



THE UNIVERSITY *of* EDINBURGH

Edinburgh Research Explorer

## Experimental investigation of drag coefficient of free-falling deformable liquid gallium droplet

**Citation for published version:**

Mohamad, M, Mackenzie Dover, C & Sefiane, K 2018, 'Experimental investigation of drag coefficient of free-falling deformable liquid gallium droplet' *European Physical Journal: Applied Physics*, vol. 84, 10903. DOI: 10.1051/epjap/2018180271

**Digital Object Identifier (DOI):**

[10.1051/epjap/2018180271](https://doi.org/10.1051/epjap/2018180271)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Peer reviewed version

**Published In:**

*European Physical Journal: Applied Physics*

**General rights**

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [openaccess@ed.ac.uk](mailto:openaccess@ed.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.



# Experimental Investigation of Drag Coefficient of Free-Falling Deformable Liquid Gallium Droplet

M. Sofwan Mohamad<sup>1,2</sup>, C.M. Mackenzie Dover<sup>1</sup> and K. Sefiane<sup>1</sup>

<sup>1</sup> Institute for Multiscale Thermofluids, School of Engineering, University of Edinburgh, Faraday Building, King's Buildings, Edinburgh United Kingdom. EH9 3DW

<sup>2</sup> School of Mechatronic Engineering, Universiti Malaysia Perlis (UniMAP), Kampus Pauh Putra, 02600 Arau, Perlis Malaysia.

Received: date / Revised version: date

**Abstract.** In this paper, the effect of shape and deformation on the drag coefficient of a free-falling liquid gallium droplet in water in a terminal state is investigated experimentally. The temperature of the dispersed and continuous liquid was varied in order to examine the effect on the liquid-metal droplets. The falling droplets were imaged using a high speed camera and a simple model was developed to predict drag coefficient over a Reynolds number range of  $10^3 < Re < 10^4$ . The drag coefficients of the deformed liquid gallium droplets were found to be larger than that associated with a solid sphere and the associated Weber number is below 4.5. It was found that the shape of all droplets in our experiment were oblate-spheroid. A correlation has been established to predict the aspect ratio of a liquid gallium droplet moving in quiescent water. The deformation is highly dependent on interfacial surface tension and inertial force, while the viscosity ratio and pressure distribution have negligible effect.

**Key words.** Droplet, Drag coefficient, Deformation, Aspect ratio, Multiphase flows

## 1 Introduction

The motion and deformation of liquid droplets is important to numerous industrial applications and has attracted considerable research interest as a result. In industrial engineering, understanding the dynamics of liquid droplets is relevant to many applications such as the liquid sprays injected in to combustion engines[1], ink-jet printers[2], micro-fluidics[3] and cooling systems[4]. Among various droplet materials, gallium and its alloys have drawn huge attention recently because of the associated physical and chemical properties such as low-melting points (liquid state at or near room temperature) and high thermal conductivities. Such characteristics offer tremendous opportunities for developing advanced technologies in newly emerging areas such as electronic cooling[5, 4], 3D printing and printed electronics[6], flexible devices[7], soft actuators for robotics[8] and adhesion[9].

Hydrodynamic drag is of major importance to countless industrial practices as it is one of the most significant parameters that govern the movement of a droplet through a liquid and reducing it could lead to a substantial energy saving. To date, a substantial amount of literature can be found on this topic. However, reported data on the dynamics of deformable droplet are limited. The study of R. Clift et al.[10], E. Loth[11] and Wegener et al.[12] presented an extensive review of theory, experimental data and relevant approximations representing the characteristics of single droplet in fluid systems. As yet, there has

been no systematic investigation on the effect of liquid gallium droplet morphology on the velocity and drag coefficient to the knowledge of the authors. In the following, the dynamics of a free falling droplet of gallium in water are studied and the effect of droplet morphology on the velocity and drag coefficient are quantified. Characterising the relationship between the dynamics of the gallium droplets and the medium it is moving through facilitates a superior control of forces that inhibit droplet motion and the correlation can be applied to a number of systems.

## 2 Experimental Setup

An experimental setup was designed to measure the terminal velocity of the falling droplets, a schematic of which is shown in Figure 1. The setup comprises a square cross section, straight-walled column with closed bottom filled with water. The column is 1000 mm in height and 70.45 mm in inner side width. These dimensions were selected such that a droplet having diameter of less than 7 mm can fall through the continuous liquid with minimal wall effects, terminal velocity could be reached and end effects could be avoided[13, 14]. The column is made of clear Perspex, allowing the motion of the falling droplets to be recorded by a high speed camera. The bottom part of the column is inclined towards a central discharge point where a liquid gallium retrieval mechanism is located. The liquid gallium retrieval mechanism consists of two ball valves.

This allowed for the liquid gallium to be removed with a minimal amount of water being discharged from the column. An electric heater is fitted at the bottom part of the column and the water temperature inside the tank is controlled by a proportional-integral-derivative (PID) controller connected to two thermocouple probes that are situated at the top and bottom of the chamber. The water in the tank can be heated up to about 80°C. In order to minimise the effect of thermal convection during experiments, the heater was turned off after heating the water to the required temperature. A few minutes after that, the temperature of water was found to be homogeneous within  $\pm 1^\circ\text{C}$ .

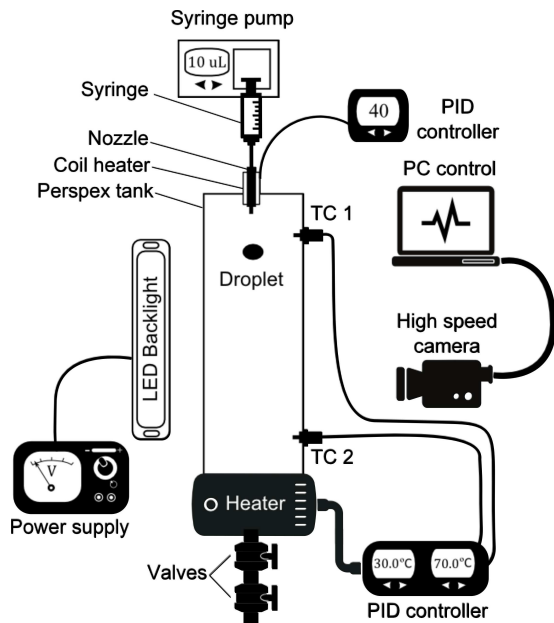


Fig. 1: Experimental set up.

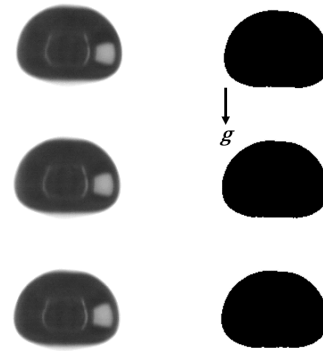
Liquid gallium was heated to the desired temperature and then fed from a syringe in to the column of water. A precise volume of liquid gallium was released gradually, directly below the water surface to create a droplet with the help of a programmable syringe pump (Cole-Parmer Touch-Screen Syringe Pump 78-8110C). The accuracy of the syringe pump is  $\pm 0.355\%$  with reproducibility of  $\pm 0.05\%$ . Droplets were increased in volume until they detached from the needle due to their weight and needles of different diameters could be attached to change the size of the examined droplet. The droplet behaviour during free fall was recorded by a high speed CMOS camera (Nanosense MKIII) with frequency up to 1024 fps at a resolution of 1280x1024 pixels. The camera was coupled with an Edmund optics double gauss lens (54-691) to produce low distortion images. A custom made 100 W LED light (Maxilux ProStrip120-High Power) was placed at about 40 mm behind the column to illuminate the test region.

The image sequences were converted into 8-bit grey scale, background-subtracted and thresholded such that

Table 1: Equivalent diameter of liquid gallium droplets used in experiment.

Volume (uL)	Equivalent diameter (mm)
17	3.19
60	4.86
90	5.56

spheres appeared as a black dot on a white background to allow for tracking and quantification of droplets using an in-house ImageJ macro. Information such as the relative position, velocity and diameter were obtained using automated analysis of this type. Figure 2 shows a series of images pre- and post-processing.



(a) Original image (b) Treated image

Fig. 2: Shape of liquid gallium droplets at terminal condition.

## 3 Results & Discussion

### 3.1 Diameter of the droplet

The diameter of the droplet can be calculated from a known dispersed volume of gallium. Volume equivalent diameter,  $d_{eq} = \sqrt[3]{\frac{3V}{4\pi}}$  can be calculated with the assumption that the droplet is a solid sphere symmetric about the major and minor axis plane of observation. Table 1 summarizes the calculated diameter of liquid gallium droplets of various volumes. The calculated equivalent diameters are a convenient parameter to represent the droplets as well as to understand the effect of deformation on drag coefficient.

### 3.2 Droplet terminal velocity

Figure 3 shows co-plots of the velocity of free falling liquid gallium droplets of various diameters in water as a function of their distance from the needle. It can be seen that

the velocity becomes terminal after 200 mm for all sizes of droplets examined. After analysing the droplet's velocity under different thermal conditions, it was observed that each droplet in the investigation could attain a terminal condition after falling a distance of 200 mm. Hence, the droplet image sequence ranging from 200 to 250 mm below the needle was selected to calculate the average droplet terminal velocity ( $u_T$ ) and drag coefficient ( $C_D$ ). In our experiments we did not observe any significant difference in  $u_T$  by varying the temperature of liquid gallium and water as depicted in Figure 4 (further discussion in Section 3.4). However, it is obvious that the  $u_T$  is dependent on the droplet size.

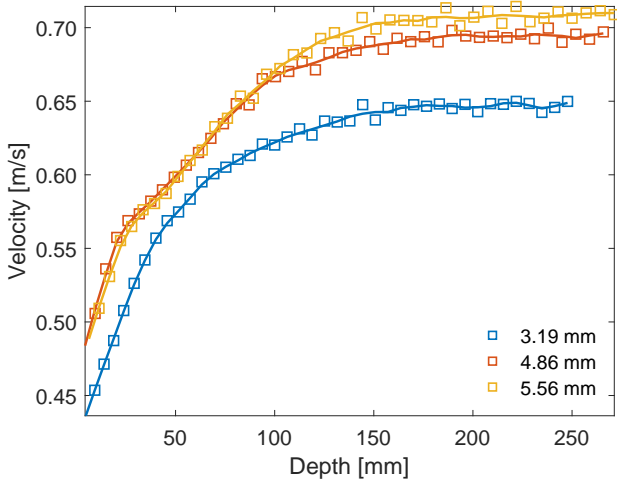


Fig. 3: Liquid gallium droplets velocity as a function of their distance from the needle in water.

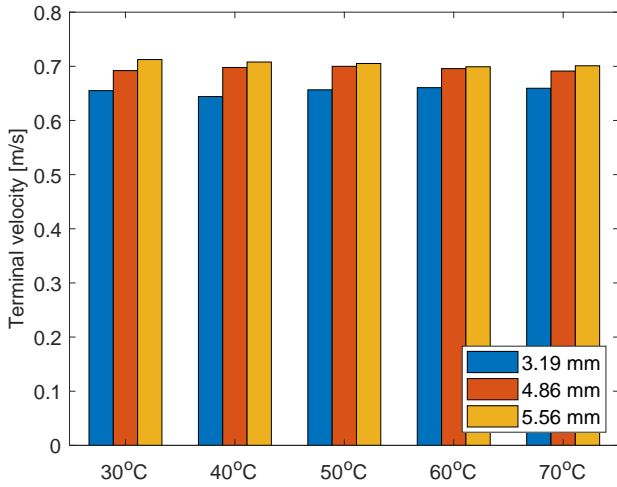


Fig. 4: Liquid gallium droplets terminal velocity.

### 3.3 Drag coefficient

Calculating the  $C_D$  was necessary to characterise the motion and the kinematics of the droplets. Firstly, a theoretical correlation to obtain  $C_D$  as a function of Reynolds number ( $Re$ ) and fluid properties was developed. Then, experimental values obtained by image processing were used to calculate the  $C_D$  and the effect of deformation is subsequently investigated. A simple correlation based on net gravitational force (gravitational-buoyancy) and drag force balance for a sphere at terminal condition are given by Eq. (1).

$$\pi(\rho_d - \rho_c)g \frac{d_{eq}^3}{6} = \frac{1}{8} C_D \pi d_{eq}^2 \rho_c u_T^2 \quad (1)$$

Note that  $d_{eq}$  is used in this equation to remove the effect of deformation. Eq. (1) can be rearranged to calculate  $C_D$  based on  $C_D = f(Re)$  as follows:

$$C_D = \frac{4(\rho_d - \rho_c)\rho_c g d_{eq}^3}{3\mu_c^2} / Re^2 \quad (2)$$

where  $Re = \frac{\rho_c u_T d}{\mu_c}$ , is the particle  $Re$ ,  $\mu_c$  and  $\rho_c$  are the continuous phase fluid viscosity and density respectively and  $d$  is the diameter of the droplet.

Next,  $C_D$  was derived from the experiment. In order to obtain the exact value of  $C_D$ , the droplet deformation was considered. Instead of  $d_{eq}$ , the droplet's actual frontal diameter obtained directly from the images through image processing was used. Figure 5 compares the  $C_D$  calculated from Eq. (2) with those from experiment along with the classical curve of  $C_D$  of a solid sphere. An acceptable agreement between theory and experiment was validated by the minimal discrepancy between theoretical and experimental values. The differences in those values, may be due to the effect of deformation that been ignored in the derivation of Eq. (2). Moreover, it is noted that as the  $Re$  increases, the experimental  $C_D$  values deviated from those calculated for an equivalent solid sphere and the difference are more pronounce for larger droplets.

### 3.4 Shape & deformation of droplet

The  $C_D$  of a falling droplet is known to be dependent upon the droplet shape. The shape of droplets experiencing free fall in an infinite medium under the influence of gravity are generally classified in three categories as shown in Figure 6. These categories are defined by the ratio of its largest horizontal dimension (equator),  $d_h$ , to its largest vertical dimension (polar),  $d_v$ , known as aspect ratio,  $E = \frac{d_h}{d_v}$ . Droplets are classified as spherical if the aspect ratio lies within 10% of unity. For an aspect ratio other than that, droplet is described deformed and can be classified as an ellipsoid, which has two sub-categories namely oblate-spheroid and prolate-spheroid.

Figure 7 shows representative photographs of different sizes of liquid gallium droplets used in our experiment as

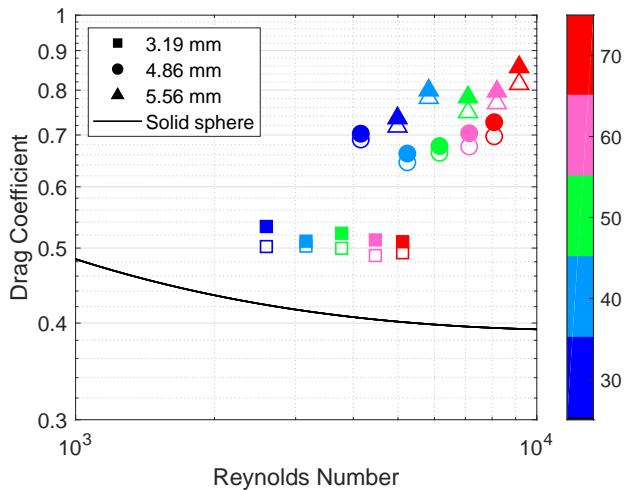


Fig. 5: The relationship between drag coefficient and Reynolds number (open symbols are experimental values and filled symbols are theoretical values).

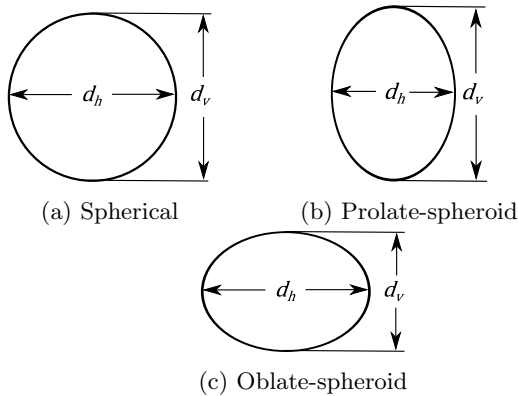


Fig. 6: Schematic diagrams of typical droplet shape.

they fall through water and Figure 8 illustrates the dimensions of those droplets obtained from image processing of the images for 5 sequences for each diameter. The gallium droplets are deformed even at the smallest diameter and the value of aspect ratio is always above 1.1 (see Figure 9), thus we can classify all of the droplets as oblate-spheroid. The deformation of the droplets from spherical shape can also be confirmed visually by looking the images in Figure 7. Due to aerodynamic and hydrostatic pressure, the droplet aspect ratio continues to increase with droplet diameter[15]. With the  $Re$  in the current investigation ranging from  $10^3$  to  $10^4$ , this observation might extend the conclusion made by Taylor and Acrivos[16] to a wider range of  $Re$ , stated that a droplet is likely to be deformed into an oblate instead of a prolate-spheroid for all cases at low  $Re$ . It is also noted that the temperature does not have a significant effect on the steady state shape of a liquid gallium droplet.

Deformation occurs because of the interplay between pressure distribution and surface tension. The pressure distribution produces local fluid-dynamic stresses, which

Table 2: Physical parameters of the continuous and dispersed fluid.

$T(^{\circ}C)$	$\rho_c$	$\mu_c$	$\rho_d$	$\mu_d$	$\sigma_{c-d}$
30	995.7	0.798	6084	139.0	698.4
40	992.2	0.653	6077	137.7	698.1
50	988.0	0.547	6071	136.5	697.9
60	983.2	0.466	6064	135.2	697.5
70	977.8	0.404	6058	134.0	697.2

are controlled by the viscosity ratio,  $\lambda$  (which controls the circulation inside droplet) and  $Re$  (which controls the importance of viscosity). Surface tension is affected by the Weber number ( $We$ ), the ratio of continuous fluid stresses, which causes deformation to the surface tension stresses, which oppose deformation. These independent non-dimensional parameters can be calculated using the following equations:

$$\lambda = \frac{\mu_d}{\mu_c} \tag{3}$$

$$We = \frac{\rho_c u^2 d}{\sigma_{c-d}} \tag{4}$$

where  $\mu_d$  is the dispersed fluid viscosity.  $\sigma_{c-d}$  is the interfacial surface tension between the dispersed and continuous fluid. This parameter could possibly be approximated by utilizing Eq. (5)[17]. Table 2 summarises the physical properties of liquid gallium and water at different temperature that are relevant to the experiment.

$$\sigma_{c-d} = \sigma_d + \sigma_c - 2\sqrt{\sigma_c \sigma_d} \tag{5}$$

Figure 10 shows the relationship between the aspect ra-

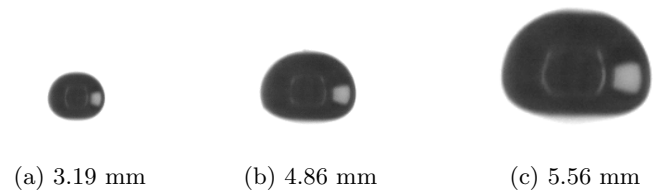


Fig. 7: Shape of liquid gallium droplets at terminal condition.

tio and  $We$  for different  $\lambda$ . In our experiments, viscosity variation is caused by varying the temperature of the liquid gallium and water as shown in Table 2, producing a range of  $\lambda$  from 166 to 329. Internal circulation inside the droplet caused by the  $\lambda$ , which produces high pressure at the leading and trailing edge of the droplet, tends to cause prolate shapes. On the other hand, fluid surrounding the droplet also produces a high-pressure zone at the trailing and leading edges of the droplet and a low-pressure zone near the equator[18]. This opposes the effect of internal circulation and tends to cause oblate droplet shapes[19]. In our experiments, the viscosity of the dispersed fluid (liquid

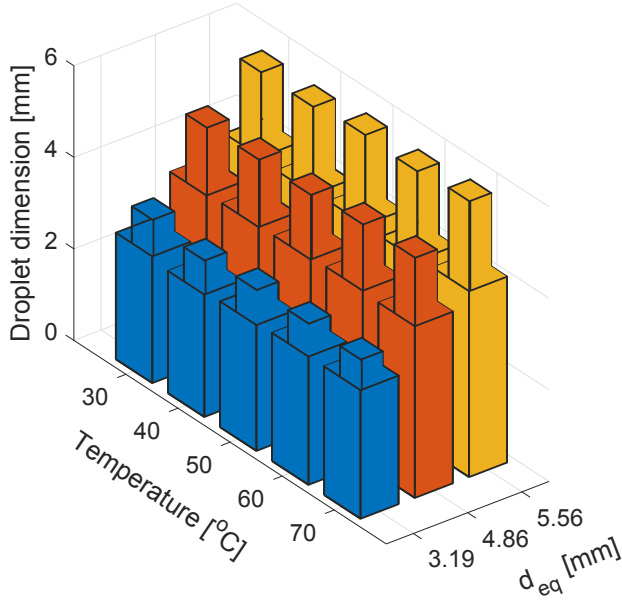


Fig. 8: Comparison of horizontal (thin bar) and vertical (wide bar) dimension of liquid gallium droplet at different temperature.

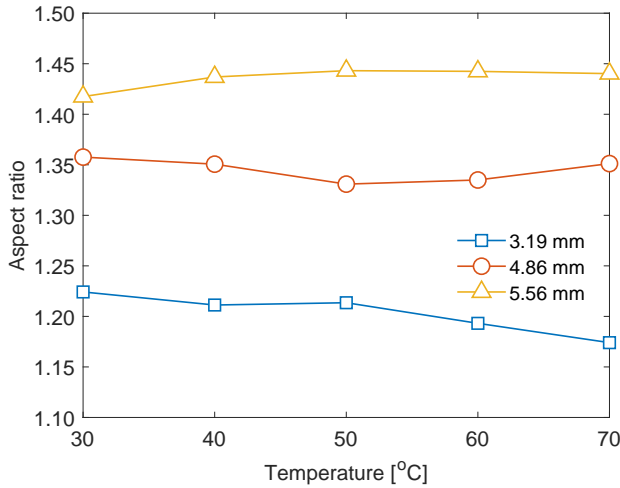


Fig. 9: Comparison of aspect ratio of liquid gallium droplet at different temperature.

gallium) is relatively large compared with the continuous fluid (water) giving a high  $\lambda$ . This condition restricts the internal circulation, hence the droplets are deformed in an oblate. As expected, droplet deformation is very much less affected by the  $\lambda$  but is rather dependent on droplet size and  $We$ [18]. The correlation is found to follow a quadratic trend line:

$$E = 1.0122e^{0.0885We} \quad (6)$$

For droplets with small values of  $We$ , the values of the aspect ratio are nearly 1.1 (nearly spherical) and tend to increase as the  $We$  increases. This trend indicates that at low  $We$ , the surface tension can counter balance the continuous fluid stresses. However, as the  $We$  increases,

the continuous fluid stresses become significant up to a point where the surface tension can no longer resist the deformation from occurring. This may also explain why no significant change in the  $u_T$  is observed. The restricted internal circulation increases the velocity gradient at the liquid-liquid interface producing higher shear stresses at the interface. The effect of lower buoyancy force due to lower water viscosity caused by increasing temperature may be cancelled by the larger viscous dissipation between the phases. The overall effect is that the  $u_T$  is unaffected by the temperature range used for this experiment.

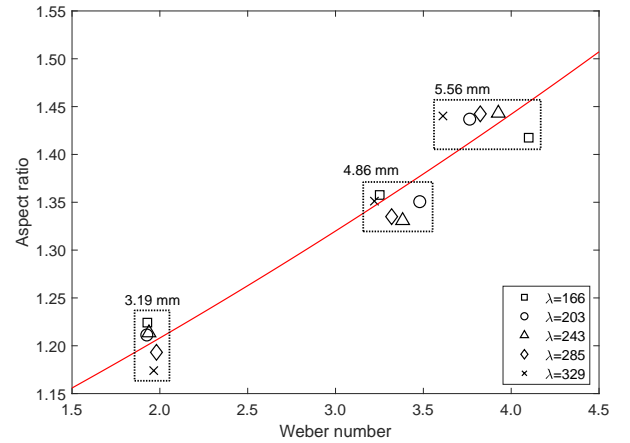


Fig. 10: The relationship between aspect ratio and Weber number for different viscosity ratio.

In order to further quantify the effect of pressure distribution on droplet deformation, the relationship between the aspect ratio and  $Re$  for different droplet sizes are plotted in Figure 11. While the data exhibits some scattering, the aspect ratio remains almost constant as a function of  $Re$  and depends more on droplet diameter. This phenomenon is most pronounced for  $4000 < Re < 8000$ , where  $Re$  for small and large diameter droplets overlap. At constant  $d_{eq}$  and  $u_T$ , only two parameters in  $Re$  were varied by temperature;  $\rho_c$  and  $\mu_c$ . Based on this, we can say that viscous force has a negligible effect on droplet deformation. Hence, it is reasonable to conclude that droplet shape and deformation are influenced mainly by interfacial surface tension and inertial force and is less affected by viscous force and pressure distribution. This conclusion is similar to the Hadamard-Rybczynski solution which states that pressure distribution due to viscous force do not produce deformation for Stokes flow over a droplet and that a droplet of liquid is only affected by viscosity minimally[16, 10].

## 4 Conclusion

An experimental study has been conducted on the effect of shape and deformation on  $u_T$  and  $C_D$  of free-falling liquid gallium droplet in water at terminal condition. Liquid

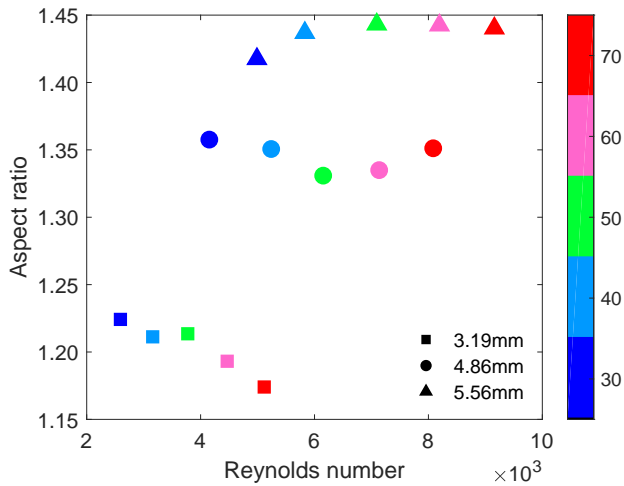


Fig. 11: The relationship between aspect ratio and Reynolds number for different size of liquid gallium droplet.

gallium and water were held at temperatures in the range of 30°C to 70°C, selected in order to produce a significant variation in physical properties (density, viscosity and surface tension). A simple theoretical  $C_D$  correlation has been proposed and then compared with the experimental data. The  $d_{eq}$  was used in the correlation to calculate the  $C_D$ . Considering the error caused by the assumption that the droplet remains spherical, the correlation agrees well with empirical data of  $C_D$  for droplets with a  $Re$  in the range of  $10^3 < Re < 10^4$ . Consequently, the effect of three independent non-dimensional parameters, namely viscosity ratio, Weber number and Reynolds number on the dynamic droplet shape, which has a direct influence on  $C_D$ , were examined. It was found that for  $We < 4.5$  the shape of droplets tend to deform into an oblate-spheroid. The oblateness was seen to increase in extent with the  $We$ . The droplet shape and deformation are strongly dependent on the interfacial surface tension and inertial force, in contrast to pressure distribution and  $\lambda$ , which have a negligible effect.

## 5 Authors contributions

All the authors were involved in the preparation of the manuscript. All the authors have read and approved the final manuscript.

## References

- G. Edara, Y. V. Murthy, K. V. Sharma, and S. Pullela, "Numerical modeling of a fuel droplet for the evaluation of ignition temperature considering transport properties," *Case Studies in Thermal Engineering*, vol. 10, no. January, pp. 121–130, 2017.
- A. Ono, T., Kawai, "Free Fall Mechanism of Micro Liquid Droplet," *Journal of Photopolymer Science and Technology*, vol. 23, no. 3, pp. 363–366, 2010.
- R. Seemann, M. Brinkmann, T. Pfohl, and S. Herminghaus, "Droplet based microfluidics," *Reports on Progress in Physics*, vol. 75, no. 1, 2012.
- J. Oh, P. Birbarah, T. Foulkes, S. L. Yin, M. Rentauskas, J. Neely, R. C. Pilawa-Podgurski, and N. Miljkovic, "Jumping-droplet electronics hot-spot cooling," *Applied Physics Letters*, vol. 110, no. 12, pp. 1–6, 2017.
- H. Ge and J. Liu, "Keeping Smartphones Cool With Gallium Phase Change Material," *Journal of Heat Transfer*, vol. 135, no. 5, p. 054503, 2013.
- Q. Wang, Y. Yu, J. Yang, and J. Liu, "Fast Fabrication of Flexible Functional Circuits Based on Liquid Metal Dual-Trans Printing," *Advanced Materials*, vol. 27, no. 44, pp. 7109–7116, 2015.
- X. Li, M. Li, L. Zong, X. Wu, J. You, P. Du, and C. Li, "Liquid Metal Droplets Wrapped with Polysaccharide Microgel as Biocompatible Aqueous Ink for Flexible Conductive Devices," *Advanced Functional Materials*, vol. 28, no. 39, pp. 1–8, 2018.
- L. Hines, K. Petersen, G. Z. Lum, and M. Sitti, "Soft Actuators for Small-Scale Robotics," *Advanced Materials*, vol. 29, no. 13, 2017.
- M. Yunusa, G. J. Amador, D. M. Drotlef, and M. Sitti, "Wrinkling Instability and Adhesion of a Highly Bendable Gallium Oxide Nanofilm Encapsulating a Liquid-Gallium Droplet," *Nano Letters*, vol. 18, no. 4, pp. 2498–2504, 2018.
- M. Clift, R., Grace, J.R., Weber, *Bubbles, Drops and Particles*, vol. 94. 1978.
- E. Loth, "Quasi-steady shape and drag of deformable bubbles and drops," *International Journal of Multiphase Flow*, vol. 34, no. 6, pp. 523–546, 2008.
- M. Wegener, N. Paul, and M. Kraume, "Fluid dynamics and mass transfer at single droplets in liquid/liquid systems," *International Journal of Heat and Mass Transfer*, vol. 71, pp. 475–495, 2014.
- P. Uhlherr and R. Chhabra, "Wall Effect for the Fall of Spheres in Cylindrical Tubes at High Reynolds Number," *The Can. J. of Chemical Engineering*, vol. 73, 1995.
- P. P. Brown and D. F. Lawler, "Sphere Drag and Settling Velocity Revisited," *Journal of Environmental Engineering*, vol. 129, no. 3, pp. 222–231, 2003.
- C. Béguin, S. Étienne, and M. J. Pettigrew, "Effect of dispersed phase fraction on the drag coefficient of a droplet or a bubble in an idealized two-phase flow," *European Journal of Mechanics, B/Fluids*, vol. 65, pp. 339–349, 2017.
- T. D. Taylor and A. Acrivos, "On the deformation and drag of a falling viscous drop at low Reynolds number," *Journal of Fluid Mechanics*, vol. 18, no. 03, pp. 466–476, 1964.
- Y. M. Jung, H. C. Oh, and I. S. Kang, "Electrical charging of a conducting water droplet in a dielectric fluid on the electrode surface," *Journal of Colloid and Interface Science*, vol. 322, no. 2, pp. 617–623, 2008.
- B. T. Helenbrook and C. F. Edwards, "Quasi-steady deformation and drag of uncontaminated liquid drops," *International Journal of Multiphase Flow*, vol. 28, no. 10, pp. 1631–1657, 2002.
- B. T. Helenbrook, "Numerical Studies of Droplet Deformation and Break-up," no. May, 2001.