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D2 to D2

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ABSTRACT: Starting from maximally supersymmetric (2+1)d Yang-Mills theory and using a duality transformation due to de Wit, Nicolai and Samtleben, we obtain the ghost-free Lorentzian 3-algebra theory that has recently been proposed to describe M2-branes. Our derivation does not invoke any properties of 3-algebras. Being derivable from SYM, the final theory is manifestly equivalent to it on-shell and should not be thought of as the IR limit that describes M2-branes, though it does have enhanced R-symmetry as well as superconformal symmetry off-shell.

KEYWORDS: String theory, M-theory, Branes.

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1. Introduction

There has been intense recent activity regarding a certain class of $\mathcal{N} = 8$ superconformal theories in three dimensions, following the work of Bagger-Lambert (BL) [1, 2, 3] and also Gustavsson [4], as these theories are potential candidates for the worldvolume description of multiple M2-branes in M-theory. These constructions rely on the introduction of an algebraic structure going under the name of a Lie 3-algebra, which is necessary for the closure of the supersymmetry algebra. The metric versions of the above theories[1] fall into two classes, depending on whether the invariant bilinear form in 3-algebra space is positive definite or indefinite: the Euclidean theories originally proposed by Bagger-Lambert and their more recent Lorentzian counterparts [6, 7, 8].

The Lorentzian 3-algebra theories have been claimed to be capturing the low-energy worldvolume dynamics of multiple parallel M2-branes but are plagued by apparent unitarity problems due the presence of ghost-like degrees of freedom in the classical action. In order to address this issue, a proposal has appeared which enlarges the theory by gauging a shift symmetry for one of two ghosts, via the introduction of appropriate gauge fields. This construction leads to a manifestly ghost-free spectrum [9, 10]. However, this results in the other ghost field being frozen to a constant vev. Then, as already observed in [8] along the lines of [11], the theory reduces precisely to maximally supersymmetric Yang-Mills in three dimensions with a gauge coupling equal to the scalar vev.[2] This and other properties

[1]See also [5] for the treatment of a non-metric proposal.

[2]A similar procedure has been carried out in the context of Janus field theory in Ref. [12].

have been used to argue [10] that the ghost-free Lorentzian 3-algebra theory is indeed the IR limit of SYM.

However,[3] such a precise reduction should make one suspicious that the ghost-free Lorentzian 3-algebra is the *same* theory as SYM rather than its infrared limit. In this letter we would like to reinforce this interpretation by reversing the procedure of [8]. We will show that starting from $\mathcal{N} = 8$ SYM one can systematically - and uniquely - recover the theory of [9, 10].

Let us summarise how we will achieve the transformation of SYM theory into the ghost-free Lorentzian 3-algebra theory. First we will use a prescription for dualising non-abelian gauge fields in the special case of three dimensions, due to de Wit, Nicolai and Samtleben (dNS) [13, 14, 15]. In this prescription the gauge field A_μ gets replaced by two non-dynamical gauge fields A_μ, B_μ with a $B \wedge F(A)$ type kinetic term, plus an extra scalar which ends up carrying the dynamical degree of freedom of the original YM gauge field. Once this is done, we observe a potential SO(8) symmetry in the theory under which the extra scalar mixes with the seven existing ones. We realise this SO(8) symmetry as a formal symmetry (acting also on coupling constants) by replacing g_{YM} with an SO(8) vector of coupling constants g_{YM}^I. Finally, the latter is promoted to a scalar field that is an SO(8) vector, whose equations of motion render it constant. We justify all these steps and note that they do not change the on-shell theory in any way. However, off-shell they give a theory with enhanced symmetries: SO(8) R-symmetry instead of SO(7) and superconformal symmetry instead of ordinary supersymmetry. We also comment on the construction of SO(8)-covariant, gauge-invariant operators and find that, unsurprisingly, these are only present off-shell and reduce to an SO(7)-covariant basis on any physical solution.

Even though the above procedure closely follows the treatment of [10], albeit in the opposite order, we believe that this angle will help to demystify the connection between the two theories and clarify that the resultant Lagrangean is nothing but a re-writing of maximally supersymmetric Yang-Mills theory. Interestingly, this re-writing allows one to recover the conformal and SO(8) symmetries off shell, which are however spontaneously broken by any physical vev of the theory. The authors of [10] propose a prescription for recovering the SO(8) R-symmetry by integrating over all values of the constrained ghost field. In our interpretation this amounts to integrating SYM theory over all values of the coupling constant. This seems unnatural at best and should also lead to violations of basic QFT axioms, such as locality, and result into problems with cluster decomposition. In that sense, starting from the theory of D2-branes, one ends up with the theory of D2-branes.

[3] This point was stressed in Ref. [9].

The M2's only emerge in the limit of taking the SYM coupling to infinity, as is of course well-known.

Our discussion has no bearing on the original "un-gauged" Lorentzian BL proposal of [6, 7, 8], which could still be a non-trivial example of an SO(8)-invariant theory. However, since this theory still needs to be demonstrated to be free of ghosts, we believe that promising candidates for the worldvolume theory of multiple M2-branes in noncompact space must lie elsewhere.

The rest of this note is organised as follows. In the next section we present our main argument in full detail. We proceed with a discussion on the SO(8)-covariant gauge-invariant operators in section 3. In section 4 we propose a potential generalisation of the dNS duality to four dimensions and speculate that it might be useful in studying 4d dualities. We close in section 5 with a discussion of our results.

2. From SYM to Lorentzian 3-algebras

We start with the maximally supersymmetric interacting super Yang-Mills Lagrangean in 2+1 dimensions based on an arbitrary Lie algebra $\mathcal{G}$:

$$\begin{aligned}\mathcal{L} =&\mathrm{Tr}\left(-\frac{1}{4g_{YM}^2}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}D_\mu X^i D^\mu X^i-\frac{g_{YM}^2}{4}[X^i,X^j][X^j,X^i]\right.\\ &\left.+\frac{i}{2}\bar\Psi\not{D}\Psi+\frac{i}{2}g_{YM}\bar\Psi\Gamma_i[X^i,\Psi]\right)\ ,\end{aligned} \tag{2.1}$$

Here A_μ is a gauge connection on $\mathcal{G}$. The field strength and the covariant derivatives are defined as:

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-[A_\mu,A_\nu] \qquad \text{and} \qquad D_\mu=\partial_\mu-[A_\mu,\cdot\,]\ . \tag{2.2}$$

The X^is are seven matrix valued scalar fields transforming as vectors under the SO(7) R-symmetry group. The Ψs are two-component spinors in (2+1)d and also 8-component spinors of SO(7).

When $\mathcal{G}$ is $\mathrm{U}(N)$ this theory is the low energy worldvolume action for multiple parallel D2-branes in flat space. For the other classical Lie algebras, it describes D-branes at orientifolds. Our goal in this note is to show that for any gauge group, this Lagrangean can be brought to the form of the Lorentzian Bagger-Lambert or 3-algebra field theory proposed in [6, 7, 8], or more precisely to the "gauged" version of the above theory described in [9, 10].

We proceed by introducing two new fields B_μ and ϕ that are adjoints of $\mathcal{G}$. In terms of these new fields the dNS duality transformation [13, 14, 15] is the replacement:

$$\mathrm{Tr}\left(-\frac{1}{4g_{YM}^2}F^{\mu\nu}F_{\mu\nu}\right)\to\mathrm{Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda}-\frac{1}{2}\left(D_\mu\phi-g_{YM}B_\mu\right)^2\right)\ . \tag{2.3}$$

We see that in addition to the gauge symmetry $\mathcal{G}$, the new action has a *noncompact abelian* gauge symmetry that we can call $\tilde{\mathcal{G}}$, which has the same dimension as the original gauge group $\mathcal{G}$.[4] This symmetry consists of the transformations:

$$\delta\phi = g_{YM} M \ , \qquad \delta B_\mu = D_\mu M \ , \tag{2.4}$$

where $M(x)$ is an arbitrary matrix, valued in the adjoint of $\mathcal{G}$. Clearly B_μ is the gauge field for the shift symmetries $\tilde{\mathcal{G}}$. Note that both in Eq. (2.3) and Eq. (2.4), the covariant derivative D_μ is the one defined in Eq. (2.2).

If one chooses the gauge $D^\mu B_\mu = 0$ to fix the shift symmetry, the degree of freedom of the original Yang-Mills gauge field A_μ can be considered to reside in the scalar ϕ. In this sense one can think of ϕ as morally the dual of the original A_μ [13, 14, 15]. Alternatively we can choose the gauge $\phi = 0$, in which case the same degree of freedom resides in B_μ. The equivalence of the RHS to the LHS of Eq. (2.3) can be conveniently seen by going to the latter gauge. Once $\phi = 0$ then B_μ is just an auxiliary field and one can integrate it out to find the usual YM kinetic term for $F_{\mu\nu}$.[5]

We can now proceed to study the dNS-duality transformed $\mathcal{N} = 8$ Yang-Mills theory. Its Lagrangean is:

$$\begin{aligned}\mathcal{L} =& \mathrm{Tr}\bigg(\frac{1}{2}\epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2}\left(D_\mu\phi - g_{YM}B_\mu\right)^2 - \frac{1}{2} D_\mu X^i D^\mu X^i \\ & - \frac{g_{YM}^2}{4}[X^i, X^j][X^j, X^i] + \frac{i}{2}\bar{\Psi}\, \not{D}\Psi + \frac{i}{2} g_{YM} \bar{\Psi}\Gamma_i [X^i, \Psi]\bigg) \ .\end{aligned} \tag{2.5}$$

The gauge-invariant kinetic terms for the eight scalar fields have a potential SO(8) invariance, which can be exhibited as follows. First rename $\phi(x) \to X^8(x)$. Then the scalar kinetic terms become $-\frac{1}{2}\hat{D}_\mu X^I \hat{D}^\mu X^I$, where:

$$\begin{aligned}\hat{D}_\mu X^i &= D_\mu X^i = \partial_\mu X^i - [A_\mu, X^i], \quad i = 1, 2, \ldots, 7 \\ \hat{D}_\mu X^8 &= D_\mu X^8 - g_{YM} B_\mu = \partial_\mu X^8 - [A_\mu, X^8] - g_{YM} B_\mu \ .\end{aligned} \tag{2.6}$$

Defining the constant 8-vector:

$$g_{YM}^I = (0, \ldots, 0, g_{YM}) \ , \quad I = 1, 2, \ldots, 8 \ , \tag{2.7}$$

[4]For the U(1) case the scalar ϕ is a periodic field. For non-simply connected gauge groups, *i.e.* for U(N), the U(1) part will still have the aforementioned periodic shift symmetry. Our discussion here applies to the SU(N) part of the theory after decoupling the U(1) supermultiplet, so compactness is not an issue.

[5]If the scalar field ϕ were not introduced, the duality would go through but there would be no gauge symmetry associated to the vector field B_μ, and it would simply end up as an auxiliary field in an uninteresting field theory. Here on the other hand, we instead get an interesting dual field theory containing all of A, B, ϕ.

the covariant derivatives can together be written:

$$\hat{D}_\mu X^I = D_\mu X^I - g^I_{YM} B_\mu \ . \tag{2.8}$$

One can now uniquely write the SYM action in a form that is SO(8)-invariant under transformations that rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$\begin{aligned}\mathcal{L} =& \mathrm{Tr}\Big(\frac{1}{2}\epsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda} - \frac{1}{2}\left(D_\mu X^I - g^I_{YM}B_\mu\right)^2 + \frac{i}{2}\bar{\Psi}\,\not{D}\Psi + \frac{i}{2}g^I_{YM}\bar{\Psi}\Gamma_{IJ}[X^J,\Psi] \\ & - \frac{1}{12}\left(g^I_{YM}[X^J,X^K] + g^J_{YM}[X^K,X^I] + g^K_{YM}[X^I,X^J]\right)^2\Big) \ .\end{aligned} \tag{2.9}$$

Here, in writing down the Yukawa-type interaction we have embedded the SO(7) Γ-matrices into SO(8) using $\Gamma^i = \tilde{\Gamma}^8\tilde{\Gamma}^i$. On the RHS the Γ-matrices are those of SO(8). One can easily see that with this definition the LHS matrices satisfy the Clifford algebra of SO(7).

The $\mathcal{N}=8$ supersymmetry transformations for the theory above can also be written in a formally SO(8)-invariant form:

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\,\Gamma^I\Psi \ , & \delta\Psi &= \left(D_\mu X^I - g^I_{YM}B_\mu\right)\Gamma^\mu\Gamma_I\epsilon - \frac{1}{2}g^I_{YM}[X^J,X^K]\,\Gamma_{IJK}\epsilon \\ \delta A_\mu &= \frac{i}{2}g^I_{YM}\bar{\epsilon}\,\Gamma_\mu\Gamma_I\Psi \ , & \delta B_\mu &= i\bar{\epsilon}\,\Gamma_\mu\Gamma_I[X^I,\Psi] \ . \end{aligned} \tag{2.10}$$

The theory in Eq. (2.9) is merely a re-writing of the dNS-transformed $\mathcal{N}=8$ SYM action. It has the nice property that g^I_{YM} can be an arbitrary 8-vector, not necessarily of the form in Eq. (2.7), with the theory depending only on its norm. To see this, simply pick an arbitrary vector g^I_{YM} with $\sqrt{g^I_{YM}g^I_{YM}} = g_{YM}$ and use the SO(8) invariance to rotate to a basis where it takes the form Eq. (2.7). It is in this basis that the field ϕ appearing in the dNS duality transformation can be identified with X^8.

At this stage the theory is only formally SO(8) invariant, as the transformations must act on the coupling constants as well as the fields. So SO(8) is not a true field-theoretic symmetry.[6] By the same token, the theory is formally conformal invariant if, in addition to scaling the fields, one scales the dimensional coupling constant vector g^I_{YM} (this fact is inherited from the original SYM in (2+1)d which already had this property). Again the conformal invariance is not a true field-theoretic symmetry – this is particularly obvious, as $\mathcal{N}=8$ SYM at finite coupling can hardly be conformal!

However at this stage we are in a position to simultaneously convert the formal SO(8) and conformal symmetries to true field-theoretic symmetries by replacing the coupling constant vector by a set of eight new scalar fields $X^I_+(x)$. The resulting theory will be on-shell equivalent to the theory in Eq. (2.9) if and only if the new scalar field $X^I_+(x)$ has

[6]An equivalent way to say this is that any particular choice of the coupling constant vector breaks SO(8) down to the SO(7) orthogonal to it.

an equation of motion that renders it constant. Then we can simply write an arbitrary classical solution as $\langle X^I_+ \rangle = g^I_{YM}$ and the theory will reduce to that in Eq. (2.9), which we have already shown to be correct.

To enforce the constancy of X^I_+ one introduces an auxiliary Lagrange multiplier. This can be implemented by adding the following term to the action:

$$\mathcal{L}_{\rm mult} = C^\mu_I \partial_\mu X^I_+ \ . \tag{2.11}$$

As pointed out in footnote 5, it is typically not useful to introduce a vector field without an associated gauge symmetry. To see what goes wrong for the case at hand, note that the above term has a new symmetry [10]:

$$\delta C^\mu_I = \epsilon^{\mu\nu\lambda}\partial_\nu \alpha_{\lambda\ I} \tag{2.12}$$

that needs to be gauge fixed. On doing this, one finds (for more details, see Ref. [10]) a standard gauge kinetic term:

$$\mathcal{L}_{\rm GF} = (\epsilon_{\mu\nu\lambda}\partial^\nu C^{\lambda I})^2 = (\partial^\nu C^{\lambda I} - \partial^\lambda C^{\nu I})^2 = (F_{\nu\lambda}(C^I))^2 \ , \tag{2.13}$$

where $F_{\nu\lambda}(C^I)$ denotes the field strength of C^I_μ. Having obtained a kinetic term, the C^I_μ's will introduce new negative norm states through their C^I_0 component.

Therefore we impose a shift symmetry for C^I_μ by introducing a new scalar field, which we call X^I_-, and writing the relevant term in the action as:

$$\mathcal{L}_{\rm mult} = (C^{\mu\ I} - \partial^\mu X^I_-)\partial_\mu X^I_+ \ . \tag{2.14}$$

This action now has an 8-dimensional abelian (shift) symmetry:

$$\delta X^I_- = \lambda^I, \quad \delta C^I_\mu = \partial_\mu \lambda^I \ , \tag{2.15}$$

which will remove the negative-norms states associated to C^I_μ.

The rest of the argument follows Ref. [10]. One needs to gauge-fix the shift symmetry and this is done by introducing the appropriate ghosts. The fermionic terms are easily obtained in a similar fashion. We introduce a superpartner Ψ_+ for X^I_+ and a Lagrange multiplier η^μ enforcing the condition $\Psi_+ = 0$. Following the above chain of arguments, one also needs to introduce a superpartner for X^I_-, called Ψ_-. The structure of these terms in $\mathcal{L}_{\rm fermion}$ will be uniquely dictated by supersymmetry.[7]

We have thus ended up with the action:

$$\begin{aligned}\mathcal{L} =&{\rm Tr}\Big(\frac{1}{2}\epsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda} - \frac{1}{2}\left(D_\mu X^I - X^I_+ B_\mu\right)^2 \\ &- \frac{1}{12}\left(X^I_+[X^J,X^K] + X^J_+[X^K,X^I] + X^K_+[X^I,X^J]\right)^2\Big) \\ &+ (C^{\mu\ I} - \partial^\mu X^I_-)\partial_\mu X^I_+ + \mathcal{L}_{\rm gauge-fixing} + \mathcal{L}_{\rm ghosts} + \mathcal{L}_{\rm fermions} \ .\end{aligned} \tag{2.16}$$

[7] The corresponding supersymmetry transformations can be found in Ref. [10].

This is precisely the Bagger-Lambert action for an arbitrary Lorentzian 3-algebra (based on a Lie algebra $\mathcal{G}$). Because our manipulations required the field X_+^I to be constant on-shell, we were led not to the original form discovered in Refs. [6, 7, 8] (which indeed has not yet been shown to be ghost-free) but directly to the modified one subsequently proposed in Refs. [9, 10] containing the gauge field C_μ^I, for which freedom from ghosts is easily demonstrated.

Gauging X_-^I to zero using the shift symmetry and eliminating C_μ^I, we obtain the desired constraint $\partial_\mu X_+^I = 0$, whose general solution is $X_+^I = g_{YM}^I$ and the action reduces to that in Eq. (2.9).[8] Hence we have arrived at the above action by a series of completely justified transformations starting from $\mathcal{N} = 8$ SYM.

It is striking that all the interactions and consequent properties of this action, which were originally derived using 3-algebras with a Lorentzian metric, have been arrived at here without any reference to 3-algebras whatsoever. As an example, the $dim\mathcal{G}$ abelian shift symmetries that arise from the 3-algebra structure, as discussed in some detail in Ref. [7], are simply a basic property of the dNS transformation in our approach. Likewise the sextic interaction that arises via 3-algebra structure constants in the original papers, appears in our work when we promote g_{YM} to an SO(8) vector of coupling constants and subsequently replace that by an SO(8) vector of scalar fields.

3. Gauge-invariant operators and SO(8) symmetry

In Ref. [10] it was noted that for this theory operators of the form $\mathrm{Tr}(X^{I_1} \cdots X^{I_n})$ are not gauge invariant under the abelian noncompact symmetry associated with B_μ. To redress this problem, the authors introduce a *nonlocal* adjoint scalar field which we call $\tilde{\phi}$, defined as:

$$\tilde{\phi}(x) = \frac{1}{D^2} D^\mu B_\mu \ . \tag{3.1}$$

They identify this with the magnetic dual to the non-abelian gauge field A_μ. They then define fields:

$$Y^I = X^I - \tilde{\phi} X_+^I \tag{3.2}$$

that are invariant under the shift gauge transformations and lead to operators $\mathrm{Tr}(Y^{I_1} \cdots Y^{I_n})$. Under the noncompact gauge transformations Eq. (2.4), $\tilde{\phi} \to \tilde{\phi} + M$. Therefore, one can choose a gauge in which $\tilde{\phi} = 0$ and the Y^Is reduce to X^Is, so one does seem to recover SO(8) covariance in this fashion but at the cost of losing manifest gauge invariance.

[8]The classical solution breaks SO(8) to SO(7) and superconformal symmetry to ordinary supersymmetry. This breaking is therefore *spontaneous*, which is an essential feature of this approach.

The above discussion can be re-interpreted in terms of the dNS duality.[9] Recall our definition of the scalar field ϕ in Eq. (2.3). Since this transforms under the shift symmetry as $\phi \to \phi + g_{YM}M$, it follows that:

$$Z(x) \equiv \phi(x) - g_{YM}\tilde{\phi}(x) = \left(\phi - g_{YM}\frac{1}{D^2}D^\mu B_\mu\right) \tag{3.3}$$

is gauge invariant. It is this field, rather than $\tilde{\phi}$, that unambiguously carries the single physical adjoint degree of freedom of the original Yang-Mills gauge field after dNS duality. Z is in general nonlocal, and becomes a local field only when we choose the gauge $\tilde{\phi} = 0$. When we apply our covariantisation procedure (promoting g_{YM} to an 8-vector and thence to the field X_+^8) we find that $Z(x)$ combines with the remaining seven adjoint scalar fields to form the 8-vector $X^I - \tilde{\phi}X_+^I$, which is precisely the set Y^I defined above.

We see, as in our previous discussions, that these operators are SO(8) covariant only off-shell (when X_+^I is still a field) but as soon as X_+^I develops a vev, the SO(8) is broken to SO(7).

4. Four-dimensional duality?

The dNS duality transformation is, as we have seen, particularly useful in (2+1)d where it allowed us to relate $\mathcal{N} = 8$ SYM to the Lorentzian 3-algebra theory. However a variant of it can be written down in $3+1$ dimensions, and we briefly describe it here in the hope that it might enhance our understanding of four-dimensional dualities. The transformation in 4d maps a Yang-Mills theory with gauge field A_μ to a theory having three fields: a gauge field A_μ, a 2-form gauge field $B_{\mu\nu}$ and a second gauge field A'_μ, the latter two being in the adjoint of the original gauge group $\mathcal{G}$.

The map is as follows:

$$\mathrm{Tr}\left(-\frac{1}{4g_{YM}^2}F^{\mu\nu}F_{\mu\nu}\right) \to \mathrm{Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}B_{\mu\nu}F_{\lambda\rho} - \frac{1}{2}\left(D_\mu A'_\nu - D_\nu A'_\mu - g_{YM}B_{\mu\nu}\right)^2\right) \tag{4.1}$$

where:

$$D_\mu A'_\nu = \partial_\mu A'_\nu - [A_\mu, A'_\nu]\ . \tag{4.2}$$

In addition to the gauge symmetry $\mathcal{G}$, this theory has an abelian "2-form" gauge symmetry $\tilde{\mathcal{G}}$ that acts as:

$$\delta A'_\mu = g_{YM}M_\mu, \quad \delta B_{\mu\nu} = D_\mu M_\nu - D_\nu M_\mu\ . \tag{4.3}$$

The duality is demonstrated by using the abelian shift symmetry to gauge A'_μ to 0 and then integrating out $B_{\mu\nu}$. It would also be natural to extend this duality to make A'_μ into

[9]We would like to acknowledge correspondence with the authors of Ref. [10], based on which this section has been revised.

a gauge field with an associated abelian symmetry. This can be done in an obvious way by introducing a scalar field ϕ' and replacing $A'_\mu \to A'_\mu - D_\mu \phi'$ everywhere in the above. The question then is whether the formulation on the RHS of Eq. (4.1), or the generalisation of it with this extra scalar field, can teach us something about supersymmetric field theories in 4d.

From the above discussion we see that the dNS transformation can have a (3+1)d analogue. However, the same does not appear to be true in any useful way for the rest of our procedure, namely lifting of the coupling constant to a field and the consequent enhancement of off-shell symmetries. It is a special feature of (2+1)d that the Yang-Mills coupling g_{YM} has the same canonical dimension as a scalar field, namely $\frac{1}{2}$ in appropriate units. In (3+1)d the former has canonical dimension 0 and the latter has dimension 1, while in (1+1)d the reverse is true. Therefore, lifting the coupling constant to a field that is rendered constant by its equation of motion seems to be natural only in (2+1)d. However it may still be worth exploring whether the 4d duality transform exhibited here has some useful application.

5. Discussion

The reduction of a proposed M2-brane field theory to Yang-Mills was proposed in Ref. [11] in the context of the Bagger-Lambert $\mathcal{A}_4$ theory. There, giving a vev to a scalar field reduces the field theory to a maximally supersymmetric YM theory plus corrections suppressed by inverse powers of g_{YM}. The corrections are actually crucial, for at any finite value of g_{YM} they imply that the theory is *not only* SYM. At infinite coupling the theory *is* only SYM, but in the IR limit, which is precisely what one wants.

With Lorentzian 3-algebras the result is different. Giving a vev to the scalar X^I_+ reduces the theory (in its ghost-free form) to *only* SYM without corrections, as first noticed in Ref. [8]. In hindsight, this is a negative indication for the usefulness of the theory. In this note we have shown that one can go directly from SYM to the Lorentzian 3-algebra theory, clearly demonstrating that the two are equivalent theories.

Last Monday, a new candidate theory for N M2-branes was announced [16]. This theory, based on $\mathrm{U}(N)\times\mathrm{U}(N)$ Chern-Simons theory with bi-fundamental matter, appears to have the property that, as with the Bagger-Lambert $\mathcal{A}_4$ theory, the Higgs mechanism of Ref. [11] gives a non-trivial reduction to SYM with corrections that are suppressed by inverse powers of g_{YM}. This is a positive feature. Indeed we suspect it could be used as a proof that the theory is a correct description of multiple M2's.

As a final point, let us observe that the Lorentzian 3-algebra theory, while not (yet) incorporating the flow to the infrared fixed point of SYM, may yet do so if an imaginative treatment of it is found. Quite simply one needs to expand the theory about the vev $\langle X_+^I \rangle = \infty$. It is not ruled out that an astute field redefinition or other modification might make this possible.

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References

[1] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev.* **D75** (2007) 045020, [hep-th/0611108].

[2] J. Bagger and N. Lambert, *Gauge Symmetry and Supersymmetry of Multiple M2-Branes*, *Phys. Rev.* **D77** (2008) 065008, [arXiv:0711.0955].

[3] J. Bagger and N. Lambert, *Comments On Multiple M2-branes*, *JHEP* **02** (2008) 105, [arXiv:0712.3738].

[4] A. Gustavsson, *Algebraic structures on parallel M2-branes*, arXiv:0709.1260.

[5] U. Gran, B. E. W. Nilsson, and C. Petersson, *On relating multiple M2 and D2-branes*, arXiv:0804.1784.

[6] J. Gomis, G. Milanesi, and J. G. Russo, *Bagger-Lambert Theory for General Lie Algebras*, arXiv:0805.1012.

[7] S. Benvenuti, D. Rodriguez-Gomez, E. Tonni, and H. Verlinde, *N=8 superconformal gauge theories and M2 branes*, arXiv:0805.1087.

[8] P.-M. Ho, Y. Imamura, and Y. Matsuo, *M2 to D2 revisited*, arXiv:0805.1202.

[9] M. A. Bandres, A. E. Lipstein, and J. H. Schwarz, *Ghost-Free Superconformal Action for Multiple M2-Branes*, arXiv:0806.0054.

[10] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk, and H. Verlinde, *The Superconformal Gauge Theory on M2-Branes*, arXiv:0806.0738.

[11] S. Mukhi and C. Papageorgakis, *M2 to D2*, *JHEP* **05** (2008) 085, [arXiv:0803.3218].

[12] Y. Honma, S. Iso, Y. Sumitomo, and S. Zhang, *Janus field theories from multiple M2 branes*, arXiv:0805.1895.

[13] H. Nicolai and H. Samtleben, *Chern-Simons vs. Yang-Mills gaugings in three dimensions*, *Nucl. Phys.* **B668** (2003) 167–178, [hep-th/0303213].

[14] B. de Wit, I. Herger, and H. Samtleben, *Gauged locally supersymmetric $D = 3$ nonlinear sigma models*, *Nucl. Phys.* **B671** (2003) 175–216, [hep-th/0307006].

[15] B. de Wit, H. Nicolai, and H. Samtleben, *Gauged supergravities in three dimensions: A panoramic overview*, hep-th/0403014.

[16] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, arXiv:0806.1218.