v}[\boldsymbol{X}^i, \boldsymbol{X}^j]\ , \quad X^{ijk} \to \mathcal{O}\Big(\tfrac{1}{v^3}\Big)\\
&\tilde{D}^\mu X^{8\dagger} \to -\tfrac{1}{v}\mathbf{f}^\mu\ , \ \tilde{D}^\mu X^{i\dagger} \to -\tfrac{1}{v}D^\mu \boldsymbol{X}^i\ , \ X^{ij8\dagger} \to \tfrac{1}{4v}[\boldsymbol{X}^i, \boldsymbol{X}^j]\ , \quad X^{ijk\dagger} \to \mathcal{O}\Big(\tfrac{1}{v^3}\Big)\ .
\end{aligned} \tag{2.31}$$

- For the fermions:

$$\begin{aligned}
&\tilde{D}^\mu \Psi \to \tfrac{1}{v}D^\mu \boldsymbol{\Psi}\ , \qquad [X^i, X^{8\dagger}, \Psi] \to -\tfrac{1}{4v}[\boldsymbol{X}^i, \boldsymbol{\Psi}]\ , \qquad [X^i, X^{j\dagger}, \Psi] \to \mathcal{O}\Big(\tfrac{1}{v^3}\Big)\\
&\tilde{D}^\mu \bar{\Psi} \to \tfrac{1}{v}D^\mu \bar{\boldsymbol{\Psi}}\ , \qquad [X^{i\dagger}, X^8, \Psi^\dagger] \to \tfrac{1}{4v}[\boldsymbol{X}^i, \boldsymbol{\Psi}]\ , \qquad [X^{i\dagger}, X^j, \Psi] \to \mathcal{O}\Big(\tfrac{1}{v^3}\Big)
\end{aligned} \tag{2.32}$$

and $\Gamma_{i8} \to \Gamma_i$.[8]

Using these rules we can readily obtain U(2), $\mathcal{N}=8$ SYM in $(2+1)$ dimensions as in Eq. (2.21). All other terms, including those involving the gauge field, are $\mathcal{O}(\frac{1}{v})$ and vanish in the limit $v \to \infty$, $k \to \infty$ with $\frac{k}{v^2} \to$ fixed, up to a total derivative. These rules will be very useful in the following section where we consider the effect of the Higgsing process on the higher derivative terms.

3. 3BI to DBI

We are now ready to move on to our main discussion and study the form of the lowest non-trivial ℓ_p corrections to the $\mathcal{A}_4$-theory. We begin by writing down the form of the higher derivative action at this order as a certain combination of dimension six operators in the notation that we established in the previous section. The main assumption we will make is that these admit an organisation in terms of the 3-algebra product. Therefore we start with the ansatz that the leading ℓ_p corrections take the most general form that can arise using Euclidean 3-algebra 'building blocks', but with arbitrary coefficients. We will then use the Higgs mechanism to uniquely determine the value of these coefficients

[7]Note here that when contracting two cubic expressions $X^{IJK}X^{IJK\dagger}$ there is an extra combinatorial factor of 3 coming from setting any of the $\{I,J,K\} = 8$. Terms of the kind X^{ijk} with $i,j,k \neq 8$ will be higher order in $\frac{1}{v}$ after the Higgsing and will not contribute in the large v limit.

[8]When contracting the Yukawa-type interaction with Γ_{IJ} there is an extra combinatorial factor of 2 because of the $I \leftrightarrow J$ symmetry. Once again terms obtained from $[X^i, X^{\dagger j}, \Psi]$ with $i,j \neq 8$ will not contribute at large v.

by matching to the leading α' corrections in the low-energy theory of two D2-branes. As explained in the introduction, these corrections are $\mathcal{O}(\ell_p^3)$ for the $\mathcal{A}_4$-theory and $\mathcal{O}(\alpha'^2)$ for the D2-brane theory.

3.1 Bosonic Part

We begin with the bosonic content of the theory. Our ansatz for the $\mathcal{A}_4$-theory will contain all the terms built out of 3-algebra 'blocks' that are gauge/Lorentz invariant, dimension six and lead to expressions contained in the D2-brane effective action upon Higgsing. However some adjustments must be made for the fact that, unlike for the D2-brane theory, our fields X^I and the corresponding triple-product X^{IJK} defined in Eq. (2.2) are *complex* – at least in the bi-fundamental formulation of Ref. [19]. As a result we first need to re-examine the definition of symmetrised trace. We propose that this definition be extended, for bi-fundamentals, to a symmetrisation of the objects while keeping the daggers in their original place. Concretely:

$$\mathrm{STr}(AB^\dagger CD^\dagger) = \tfrac{1}{12}\mathrm{Tr}\Big[A\big(B^\dagger CD^\dagger + B^\dagger DC^\dagger + C^\dagger DB^\dagger + C^\dagger BD^\dagger + D^\dagger BC^\dagger + D^\dagger CB^\dagger\big) + \text{h.c.}\Big] \tag{3.1}$$

Note that this reduces to the conventional definition for Hermitian fields, for which adding the complex conjugate is not necessary.

There is one simplification in the $\mathcal{A}_4$-theory that should be noted at this stage. It corresponds to an identity arising from the low rank of the gauge group, $\mathrm{SU}(2)\times\mathrm{SU}(2)$. This identity is straightforward to verify and states that all three possible contractions in the $(X^{IJK})^4$ terms are proportional to each other:

$$\begin{aligned}\mathrm{STr}\left[X^{IJK}X^{IJL\dagger}X^{MNK}X^{MNL\dagger}\right] &= 2\,\mathrm{STr}\left[X^{IJM}X^{KLM\dagger}X^{IKN}X^{JLN\dagger}\right]\\ &= \tfrac{1}{3}\,\mathrm{STr}\left[X^{IJK}X^{IJK\dagger}X^{LMN}X^{LMN\dagger}\right]. \end{aligned}\tag{3.2}$$

Using this, we can write down the following general ansatz for the $\mathcal{O}(\ell_p^3)$ corrections to the $\mathcal{A}_4$-theory:

$$\begin{aligned}(\tilde{D}X)^4:&\quad k^2\,\mathrm{STr}\left[\mathbf{a}\,\tilde{D}^\mu X^I\,\tilde{D}_\mu X^{J\dagger}\,\tilde{D}^\nu X^J\,\tilde{D}_\nu X^{I\dagger} + \mathbf{b}\,\tilde{D}^\mu X^I\,\tilde{D}_\mu X^{I\dagger}\,\tilde{D}^\nu X^J\,\tilde{D}_\nu X^{J\dagger}\right]\\ X^{IJK}(\tilde{D}X)^3:&\quad k^2\,\varepsilon^{\mu\nu\lambda}\,\mathrm{STr}\left[\mathbf{c}\,X^{IJK}\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J\,\tilde{D}_\lambda X^{K\dagger}\right]\\ (X^{IJK})^2(\tilde{D}X)^2:&\quad k^2\,\mathrm{STr}\left[\mathbf{d}\,X^{IJK}\,X^{IJK\dagger}\,\tilde{D}_\mu X^L\,\tilde{D}^\mu X^{L\dagger} + \mathbf{e}\,X^{IJK}\,X^{IJL\dagger}\,\tilde{D}_\mu X^K\,\tilde{D}^\mu X^{L\dagger}\right]\\ (X^{IJK})^4:&\quad k^2\,\mathrm{STr}\left[\mathbf{f}\;X^{IJK}\,X^{IJK\dagger}\,X^{LMN}\,X^{LMN\dagger}\right],\end{aligned}\tag{3.3}$$

where $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}$ are constants which we will determine. The sum of all terms above will be denoted $\Delta\mathcal{L}$.

Note the absence of pure gauge field terms in Eq. (3.3). Higher dimension combinations of CS terms would break invariance under large gauge transformations. Higher powers of the field strength would explicitly break supersymmetry, which is expected to remain maximal in the ℓ_p expansion.

The next step would be to Higgs the theory in Eq. (3.3) and compare with the derivative-corrected D2-brane theory. We have already written down some 'effective Higgs rules' in Section 2.2. However, the rules themselves could in principle be modified once higher-derivative corrections are included. Fortunately, as we now show, to the lowest nontrivial order (which is the order at which we are working) these rules in fact need no modification.

Combining Eqs. (2.11) and (3.3), the equation of motion for the gauge field B_μ is now of the form:

$$B_\mu = -\frac{1}{v^2} f_\mu - \frac{1}{v} D_\mu \boldsymbol{x}^8 - \frac{\ell_p^3}{2v^2}\frac{\delta(\Delta\mathcal{L})}{\delta B^\mu} \,. \tag{3.4}$$

We now wish to substitute this back into the B_μ-dependent part of the action:

$$2vB^\mu D_\mu \boldsymbol{x}^8 + v^2 B^\mu B_\mu + \ell_p^3\, \Delta\mathcal{L}(B) \,. \tag{3.5}$$

It is easily seen that the result is:

$$-D^\mu \boldsymbol{x}^8 D_\mu \boldsymbol{x}^8 + \frac{1}{v^2} f^\mu f_\mu + \ell_p^3\, \Delta\mathcal{L}\Big|_{B_\mu = -\frac{1}{v^2} f_\mu} \,. \tag{3.6}$$

In the process, two complicated terms at order ℓ_p^3 have cancelled out, considerably simplifying the computation. The last term above is what one gets by substituting Eq. (2.13) into $\Delta\mathcal{L}$. It follows that we can apply the Higgs rules Eq. (2.31) as they are, directly to the four-derivative action.

Through the substitutions Eq. (2.31) the various pieces become:

$$\begin{aligned}
S^b_{\mathbf{a}} &= \mathbf{a}\, \left(\tfrac{k}{v^2}\right)^2 \int d^3x\ \mathrm{STr}\left[D^\mu \boldsymbol{X}^i D_\mu \boldsymbol{X}^j D^\nu \boldsymbol{X}^i D_\nu \boldsymbol{X}^j + 2D^\mu \boldsymbol{X}^i D_\nu \boldsymbol{X}^i \mathbf{f}^\mu \mathbf{f}_\nu + \mathbf{f}^\mu \mathbf{f}_\mu \mathbf{f}^\nu \mathbf{f}_\nu\right]\\
S^b_{\mathbf{b}} &= \mathbf{b}\, \left(\tfrac{k}{v^2}\right)^2 \int d^3x\ \mathrm{STr}\left[D^\mu \boldsymbol{X}^i D_\mu \boldsymbol{X}^i D^\nu \boldsymbol{X}^j D_\nu \boldsymbol{X}^j + 2D^\mu \boldsymbol{X}^i D_\mu \boldsymbol{X}^i \mathbf{f}^\nu \mathbf{f}_\nu + \mathbf{f}^\mu \mathbf{f}_\mu \mathbf{f}^\nu \mathbf{f}_\nu\right]\\
S^b_{\mathbf{c}} &= \mathbf{c}\, \left(\tfrac{k}{v^2}\right)^2 \int d^3x\ \mathrm{STr}\left[\tfrac{3}{4}\varepsilon^{\mu\nu\lambda} D_\mu \boldsymbol{X}^i \mathbf{f}_\nu D_\lambda \boldsymbol{X}^j \boldsymbol{X}^{ji}\right]\\
S^b_{\mathbf{d}} &= \mathbf{d}\, \left(\tfrac{k}{v^2}\right)^2 \int d^3x\ \mathrm{STr}\left[\tfrac{3}{16} D^\mu \boldsymbol{X}^i D_\mu \boldsymbol{X}^i \boldsymbol{X}^{jk} \boldsymbol{X}^{jk} + \tfrac{3}{16}\mathbf{f}^\mu \mathbf{f}_\mu \boldsymbol{X}^{ij} \boldsymbol{X}^{ij}\right]\\
S^b_{\mathbf{e}} &= \mathbf{e}\, \left(\tfrac{k}{v^2}\right)^2 \int d^3x\ \mathrm{STr}\left[\tfrac{1}{8} D^\mu \boldsymbol{X}^i \boldsymbol{X}^{ij} \boldsymbol{X}^{kj} D_\mu \boldsymbol{X}^k + \tfrac{1}{16}\mathbf{f}^\mu \mathbf{f}_\mu \boldsymbol{X}^{ij} \boldsymbol{X}^{ij}\right]\\
S^b_{\mathbf{f}} &= \mathbf{f}\, \left(\tfrac{k}{v^2}\right)^2 \int d^3x\ \mathrm{STr}\left[\tfrac{9}{256} \boldsymbol{X}^{ij} \boldsymbol{X}^{ji} \boldsymbol{X}^{kl} \boldsymbol{X}^{lk}\right]
\end{aligned} \tag{3.7}$$

plus terms in $\mathcal{O}(\frac{1}{v})$, where we are using $\boldsymbol{X}^{ij} = [\boldsymbol{X}^i, \boldsymbol{X}^j]$. The cancellations between the $\boldsymbol{x}^8$'s continue to be trivially present at this order. This is hardly surprising if the Higgs mechanism is to work, since these Goldstone degrees of freedom need to disappear from the action. Putting back the factor ℓ_p^3 in the above terms and using

$$(2\pi)^2\ell_p^3\left(\frac{k}{2\pi v^2}\right)^2 = \frac{(2\pi\alpha')^2}{g_{YM}^2} \tag{3.8}$$

it is now straightforward to compare with the appropriate terms coming from the D2-brane theory.

The precise form of the low-energy effective action for multiple parallel D-branes is still not known to all orders. However, up to order α'^2 it has been explicitly obtained using open string scattering amplitude calculations (see *e.g.* [21, 22] and references therein) and the result agrees with Tseytlin's proposal for a DBI action with a symmetrised prescription for the trace [20]. Starting from D9-branes, the prescription requires to symmetrise over the gauge field strengths. For lower dimensional branes, T-duality requires that this carries on to scalar covariant derivatives and scalar commutators [26, 27]. This proposal fails at order α'^4 [28] but is good enough for our purposes.

The form of the relevant action for 2 D2-branes is given at this order by an appropriately modified, dimensionally reduced version of the D9-brane answer provided in [21]:[9]

$$\begin{aligned} S^b_{\alpha'^2} = \frac{(2\pi\alpha')^2}{g_{YM}^2}\int d^3x\, \mathrm{STr}\Big[&\tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\nu\rho}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\sigma\mu} - \tfrac{1}{16}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\rho\sigma}\boldsymbol{F}_{\rho\sigma} - \tfrac{1}{4}D_\mu\boldsymbol{X}^iD^\mu\boldsymbol{X}^iD_\nu\boldsymbol{X}^jD^\nu\boldsymbol{X}^j \\ &+ \tfrac{1}{2}D_\mu\boldsymbol{X}^iD^\nu\boldsymbol{X}^iD_\nu\boldsymbol{X}^jD^\mu\boldsymbol{X}^j + \tfrac{1}{4}\boldsymbol{X}^{ij}\boldsymbol{X}^{jk}\boldsymbol{X}^{kl}\boldsymbol{X}^{li} - \tfrac{1}{16}\boldsymbol{X}^{ij}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{kl} \\ &- \boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\nu\rho}D_\rho\boldsymbol{X}^iD^\mu\boldsymbol{X}^i - \tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}D_\rho\boldsymbol{X}^iD^\rho\boldsymbol{X}^i - \tfrac{1}{8}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}\boldsymbol{X}^{kl}\boldsymbol{X}^{kl} \\ &- \tfrac{1}{4}D_\mu\boldsymbol{X}^iD^\mu\boldsymbol{X}^i\boldsymbol{X}^{kl}\boldsymbol{X}^{kl} - \boldsymbol{X}^{ij}\boldsymbol{X}^{jk}D^\mu\boldsymbol{X}^kD_\mu\boldsymbol{X}^i - \boldsymbol{F}_{\mu\nu}D^\nu\boldsymbol{X}^iD^\mu\boldsymbol{X}^j\boldsymbol{X}^{ij}\Big]\,, \end{aligned} \tag{3.9}$$

and note that for U(2) one has the additional simplification:

$$\mathrm{STr}\left[\boldsymbol{X}^{ij}\boldsymbol{X}^{jk}\boldsymbol{X}^{kl}\boldsymbol{X}^{li}\right] = \tfrac{1}{2}\mathrm{STr}\left[\boldsymbol{X}^{ij}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{kl}\right]. \tag{3.10}$$

It is then straightforward to compare the coefficients for all of these terms to finally obtain:

$$\begin{aligned} \mathbf{a} &= \tfrac{1}{2}\,, \quad \mathbf{b} = -\tfrac{1}{4}\,, \quad \mathbf{c} = -\tfrac{4}{3}\,, \\ \mathbf{d} &= -\tfrac{4}{3}\,, \quad \mathbf{e} = 8\,, \quad \mathbf{f} = \tfrac{16}{9}\,. \end{aligned} \tag{3.11}$$

It is important to note that the fixing of coefficients by the above comparison is non-trivial. There are 3-algebra terms of Eq. (3.7) that after Higgsing give rise to terms in the D2 action Eq. (3.9) coming from different index contractions (that is, ultimately, different index contractions of the D9-brane theory before dimensional reduction). Also in some places, two terms in the 3-algebra theory lead to the same term in the D2 action. Hence, it was not obvious at the outset that there would be any values of the coefficients in the above expression that would lead to the D2 theory upon Higgsing. The fact that we find a consistent and unique set of coefficients is therefore very satisfying.

[9] Note that the coefficients here are twice their value given in [21], because the normalisation of the trace used there is $\mathrm{Tr}\,(T^aT^b) = \delta^{ab}$ while we consistently use $\mathrm{Tr}\,(\frac{\sigma^a}{2}\frac{\sigma^b}{2}) = \frac{1}{2}\delta^{ab}$.

3.2 Fermionic Part

The fermions will follow the above discussion closely. The most general form for this part of the action at four-derivative order is:[10]

$$
\begin{aligned}
S^f_{\ell_p^3} = \ell_p^3 k^2 \int d^3x\, \mathrm{STr}\Big[& \hat{\mathbf{a}}\ \bar{\Psi}^\dagger \Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{KL}[X^I, X^{J\dagger}, \Psi] + \hat{\mathbf{b}}\ \bar{\Psi}^\dagger\Gamma^\mu \tilde{D}^\nu\Psi\Psi^\dagger\Gamma_\nu\tilde{D}_\mu\Psi \\
& + \hat{\mathbf{c}}\ \bar{\Psi}^\dagger\Gamma^\mu[X^I, X^{J\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\mu\Psi + \hat{\mathbf{d}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}\tilde{D}_\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^J \\
& + \hat{\mathbf{e}}\ \bar{\Psi}^\dagger\Gamma_\mu\tilde{D}^\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}_\nu X^I + \hat{\mathbf{f}}\ \bar{\Psi}^\dagger\Gamma^{IJKL}\tilde{D}_\nu\Psi\ X^{IJK\dagger}\tilde{D}^\nu X^L \\
& + \hat{\mathbf{g}}\ \bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\nu\Psi\ X^{IJK\dagger}\tilde{D}^\nu X^K + \hat{\mathbf{h}}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^J, X^{K\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}_\mu X^K \\
& + \hat{\mathbf{i}}\ \bar{\Psi}^\dagger\Gamma^{\mu\nu}[X^I, X^{J\dagger}, \Psi]\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J + \hat{\mathbf{j}}\ \bar{\Psi}^\dagger\Gamma_{\mu\nu}\Gamma^{IJ}[X^J, X^{K\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^K \\
& + \hat{\mathbf{k}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\tilde{D}^\mu X^{I\dagger}X^{JKL} + \hat{\mathbf{l}}\ \bar{\Psi}^\dagger\Gamma_\mu[X^I, X^{J\dagger}, \Psi]\tilde{D}^\mu X^{K\dagger}X^{IJK} \\
& + \hat{\mathbf{m}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJKL}[X^L, X^{M\dagger}, \Psi]X^{IJK\dagger}\tilde{D}^\mu X^M + \hat{\mathbf{n}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]X^{IJK\dagger}\tilde{D}^\mu X^L \\
& + \hat{\mathbf{o}}\ \bar{\Psi}^\dagger\Gamma^{IJKL}[X^M, X^{N\dagger}, \Psi]X^{IJL\dagger}X^{KMN} + \hat{\mathbf{p}}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]X^{IJM\dagger}X^{KLM} \\
& + \text{h.c. with same coefficients}\Big]\,.
\end{aligned}
\tag{3.12}
$$

It is a straightforward, but rather tedious, exercise to use the Higgs rules and compare with the fermionic terms in the D2-brane effective action at order $\mathcal{O}(\alpha'^2)$, as given for U(2) in [21]:[11]

$$
\begin{aligned}
S^f_{\alpha'^2} = \frac{(2\pi\alpha')^2}{g^2_{YM}} \int d^3x\, \mathrm{STr}\Big(& -\tfrac{1}{8}\bar{\boldsymbol{\Psi}}\Gamma^\mu D^\nu\boldsymbol{\Psi}\ \bar{\boldsymbol{\Psi}}\Gamma_\nu D_\mu\boldsymbol{\Psi} - \tfrac{1}{4}\bar{\boldsymbol{\Psi}}\Gamma^i D^\nu\boldsymbol{\Psi}\ \bar{\boldsymbol{\Psi}}\Gamma_\nu[\boldsymbol{X}^i, \boldsymbol{\Psi}] \\
& -\tfrac{1}{8}\bar{\boldsymbol{\Psi}}\Gamma^i[\boldsymbol{X}^j, \boldsymbol{\Psi}]\ \bar{\boldsymbol{\Psi}}\Gamma^j[\boldsymbol{X}^i, \boldsymbol{\Psi}] + \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_\mu D^\nu\boldsymbol{\Psi}\ \boldsymbol{F}^{\mu\rho}\boldsymbol{F}_{\rho\nu} - \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_\mu D^\nu\boldsymbol{\Psi}\ D^\mu\boldsymbol{X}^l D_\nu\boldsymbol{X}^l \\
& -\tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma^i D^\nu\boldsymbol{\Psi}\ D^\rho\boldsymbol{X}^i\boldsymbol{F}_{\rho\nu} - \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma^i D^\nu\boldsymbol{\Psi}\ \boldsymbol{X}^{il}D_\nu\boldsymbol{X}^l + \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_\mu[\boldsymbol{X}^j, \boldsymbol{\Psi}]\ \boldsymbol{F}^{\mu\rho}D_\rho\boldsymbol{X}^j \\
& -\tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma^i[\boldsymbol{X}^j, \boldsymbol{\Psi}]\ D^\rho\boldsymbol{X}^i D_\rho\boldsymbol{X}^j + \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_\mu[\boldsymbol{X}^j, \boldsymbol{\Psi}]D^\mu\boldsymbol{X}^l\boldsymbol{X}^{lj} + \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma^i[\boldsymbol{X}^j, \boldsymbol{\Psi}]\boldsymbol{X}^{il}\boldsymbol{X}^{lj} \\
& -\tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu\rho}D_\sigma\boldsymbol{\Psi}\ \boldsymbol{F}^{\mu\nu}\boldsymbol{F}^{\rho\sigma} - \tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu\rho}[\boldsymbol{X}^k, \boldsymbol{\Psi}]\ \boldsymbol{F}^{\mu\nu}D^\rho\boldsymbol{X}^k + \tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu l}D_\sigma\boldsymbol{\Psi}\ \boldsymbol{F}^{\mu\nu}D^\sigma\boldsymbol{X}^l \\
& -\tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu l}[\boldsymbol{X}^k, \boldsymbol{\Psi}]\ \boldsymbol{F}^{\mu\nu}\boldsymbol{X}^{lk} - \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_{\mu j\rho}D_\sigma\boldsymbol{\Psi}\ D^\mu\boldsymbol{X}^j\boldsymbol{F}^{\rho\sigma} - \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_{\mu j\rho}[\boldsymbol{X}^k, \boldsymbol{\Psi}]\ D^\mu\boldsymbol{X}^j D^\rho\boldsymbol{X}^k \\
& +\tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_{\mu jl}D_\sigma\boldsymbol{\Psi}\ D^\mu\boldsymbol{X}^j D^\sigma\boldsymbol{X}^l - \tfrac{i}{2}\bar{\boldsymbol{\Psi}}\Gamma_{\mu jl}[\boldsymbol{X}^k, \boldsymbol{\Psi}]\ D^\mu\boldsymbol{X}^j\boldsymbol{X}^{lk} - \tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{ij\rho}D_\sigma\boldsymbol{\Psi}\ \boldsymbol{X}^{ij}\boldsymbol{F}^{\rho\sigma} \\
& -\tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{ij\rho}[\boldsymbol{X}^k, \boldsymbol{\Psi}]\ \boldsymbol{X}^{ij}D^\rho\boldsymbol{X}^k + \tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{ijl}D_\sigma\boldsymbol{\Psi}\ \boldsymbol{X}^{ij}D^\sigma\boldsymbol{X}^l - \tfrac{i}{4}\bar{\boldsymbol{\Psi}}\Gamma_{ijl}[\boldsymbol{X}^k, \boldsymbol{\Psi}]\ \boldsymbol{X}^{ij}\boldsymbol{X}^{lk}\Big)\,.
\end{aligned}
\tag{3.13}
$$

During the Higgs reduction and comparison of coefficients we use that since in 2+1 dimensions $\Gamma_{\mu\nu\lambda} = \varepsilon_{\mu\nu\lambda}\,\mathbb{1}_{2\times 2}$ and $\boldsymbol{F}_{\mu\nu} = \varepsilon_{\mu\nu\lambda}\mathbf{f}^\lambda$, then:

$$
\mathrm{STr}\Big[\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu\rho}D_\sigma\boldsymbol{\Psi}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}^{\rho\sigma}\Big] \sim \mathrm{STr}\Big[\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu\rho}D_\sigma\boldsymbol{\Psi}\varepsilon^{\mu\nu\kappa}\varepsilon^{\rho\sigma\lambda}\mathbf{f}_\kappa\mathbf{f}_\lambda\Big] \sim \mathrm{STr}\Big[\bar{\boldsymbol{\Psi}}D_\sigma\boldsymbol{\Psi}\varepsilon^{\sigma\kappa\lambda}\mathbf{f}_\kappa\mathbf{f}_\lambda\Big] = 0 \tag{3.14}
$$

[10] The issue of uniqueness is significantly more complicated, as compared to the bosonic case, as there are many more terms that one could write down in addition to the ones presented in Eq. (3.12). However, it can be shown that these other terms can be re-expressed by combinations already present in our ansatz. We defer the presentation of these arguments to Appendix C.

[11] Again, the coefficients here are twice their value given in [21], for reasons of normalisation that we have already explained.

because of the STr. We also set the on-shell terms $\alpha'^2(\Gamma_\mu D^\mu \boldsymbol{\Psi} + \Gamma^i[\boldsymbol{X}^i, \boldsymbol{\Psi}])$ to zero, as in [23], since this can be achieved by appropriate field redefinitions. The result for the fermionic coefficients is:

$$\begin{aligned}
&\hat{\mathbf{a}} = -\tfrac{1}{4}\,, \quad \hat{\mathbf{b}} = -\tfrac{1}{16}\,, \quad \hat{\mathbf{c}} = \tfrac{1}{4}\,, \quad \hat{\mathbf{d}} = \tfrac{i}{4}\,, \quad \hat{\mathbf{e}} = -\tfrac{i}{4}\,, \quad \hat{\mathbf{f}} = \tfrac{i}{6}\\
&\hat{\mathbf{g}} = -\tfrac{i}{2}\,, \quad \hat{\mathbf{h}} = -i\,, \quad \hat{\mathbf{i}} = i\,, \quad \hat{\mathbf{j}} = i\,, \quad \hat{\mathbf{k}} = -2i\\
&\hat{\mathbf{l}} = 2i\,, \quad \hat{\mathbf{m}} = -\tfrac{2i}{3}\,, \quad \hat{\mathbf{n}} = 2i\,, \quad \hat{\mathbf{o}} = \tfrac{4i}{3}\,, \quad \hat{\mathbf{p}} = 4i\,.
\end{aligned} \tag{3.15}$$

4. The four-derivative corrections in 3-algebra form

In this section we will re-cast our results in 3-algebra language. There are several important reasons to do so. One is that we will uncover some new properties of 3-algebras, arising from the fact that at order ℓ_p^3 we encounter traces of as many as four 3-algebra generators for the first time.

Another reason is that corrections of order ℓ_p^3 are already known [23, 24] for the special case of Lorentzian 3-algebras. By re-writing the derivative corrections of $\mathcal{A}_4$-theory in terms of 3-algebra quantities, we will be able to compare them with the results of Refs. [23, 24]. Indeed, it is natural to hope that all BLG theories (including both $\mathcal{A}_4$ and Lorentzian sub-classes) originate from a common 3-algebra formulation, even though they were obtained using completely different procedures. As we now have all the necessary data for determining what that formulation is, we will compare the two classes of theories explicitly. After dealing with some issues of normalisation we will find that there is indeed complete agreement.

Yet another reason to re-express our results in 3-algebra language is to open the possibility of extending this investigation to the $\mathcal{N} = 6$ 3-algebras of Refs. [16, 17] which encode, among other things, the ABJM field theory. In the final subsection we will make some general comments on how this might be done.

Let us first remind the reader of the original formulation for BLG 3-algebra theories. Following [1], the maximally ($\mathcal{N} = 8$) supersymmetric 3-algebra field theory in 2 + 1 dimensions involves a set of bosonic fields $X^{Ia}, A_\mu{}^{ab}$, with $I = 1, ..., 8$, and 32-component spinors Ψ^a, where $a = 1, ..., \dim_{\mathcal{A}}$, with $\dim_{\mathcal{A}}$ the dimension of the 3-algebra. $A_\mu{}^{ab}$ is anti-symmetric in a and b. To write the action one introduces the 4-index structure constants $f^{abc}{}_d$ associated to the totally anti-symmetric three-bracket:

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \tag{4.1}$$

and a generalisation of the trace taken over the three-algebra indices, which provides an appropriate 3-algebra metric:

$$h^{ab} = \mathrm{Tr}(T^a T^b)\,. \tag{4.2}$$

The structure constants satisfy the so-called "fundamental identity":

$$f^{[abc}{}_g f^{e]fg}{}_d = 0 \tag{4.3}$$

and are also completely anti-symmetric under the exchange of indices:

$$f^{abcd} = f^{[abcd]}\ . \tag{4.4}$$

The action can then be written as:

$$\begin{aligned} S_{\rm BLG} = \int d^3x\, \Big[\mathrm{Tr}\Big(&-\frac{1}{2}\tilde{D}_\mu X^I \tilde{D}^\mu X^I + \tfrac{i}{2}\bar{\Psi}\tilde{\not{D}}\Psi \\ &+ \tfrac{i}{4}\bar{\Psi}\Gamma^{IJ}[X^I, X^J, \Psi] - \tfrac{1}{12}[X^I, X^J, X^K][X^I, X^J, X^K]\Big) \\ &+ \tfrac{1}{2}\,\varepsilon^{\mu\nu\lambda}(\tilde{A}_\mu{}^a{}_b\,\partial_\nu A_\lambda{}^b{}_a + \tfrac{2}{3}\,A_\mu{}^a{}_b\tilde{A}_\nu{}^b{}_c\tilde{A}_\lambda{}^c{}_a)\Big] \end{aligned} \tag{4.5}$$

where $\tilde{A}^{cd}_\mu = f_{ab}{}^{cd}A^{ab}_\mu$ and:

$$\tilde{D}_\mu X^{Ia} = \partial_\mu X^{Ia} + \tilde{A}_\mu{}^a{}_b X^{Ib}\ . \tag{4.6}$$

Note that the Tr here is the abstract 3-algebra trace defined in Eq. (4.2). The fields are invariant under the gauge transformations:

$$\delta X^{Ia} = -\tilde{\Lambda}^a{}_b X^{Ib} \tag{4.7}$$

$$\delta\Psi^a = -\tilde{\Lambda}^a{}_b\Psi^b \tag{4.8}$$

$$\delta(\tilde{A}^{cd}_\mu) = \tilde{D}_\mu\tilde{\Lambda}^{cd} \tag{4.9}$$

and the supersymmetries:

$$\delta X^{Ia} = i\,\bar{\epsilon}\,\Gamma^I\Psi^a \tag{4.10}$$

$$\delta\Psi^a = D_\mu X^{Ia}\Gamma^\mu\Gamma^I\epsilon + \frac{1}{6}f^a{}_{bcd}X^{Ib}X^{Jc}X^{Kd}\Gamma^{IJK}\epsilon \tag{4.11}$$

$$\delta(\tilde{A}^{cd}_\mu) = if_{ab}{}^{cd}X^{Ia}\,\bar{\epsilon}\,\Gamma_\mu\Gamma_I\Psi^b \tag{4.12}$$

where $\Gamma_{012}\epsilon = \epsilon$ and $\Gamma_{012}\Psi^a = -\Psi^a$.

4.1 $\mathcal{A}_4$ 3-algebra theory

For a Euclidean 3-algebra metric, $h^{ab} = \delta^{ab}$, the possible BLG theories are the $\mathcal{A}_4$-theory with $a = 1, ...4$, and direct products thereof [29, 30]. Already at the lowest (quadratic) order it is easy to see how one can convert the above 3-algebra formulation to the bi-fundamental action of [19] after noting a subtlety in the definition of the trace between the two cases. Whereas $\mathrm{Tr}(T^aT^b) = \delta^{ab}$ in 3-algebra notation, one has for the SU(2) generators $T^i = \frac{\sigma^i}{2}$, that the trace is $\mathrm{Tr}(\frac{\sigma^i}{2}\frac{\sigma^j}{2}) = \frac{1}{2}\delta^{ij}$. Taking this into account it is straightforward to convert Eq. (4.5) into Eq. (2.1) and vice-versa. The powers of $\frac{1}{f^2} = (\frac{k}{2\pi})^2$ will appear once one re-scales the fields appropriately by $(X, \Psi) \to \frac{1}{\sqrt{f}}(X, \Psi)$.

This is useful, since we have obtained the four-derivative action in bi-fundamental notation and we now want to express it in 3-algebra form. In doing so one also has to deal with evaluating the symmetrised trace of four 3-algebra generators. Symmetry restricts its form to be:

$$\mathrm{STr}\Big(T^a T^b T^c T^d\Big) = m\, h^{(ab}h^{cd)}\ , \tag{4.13}$$

where m is a yet undetermined numerical coefficient. However, the Lorentzian 3-algebras can help us determine the latter as follows. Lorentzian 3-algebras include a set of generators corresponding to a compact subgroup of the theory's whole symmetry group. One is then free to choose them as the generators of any semi-simple Lie algebra, *e.g.* SU(2). In turn, tracing over the latter leads to a flat Euclidean block in the 3-algebra metric, $h^{ij} = \delta^{ij}$. In any four-derivative Lorentzian 3-algebra action there will be terms with components for which the generators in Eq. (4.13) run over this subset. In that case, and once again taking into consideration the appropriate definition of the trace, one can explicitly evaluate the following expression for the particular case of SU(2):

$$\mathrm{STr}\Big(T^i T^j T^k T^l\Big) = 2\ \mathrm{STr}\Big(\tfrac{\sigma^i}{2}\tfrac{\sigma^j}{2}\tfrac{\sigma^k}{2}\tfrac{\sigma^l}{2}\Big) = \tfrac{1}{4}\,\delta^{(ij}\delta^{kl)} \tag{4.14}$$

and this fixes $m = \frac{1}{4}$.

Equipped with the above fact, we can finally convert our results and we write for the bosonic part of the $\mathcal{A}_4$-theory in 3-algebra form:

$$\begin{aligned} S^b_{\ell_p^3} = (2\pi)^2\ell_p^3 \int d^3x\ \mathrm{STr}\Big[&\tfrac{1}{4}\Big(\tilde{D}^\mu X^I \tilde{D}_\mu X^J \tilde{D}^\nu X^J \tilde{D}_\nu X^I - \tfrac{1}{2}\tilde{D}^\mu X^I \tilde{D}_\mu X^I \tilde{D}^\nu X^J \tilde{D}_\nu X^J\Big) \\ &- \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda}\Big(X^{IJK}\tilde{D}_\mu X^I \tilde{D}_\nu X^J \tilde{D}_\lambda X^K\Big) \\ &+ \tfrac{1}{4}\Big(X^{IJK}X^{IJL}\tilde{D}^\mu X^K \tilde{D}_\mu X^L - \tfrac{1}{6}X^{IJK}X^{IJK}\tilde{D}^\mu X^L \tilde{D}_\mu X^L\Big) \\ &+ \tfrac{1}{288}\Big(X^{IJK}X^{IJK}X^{LMN}X^{LMN}\Big)\Big]\ , \end{aligned} \tag{4.15}$$

where now:

$$X^{IJK} = [X^I, X^J, X^K]\ . \tag{4.16}$$

4.2 Lorentzian 3-algebra theory

In Ref. [23] the equivalent four derivative terms were constructively obtained for Lorentzian 3-algebra theories and it was conjectured there that the $\mathcal{A}_4$-theory should also be expressed in the terms of the same 3-algebra structures at four derivative order. We will soon verify this conjecture.

Let us start by quoting the result found in Ref. [23] for the higher-derivative corrections to Lorentzian 3-algebra theories. To avoid confusion with the Euclidean signature theories that are the focus of this paper, we will consistently denote all Lorentzian 3-algebra variables with a hat symbol on top. Accordingly, the field variables in Ref. [23] are eight adjoint scalars $\hat{X}^I$ and fermions $\hat{\lambda}$, as well as sixteen gauge-singlet scalars and fermions $\hat{X}^I_\pm, \hat{\lambda}_\pm$ and a pair of gauge fields $\hat{A}_\mu, \hat{B}_\mu$.

Due to constraints, the fields $\hat{X}^I_-, \hat{\lambda}_-$ decouple and the fields $\hat{X}^I_+, \hat{\lambda}_+$ are fixed to be a constant and zero, respectively. It was shown that the bosonic part of the ℓ_p^3 correction can be written entirely in terms of the building blocks:

$$\begin{aligned}
\hat{D}_\mu \hat{X}^I &= \partial_\mu \hat{X}^I - [\hat{A}_\mu, \hat{X}^I] - \hat{B}_\mu \hat{X}^I_+ \\
\hat{X}^{IJK} &= \hat{X}^I_+[\hat{X}^J, \hat{X}^K] + \hat{X}^J_+[\hat{X}^K, \hat{X}^I] + \hat{X}^K_+[\hat{X}^I, \hat{X}^J]\,.
\end{aligned}$$

To simplify formulae, we have converted the results of Ref. [23] into symmetrised-trace form. Then Eq.(3.14) of that paper[12] is the sum of the following four terms (we only write the $\mathcal{O}(\ell_p^3)$ corrections, dropping the leading terms):

$$\begin{aligned}
(\hat{D}\hat{X})^4: &\quad \tfrac{1}{4}\,\mathrm{STr}\left(\hat{D}^\mu \hat{X}^I \hat{D}_\mu \hat{X}^J \hat{D}^\nu \hat{X}^J \hat{D}_\nu \hat{X}^I - \tfrac{1}{2}\hat{D}^\mu \hat{X}^I \hat{D}_\mu \hat{X}^I \hat{D}^\nu \hat{X}^J \hat{D}_\nu \hat{X}^J\right) \\
\hat{X}^{IJK}(\hat{D}\hat{X})^3: &\quad -\tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda}\,\mathrm{STr}\left(\hat{X}^{IJK}\hat{D}_\mu \hat{X}^I \hat{D}_\nu \hat{X}^J \hat{D}_\lambda \hat{X}^K\right) \\
(\hat{X}^{IJK})^2(\hat{D}\hat{X})^2: &\quad \tfrac{1}{4}\,\mathrm{STr}\left(\hat{X}^{IJK}\hat{X}^{IJL}\hat{D}^\mu \hat{X}^K \hat{D}_\mu \hat{X}^L - \tfrac{1}{6}\hat{X}^{IJK}\hat{X}^{IJK}\hat{D}^\mu \hat{X}^L \hat{D}_\mu \hat{X}^L\right) \\
(\hat{X}^{IJK})^4: &\quad \tfrac{1}{24}\,\mathrm{STr}\left(\hat{X}^{IJM}\hat{X}^{KLM}\hat{X}^{IKN}\hat{X}^{JLN} - \tfrac{1}{12}\hat{X}^{IJK}\hat{X}^{IJK}\hat{X}^{LMN}\hat{X}^{LMN}\right)\,.
\end{aligned} \tag{4.17}$$

Here, the trace is defined using $\mathrm{Tr}\,(T^a T^b) = \delta^{ab}$ where a, b are now adjoint Lie algebra indices.

Note that the above expression involves all possible terms one can write down at this order using $\hat{D}_\mu \hat{X}^I$ and $\hat{X}^{IJK}$ as building blocks, with one apparent exception: The $(\hat{X}^{IJK})^4$ terms could have contained one more distinct index contraction, namely the one with $\hat{X}^{IJK}\hat{X}^{IJL}\hat{X}^{MNK}\hat{X}^{MNL}$. However, it is easy to demonstrate the identity:

$$\begin{aligned}
\mathrm{STr}\Big(\hat{X}^{IJK}\hat{X}^{IJL}\hat{X}^{MNK}\hat{X}^{MNL}\Big) = \mathrm{STr}\Big(&\tfrac{4}{3}\hat{X}^{IJM}\hat{X}^{KLM}\hat{X}^{IKN}\hat{X}^{JLN} \\
&+\tfrac{1}{9}\hat{X}^{IJK}\hat{X}^{IJK}\hat{X}^{LMN}\hat{X}^{LMN}\Big)\,,
\end{aligned} \tag{4.18}$$

as a result of which only two of the three possible $\mathcal{O}(\hat{X}^{IJK})^4$ terms are independent.

[12]We have corrected a few of the coefficients.

4.3 Final answer for the BLG theory

We would finally like to recover the four-derivative action to BLG theory for general 3-algebras. A reasonable guess would be to see whether Eq. (4.15) provides the answer by simply replacing the $\mathcal{A}_4$ structure constants and metric with their Lorentzian counterparts inside the expressions. One then finds that all terms and coefficients in Eq. (4.17) can be readily obtained except for $\mathcal{O}(\hat{X}^{IJK})^4$. This discrepancy is easily traced back to the difference between the identities obeyed by quartic powers of triple-products in the two cases and is resolved by noticing that Eq. (3.2) is actually a special case of Eq. (4.18), due to the particularly simple nature of the $\mathcal{A}_4$ structure constants ϵ^{abcd}. Therefore within the class of BLG theories we are considering, the following identity is the most general one to be always satisfied:

$$\mathrm{STr}\Big(X^{IJK}X^{IJL}X^{MNK}X^{MNL}\Big) = \mathrm{STr}\Big(\tfrac{4}{3}X^{IJM}X^{KLM}X^{IKN}X^{JLN} + \tfrac{1}{9}X^{IJK}X^{IJK}X^{LMN}X^{LMN}\Big)\,. \qquad (4.19)$$

This raises the interesting question, which we leave for a future investigation, of whether this identity is also obeyed by other indefinite-signature BLG theories, notably those with multiple time-like directions as discussed in [31, 32, 33]. If the answer turns out to be in the affirmative, then we would have found a new relation for quartic products of structure constants that holds for a generic $\mathcal{N}=8$ 3-algebra.

With these observations we can at last write a common expression for both $\mathcal{A}_4$ and Lorentzian BLG theories:

$$\begin{aligned} S^b_{\mathrm{BLG},\ell_p^3} = \ell_p^3\int d^3x\ \mathrm{STr}\Big[&\tfrac{1}{4}\Big(\tilde{D}^\mu X^I\tilde{D}_\mu X^J\tilde{D}^\nu X^J\tilde{D}_\nu X^I - \tfrac{1}{2}\tilde{D}^\mu X^I\tilde{D}_\mu X^I\tilde{D}^\nu X^J\tilde{D}_\nu X^J\Big)\\ &-\tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda}\Big(X^{IJK}\tilde{D}_\mu X^I\tilde{D}_\nu X^J\tilde{D}_\lambda X^K\Big)\\ &+\tfrac{1}{4}\Big(X^{IJK}X^{IJL}\tilde{D}^\mu X^K\tilde{D}_\mu X^L - \tfrac{1}{6}X^{IJK}X^{IJK}\tilde{D}^\mu X^L\tilde{D}_\mu X^L\Big)\\ &+\tfrac{1}{24}\Big(X^{IJM}X^{KLM}X^{IKN}X^{JLN} - \tfrac{1}{12}X^{IJK}X^{IJK}X^{LMN}X^{LMN}\Big)\Big]\,. \end{aligned} \qquad (4.20)$$

It is very satisfactory that one can obtain the precise coefficients of Eq. (3.11) as well as Eq. (4.17) from this expression upon specifying the 3-algebra.

Similarly we can write down the corrections for the terms including fermions in $\mathcal{N}=8$ 3-algebra form:

$$\begin{aligned} S^f_{\mathrm{BLG},\ell_p^3} = (2\pi)^2\ell_p^3\int d^3x\ \mathrm{STr}\Big[&-\tfrac{1}{64}\bar{\Psi}\Gamma^{IJ}[X^K,X^L,\Psi]\bar{\Psi}\Gamma^{KL}[X^I,X^J,\Psi] - \tfrac{1}{16}\bar{\Psi}\Gamma^\mu\tilde{D}^\nu\Psi\Psi\Gamma_\nu\tilde{D}_\mu\Psi\\ &+\tfrac{1}{16}\bar{\Psi}\Gamma^\mu[X^I,X^J,\Psi]\bar{\Psi}\Gamma^{IJ}\tilde{D}_\mu\Psi + \tfrac{i}{4}\bar{\Psi}\Gamma_\mu\Gamma^{IJ}\tilde{D}_\nu\Psi\tilde{D}^\mu X^I\tilde{D}^\nu X^J\\ &-\tfrac{i}{4}\bar{\Psi}\Gamma_\mu\tilde{D}^\nu\Psi\tilde{D}^\mu X^I\tilde{D}_\nu X^I + \tfrac{i}{24}\bar{\Psi}\Gamma^{IJKL}\tilde{D}_\nu\Psi\ X^{IJK}\tilde{D}^\nu X^L\\ &-\tfrac{i}{8}\bar{\Psi}\Gamma^{IJ}\tilde{D}_\nu\Psi\ X^{IJK}\tilde{D}^\nu X^K - \tfrac{i}{4}\bar{\Psi}\Gamma^{IJ}[X^J,X^K,\Psi]\tilde{D}^\mu X^I\tilde{D}_\mu X^K \end{aligned}$$

$$
\begin{aligned}
&+\tfrac{i}{4}\bar{\Psi}\Gamma^{\mu\nu}[X^I,X^J,\Psi]\tilde{D}_\mu X^I\tilde{D}_\nu X^J+\tfrac{i}{4}\bar{\Psi}\Gamma_{\mu\nu}\Gamma^{IJ}[X^J,X^K,\Psi]\tilde{D}^\mu X^I\tilde{D}^\nu X^K\\
&-\tfrac{i}{8}\bar{\Psi}\Gamma_\mu\Gamma^{IJ}[X^K,X^L,\Psi]\tilde{D}^\mu X^I X^{JKL}+\tfrac{i}{8}\bar{\Psi}\Gamma_\mu[X^I,X^J,\Psi]\tilde{D}^\mu X^K X^{IJK}\\
&-\tfrac{i}{24}\bar{\Psi}\Gamma_\mu\Gamma^{IJKL}[X^L,X^M,\Psi]X^{IJK}\tilde{D}^\mu X^M+\tfrac{i}{8}\bar{\Psi}\Gamma_\mu\Gamma^{IJ}[X^K,X^L,\Psi]X^{IJK}\tilde{D}^\mu X^L\\
&+\tfrac{i}{48}\bar{\Psi}\Gamma^{IJKL}[X^M,X^N,\Psi]X^{IJL}X^{KMN}+\tfrac{i}{16}\bar{\Psi}\Gamma^{IJ}[X^K,X^L,\Psi]X^{IJM}X^{KLM}\Big]\,,
\end{aligned}
\tag{4.21}
$$

where:

$$
[X^I,X^J,\Psi]=X^I_aX^J_b\Psi_c[T^a,T^b,T^c]\,. \tag{4.22}
$$

The above reduces to both Eq. (3.12) with the coefficients as given in Eq. (3.15) and the analogous result valid for BLG theories with Lorentzian signature as given in [23].

The expressions Eq. (4.20) and Eq. (4.21) are the main results of this paper.

4.4 Towards $\mathcal{N}=6$ 3-algebra theories at four-derivative order

It is natural to try and see whether the above can be extended to the case of $\mathcal{N}=6$ 3-algebra theories, which include the ABJM model [15]. Finding such an extension is of great interest as these theories have a clear spacetime interpretation in M-theory.

One approach would be to work directly with the ABJM theory and repeat the analysis of Section 2. The straightforward application of the Higgs mechanism to the $\mathrm{U}(N)\times\mathrm{U}(N)$ ABJM theory was shown to yield a U(N) YM action in Refs. [34, 35]. In ABJM the matter fields are complex, $Z^A=(X^A+iX^{A+4})$, where $A=1,\ldots4$, since the R-symmetry group is $\mathrm{SU}(4)\simeq\mathrm{SO}(6)$. In order to Higgs the theory one then gives a vev to the real component of, say, Z^4. A difference between this case and the treatment of Section 2 is that the gauge fields are already in $\mathrm{U}(N)$, as opposed to $\mathrm{SU}(N)$. Hence, if everything were to work in exactly the same way as for $\mathcal{A}_4$ one would end up with an extra U(1) degree of freedom.

However, it is easy to verify that there is an extra Goldstone mode in the problem: It is not only the *traceless* part of X^4 (the real component of Z^4) that cancels out during the calculation but also the *trace* part of X^8 (the imaginary part of Z^4). Hence the number of degrees of freedom works out right. Moreover, there is no need to perform an Abelian duality in this context.[13]

Nevertheless, trying to construct and apply effective Higgs rules for this case is cumbersome and becomes even more so at four-derivative order. This is related to the fact that the ABJM matter fields are complex with 8 real components yet reduce to real YM fields with 7 real components, hence calling for a separate treatment of $Z^{1,2,3}$ and Z^4. As a result, the 'direct' extension is not that straightforward and we will not attempt to carry it out here.

[13] As far as we know this point was not noted in Refs. [34, 35].

Another way to proceed would be to take advantage of the 3-algebra formulation that we have just uncovered and try to generalise the answer to the $\mathcal{N}=6$ 3-algebra theories of Refs. [16, 17]. In the latter case the generators are complex, as are the structure constants which are further only partially anti-symmetric under the exchange of their indices:[14]

$$[T^a, T^b; \bar{T}^{\bar{c}}] = f^{ab\bar{c}}{}_d T^d \ , \tag{4.23}$$

with:

$$f^{ab\bar{c}\bar{d}} = -f^{ba\bar{c}\bar{d}} \qquad \text{and} \qquad f^{ab\bar{c}\bar{d}} = f^{*\bar{c}\bar{d}ab} \ , \tag{4.24}$$

as well as:

$$h^{\bar{a}b} = \mathrm{Tr}(\bar{T}^{\bar{a}} T^b) \ . \tag{4.25}$$

The generators satisfy a complex version of the "fundamental identity":

$$f^{ef\bar{g}}{}_b f^{cb\bar{a}}{}_d + f^{fe\bar{a}}{}_d f^{cb\bar{g}}{}_d + f^{*\bar{g}\bar{a}f}{}_{\bar{b}} f^{ce\bar{b}}{}_d + f^{*\bar{a}\bar{g}e}{}_{\bar{b}} f^{cf\bar{b}}{}_d = 0. \tag{4.26}$$

Since we wish to be illustrative, we only focus on the bosonic piece of the $\mathcal{N}=6$ 3-algebra action, which is:

$$\begin{aligned} S^b_{\mathcal{N}=6} = \int d^3x \, \Big[\mathrm{Tr} \Big(-\tilde{D}_\mu \bar{Z}_A \tilde{D}^\mu Z^A - \tfrac{2}{3} \Upsilon^{CD}_B \bar{\Upsilon}^B_{CD} \Big) \\ + \tfrac{1}{2} \varepsilon^{\mu\nu\lambda} (\tilde{A}_\mu{}^a{}_b \, \partial_\nu A_\lambda{}^b{}_a + \tfrac{2}{3} A_\mu{}^a{}_b \tilde{A}_\nu{}^b{}_c \tilde{A}_\lambda{}^c{}_a) \Big] \ , \end{aligned} \tag{4.27}$$

with

$$\Upsilon^{CD}_{Bd} = f^{ab\bar{c}}{}_d Z^C_a Z^D_b \bar{Z}_{B\bar{c}} - \tfrac{1}{2} \delta^C_B f^{ab\bar{c}}{}_d Z^E_a Z^D_b \bar{Z}_{E\bar{c}} + \tfrac{1}{2} \delta^D_B f^{ab\bar{c}}{}_d Z^E_a Z^C_b \bar{Z}_{E\bar{c}} \ . \tag{4.28}$$

Without going into all details about this theory (the interested reader should refer to [16]), we would like to highlight some relevant points. The supervariation of the fermion in this model can be expressed as:

$$\delta\psi_{Bd} = \not{D} Z^A_d \epsilon_{AB} + \Upsilon^{CD}_{Bd} \epsilon_{CD} \ , \tag{4.29}$$

hence Υ^{CD}_B is the natural generalisation of the $\mathcal{N}=8$ triple-product appearing in Eq. (4.11).

Note that at lowest order the sextic scalar potential appears without tracing any of the SU(4) indices in a given Υ, although in principle one could also have had terms of the type $\Upsilon^{CD}_C \bar{\Upsilon}^B_{BD}$. The reason behind this is the supersymmetry of the theory and is made manifest through Eq. (4.29). We expect that this structure will carry on for all 3-algebra theories even when Eq. (4.29) and Eq. (4.11) receive ℓ_p^3 corrections; in fact, this seems necessary if

[14]A different generalisation of 3-algebra theories for which the structure constants are not totally anti-symmetric was considered in [36].

we want Eq. (4.20)-Eq. (4.21) to be invariant under the $\mathcal{N}=8$ supersymmetry variations. This suggests that all higher derivative corrections in $\mathcal{N}=6$ ought to be expressed in terms of Υ^{CD}_{B} building blocks, in the same spirit as per our $\mathcal{N}=8$ example.

Let us investigate how far one can go with such an ansatz. At lowest order, the $\mathcal{N}=8$ 3-algebra action emerges as a special case of $\mathcal{N}=6$ when the structure constants are totally anti-symmetric. It is natural to assume that the same should also hold for higher derivative terms. Hence, Eq. (4.20) can serve as a 'boundary condition' for the higher order $\mathcal{N}=6$ action. With that condition in mind, it is easy to see that the form of the $(\tilde{D}Z)^4$ and $\Upsilon(\tilde{D}Z)^3$ terms of interest are uniquely determined, including the numerical coefficients.

Things start to potentially differ for $(\Upsilon)^2(\tilde{D}Z)^2$ and $(\Upsilon)^4$ terms, where one has several index contractions available leading to the same $\mathcal{N}=8$ terms as a special case. This would suggest at first sight that it will be impossible to determine these coefficients uniquely through Higgsing. However, we believe that there will be generalisations of the identity Eq. (4.19) to $\mathcal{N}=6$, that relate terms with different index contractions. Hence, we still hope that the Higgs mechanism will be powerful enough to also dictate the form of the $\mathcal{N}=6$ 3-algebra theory. Progress in that direction would probably involve first understanding the origin of Eq. (4.19) directly from the BLG 3-algebra point of view, as opposed to our approach which involved studying its particular representations.

5. Conclusions

In this paper we have derived an extension to the BLG 3-algebra theory at four-derivative order, which Higgses uniquely to the four-derivative correction of the D2-brane effective worldvolume theory. Our result applies equally to the $\mathcal{A}_4$ Euclidean theory and the Lorentzian 3-algebra theory, with the latter result having been already obtained in Refs. [23, 24]. We find it satisfying that both classes of BLG theories have the same four-derivative corrections, depending only on 3-algebra quantities.

An open question raised by this investigation is to determine whether our result applies to *all* BLG theories. While the $\mathcal{A}_4$-theory (and its direct sums) exhausts the Euclidean signature ones, on the Lorentzian signature side we have only looked at the theories with one time-like direction in 3-algebra space because of their more immediate physical relevance. However there do exist theories with two or more time-like directions [31, 32, 33] that we have not covered in our analysis. If our result can be shown to apply also to these theories then it would be truly universal for $\mathcal{N}=8$ BLG theory and it might lead to a deeper understanding of the relevant 3-algebras.

Even though the Higgs mechanism constrains the four-derivative BLG action uniquely, it is crucial to explicitly check that it is invariant under the set of supersymmetry transformations. This should be done both for the Lorentzian as well as the $\mathcal{A}_4$ cases. For the former there is a constructive method to carry out the supersymmetry analysis, starting with the corresponding analysis for D2-branes and using the methods of Refs. [12, 23]. For the latter, one has to use the Higgs mechanism as a guide. However ultimately the results for the two cases should converge into a common formula valid for all BLG theories or at least for the two classes of BLG theories studied here.

One would like to extend our method to find derivative corrections involving more than four derivatives (equivalently, to order higher than ℓ_p^3). On the Lorentzian side, Ref. [24] has proposed an action to all orders in ℓ_p that reduces to the action of Refs. [26, 27] after Higgsing. Something similar can surely be done for the $\mathcal{A}_4$-theory. However it is important to keep in mind that the D-brane action of Refs. [26, 27] works for certain purposes such as finding classical solutions, but cannot be considered correct as far as generating string amplitudes is concerned (since it is known that the symmetrised trace prescription does not work beyond four derivatives).

Our findings also support the idea of a spacetime realisation for the $\mathcal{A}_4$-theory. In Refs. [5, 6] a proposal for such an interpretation was made in terms 2 M2-branes on a yet unknown 'orbifold' of M-theory, dubbed as an "M-fold", which preserves maximal, $\mathcal{N} = 8$, supersymmetry and has a moduli space $(\mathbb{R}^8 \times \mathbb{R}^8)/\mathbb{D}_{2k}$, where $\mathbb{D}_{2k}$ is the dihedral group of order $4k$. The fact that we are able to find an $\mathcal{O}(\ell_p^3)$ correction to the action, from which one can recover the precise α'^2 corrections to the D2-brane theory by Higgsing, is encouraging and strongly suggests that such a spacetime description should exist.

It has to be noted in this context that one expects ℓ_p corrections in M-theory to give rise to both the α' as well as g_s corrections in string theory. While the action we have found reproduces the first α' correction by construction, it is not clear what part of the corresponding g_s correction (if any) it reproduces, since in general g_s corrections are expected to be non-local. It would therefore be nice to understand which aspects of the membrane dynamics are captured by the higher derivative action that we have constructed. As indicated in the Introduction, at large k one expects to be safe because the $\mathcal{A}_4$-theory is weakly coupled, so this caveat only applies when we take k small.[15]

Finally we discussed possible generalisations of our result to $\mathcal{N} = 6$ 3-algebras and the ABJM theory. Here we did not find a complete result, but have sketched how one can approach the problem. It is of considerable interest to explicitly pursue this direction for two reasons: these models have a well-understood spacetime interpretation at finite k in terms of membranes at a geometric orbifold, and one can also use them to perform

[15]We would like to acknowledge the participants of the Indian Strings Meeting in Pondicherry (ISM08) in December 2008 for useful comments on this point.

precision calculations at large k via the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence. We hope to report on this in more detail in the future.

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A. A note on spinor conventions

Throughout this paper we have used 32-component spinors Ψ for our 3-algebra theories. These are acted upon by Γ-matrices of $\mathrm{SO}(10,1)$. The latter can be arranged in terms of the following $\mathrm{SO}(2,1)\times\mathrm{SO}(8)$ decomposition:

$$\Gamma^M = \{\hat{\gamma}^\mu\otimes\gamma^9,\ 1\!\!1_{2\times 2}\otimes\gamma^I\}\ , \tag{A.1}$$

where $\mu = 0,1,2$ and $I = 1,...,8$, $\gamma^9 = \gamma^1\dots\gamma^8$ is the $\mathrm{SO}(8)$ chirality matrix, while the Γ-matrices satisfy the Clifford algebra $\{\Gamma^M,\Gamma^N\} = 2\eta^{MN}$. The $\mathrm{SO}(2,1)$ $\hat{\gamma}$-matrices obey the following identities, defined with weight one:

$$\begin{aligned}
\hat{\gamma}_{\mu\nu} &= \tfrac{1}{2}(\hat{\gamma}_\mu\hat{\gamma}_\nu - \hat{\gamma}_\nu\hat{\gamma}_\mu)\\
\hat{\gamma}_{\mu\nu\lambda} &= \hat{\gamma}_\mu\hat{\gamma}_\nu\hat{\gamma}_\lambda - \hat{\gamma}_\mu\eta_{\nu\lambda} + \hat{\gamma}_\nu\eta_{\mu\lambda} - \hat{\gamma}_\lambda\eta_{\mu\nu}\\
\hat{\gamma}_\mu\hat{\gamma}_{\nu\lambda} &= \hat{\gamma}_{\mu\nu\lambda} + \hat{\gamma}_\lambda\eta_{\mu\nu} - \hat{\gamma}_\nu\eta_{\mu\lambda}\\
\hat{\gamma}_{\nu\lambda}\hat{\gamma}_\mu &= \hat{\gamma}_{\nu\lambda\mu} + \hat{\gamma}_\nu\eta_{\mu\lambda} - \hat{\gamma}_\lambda\eta_{\mu\nu}\\
\varepsilon_{\mu\nu\lambda}\,1\!\!1_{2\times 2} &= \hat{\gamma}_{\mu\nu\lambda}\\
\varepsilon_{\mu\nu\lambda}\hat{\gamma}^\lambda &= \hat{\gamma}_{\mu\nu}\\
\varepsilon_{\rho\sigma\nu}\hat{\gamma}^{\nu\mu} &= 2\delta^\mu_{[\sigma}\hat{\gamma}_{\rho]}\\
\hat{\gamma}_0\hat{\gamma}_0 &= -1\ .
\end{aligned} \tag{A.2}$$

Moreover, the 3-algebra spinors are Goldstinos of the symmetry breaking Eq. (A.1) and hence obey the following chirality condition [3, 19]:

$$\Gamma_{012}\Psi = -\Psi\ , \tag{A.3}$$

which translates to:

$$\begin{aligned}
\Gamma_{012}\Psi &= (\hat{\gamma}_{012}\otimes\gamma_9)\Psi\\
&= (\varepsilon_{012}\,1\!\!1_{2\times 2}\otimes\gamma_9)\Psi\\
&= -(\,1\!\!1_{2\times 2}\otimes\gamma_9)\Psi
\end{aligned}$$

$$
\begin{aligned}
&\equiv -\Gamma_9\Psi \\
&= -\Psi \\
\Rightarrow \Gamma_9\Psi &= \Psi\ . \qquad\qquad (\mathrm{A.4})
\end{aligned}
$$

We are working with conventions where $\varepsilon_{012} = -1$, that is $\{\hat{\gamma}_0, \hat{\gamma}_1, \hat{\gamma}_2\} = \{\sigma_1, -i\sigma_2, \sigma_3\}$ and σ the usual Pauli matrices. One can then use Γ_9 to get the 11d identities:

$$
\begin{aligned}
\Gamma_{\mu\nu} &= \tfrac{1}{2}(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu) \\
\Gamma_{\mu\nu\lambda} &= \Gamma_\mu\Gamma_\nu\Gamma_\lambda - \Gamma_\mu\eta_{\nu\lambda} + \Gamma_\nu\eta_{\mu\lambda} - \Gamma_\lambda\eta_{\mu\nu} \\
\Gamma_\mu\Gamma_{\nu\lambda} &= \Gamma_{\mu\nu\lambda} + \Gamma_\lambda\eta_{\mu\nu} - \Gamma_\nu\eta_{\mu\lambda} \\
\Gamma_{\nu\lambda}\Gamma_\mu &= \Gamma_{\nu\lambda\mu} + \Gamma_\nu\eta_{\mu\lambda} - \Gamma_\lambda\eta_{\mu\nu} \\
\hat{\varepsilon}_{\mu\nu\lambda} &\equiv \varepsilon_{\mu\nu\lambda}\Gamma^9 = \Gamma_{\mu\nu\lambda} \\
\hat{\varepsilon}_{\mu\nu\lambda}\Gamma^\lambda &= \Gamma_{\mu\nu} \\
\hat{\varepsilon}_{\rho\sigma\nu}\Gamma^{\nu\mu} &= 2\delta^\mu_{[\sigma}\Gamma_{\rho]} \\
\Gamma_9\Gamma_9 &= 1 \\
\Gamma_0\Gamma_0 &= -1\ . \qquad\qquad (\mathrm{A.5})
\end{aligned}
$$

We have implemented the above identities in Subsection 3.2. Note that while Γ^9 anti-commutes with the Γ^i's, it commutes with the Γ^μ's.

B. Explicit Higgsing of the fermionic terms

Here we give a complete list for the explicit Higgsing of the fermionic terms that we presented in Eq. (3.12). Applying the Higgs rules of Section 2.2 these give:

$$
\begin{aligned}
\hat{\mathbf{a}}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^K, X^{L\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{KL}[X^I, X^{J\dagger}, \Psi] &\to \tfrac{\hat{\mathbf{a}}}{v^4}\tfrac{1}{2}\bar{\mathbf{\Psi}}\Gamma^i[\boldsymbol{X}^j, \mathbf{\Psi}]\bar{\mathbf{\Psi}}\Gamma^j[\boldsymbol{X}^i, \mathbf{\Psi}] \\
\hat{\mathbf{b}}\ \bar{\Psi}^\dagger\Gamma^\mu\tilde{D}^\nu\Psi\bar{\Psi}^\dagger\Gamma_\nu\tilde{D}_\mu\Psi &\to 2\tfrac{\hat{\mathbf{b}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^\mu D_\nu\mathbf{\Psi}\bar{\mathbf{\Psi}}\Gamma^\nu D_\mu\mathbf{\Psi} \\
\hat{\mathbf{c}}\ \bar{\Psi}^\dagger\Gamma^\mu[X^I, X^{J\dagger}, \Psi]\bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\mu\Psi &\to -\tfrac{\hat{\mathbf{c}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^\mu[\boldsymbol{X}^i, \mathbf{\Psi}]\bar{\mathbf{\Psi}}\Gamma^i D_\mu\mathbf{\Psi} \\
\hat{\mathbf{d}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}\tilde{D}_\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^J &\to \tfrac{\hat{\mathbf{d}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma^i D_\nu\mathbf{\Psi}D^\nu\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
&\quad +2\tfrac{\hat{\mathbf{d}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^\mu\Gamma^{ij}D_\nu\mathbf{\Psi}D_\mu\boldsymbol{X}^i D^\nu\boldsymbol{X}^j \\
&\quad +2\tfrac{\hat{\mathbf{d}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^{\mu\rho}\Gamma^i D^\sigma\mathbf{\Psi}D_\mu\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
&\quad -2\tfrac{\hat{\mathbf{d}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^i D^\sigma\mathbf{\Psi}D^\rho\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
&\quad +\tfrac{\hat{\mathbf{d}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma_\mu\Gamma^i \not{D}\mathbf{\Psi}D^\mu\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
\hat{\mathbf{e}}\ \bar{\Psi}^\dagger\Gamma_\mu\tilde{D}^\nu\Psi\tilde{D}^\mu X^{I\dagger}\tilde{D}_\nu X^I &\to -2\tfrac{\hat{\mathbf{e}}}{v^4}\bar{\mathbf{\Psi}}\Gamma_\mu D^\nu\mathbf{\Psi}\boldsymbol{F}^{\mu\rho}\boldsymbol{F}_{\rho\nu} \\
&\quad +2\tfrac{\hat{\mathbf{e}}}{v^4}\bar{\mathbf{\Psi}}\Gamma_\mu D^\nu\mathbf{\Psi}D^\mu\boldsymbol{X}^i D_\nu\boldsymbol{X}^i \\
&\quad -\tfrac{\hat{\mathbf{e}}}{v^4}\bar{\mathbf{\Psi}}\not{D}\mathbf{\Psi}\boldsymbol{F}^{\rho\sigma}\boldsymbol{F}_{\rho\sigma} \\
\hat{\mathbf{f}}\ \bar{\Psi}^\dagger\Gamma^{IJKL}\tilde{D}_\nu\Psi\ X^{IJK\dagger}\tilde{D}^\nu X^L &\to \tfrac{\hat{\mathbf{f}}}{v^4}\tfrac{3}{2}\bar{\mathbf{\Psi}}\Gamma^{ijl}D_\nu\mathbf{\Psi}\boldsymbol{X}^{ij}D^\nu\boldsymbol{X}^l \\
\hat{\mathbf{g}}\ \bar{\Psi}^\dagger\Gamma^{IJ}\tilde{D}_\nu\Psi\ X^{IJK\dagger}\tilde{D}^\nu X^K &\to \tfrac{\hat{\mathbf{g}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^i D_\nu\mathbf{\Psi}\boldsymbol{X}^{ij}D^\nu\boldsymbol{X}^j \\
&\quad +\tfrac{\hat{\mathbf{g}}}{v^4}\tfrac{1}{2}\bar{\mathbf{\Psi}}\Gamma^{ij}\Gamma^\rho D^\sigma\mathbf{\Psi}\boldsymbol{X}^{ij}\boldsymbol{F}_{\rho\sigma}
\end{aligned}
$$

$$
\begin{array}{rcl}
 & & -\frac{\hat{\mathbf{g}}}{4v^4}\bar{\mathbf{\Psi}}\Gamma^{ij}\Gamma^{\rho\sigma}\not{D}\mathbf{\Psi}\boldsymbol{X}^{ij}\boldsymbol{F}_{\rho\sigma} \\
\hat{\mathbf{h}}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^J,X^{K\dagger},\Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}_\mu X^K & \to & \frac{1}{2}\frac{\hat{\mathbf{h}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^i[\boldsymbol{X}^k,\mathbf{\Psi}]D_\mu\boldsymbol{X}^i D^\mu\boldsymbol{X}^k \\
 & & +\frac{\hat{\mathbf{h}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}\Gamma^{\mu\rho\sigma}[\boldsymbol{X}^i,\mathbf{\Psi}]D_\mu\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
 & & -\frac{\hat{\mathbf{h}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}\Gamma_{\mu\rho\sigma}\Gamma^i(\Gamma^j[\boldsymbol{X}^j,\mathbf{\Psi}])D^\mu\boldsymbol{X}^i\boldsymbol{F}^{\rho\sigma} \\
 & & -\frac{\hat{\mathbf{h}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}(\Gamma^j[\boldsymbol{X}^j,\mathbf{\Psi}])\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\rho\sigma} \\
\hat{\mathbf{i}}\ \bar{\Psi}^\dagger\Gamma^{\mu\nu}[X^I,X^{J\dagger},\Psi]\tilde{D}_\mu X^{I\dagger}\tilde{D}_\nu X^J & \to & \hat{\mathbf{i}}\bar{\mathbf{\Psi}}\Gamma_\rho[\boldsymbol{X}^i,\mathbf{\Psi}]D_\mu\boldsymbol{X}^i\boldsymbol{F}^{\rho\mu} \\
\hat{\mathbf{j}}\ \bar{\Psi}^\dagger\Gamma_{\mu\nu}\Gamma^{IJ}[X^J,X^{K\dagger},\Psi]\tilde{D}^\mu X^{I\dagger}\tilde{D}^\nu X^K & \to & +\frac{\hat{\mathbf{j}}}{v^4}\frac{1}{2}\bar{\mathbf{\Psi}}\Gamma^{\mu\nu}\Gamma^i[\boldsymbol{X}^k,\mathbf{\Psi}]D_\mu\boldsymbol{X}^i D_\nu\boldsymbol{X}^k \\
 & & +\frac{\hat{\mathbf{j}}}{v^4}\frac{1}{2}\bar{\mathbf{\Psi}}\Gamma^\sigma[\boldsymbol{X}^i,\mathbf{\Psi}]D^\rho\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
 & & -\frac{\hat{\mathbf{j}}}{v^4}\frac{1}{2}\bar{\mathbf{\Psi}}\Gamma^\rho\Gamma^i(\Gamma^j[\boldsymbol{X}^j,\mathbf{\Psi}])D^\sigma\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma} \\
\hat{\mathbf{k}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K,X^{L\dagger},\Psi]\tilde{D}^\mu X^{I\dagger}X^{JKL} & \to & \frac{\hat{\mathbf{k}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}\Gamma^\mu\Gamma^{ij}[\boldsymbol{X}^k,\mathbf{\Psi}]D_\mu\boldsymbol{X}^i\boldsymbol{X}^{jk} \\
 & & -\frac{1}{8}\frac{\hat{\mathbf{k}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma^j[\boldsymbol{X}^k,\mathbf{\Psi}]\boldsymbol{X}^{jk}\boldsymbol{F}_{\rho\sigma} \\
\hat{\mathbf{l}}\ \bar{\Psi}^\dagger\Gamma_\mu[X^I,X^{J\dagger},\Psi]\tilde{D}^\mu X^{K\dagger}X^{IJK} & \to & -\frac{\hat{\mathbf{l}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}\Gamma_\mu[\boldsymbol{X}^i,\mathbf{\Psi}]D^\mu\boldsymbol{X}^k\boldsymbol{X}^{ik} \\
\hat{\mathbf{m}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJKL}[X^L,X^{M\dagger},\Psi]X^{IJK\dagger}\tilde{D}^\mu X^M & \to & \frac{\hat{\mathbf{m}}}{v^4}\frac{3}{8}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma i}[\boldsymbol{X}^j,\mathbf{\Psi}]\boldsymbol{X}^{ij}\boldsymbol{F}_{\rho\sigma} \\
 & & -\frac{\hat{\mathbf{m}}}{v^4}\frac{3}{16}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma^{ij}(\Gamma^l[\boldsymbol{X}^l,\mathbf{\Psi}])\boldsymbol{X}^{ij}\boldsymbol{F}_{\rho\sigma} \\
\hat{\mathbf{n}}\ \bar{\Psi}^\dagger\Gamma_\mu\Gamma^{IJ}[X^K,X^{L\dagger},\Psi]X^{IJK\dagger}\tilde{D}^\mu X^L & \to & -\frac{\hat{\mathbf{n}}}{v^4}\frac{1}{8}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma^j[\boldsymbol{X}^k,\Psi]\boldsymbol{X}^{jk}\boldsymbol{F}_{\rho\sigma} \\
 & & -\frac{\hat{\mathbf{n}}}{v^4}\frac{1}{8}\bar{\mathbf{\Psi}}\Gamma^\mu\Gamma^{ij}[\boldsymbol{X}^l,\mathbf{\Psi}]\boldsymbol{X}^{ij}D_\mu\boldsymbol{X}^l \\
\hat{\mathbf{o}}\ \bar{\Psi}^\dagger\Gamma^{IJKL}[X^M,X^{N\dagger},\Psi]X^{IJL\dagger}X^{KMN} & \to & -\frac{\hat{\mathbf{o}}}{v^4}\frac{3}{16}\bar{\mathbf{\Psi}}\Gamma^{ijk}[\boldsymbol{X}^m,\mathbf{\Psi}]\boldsymbol{X}^{ij}\boldsymbol{X}^{km} \\
\hat{\mathbf{p}}\ \bar{\Psi}^\dagger\Gamma^{IJ}[X^K,X^{L\dagger},\Psi]X^{IJM\dagger}X^{KLM} & \to & -\frac{\hat{\mathbf{p}}}{v^4}\frac{1}{8}\bar{\mathbf{\Psi}}\Gamma^j[\boldsymbol{X}^k,\mathbf{\Psi}]\boldsymbol{X}^{jm}\boldsymbol{X}^{km}\ ,
\end{array}
\tag{B.1}
$$

where on each right hand side of the above we have included a factor of 2 contribution from also taking into account the Higgsing of the Hermitian conjugates. We have made heavy use of the Γ-matrix identities from Appendix A.

Note that terms containing parts of the on-shell terms, $\alpha'^2(\Gamma^\mu D_\mu\mathbf{\Psi}+\Gamma^i[X^i,\mathbf{\Psi}])$, will combine and cancel out:

$$-\frac{\hat{\mathbf{e}}}{v^4}\bar{\mathbf{\Psi}}\not{D}\mathbf{\Psi}\boldsymbol{F}^{\rho\sigma}\boldsymbol{F}_{\rho\sigma}-\frac{\hat{\mathbf{h}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}(\Gamma^j[\boldsymbol{X}^j,\mathbf{\Psi}])\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\rho\sigma}=0 \tag{B.2}$$

$$-\frac{\hat{\mathbf{g}}}{4v^4}\bar{\mathbf{\Psi}}\Gamma^{ij}\Gamma^{\rho\sigma}\not{D}\mathbf{\Psi}\boldsymbol{X}^{ij}\boldsymbol{F}_{\rho\sigma}-\frac{\hat{\mathbf{m}}}{v^4}\frac{3}{16}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma^{jk}(\Gamma^l[\boldsymbol{X}^l,\mathbf{\Psi}])\boldsymbol{X}^{jk}\boldsymbol{F}_{\rho\sigma}=0 \tag{B.3}$$

$$\begin{aligned}
&\frac{\hat{\mathbf{d}}}{v^4}\bar{\mathbf{\Psi}}\Gamma^{\rho\sigma}\Gamma_\mu\Gamma^i\not{D}\mathbf{\Psi}D^\mu\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma}-\frac{\hat{\mathbf{h}}}{v^4}\frac{1}{4}\bar{\mathbf{\Psi}}\Gamma_{\mu\rho\sigma}\Gamma^i(\Gamma^j[\boldsymbol{X}^j,\mathbf{\Psi}])D^\mu\boldsymbol{X}^i\boldsymbol{F}^{\rho\sigma}\\
&\qquad-\frac{\hat{\mathbf{j}}}{v^4}\frac{1}{2}\bar{\mathbf{\Psi}}\Gamma^\rho\Gamma^i(\Gamma^j[\boldsymbol{X}^j,\mathbf{\Psi}])D^\sigma\boldsymbol{X}^i\boldsymbol{F}_{\rho\sigma}=0
\end{aligned} \tag{B.4}$$

for the values of the coefficients given in Eq. (3.15), $\hat{\mathbf{d}}=\frac{i}{4}$, $\hat{\mathbf{e}}=-\frac{i}{4}$, $\hat{\mathbf{g}}=-\frac{i}{2}$, $\hat{\mathbf{h}}=-i$, $\hat{\mathbf{j}}=i$ and $\hat{\mathbf{m}}=-\frac{2i}{3}$.

C. Uniqueness of the four-derivative fermion ansatz

When dealing with the fermionic part of the action one might worry about the uniqueness claim of our proposal, since it looks as if there are many additional terms that could lead to the operators present in the α'^2-corrected D2-brane action upon Higgsing. In order to address that, we give below the most general set of expressions obtained by 'uplifting' the terms containing fermions in the D2 action at order α'^2. The 'uplifting' procedure involves writing down the most general 3-algebra expression that could reduce to a particular D2 term by Higgsing. The list excludes 'on-shell' terms, that is $\Gamma^\mu D_\mu \Psi$ and $\Gamma^{IJ}[X^I, X^J, \Psi]$, which we will set to zero by using the lowest order 3-algebra equations of motion. These terms would also have led to on-shell-type terms in the D2 theory, which we know are absent, so we can safely set their coefficients to zero.

In the following, the terms that appear in the main part of this paper have been identified. The ones that did not have been enumerated and we will show why they do not contribute to Eq. (3.12). Ignoring signs and numerical factors we have:

$$
\begin{aligned}
\bar{\boldsymbol{\Psi}}\Gamma^\mu D^\nu \boldsymbol{\Psi}\bar{\boldsymbol{\Psi}}\Gamma_\nu D_\mu \boldsymbol{\Psi} \rightarrow\ & \bar{\Psi}\Gamma^\mu D^\nu \Psi\bar{\Psi}\Gamma_\nu D_\mu\Psi \sim \text{term } \hat{\mathbf{b}} \\
\bar{\boldsymbol{\Psi}}\Gamma^i D^\nu \boldsymbol{\Psi}\bar{\boldsymbol{\Psi}}\Gamma_\nu [\boldsymbol{X}^i, \boldsymbol{\Psi}] \rightarrow\ & \bar{\Psi}\Gamma^{IJ} D^\nu \Psi\bar{\Psi}\Gamma_\nu [X^I, X^J, \Psi] \sim \text{term } \hat{\mathbf{c}} \\
\bar{\boldsymbol{\Psi}}\Gamma^i [\boldsymbol{X}^j, \boldsymbol{\Psi}]\bar{\boldsymbol{\Psi}}\Gamma^j [\boldsymbol{X}^i, \boldsymbol{\Psi}] \rightarrow\ & \bar{\Psi}\Gamma^{IM}[X^J, X^N, \Psi]\bar{\Psi}\Gamma^{JN}[X^I, X^M, \Psi] \sim \text{term } \hat{\mathbf{a}} \\
& \bar{\Psi}\Gamma^{IN}[X^J, X^N, \Psi]\bar{\Psi}\Gamma^{JM}[X^I, X^M, \Psi] \qquad \text{(C.1)} \\
& \bar{\Psi}\Gamma^{IM}[X^J, X^N, \Psi]\bar{\Psi}\Gamma^{JM}[X^I, X^N, \Psi] \qquad \text{(C.2)} \\
\bar{\boldsymbol{\Psi}}\Gamma_\mu D^\nu \boldsymbol{\Psi} D^\mu \boldsymbol{X}^l D_\nu \boldsymbol{X}^l \rightarrow\ & \bar{\Psi}\Gamma_\mu D^\nu \Psi D^\mu \boldsymbol{X}^L D_\nu \boldsymbol{X}^L \sim \text{term } \hat{\mathbf{e}} \\
\bar{\boldsymbol{\Psi}}\Gamma^i D^\nu \boldsymbol{\Psi} D^\rho \boldsymbol{X}^i \boldsymbol{F}_{\rho\nu} \rightarrow\ & \bar{\Psi}\Gamma^{IJ\rho\nu\lambda} D^\nu \Psi D^\rho X^I D_\lambda X^J \sim \text{term } \hat{\mathbf{d}} \\
\bar{\boldsymbol{\Psi}}\Gamma^i D^\nu \boldsymbol{\Psi} \boldsymbol{X}^{il} D_\nu \boldsymbol{X}^l \rightarrow\ & \bar{\Psi}\Gamma^{IM} D^\nu \Psi X^{ILM} D_\nu X^L \sim \text{term } \hat{\mathbf{g}} \\
\bar{\boldsymbol{\Psi}}\Gamma_\mu [\boldsymbol{X}^j, \boldsymbol{\Psi}] \boldsymbol{F}^{\mu\rho} D_\rho \boldsymbol{X}^j \rightarrow\ & \bar{\Psi}\Gamma^{\rho\lambda}[X^J, X^I, \Psi] D_\lambda X^I D_\rho X^J \sim \text{term } \hat{\mathbf{i}} \\
& \bar{\Psi}\Gamma^{\rho\lambda KL}[X^J, X^K, \Psi] D_\lambda X^L D_\rho X^J \sim \text{term } \hat{\mathbf{j}} \\
\bar{\boldsymbol{\Psi}}\Gamma^i [\boldsymbol{X}^j, \boldsymbol{\Psi}] D^\rho \boldsymbol{X}^i D_\rho \boldsymbol{X}^j \rightarrow\ & \bar{\Psi}\Gamma^{IM}[X^J, X^M, \Psi] D^\rho X^I D_\rho X^J \sim \text{term } \hat{\mathbf{h}} \\
\bar{\boldsymbol{\Psi}}\Gamma_\mu [\boldsymbol{X}^j, \boldsymbol{\Psi}] D^\mu \boldsymbol{X}^l \boldsymbol{X}^{lj} \rightarrow\ & \bar{\Psi}\Gamma_\mu [X^J, X^M, \Psi] D^\mu X^L X^{LJM} \sim \text{term } \hat{\mathbf{l}} \\
& \bar{\Psi}\Gamma_\mu \Gamma^{MN}[X^J, X^M, \Psi] D^\mu X^L X^{LJN} \qquad \text{(C.3)} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu\rho}[\boldsymbol{X}^k, \boldsymbol{\Psi}] \boldsymbol{F}^{\mu\nu} D^\rho \boldsymbol{X}^k \rightarrow\ & \bar{\Psi}\Gamma^{MN}[X^K, X^M, \Psi] D_\rho X^N D^\rho X^K \sim \text{term } \hat{\mathbf{h}} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu l} D_\sigma \boldsymbol{\Psi} \boldsymbol{F}^{\mu\nu} D^\sigma X^l \rightarrow\ & \bar{\Psi}\Gamma^{\mu LM} D_\sigma \Psi D_\mu X^M D^\sigma X^L \sim \text{term } \hat{\mathbf{d}} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\rho\mu j} D_\sigma \boldsymbol{\Psi} D^\mu \boldsymbol{X}^j \boldsymbol{F}^{\rho\sigma} \rightarrow\ & \bar{\Psi}\Gamma^{\mu IJ} D^\nu \Psi D_\nu X^I D_\mu X^J \sim \text{term } \hat{\mathbf{d}} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\mu j\rho}[\boldsymbol{X}^k, \boldsymbol{\Psi}] D^\mu \boldsymbol{X}^j D^\rho \boldsymbol{X}^k \rightarrow\ & \bar{\Psi}\Gamma_{\mu\rho JM}[X^K, X^M, \Psi] D^\mu X^J D^\rho X^K \sim \text{term } \hat{\mathbf{j}} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\mu jl} D_\sigma \boldsymbol{\Psi} D^\mu \boldsymbol{X}^j D^\sigma \boldsymbol{X}^l \rightarrow\ & \bar{\Psi}\Gamma_{\mu JL} D_\sigma \Psi D^\mu X^J D^\sigma X^L \sim \text{term } \hat{\mathbf{d}} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\mu jl}[\boldsymbol{X}^k, \boldsymbol{\Psi}] D^\mu \boldsymbol{X}^j \boldsymbol{X}^{lk} \rightarrow\ & \bar{\Psi}\Gamma_{\mu JL}[X^K, X^M, \Psi] D^\mu X^J X^{LKM} \sim \text{term } \hat{\mathbf{k}} \\
& \bar{\Psi}\Gamma_\mu \Gamma^{JL}\Gamma^{MN}[X^K, X^M, \Psi] D^\mu X^J X^{LKN} \qquad \text{(C.4)} \\
\bar{\boldsymbol{\Psi}}\Gamma_{ij\rho} D_\sigma \boldsymbol{\Psi} \boldsymbol{X}^{ij} \boldsymbol{F}^{\rho\sigma} \rightarrow\ & \bar{\Psi}\Gamma^{\rho\sigma}\Gamma^M \Gamma^{IJK} D_\sigma \Psi X^{IJK} D_\rho X^M \qquad \text{(C.5)} \\
& \bar{\Psi}\Gamma^{\rho\sigma}\Gamma^{IJ} D_\sigma \Psi X^{IJM} D_\rho X^M \qquad \text{(C.6)} \\
\bar{\boldsymbol{\Psi}}\Gamma^{ij}\Gamma_\rho [\boldsymbol{X}^k, \boldsymbol{\Psi}] \boldsymbol{X}^{ij} D^\rho \boldsymbol{X}^k \rightarrow\ & \bar{\Psi}\Gamma^{IJ}\Gamma_\rho [X^K, X^M, \Psi] X^{IJM} D^\rho X^K \sim \text{term } \hat{\mathbf{h}} \\
& \bar{\Psi}\Gamma^{IJN}\Gamma_\rho \Gamma^M [X^K, X^M, \Psi] X^{IJN} D^\rho X^K \sim \text{term } \hat{\mathbf{m}} \\
\bar{\boldsymbol{\Psi}}\Gamma_{ijl} D_\sigma \boldsymbol{\Psi} \boldsymbol{X}^{ij} D^\sigma \boldsymbol{X}^l \rightarrow\ & \bar{\Psi}\Gamma^{IJK}\Gamma^M D_\sigma \Psi X^{IJK} D^\sigma X^M \sim \text{term } \hat{\mathbf{f}}
\end{aligned}
$$

$$
\begin{aligned}
\bar{\boldsymbol{\Psi}}\Gamma^i[\boldsymbol{X}^j,\boldsymbol{\Psi}]\boldsymbol{X}^{il}\boldsymbol{X}^{lj} \rightarrow\ & \bar{\Psi}\Gamma^{IM}[X^J,X^K,\Psi]X^{ILM}X^{LJK} \sim \text{term } \hat{\mathbf{p}} \\
& \bar{\Psi}\Gamma^{IM}[X^J,X^M,\Psi]X^{ILK}X^{LJK} && \text{(C.7)} \\
& \bar{\Psi}\Gamma^{IM}[X^J,X^N,\Psi]X^{ILN}X^{LJM} && \text{(C.8)} \\
& \bar{\Psi}\Gamma^{IK}\Gamma^{MN}[X^J,X^N,\Psi]X^{ILK}X^{LJM} && \text{(C.9)} \\
\bar{\boldsymbol{\Psi}}\Gamma_{\mu\nu l}[\boldsymbol{X}^k,\boldsymbol{\Psi}]\boldsymbol{F}^{\mu\nu}\boldsymbol{X}^{lk} \rightarrow\ & \bar{\Psi}\Gamma^{\mu}\Gamma^{LN}[X^K,X^M,\Psi]D_\mu X^N X^{LKM} \sim \text{term } \hat{\mathbf{k}} \\
& \bar{\Psi}\Gamma^{\mu}\Gamma^{LN}[X^K,X^M,\Psi]D_\mu X^M X^{LKN} \sim \text{term } \hat{\mathbf{n}} \\
& \bar{\Psi}\Gamma^{\mu}\Gamma^{LM}[X^K,X^M,\Psi]D_\mu X^N X^{LKN} && \text{(C.10)} \\
& \bar{\Psi}\Gamma^{\mu}\Gamma^{LN}\Gamma^{M}\Gamma^{P}[X^K,X^M,\Psi]D_\mu X^P X^{LKN} && \text{(C.11)} \\
\bar{\boldsymbol{\Psi}}\Gamma_{ijl}[\boldsymbol{X}^k,\boldsymbol{\Psi}]\boldsymbol{X}^{ij}\boldsymbol{X}^{lk} \rightarrow\ & \bar{\Psi}\Gamma^{IJLN}[X^K,X^M,\Psi]X^{IJN}X^{LKM} \sim \text{term } \hat{\mathbf{o}} \\
& \bar{\Psi}\Gamma^{IJL}\Gamma^{N}[X^K,X^N,\Psi]X^{IJM}X^{LKM} && \text{(C.12)} \\
& \bar{\Psi}\Gamma^{IJL}\Gamma^{N}[X^K,X^M,\Psi]X^{IJM}X^{LKN} && \text{(C.13)} \\
& \bar{\Psi}\Gamma^{IJL}\Gamma^{N}\Gamma^{P}\Gamma^{M}[X^K,X^M,\Psi]X^{IJN}X^{LKP} && \text{(C.14)}
\end{aligned}
$$

The enumerated terms do not contribute as they are either related to terms already present in the ansatz (up to 'on-shell' terms) or Higgs to terms not present in the D2 theory and should therefore have a zero coefficient. We have used the following ϵ-tensor identity in showing the equivalence of several terms by re-shuffling SO(8) indices amongst products of 3-brackets:

$$\epsilon^{a[bcd}\epsilon^{e]fgh} = 0\ , \tag{C.15}$$

where the above indices are gauge indices and one should also remember that there is a STr in front of each expression. This leads to the fermionic analogues of Eq. (3.2), the origin of which also lies in the above identity and the implementation of the STr prescription. In more detail we have:

- (C.1) gives an on-shell term upon setting $I = J = 8$
- (C.2) gives a term that doesn't exist in D2 for $N = 8 \neq M$
- (C.3) is equivalent to $\hat{\mathbf{n}}$
- (C.4) Higgses to a term not present in D2 for $K = 8$
- (C.5) Higgses to a term not present in D2 for $M \neq 8$
- (C.6) by expanding $\Gamma^{IJ} = \Gamma^I\Gamma^J - \delta^{IJ}$ reduces to $\hat{\mathbf{g}}$ and an on-shell term
- (C.7) is equivalent to $\hat{\mathbf{p}}$ up to an on-shell term
- (C.8) is equivalent to $\hat{\mathbf{p}}$ up to an on-shell term

- (C.9) is equivalent to $\hat{\mathbf{o}}$ up to an on-shell term
- (C.10) is the same as (C.5)
- (C.11) Higgses to a term not present in D2 for $K = 8$
- (C.12) is equivalent to $\hat{\mathbf{o}}$ up to an on-shell term
- (C.13) is equivalent to $\hat{\mathbf{o}}$ up to an on-shell term
- (C.14) Higgses to a term not present in D2 for $K = 8$

Therefore, the only independent terms are the ones with coefficients $\hat{\mathbf{a}}, ..., \hat{\mathbf{p}}$ that we have already included in Eq. (3.12).

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