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RELATIVE SOURCE LOCATION USING CODA WAVE INTERFEROMETRY: METHOD,
CODE PACKAGE, AND APPLICATION TO MINING INDUCED EARTHQUAKES

Running title: Source location with CWI: method & code

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ABSTRACT

A wide range of applications requires relative locations of sources of energy to be known accurately. Most conventional location methods are either subject to errors that depend strongly on inaccuracy in the model of propagation velocity used, or demand a well-distributed network of surrounding seismic stations in order to produce reliable results. A new source location method based on coda wave interferometry (CWI) is relatively insensitive to the number of seismic stations and to the source-to-station azimuthal coverage. It therefore opens new avenues for research, for applications in areas with unfavourable recording geometries, and for applications which require a complementary method. This method uses CWI to estimate distances between pairs of seismic events with similar source mechanism recorded at the same station. These separation estimates are used to solve for the locations of clusters of events relative to one another within a probabilistic framework through optimization. It is even possible to find relative locations of clusters of events with one single-channel station. Given these advantages, it is likely that one reason that the method is not used more widely is the lack of reliable code that implements this multistage method. We therefore present a well-commented MATLAB code that does so, and present examples of its applications. It can be used with seismic data from a single station channel, and enables data recorded by different channels and stations to be used simultaneously. It is therefore possible to combine data from permanent yet sparse networks, and from temporary arrays closer to the source region. We use the code to apply the location method to a selected data set of the New Ollerton earthquakes, England, to demonstrate the validity of the code. The worked example is provided within the package. A way to assess the quality of the location results is also provided.

INTRODUCTION

Finding accurate locations of seismic energy sources is essential for a wide range of seismological, industrial and other applications. Examples include the study of earthquake interaction and recurrence (Marzocchi and Lombardi, 2008; Chen et al., 2013), discriminating earthquake fault and auxiliary planes using aftershocks or foreshocks (Got et al., 2004), modeling earthquake hazards (Frankel et al., 2000), monitoring seismic activity during and after hydraulic fracturing (Kumar et al., 2017) or underground mining (Ge, 2005), and attempts to forecast earthquakes (Gerstenberger et al., 2004). In other areas of application, finding locations of different types of sources can be important, for example in ocean acoustic (Dosso and Wilmut, 2009; 2011; Verlinden et al., 2015), in disaster rescue (Mae et al., 2017; Kawaguchi and Fukuda, 2017), and in military applications (Sheng and Hu, 2005). The quality of absolute locations found in each case depends heavily on the velocity model used, the number of stations available, and source-to-receiver distances and azimuths. In seismological applications, earthquake location uncertainties are therefore usually of the order of kilometers or hundreds of meters, and from here on we focus only on seismological applications.

To achieve higher accuracy, relative source location methods are often used. These typically ignore absolute locations and instead estimate the locations of multiple event positions relative to each other directly from the differences in their recorded arrival-times of their radiated energy at receivers which are obtained either from catalog (preprepared) data or from temporal crosscorrelation of their various recorded waveforms (Deichmann and Garcia-Fernandez, 1992; Waldhauser and Ellsworth, 2000; Sgattoni et al., 2016). Events located in this way are often clustered within a region smaller than a quarter of the wavelength of radiating energy at the

dominant frequency; the range of source-to-receiver distances, and velocity variations outside the source region therefore affect waveforms from all sources in a similar manner so that errors in their relative location associated with velocity variations are largely avoided.

Conventional relative location methods are often able to reduce seismic source relative location errors to less than ~100m provided that a well-designed seismic network with a large number of stations is available (Ito, 1985, 1990; Deichmann and Garcia-Fernandez, 1992; Waldhauser et al., 1999). The master event location method (Fremont and Malone, 1987; Deichmann and Garcia-Fernandez, 1992) takes one event in an event cluster as the master event, crosscorrelates its seismogram waveforms with those of the other events, and relocates each of them relative to their master event through their relative traveltimes differences. Thus the spatial extension of the cluster is limited to about a quarter of a dominant wavelength as the waveforms of all other events need to be closely comparable to those of the master event to avoid cycle skipping in their crosscorrelation. Grigoli et al. (2016) shows that the spatial extension limit can be overcome to some extent by using a multi-master event strategy if a large number of seismic stations are available for waveform stacking. The popular double-difference (DD) location method (Waldhauser and Ellsworth, 2000; Bai et al., 2006) overcomes this limitation by linking different events and clusters using differential traveltimes obtained from catalog data, thus extending the feasible relative distance range. The DD method determines event locations by minimizing the double-difference (residual between observed and theoretical differential traveltimes of a pair of events recorded at a common station) of pair-wise events by adjusting the vector difference between their hypocenters. The system can easily become ill-conditioned, hence it is solved as a damped least squares problem. The solution is subject to the choice of the damping factor, which depends on the condition number of the system (Waldhauser, 2001; Zhao et al., 2017). The DD

location results are also influenced by the number of seismic stations available: results deteriorate when the number falls below seven, and the method fails to produce results when the number is smaller than four (Robinson et al., 2013).

A novel location method (Robinson et al., 2013) based on coda wave interferometry (CWI) is a feasible alternative to these conventional location methods when there are few seismic stations, or where the source-station azimuth range is unfavorable, as well as in cases where a different method is needed for a quality check of other methods. The CWI technique (Snieder, 2006) makes use of the multiply scattered waves recorded in waveform codas which have travelled through a much larger volume of the medium, and hence contain more azimuthal information than the first or early arrivals used by conventional methods. As a result, the CWI-based location method is even able to locate a cluster of events with one single-channel station (Robinson et al., 2013; Zhao et al., 2017).

Coda refers to the multiply scattered waves comprising the later parts of recorded seismograms, and is extremely sensitive to differences between pairs of seismic sources (Snieder and Vrijlandt 2005; Robinson et al., 2007) or changes in the medium of propagation (Roberts et al., 1992; Snieder et al., 2002; Ratdomopurbo and Poupinet, 1995; Gret et al., 2005; 2006). CWI measures the differences in the coda of waveforms recorded at the same station before and after some change occurs, to estimate the differences between the two seismic states. For example, it has been used in a laboratory to measure changes due to the nonlinear dependence of the seismic velocity structure of granite due to temperature variations that are too small for other methods to detect (Snieder et al., 2002); to monitor velocity changes associating volcano activities (Gret et al., 2005; Wegler et al., 2006; Brenguier et al., 2008a; Mordret et al., 2010; Baptie, 2010); to study changes in fault zones (Wang et al., 2008; Brenguier et al., 2008b; de Angelis, 2009); as well as in

geoengineering to monitor stress changes in concrete structures, such as bridges (Stahler et al., 2011) and buildings (Larose et al., 2006).

By comparing pairs of similar earthquakes, the source location algorithm that we present here uses CWI to estimate the inter-source separations of a cluster of events with similar source mechanisms, and then uses the separation data as input to a location algorithm. Different types of changes in seismic states (velocity change, scatterer displacement, source displacement, and source mechanism change) leave different footprints on seismic coda when compared in a statistical manner (Snieder 2006). Although theoretically the three types of changes could therefore be discriminated, such tests and discussion are beyond the scope of this particular work: here we assume differences in coda are due only to differences in the source locations of different seismic events. In such case, clusters of events can even be located relative to one another in 3D using a single seismic receiver (Robinson et al., 2013; Zhao et al., 2017).

Despite the advantages of this method, it is curious to observe that uptake in its use has been limited to the above two papers. In part we suspect that this is because the relative location algorithm requires unfamiliar methods to be used, and is therefore also partly due to the lack of readily available, easily editable code with which practitioners can gain familiarity and confidence. We provide such a code.

In this article, we present the CWI-based source location algorithm developed by Robinson et al. (2013) with the improvements proposed by Zhao et al. (2017). We then introduce a way to assess the location results obtained from the nonlinear optimization solution. We also describe the accompanying computer code, written in MATLAB, that combines these theories and methods and estimates relative source locations using a three step routine: 1) classifying events into clusters based on waveform similarity; 2) estimating inter-source separation distances with CWI; and 3)

estimating the relative source locations from the separation data. In what follows we first introduce the theory of CWI and the location algorithm in Sections 2 and 3, and then give a brief description of the core functions and scripts of the code package in Section 4. The method used to classify events (Ottemoller et al., 2017) is also described in Section 4. Applications of the code to synthetic examples and to mining induced events from New Ollerton, England are illustrated in Section 5 and Section 6, respectively, and these example data sets are included within the code package.

ESTIMATING INTER-SOURCE SEPARATIONS WITH CWI

CWI estimates the inter-source separation between a pair of events by comparing the coda of the two seismograms recorded by each seismic station channel. The theory is based on path summation of scattered waves (Snieder, 1999) – the assumption that the total wavefield at a given location can be written as the superposition of waves traveling along all possible trajectories through the medium

$$u^1(t) = \sum_T A_T(t), \quad (1)$$

where $u^1(t)$ is the total wavefield from event 1, T represents a wave trajectory, and A_T is the contribution to the total wavefield of waves that travel along trajectory T . The trajectory of each scattered wave consists of the path from the source to the first scatterer encountered, and the path followed thereafter. For a second event that is close to event 1 and has very similar source mechanism, CWI assumes that the paths to the first scatterer on each trajectory change, but that the subsequent trajectory does not because it depends on the medium rather than on the sources. Since the subsequent scattering trajectories create a complex mixture of any differences in travel times to the first scatters, for small changes in source location the dominant differences in recorded

waveforms at the same seismic station occur in coda wave arrival times (Snieder, 2006). The wavefield of event 2 can be written as

$$u^2(t) = \sum_T A_T(t - \tau_T), \quad (2)$$

where τ_T is the travel-time difference of waves traveling along trajectory T to the first scatterer due to the difference in source position. If we assume proximity between the two source locations and similarity in source mechanisms, and the two waveforms will be similar. Any differences can be quantified by the normalized crosscorrelation of the two waveforms in a time-window defined by a central time t and a half-width t_ω , computed for a sequence of time-windows in the coda

$$R(t_s) = \frac{\int_{t-t_\omega}^{t+t_\omega} u^{(1)}(t')u^{(2)}(t' + t_s)dt'}{\sqrt{\int_{t-t_\omega}^{t+t_\omega} u^{(1)2}(t')dt' \int_{t-t_\omega}^{t+t_\omega} u^{(2)2}(t')dt'}}. \quad (3)$$

The distribution of any traveltimes differences τ_T in each time-window contains information about the source separation δ . Snieder et al. (2006) estimate the standard deviation of the traveltimes difference, φ_τ , from the maximum of the correlation coefficient R_{max} , and show that φ_τ is related to the source separation δ by

$$\varphi_\tau^2 = \frac{1}{2} \frac{\delta^2}{v^2} \quad \text{for isotropic sources in 2D acoustic media} \quad (4a)$$

$$\varphi_\tau^2 = \frac{1}{3} \frac{\delta^2}{v^2} \quad \text{for isotropic sources in 3D acoustic media} \quad (4b)$$

$$\varphi_\tau^2 = \frac{6/\alpha^8 + 7/\beta^8}{7(2/\alpha^6 + 3/\beta^6)} \delta^2 \quad \text{for double couple sources on the same fault plane} \quad (4c)$$

where α and β are the P- and S-wave velocity in the vicinity of the two sources (Snieder and Vrijlandt, 2005). The waves arriving in different time-windows have traveled along different paths, so separation results derived from each time-window of the seismograms are therefore independent

and their distribution can be used to estimate uncertainty. It has also been shown that estimates of inter-source separations from different station channels are highly consistent (Snieder and Vrijlandt, 2005; Robinson et al., 2011; Zhao et al., 2017). Note we have assumed that the difference in two waveforms is only due to the second event having a different location with the first, however in reality the change in source location may be accompanied by a velocity change or scatterer displacement in the medium of propagation. In cases where other types of changes also occur, the estimation of source separations would be overestimated, especially for event pairs with small source separations, because all perturbations contribute negatively to the correlation coefficient (equation 3) hence positively to source separation estimates (equation 4).

As we move through a seismogram towards later times, seismic coda becomes suitable for CWI where the waves are sufficiently scattered so that a time-window contains waves leaving the source from many different directions; the suitable section ends where the noise level exceeds that of the signal. There is therefore a limited length of coda that can be used. In turn, this constrains the choices of number and length of time-windows used in equation 3, and clearly there is a trade-off between the two. From a theoretical point of view, if we insert wave representations 1 and 2 into equation 3, we can see that computation of $R(t_s)$ gives rise to double sums. CWI assumes the cross terms between the two summations are negligible compared to the diagonal terms, but their ratio is inversely proportional to the length of the time-windows (Snieder 2006). Hence, for this assumption to be reasonable, the time-windows need to have sufficient length. However, time-windows cannot be unrestrictedly long as otherwise cycle skipping may occur in the crosscorrelation in equation 3. Also, in order to obtain usable standard deviations on separation estimates, at least four time-windows are needed for each pair of waveforms. Given all of these

constraints, it is not a trivial task to select the number, length, and start time of windows used to implement CWI.

To avoid the vagaries of trial and error, the code package allows the separation-uncertainty matrix of Zhao et al. (2017) to be used to find the most suitable combination of the number, length and start time of windows systematically. For data from each station channel, a three-dimensional separation-uncertainty matrix is computed that includes a regularly sampled subset of all possible combinations of the three parameters: number, length and start time of windows. Matrix elements are computed for one combination of the three parameters by first calculating $\Omega_{i,j} = \sum_N \sigma_{i,j}/N$,

where $\sigma_{i,j} = \sqrt{\sum_{k=1}^l (\delta_{i,j,k} - \bar{\delta}_{i,j})^2 / l}$ is the standard deviation of separation estimates from l coda time windows for events i and j , $\delta_{i,j,k}$ is the separation estimate from the k th window and $\bar{\delta}_{i,j}$ is the mean separation over l windows, and N is the number of event pairs on that station channel. The value of $\Omega_{i,j}$ reflects the uncertainty of separation estimates derived from one recording channel for each source pair. Averaging over all source pairs gives a final uncertainty estimate for that combination of parameters. Thus, systematically searching for the combination of number, length, and start time of windows that gives the lowest estimate of separation uncertainties from CWI becomes an automated process.

It is essential to note that the CWI technique has a tendency to underestimate larger source separations due to cycle skipping in the correlation of coda in equation 3. This trend can be quantified by two empirical relations between the true separation $\tilde{\delta}_t$ and the mean $\mu = \mu(\tilde{\delta}_t)$ and standard deviation $\sigma = \sigma(\tilde{\delta}_t)$ of CWI separation estimates (Figure 1 a, b), where the tilde above the separation indicates that the quantity is normalized by the dominant wavelength λ_d in the recorded data: $\tilde{\delta}_t = \delta_t / \lambda_d$. The applicable range of CWI is visualized in Figure 1: CWI fails to

identify any increase in length when the true separation is larger than $0.55\lambda_d$. The empirical functions that capture this behaviour are derived from a multitude of synthetic experiments with a large range true separations in different Gaussian random media, by fitting the rational functional forms

$$\mu(\tilde{\delta}_t) = a_1 \frac{a_2 \tilde{\delta}_t^{a_4} + a_3 \tilde{\delta}_t^{a_5}}{a_2 \tilde{\delta}_t^{a_4} + a_3 \tilde{\delta}_t^{a_5} + 1}, \quad (5a)$$

$$\sigma(\tilde{\delta}_t) = b_1 \frac{b_2 \tilde{\delta}_t^{b_4} + b_3 \tilde{\delta}_t^{b_5}}{b_2 \tilde{\delta}_t^{b_4} + b_3 \tilde{\delta}_t^{b_5} + 1} + c, \quad (5b)$$

where the values of the constants are listed in Table 1 (Robinson et al., 2011, 2013). The location algorithm introduced in the next section takes account of these known biases of CWI-estimated source separations, and is able to correct for them to a significant extent in relative location results.

SOURCE LOCATION

The source location algorithm estimates relative locations from the separation estimates and their uncertainties using a probabilistic framework. Robinson et al. (2011) describe the probabilistic nature of CWI estimates using the conditional probability density function (pdf) $P(\tilde{\delta}_t | \tilde{\delta}_{CWI})$, which is the probability of the true separation being $\tilde{\delta}_t$ given that the estimate from CWI is $\tilde{\delta}_{CWI}$. According to Bayes' theorem, this so-called posterior probability of $\tilde{\delta}_t$ is proportional to the likelihood of observing $\tilde{\delta}_{CWI}$ in the case that the true separation is $\tilde{\delta}_t$, multiplied by the prior probability of $\tilde{\delta}_t$ being true

$$P(\tilde{\delta}_t | \tilde{\delta}_{CWI}) \propto P(\tilde{\delta}_{CWI} | \tilde{\delta}_t) \times P(\tilde{\delta}_t). \quad (6)$$

The prior pdf $P(\tilde{\delta}_t)$ is used to describe information about source separation or event location known prior to and independently from the CWI location process, which here is considered to be a uniform distribution with wide bounds. The likelihood function $P(\tilde{\delta}_{CWI}|\tilde{\delta}_t)$ is approximated by a positively bounded Gaussian pdf, whose mean and standard deviation are respectively represented by the empirical functional relation $\mu(\tilde{\delta}_t)$ and $\sigma(\tilde{\delta}_t)$, given true separation $\tilde{\delta}_t$.

For a cluster of events, equation 6 holds for each event pair. Robinson et al. (2013) incorporate the separations between multiple event pairs by multiplying the formulae for all available event pairs to establish their joint posterior pdf, assuming that they are independent of each other

$$P(\mathbf{e}_1, \dots, \mathbf{e}_n | \tilde{\delta}_{CWI}) = c \prod_{i=1}^n P(\mathbf{e}_i) \times \prod_{i=1}^{n-1} \prod_{j=i+1}^n P(\tilde{\delta}_{CWI,ij} | \mathbf{e}_i, \mathbf{e}_j), \quad (7)$$

where c is a constant, n is the number of events, and $\mathbf{e}_i = (x_i, y_i, z_i)$ is the location of event i . Within the last term we use the Euclidean distance $\delta_{CWI,ij} = \|\mathbf{e}_i - \mathbf{e}_j\|_2$ for source separation between the i 'th and j 'th earthquakes. Throughout the evaluation of the joint pdf, the separation quantities are used in normalized form; that is, they are divided by the dominant wavelength λ_d . However, the dominant frequency, and hence the dominant wavelengths of the set of events, could extend over a large range, and is also subject to limitations of recording instruments. Using the average dominant wavelength over different station channels therefore introduces inaccuracy to the location process. To this end, our code package instead uses the joint pdf introduced by Zhao et al. (2017), which applies an individual likelihood for each channel when data from multiple channels are used, so that the separations computed during the evaluation of the joint pdf are normalized by the actual dominant wavelength from that channel

$$P(\mathbf{e}_1, \dots, \mathbf{e}_n | \tilde{\delta}_{CWI}) = c \prod_{i=1}^n P(\mathbf{e}_i) \times \prod_{k=1}^m \prod_{i=1}^{n-1} \prod_{j=i+1}^n P_k(\tilde{\delta}_{CWI,ij}^k | \mathbf{e}_i, \mathbf{e}_j), \quad (8)$$

where m is the number of channels used, k is the index over m channels, and $P_k(\tilde{\delta}_{CWI,ij}^k | \mathbf{e}_i, \mathbf{e}_j)$ is the probability of observing $\tilde{\delta}_{CWI,ij}^k$ given source locations \mathbf{e}_i and \mathbf{e}_j , where $\tilde{\delta}_{CWI,ij}^k$ is normalized by the dominant wavelength of the k th channel. The maximum of the joint posterior pdf (equation 8) occurs at the most probable combination of event locations. Hence, the event locations can be estimated by solving an optimization problem. Taking the negative logarithm of equation 8, the multiplications are converted to summations that are more numerically stable

$$-\ln [P(\mathbf{e}_1, \dots, \mathbf{e}_n | \tilde{\delta}_{CWI})] = -\ln[c] - \sum_{i=1}^n \ln[P(\mathbf{e}_i)] - \sum_{k=1}^m \sum_{i=1}^{n-1} \sum_{j=i+1}^n \ln [P_k(\tilde{\delta}_{CWI,ij} | \mathbf{e}_i, \mathbf{e}_j)]. \quad (9)$$

Maximization of equation 8 is equivalent to minimizing equation 9, where $\ln[c]$ and $\ln [P(\mathbf{e}_i)]$ can be ignored as they are constant (for Uniform priors). Thus, the event locations $\mathbf{e}_1, \dots, \mathbf{e}_n$ can be found by minimizing the objective function

$$L(\mathbf{e}_1, \dots, \mathbf{e}_n) = - \sum_{k=1}^m \sum_{i=1}^{n-1} \sum_{j=i+1}^n \ln [P_k(\tilde{\delta}_{CWI,ij} | \mathbf{e}_i, \mathbf{e}_j)]. \quad (10)$$

In the code package, objective function L is minimized using a conjugate gradient method, the Polak-Ribiere technique (Navon and Legler, 1987; Press et al. 1987).

CODE DESCRIPTION

The accompanying relative source location code is written in MATLAB with well-commented functions and scripts. These use seismic data recorded with single or multiple station channels to find relative source locations. The package consists of three parts, each of which contains codes conducting one step of the location method: clustering events, estimating inter-

source separations, and estimating source locations. The entire process can be implemented by executing the script `main_running_script.m` section by section, with interactive operations involved occasionally. This section provides descriptions of the core functions and scripts used in each step. The sub- and auxiliary functions and scripts are explained in the user guide that is included within the package.

Clustering

The theory of CWI requires that events be constrained to have identical source mechanisms, so events first need to be classified into sets of similar mechanism. The similarity in pairs of sources is assessed by the similarity of their waveforms recorded by the same seismic station channel, which is measured by their crosscorrelation. The package classifies events in two steps, computing crosscorrelations then identifying clusters, with scripts `sort_cr.m` and `clustering.m`, respectively.

`sort_cr.m` reads in seismic data in SAC form (Helffrich et al., 2013). It first selects events and station channels for location according to criteria set by the user: `MIN_channel` is the minimum number of recording channels for an event to be considered, and `MIN_event_per_channel` is the minimum number of events recorded by a channel for that channel to be used. It then calculates the normalized crosscorrelation $R(t_s)$ of all available pairs of the selected events recorded by each selected station channel, and finds the maximum of each crosscorrelation R_{max} . For each event pair, the values of R_{max} are averaged over all selected station channels that have recorded both events, then these average maximum crosscorrelation $cravg_{max}$ are sorted in descending order.

`clustering.m` follows the method of Ottemoller et al. (2017) to identify clusters. It starts

with the event pair with the highest $cravg_{max}$ value, making them the first two events of the first cluster. It then searches through the sorted list of $cravg_{max}$, adding events that are linked to the current cluster. A link to a cluster is defined as one of the events in the pair being correlated ($cravg_{max}$ higher than `MIN_corr`, the threshold of $cravg_{max}$ for an event pair to be included) with any event that is already in the cluster. The search restarts from the first unclassified pair (the unclassified pair with the highest $cravg_{max}$) every time a new event is added to the cluster to avoid overlooking any linked events. The search loops until there is no event that can be added to the current cluster, after which it starts a new cluster from the two events of the first unclassified pair in the sorted list. The search ends when all events are classified. Clusters with fewer than `MIN_E_per_CLUSTER` events will not be identified as being part of a cluster.

Estimating inter-source separations

For each identified cluster, seismic data are processed in three sub-steps to estimate inter-source distances: picking waveform first arrivals; determining the combination of number, length, and start time of windows for implementing CWI; and estimating source separations with CWI. Users are free to conduct first arrival picking with their preferred method, however this package provides a user-friendly way for the task to be carried out in a graphical, interactive manner. The core functions and scripts are:

- `first_arrival_pick.m`: a script that allows users to pick the first arrivals of a series of waveforms interactively.
- `separation_parameters.m`: a script that finds the combination of number, length, and start time of windows to implement CWI with `CWI_sep.m` (see below) that gives the lowest separation uncertainties.

- `separations.m`: a function that estimates the inter-source separations of all event pairs in a given cluster recorded by a given station channel.
- `CWI_sep.m`: one of the core functions in the package called by multiple functions and scripts. It applies coda wave interferometry (Snieder, 2006) to estimate the separation between one pair of sources with similar mechanisms for isotropic sources in a 2D or 3D acoustic medium, or double-couple sources in an elastic medium (Snieder and Vrijlandt, 2005). The two improvements introduced by Robinson et al. (2011) are applied: 1) removing the Taylor series approximation of the waveform autocorrelation, and 2) applying a restricted range when searching for the maximum correlation value R_{\max} to avoid cycle skipping.

The result of applying these functions is a data set of inter-source separations estimated by CWI, which are ready to be used to estimate relative source locations.

Estimating source locations

The relative locations of a cluster of events are solved by minimizing the objective function L (equation 10). To start the minimization, a set of initial event locations is needed, which can either be generated with function `initialize_locations.m` in the package, or be provided by user. The main location function `Source_Location.m` first evaluates L at the given initial event locations and computes its gradient, whose negative is the steepest descent direction and is used as the initial search direction. The function searches for the minimum along the search direction using a two-tier line search algorithm by calling `line_search.m`. First this routine conducts a brute force search by evaluating the objective function at regularly spaced points within a bracketed range to find bounds on a finer range search, and then conducts a second similar brute force search to find the approximate minimum within those bounds and finally sharpens the result

by fitting a parabola using the approximate minimum and an adjacent point on each side. The event locations are then updated to the minimum found, and the value of L is reduced. The function then calculates the next search direction, a direction orthogonal to the gradient at the current position and conjugate to the last search direction, finds the minimum along the new search direction, and updates the event locations. This process iterates until one of three conditions is met: 1) the value for any non-zero step length is larger than that obtained in the previous iteration; 2) the reduction in the value of L in an iteration is smaller than a threshold; or 3) the maximum allowed number of iterations is reached. Starting with a different set of initial locations is recommended if the iterations are terminated due to the third criterion, as some initializations may lead to convergence more rapidly than others.

For this part of the method, the core functions and scripts are:

- `Source_Location.m`: the main location function that estimates the relative location of a cluster of events using inter-source separation data (their mean and standard deviations) estimated with CWI.
- `ln_joint_likelihood.m`: a function that evaluates function L (equation 10) – the negative logarithm of the joint likelihood function for a cluster of events.
- `gradient.m`: a function that computes the gradient of function L numerically at a given set of event locations.
- `line_search.m`: a function that searches for the minimum of a one-dimensional function along a given direction.
- `initialize_locations.m`: a function that generates a set of initial locations for the minimization. It first randomly generates a set of locations, and then adjusts the order of these event locations to conform as well as possible to the input CWI separation estimates (so as to

ensure the smallest sum of square residuals between the source separations given by the initial event locations and the mean of CWI separation estimates). This re-ordering procedure moves the initialization of optimization closer to the minimum, thus improving the speed of convergence.

- `rotate_cluster.m`: a function that rotates a cluster of points about its center in two orthogonal directions in the 3D space by given angles.

SYNTHETIC EXAMPLES

We use synthetic experiments to demonstrate the validity of the method and code; the data for these experiments is included in the code package so that users can verify their version of codes after editing to fit their specific needs. Applications of inter-source separation estimation and source location are shown separately in this section, to identify the abilities and limitations of each of the two successive steps. Whereas the theory of CWI assumes point scatterers (Snieder, 2006) with a constant background velocity, we use the Marmousi2 model (Martin et al., 2002; Irons, 2005) to test the method in a more realistic representation of Earth's velocity structure. The Marmousi 2 model is an elastic extension of the classic Marmousi model (Versteeg, 1994), which is based on a profile through the North Quenguela trough in the Cuanza basin and contains a large number of horizontally layered horizons and a series of normal faults. The new model also has more structural complexities and structural features compared to the original, hence is used in our example. We then demonstrate the performance of the optimization algorithm used to solve for

event locations using CWI separation estimates. In the subsequent section we apply all three steps in the method (clustering, CWI and source location) to real earthquake data.

Estimating inter-source separations

We modeled the waveforms from two identical isotropic sources in the Marmousi2 model (Figure 2) which has an average velocity of 1250m/s. The recording network consists of a surface array with 15 receivers and two borehole arrays with 12 receivers each. The two sources are 64.7m apart with a dominant frequency of 5 Hz, and the dominant wavelength λ_d of the waveforms is 250m. We apply function `CWI_sep.m` to obtain estimates of the separation between the two sources. The estimates from different time-windows fluctuate around their mean at narrow distances as shown in Figure 2b for receiver R1, which gives a mean of 59.15m and a standard deviation of 6.40m. This result agrees with the empirical relations (Figure 1), that for a true separation $\delta_t = 64.7m$ (i.e., $\tilde{\delta}_t = 0.259\lambda_d$) the separation mean $\mu(\delta_t)$ and standard deviation $\sigma(\delta_t)$ are estimated to be $0.237\lambda_d$ and $0.026\lambda_d$. The separation estimates are consistent among different receivers in both the surface array and borehole arrays, with uncertainties similar to that given by the empirical relations as shown in Figure 2c. For most receivers, the true separation is contained within one standard deviation of the mean of their associated separation estimates.

Estimating source locations

To demonstrate the ability of the code to solve for locations of a cluster of events using CWI separation estimates, we randomly distribute 50 sources in a cube with side length of 300m, shown as hollow circles in Figure 3. The dominant wavelength is $\lambda_d = 534m$, and the maximum source separation λ_{max} is 424m (i.e., $0.8\lambda_d$). For the purpose of this example, we create CWI separation

data (separation mean and standard deviations) using the empirical relations (equation 5), where the true separations are computed as $\delta_t = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$ for events $e = (x, y, z)$ and $e' = (x', y', z')$. The separation data are thus exactly consistent with the known biases and level of uncertainty of the CWI technique so that in this example we isolate the performance of the optimization algorithm that estimates source locations.

To implement the location process, initial locations for the 50 events are randomly distributed within the 3D cube using `initialize_locations.m`, as shown in Figure 3a. Thereafter the optimization took 27 iterations to converge with estimated locations shown as solid circles in Figure 3b. The improvement in event locations from the optimization is readily observed by comparing the result with their initial locations (red triangles in Figure 3a). The optimization does not lead the estimated locations to exactly the true event locations, due to the uncertainty $\sigma(\tilde{\delta}_t)$ in the separation data used. The average location error is 27m, corresponding to $0.05\lambda_d$. Figure 4 compares source separations calculated from the relocated event locations (red) to the separation data (blue) used as input to the optimization. This shows that although the input separation data deviate significantly from their actual value (the dashed line $y = x$) where the true separation is larger than $0.55\lambda_d$, the recovered source separations are only slightly underestimated. It is thus proven that the location algorithm is able to correct biases in the CWI estimates to a large extent.

Optimization techniques that guarantee convergence to the global minimum of a complicated nonlinear objective function do not currently exist. To this end, we implement the optimization multiple times with different random initializations of event locations. We illustrate the change of objective function value during the optimization process for all implementations in Figure 5. The 6 optimizations start with different objective function values because of their different random initial event locations. The value decreases rapidly for the first 10 iterations and then slows down

as the algorithm approaches the various minima. All cases converge to the same minimum of 6738, except for the 4th, which gets stuck at a local minimum of 6753. For this synthetic example, the true (global) minimum of the objective function is known because the true event locations are known, and we find that errors in the minimum found with optimization is below 1, which is negligible.

However, when applying the algorithm to real events where the correct minimum is unknown, using several optimizations from random event initializations can add confidence to the result to which most implementations converge.

APPLICATION TO NEW OLLERTON EARTHQUAKES, ENGLAND

New Ollerton, near Nottingham, England is a region where historical (micro-) seismic activity is related to coal mining. After some small earthquakes were detected, the British Geological Survey (BGS) deployed a temporary recording network in early 2014 to monitor further activities (Figure 6). These events have magnitude of 0.7-1.8ML, and the waveforms in standard SAC (Seismic Analysis Code) format (Helffrich et al., 2013) are filtered to 2-20Hz. Our dataset contains 118 SAC files, with 41 events recorded by 5 different channels of seismic stations. The code allows users to set customized criteria for selecting events and channels to be used to estimate locations. In this example, we required that events have been recorded by at least 2 station channels to be considered for clustering and then for location in the later steps. We also required that only station channels that have recorded at least 10 events contained in the total dataset (all 118 SAC files) should be used. These criteria are to ensure the robustness of data chosen for source location, presuming fewer data of higher quality gives more reliable results than more data of inconsistent

quality. As a result, 34 events and 3 channels (channel NOLCZ, NOLFE, and NOLFN) are selected to be used for source location based on the above criteria.

With the threshold on the correlation coefficient of linked events set to $\text{MIN_corr}=0.9$, and the minimum number of events to form a cluster set to 5, the selected events are classified into two clusters with 11 and 23 events, respectively, with no unclassified events. Note that setting MIN_corr to 0.9 does not ensure that all events classified as being in the same cluster have an average maximum correlation coefficient cavg_{max} over all station channels with one another, but rather it ensures that each event in a cluster has a $\text{cavg}_{max} \geq 0.9$ with at least *one* other event in the same cluster. Hence, it is recommended to set a high MIN_corr to ensure a sufficient level of waveform similarity for CWI to be applied. Figure 7a shows the waveform correlation matrix of 23 events in cluster 2 recorded by station channel NOLFN: there are some waveform pairs whose R_{max} is very close to 1, while the lowest R_{max} in the cluster is only slightly greater than 0.6. Figure 7b and 7c show comparisons of waveform pairs with the highest and lowest R_{max} , respectively.

For each cluster and each selected channel, we use the separation-uncertainty matrix to find the combination of number, length, and start time of windows that give the lowest uncertainties of CWI separation estimates. For example, for cluster 1 recorded by channel 1 (NOLCZ), the lowest average uncertainty of separation estimates is $\Omega = 11.08m$, when using 4 time-windows with a length of 2.5s, starting from 19s, indicated by the darkest blue grid cell at the bottom-left of Figure 8. The separation-uncertainty matrix provides a guideline to choose an appropriate combination of parameters to use for CWI. Usually we find that when using other combinations of parameters, the results do not change significantly as long as we do not deviate too far from the optimal values found from the matrix.

We estimated source locations for each cluster using separation estimates obtained from all channels, and from individual channels. For each location optimization, 10 different random initializations are used to evaluate robustness of our location results. Figure 9a and 9b show the progress of each implementation of cluster 1 and 2 using data from channel NOLCZ, with the horizontal zoomed panels showing details of the eventual convergence. All cases converged to similar levels to within reasonable numerical errors. The convergence for cluster 2 seems less consistent than those for cluster 1. This is as expected: cluster 1 contains 55 event pairs from 11 events, hence its objective function L involves the sum of the logarithm of 55 pair-wise likelihood functions (equation 6), whereas cluster 2 comprises 23 events so function L involves the sum of the logarithm of 253 pair-wise likelihood functions. The consistent convergence level suggests that the minimum found using different random initializations should be close to the global minimum of function L , and therefore the relative source locations have been found.

The inter-source separations obtained from the optimizations are consistent among individual channels (red, blue and green) with small residuals, and they are very similar to the results obtained using all three channels (black) as shown in Figure 10a. The location process corrects for the underestimation bias of the CWI technique, as we see by comparing the recovered separations (red) and the original CWI separation data (black) used as input to the optimization (Figure 10b). Figure 11 illustrates the location results of cluster 2 using data from individual channels, and data from all three channels, projected onto three orthogonal planes. The patterns show a high level of consistency among single channels and multiple channels. All channel combinations predict a characteristic horseshoe type structure for the cluster. Thus, we show once again that the CWI source location technique is able to give reliable relative location estimates even from single recording channels. For comparison, the double-difference (DD) location result using data from

11 available seismic station channels is shown in Figure 12, where the two methods constrain the events to a similar level of clustering.

DISCUSSION

A wide range of seismic applications requires accurate relative source locations. The popular double-difference method (Waldhauser and Ellsworth, 2000) produces high resolution location results when a large number of seismic stations are available, however its performance deteriorates when this requirement is not met, and results are subject to the choice of damping factor when solving the least squares problem. The novel location method based on coda wave interferometry opens a new avenue for research and applications in areas where a dense recording system with good source-station azimuthal coverage is unavailable, or where an independent method is useful to test compatibility of robustness of existing methods.

While we have developed and used the code package for seismic applications, there are numerous other areas in which energy source location estimates are useful or necessary as summarized in the Introduction to this article. The CWI method could potentially be applied to any of them, provided that the medium of wave propagation scatters the wavefield strongly in many directions. This is necessary in order to ensure that the recording of coda at each receiver contains energy that left each source at a wide range of angles. This range of angles is the equivalent of the standard requirement of a wide aperture between each source and the set of receivers for conventional location methods. However, the equivalence is not direct since all of the latter methods use deterministic physics to optimize locations by matching synthetic and real

data, whereas CWI uses statistical (nondeterministic) relations to estimate source separations. This difference means that CWI can also be used to provide an independent test of the efficiency of other source location methods, and also that CWI uses far more recorded data than these methods.

The CWI based method was first developed ~10 years ago (Snieder and Vrijlandt, 2005; Robinson et al. 2011) but the uptake of the method has been slow, with only two known applications to earthquake location (Robinson et al., 2013; Zhao et al., 2017). This may partly be explained by the lack of a readily available code package to implement the method, a deficiency that we rectify herein. Harder is to change the attitude that seismologists know the structure of the medium (Earth) well enough that deterministic methods can be used. Indeed, CWI as used here takes the opposite view: that we do not know the structure well, since the existence of strong coda in most recordings of crustal earthquakes shows that there must be strong scattering from unknown structure. The examples presented here show that even in such cases, relative locations can be found over length scales of a fraction of a wavelength, which should go some way towards convincing others that this point of view is both valid, and useful.

CONCLUSION

We present a MATLAB package that estimates relative source locations using source separations estimated with the CWI technique. The location method takes account of known biases in CWI separation estimates, and is capable of correcting them to a significant extent. It is able to locate events with a single station channel as demonstrated in our synthetic and real-data examples, but also to combine data from seismic arrays. The code package that accompanies this article

provides a main script that allows users to conduct the three-step method (classifying events, estimating source separations, and estimating relative source locations) while maintaining the flexibility for users to edit the code based on their own needs. This computationally inexpensive code can be run on a standard laptop for the size of event cluster demonstrated herein.

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Constant in $\mu = \mu(\tilde{\delta}_t)$ and $\sigma = \sigma(\tilde{\delta}_t)$

$\mu = \mu(\tilde{\delta}_t)$	$\sigma = \sigma(\tilde{\delta}_t)$
$a_1 = 0.4661$	$b_1 = 0.1441$
$a_2 = 48.9697$	$b_2 = 101.0376$
$a_3 = 2.4693$	$b_3 = 120.3864$
$a_4 = 4.2467$	$b_4 = 2.8430$
$a_5 = 1.1619$	$b_5 = 6.0823$
	$c = 0.017$

Table 1: Constants in the empirical relations in equations 5a and 5b for $\mu = \mu(\tilde{\delta}_t)$ and $\sigma = \sigma(\tilde{\delta}_t)$ (Robinson et al., 2013).

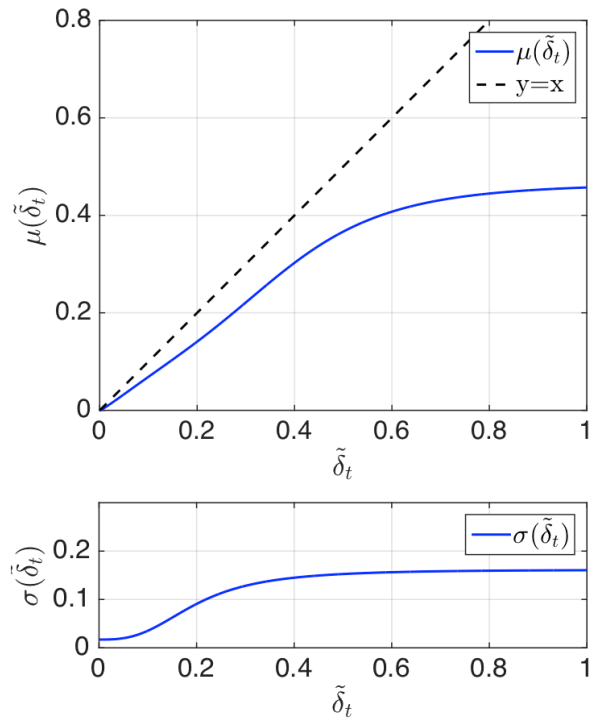


Figure 1: Empirical functions showing bias and uncertainties in separation estimates from the CWI technique. The upper panel shows the empirical relation between the mean of the source separation estimates $\mu = \mu(\tilde{\delta}_t)$ and the true separation $\tilde{\delta}_t$. All separations are normalized by the dominant wavelengths of the coda waveforms. The dashed line $y = x$ shows the case where the mean of the CWI separation estimates are identical to the true separations. The lower panel shows the empirical relation between the standard deviation $\sigma(\tilde{\delta}_t)$ of the separation estimates and the true separation $\tilde{\delta}_t$.

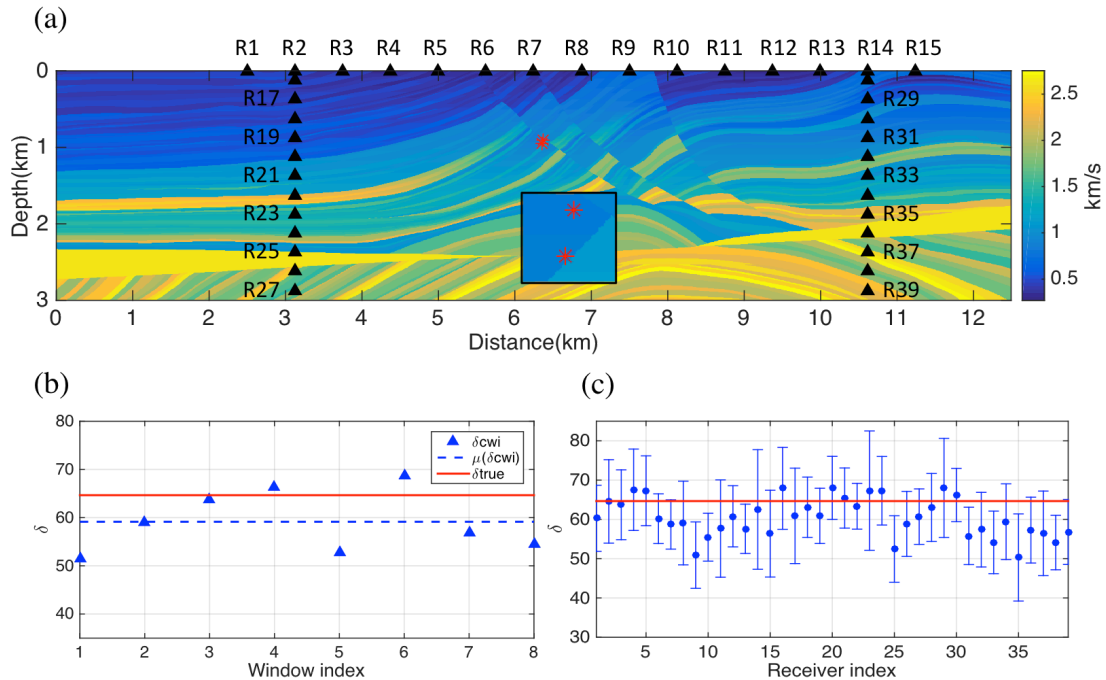


Figure 2: Panel (a) shows the Marmousi2 S-wave velocity model (Irons, 2005) used for the synthetic example. Triangles and stars are receivers and sources respectively; the small square panel shows the source region magnified. Panel (b) shows the separation estimates (triangles) of each time-window from receiver R1 only, with the mean indicated by the dashed line and the true separation between the two sources by the solid line. Panel (c) shows separation estimates from all individual receivers with error bars showing the mean plus/minus one standard deviation; the red line is the true separation between the two events.

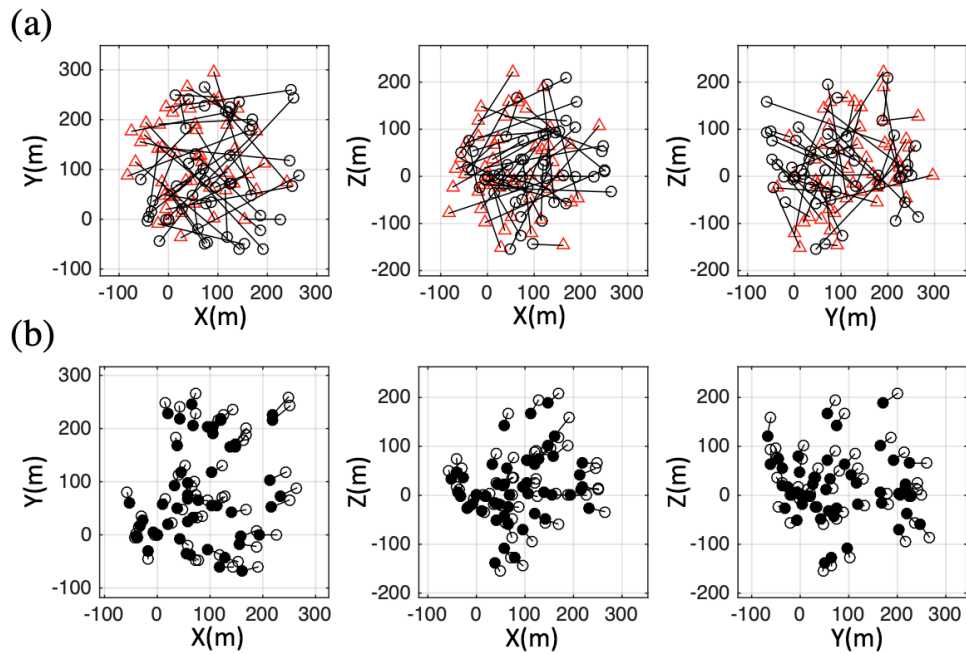


Figure 3: Planar projections of event locations, where axes X, Y and Z point in three orthogonal directions. Panels (a) compare the events' actual locations (circles) and their initial locations (triangles) before optimization, and the black bars show their differences. Panels (b) shows the events' actual locations (hollow circles) and the location results obtained (solid circles), with lines between the hollow and solid circles representing post-optimization location errors.

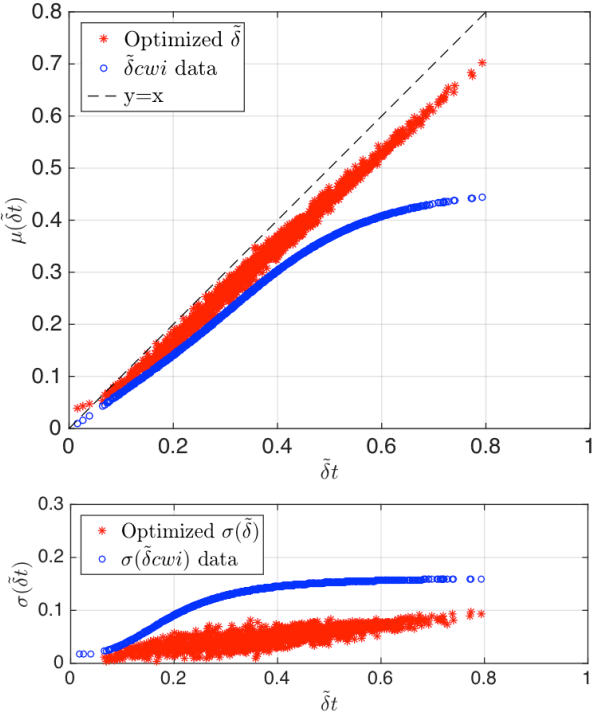


Figure 4: A comparison of input inter-source separations (blue circles) and separations calculated from the location result after optimization (red asterisks). The upper and lower panels show the separation data and their standard deviations, respectively.

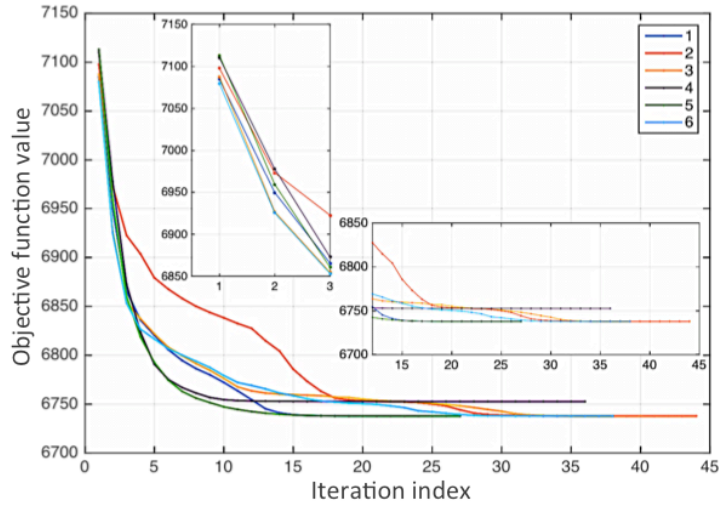


Figure 5: Illustration of the minimization process of each optimization from different initializations of random event locations using different colors to indicate each example optimization. The magnified panels show details of how the values of the objective function change with iteration number at the beginning and end of each optimization. The objective function is given in equation 10.

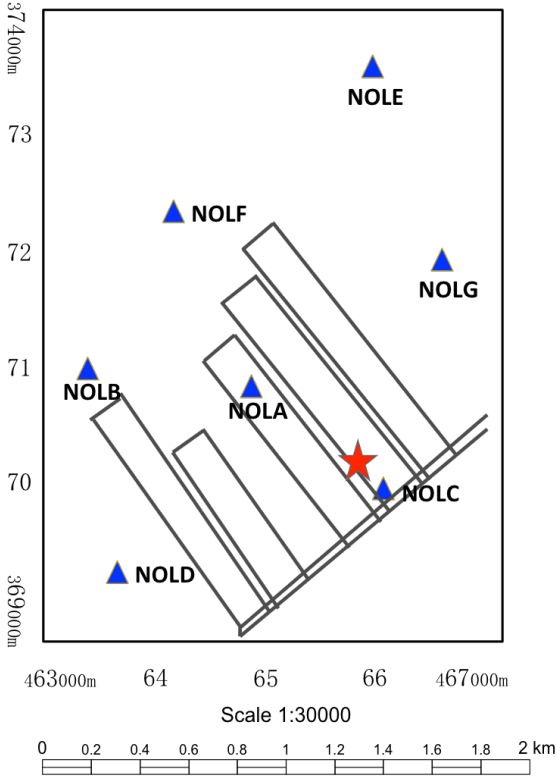


Figure 6: Map of the source region near Thorsby colliery, New Ollerton, Nottinghamshire, England. The rectangles indicate the locations of subsurface mining galleries, the star shows the area where the microearthquakes are likely to have occurred, and the triangles are temporary seismic stations.

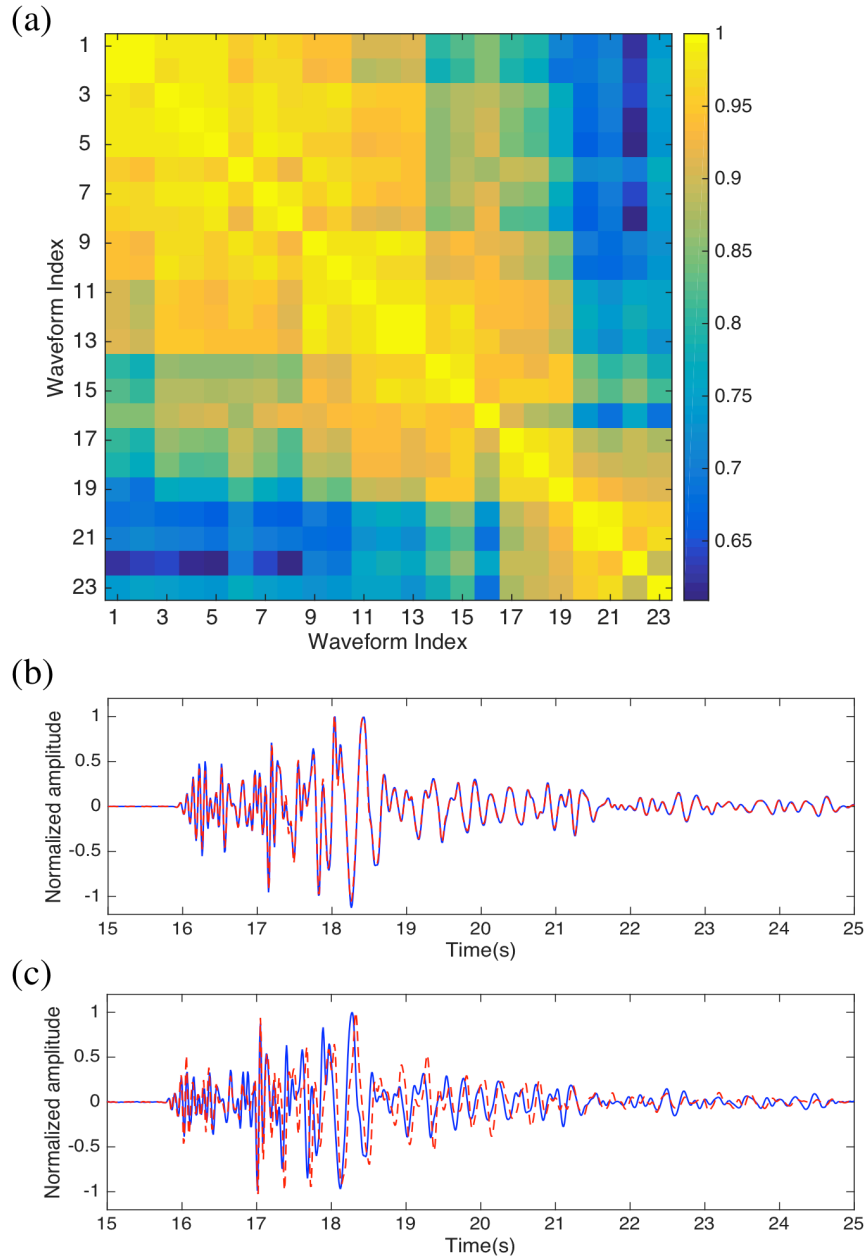


Figure 7: Illustration of waveform similarity. Panel (a) shows the correlation matrix of waveforms of the 23 events in cluster 2 recorded by station channel NOLFN. Each cell represents the maximum of correlation coefficient R_{max} of the corresponding event pair, whose value is indicated by cell color. Panel (b) and (c) show the waveform pair with the highest and lowest R_{max} respectively.

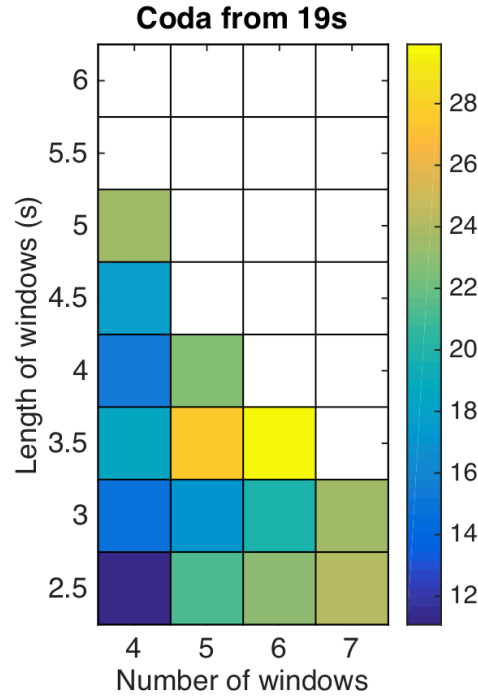


Figure 8: A slice of the 3D separation uncertainty matrix. Colors indicate the values of the average standard deviations resulting from the corresponding parameter combination of number of windows, window length and start time of windows, used to divide the coda. White cells indicate parameter combinations that are not supported by the available data. For all scenarios represented in this slice, the start of the time-windows is 19s. The parameter combination giving the lowest average separation uncertainty is 4 windows with a length of 2.5s, starting from 19s.

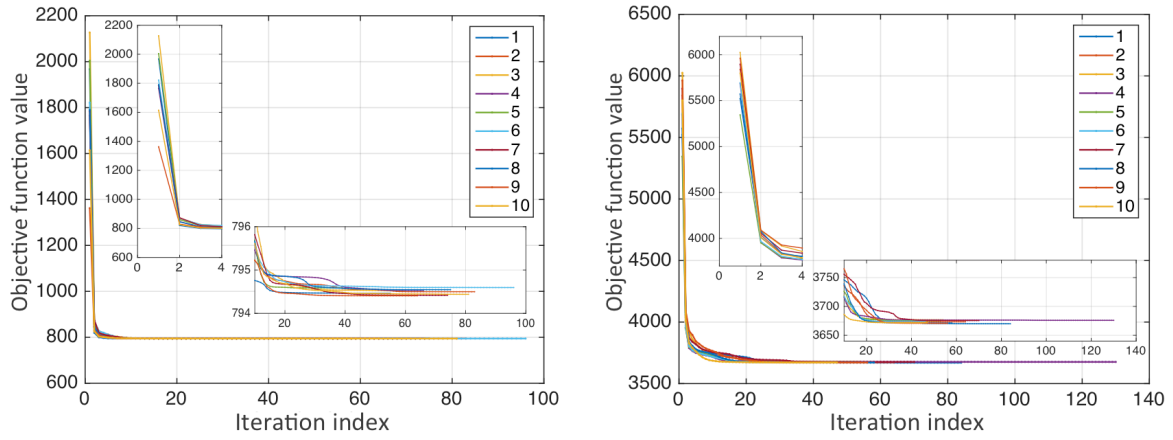


Figure 9: Illustrations of the minimization process using different colors for each optimization with different random initializations for cluster 1 (left) and cluster 2 (right) using data from channel NOLCZ. The magnified panels show details of how the values of the objective function change with iteration number at the beginning and end of each optimization. The objective function is given in equation 10.

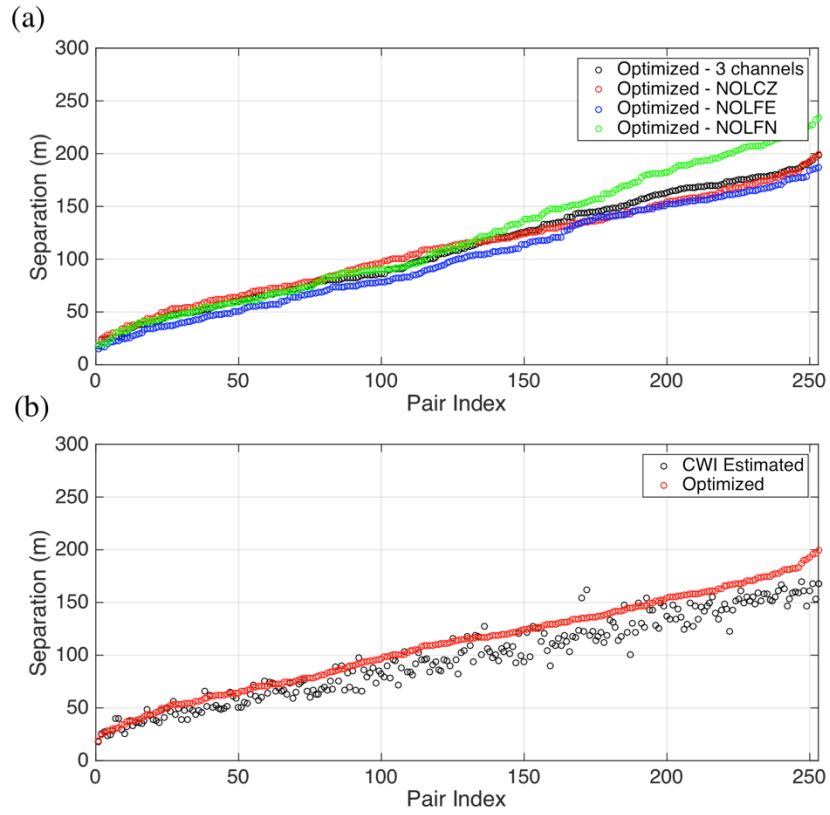


Figure 10: CWI estimates of source separations and their optimized counterparts (the latter calculated from the estimated event locations) of cluster 2. Panel (a) shows the optimized separations using all three channels (black), and using single channels NOLCZ, NOLFE and NOLFN (red, blue, and green). Panel (b) compares the optimized separations of channel NOLCZ (red) with the original CWI separation estimates (black).

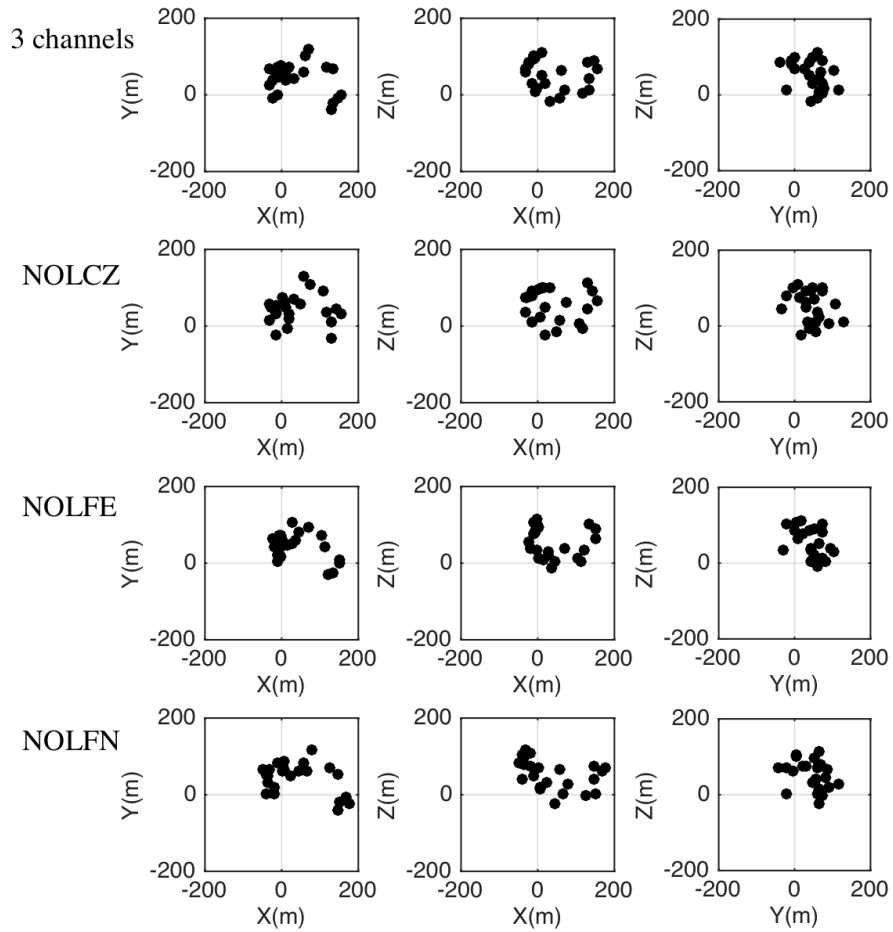


Figure 11: Planar projections of relative event location results of cluster 2, using all three channels (top row), and using single channels NOLCZ, NOLFE, and NOLFN (successive rows). Axes X, Y and Z point in orthogonal directions.

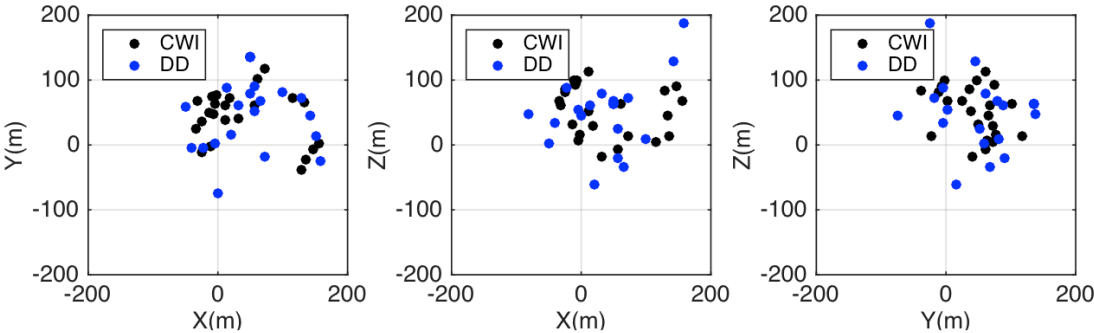


Figure 12: Comparison of the location results of cluster 2 using CWI (black) and DD (blue) methods with data from 3 and 11 station channels.