Optimal Subsidies for Development Supply Chains

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Problem definition: When donors subsidize products for sale to low-income families, questions faced by the donor include who should be subsidized in the supply chain and to what extent, and whether retail competition, substitutable products, or demand uncertainty matter.

Academic/practical relevance: By introducing and analyzing "development supply chains" in which transactions are commercial but subsidies are needed for affordability, we explore different supply chain structures, with product substitution and retail competition motivated by a field study in Haiti of supply chains of subsidized solar lanterns.

Methodology: We incorporate product substitution, retail competition, and demand uncertainty in a three-echelon supply chain model with manufacturers, retailers and consumers. This model has transactions among the donor, manufacturers, retailers and consumers as a 4-stage Stackelberg game and we solve different variations of this game by using backward induction.

Results: The donor can subsidize the manufacturer, retailer or the customer, as long as the total subsidy per unit across these echelons is at the optimal level. Having more product choice, especially with product-specific subsidies, and having more retail-channel choice can increase the number of beneficiaries adopting the products; this increase becomes more pronounced as demand becomes more uncertain.

Managerial implications: Donors must coordinate across different programs along the entire supply chain; demand uncertainly only accentuates the need to do so. They must also encourage more retailers to enter the market, to sell a diverse set of substitutable products, and to offer product-specific subsidies.

Key words: Subsidies, development supply chains, Haiti, socially responsible products, solar lanterns

1. Introduction

After a 7.0-magnitude earthquake struck Haiti on January 12, 2010, more than 200,000 people were killed, more than 300,000 injured, and 1.5 million people rendered homeless. In our field study of solar lantern distribution in Haiti during 2014-2016, we found donors created subsidy programs for selling 'essential' products such as solar lanterns to the poor using what we call *development supply chains*, to distinguish from *humanitarian* or *commercial* supply chains: We observed that there was no agreement among donors or other stakeholders as to where subsidies should be provided in the supply chain and whether or not competing products or supply chain entities should be subsidized.

This observation motivated these research questions: *Where in the supply chain and how much should the donors subsidize, keeping in mind the supply chain structure, product choice, retail competition, and demand uncertainty*? We assume the donor's goal is to maximize the number of beneficiaries who can afford the product subject to a budget. By considering a three-echelon supply chain with manufacturers, retailers, and customers, we analyze different variants of a 4-stage Stackelberg game to seek answers by considering the following settings: (1) the *base setting* with one retailer selling a single product; (2) a *choice setting* with one retailer selling two substitutable products; and, (3) a *competition setting* with two competing retailers selling two substitutable products separately. Furthermore, we add (4) *endogenous wholesale price*, and (5) *demand uncertainty* to these three settings.

Comparing the corresponding equilibrium outcomes associated with different supply chain settings, we find there is a unique optimal total subsidy level for each product in each setting. Also, we find that it is optimal to subsidize any of the manufacturers, the retailers, or the beneficiaries, as long as the total subsidy per unit is maintained at the optimal level. Moreover, we learn that a donor can stimulate more beneficiaries to adopt the lanterns without increasing the budget by: (1) encouraging more heterogeneous products with different valuations; (2) offering product-specific subsidies; and (3) encouraging more retailers entering the market. These results become more pronounced when the underlying market demand becomes more uncertain. In all settings with optimal subsidies, the retailers' profits are positive so subsidy programs, if optimal, create economic value for the micro-retailers, meeting development goals of the donor. With these findings, we seek to contribute to the literature on the use of subsidies in supply chains with our focus on supply chain structure. We also seek to contribute to the humanitarian operations literature by presenting and analyzing subsidies for post-disaster recovery and development.

Section 2 provides background for this work based on our field study. Section 3 analyzes subsidies (per lantern) in three supply chain settings as the base model in which the wholesale price is exogenous. Sections 4 and 5 extend the base model for two settings where: (1) the wholesale price is endogenously determined by the manufacturer; and (2) the market size is uncertain. Section 6 highlights our contribution to the literature and implications for practice as conclusion. Proofs for theorems appear in the (online) Appendix.

2. Background Information

The development supply chains for solar lanterns we studied in Haiti (2014-16) have entities at three echelons: (1) *OEMs (e.g., d.light)* who source the lanterns mainly from China, (2) *Importer/distributor/retailer* who import the lanterns into Haiti and supply to distributors who sell through retail chains or micro-retailers (who may be funded by micro-finance institutions), and (3) *consumers* or beneficiaries. Some importers distribute multiple brands of solar lanterns through multiple channels so product substitution and retail competition are present. Donors such as USAID offer unit subsidies indirectly to micro-entrepreneurs by funding MFIs who offer lower interest rates than market on loans to micro-entrepreneurs to buy solar lanterns from distributors. In our field study, we found that donors typically provided lump-sum grants to OEMs, importers or distributors, but donor practice in Bangladesh or India is to offer (unit) subsidies sold via cash vouchers. This papers focuses on unit subsidies and we refer the reader to XXXX (2017) for the analysis associated with lump-sum grants. Three key findings, which set the stage for modeling after this section, are:

1. *The wholesale price*: The wholesale price of solar lanterns sold in Haiti can be pre-specified (i.e., exogenous to the setting) or negotiated (i.e., endogenous to the setting). For certain brands of solar lanterns imported directly from China, the price tends to be pre-specified for all countries. One senior Manager of Solar Product Company told us that: "The norm is just like 'business is business' in China, here's our price." However, the wholesale price of other brands is negotiable. Indeed, as one interviewee (Founder and CEO of a Solar Product Company) put it, "[Negotiation] is country by country There are times that we've negotiated pretty big distribution agreements that have a minimum one container per month."

2. *Number of brands*: Some importers focus on a single brand while others prefer multiple brands of solar lanterns with different perceived quality levels and price points. One importer in Haiti told us that: "We always try to buy products approved by [World Bank-funded] Lighting Global <www.lightingglobal.org>. It is [about] forming a relationship with one manufacturer, testing in-country to see how well [the products] are accepted, and then negotiating prices." However, the CEO of a different importer of solar lanterns said that: "It's best to have a range of products – we have 12 different products for different end customers at different price points and different feature sets."

3. *Donors' objectives and budget*: Donors use a planned budget to maximize the total adoption. For instance, World Bank had a solar lantern project with a budget of $8.62 million in Haiti. A senior manager at a donor organization said, "What counts is good quality products that are affordable, and that they make a positive change to the beneficiaries." A common goal is to maximize the number of beneficiaries adopting solar lanterns subject to the donor's budget.

Despite the common goal of maximizing the total adoption of solar lanterns, we found that donors have divergent views on where to provide subsidies in the supply chain and what to subsidize. Even within the same donor organization such as USAID, different units offer financial supports at different echelons in the supply chain. This finding motivated us to better understand where and how much should the donor subsidize in a (development) supply chain. To do so, we analyze the following three supply chain settings, which we observed in our field study:

Supply chain setting 1: Selling a single product through a single retailer. In Setting 1, a major distributor (Eneji Pwop) imports and sells Nokero's solar lanterns directly to consumers through micro-entrepreneurs (Figure 1). Nokero is a US-based company who sources its production from China and sells its solar lanterns in over 120 countries. Eneji Pwop received indirect subsidies via zero- or low-interest loans provided by Kiva.org (a non-profit crowdfunding-based impact investor). These micro-entrepreneurs used micro loans from Kiva to purchase products and resell to low-income end customers around Haiti.

Supply chain setting 2: Selling two substitutable products through a single retailer. Besides Nokero's solar lanterns, Eneji Pwop sells solar lanterns from another OEM, Greenlight Planet, a "for-profit" social business (Figure 2). Greenlight Planet contracts with a manufacturer in China to produce

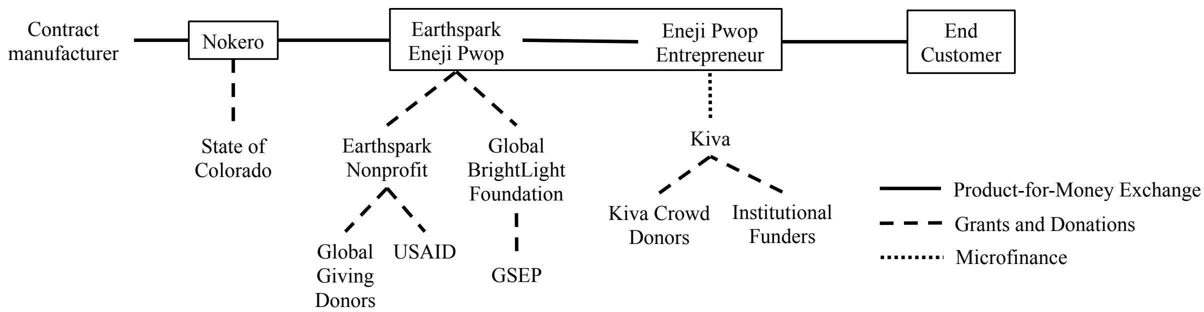

Figure 1 **One retailer, Eneji Pwop, selling product from Nokero**

solar lanterns and solar home systems for sale in 54 countries. Greenlight Planet has received investments from a number of impact investors, including Bamboo Finance, the Overseas Private Investment Corporation (OPIC), and Ashden. Eneji Pwop received indirect subsidies from Kiva.org as stated above. (Notice from Figure 2 that the Greenlight Planet solar lanterns are distributed by Total Haiti - a division of Total, the French oil-and-gas multinational.)

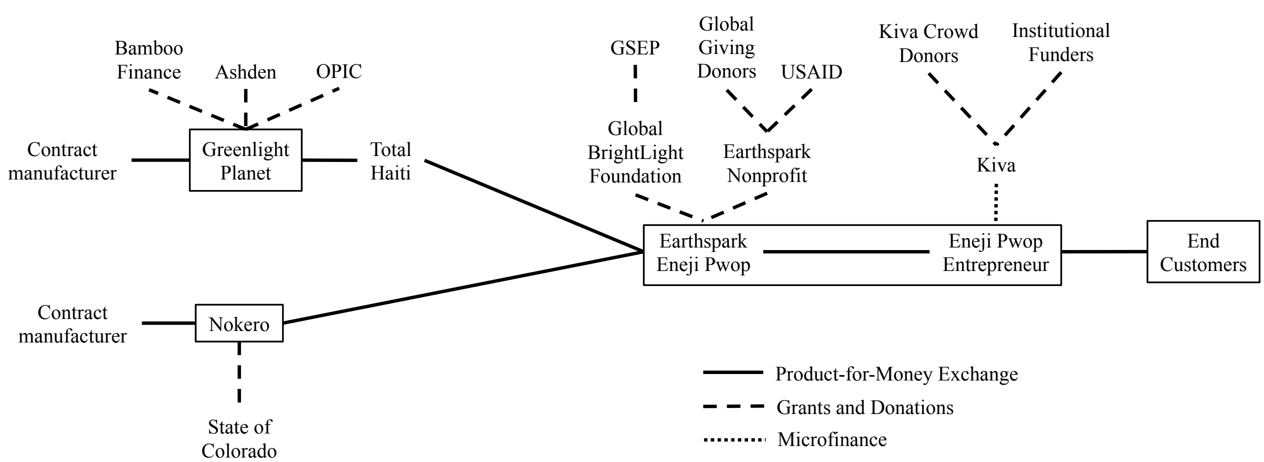

Figure 2 **One retailer, Earthspark Eneji Pwop, selling products from Greenlight Planet and Nokero**

Supply chain setting 3: Selling two substitutable products separately through two competing retailers. In Haiti, MicamaSoley and Sogexpress are major distributors of d.light and Ekotek solar lanterns, respectively (Figure 3). Solar lanterns from d.light have higher perceived quality because they are *certified* by Lighting Global. MicamaSoley sells d.light solar lanterns at wholesale prices to micro-entrepreneur women who accept Fonkoze (or other subsidized) micro-loans. Fonkoze is a large Washington DC-based non-profit MFI who uses donor grants to supplement revenues from loans, enabling it to offer micro-loans at low interest rates. Sogexpress distributes Ekotek's solar lanterns using a similar setup.

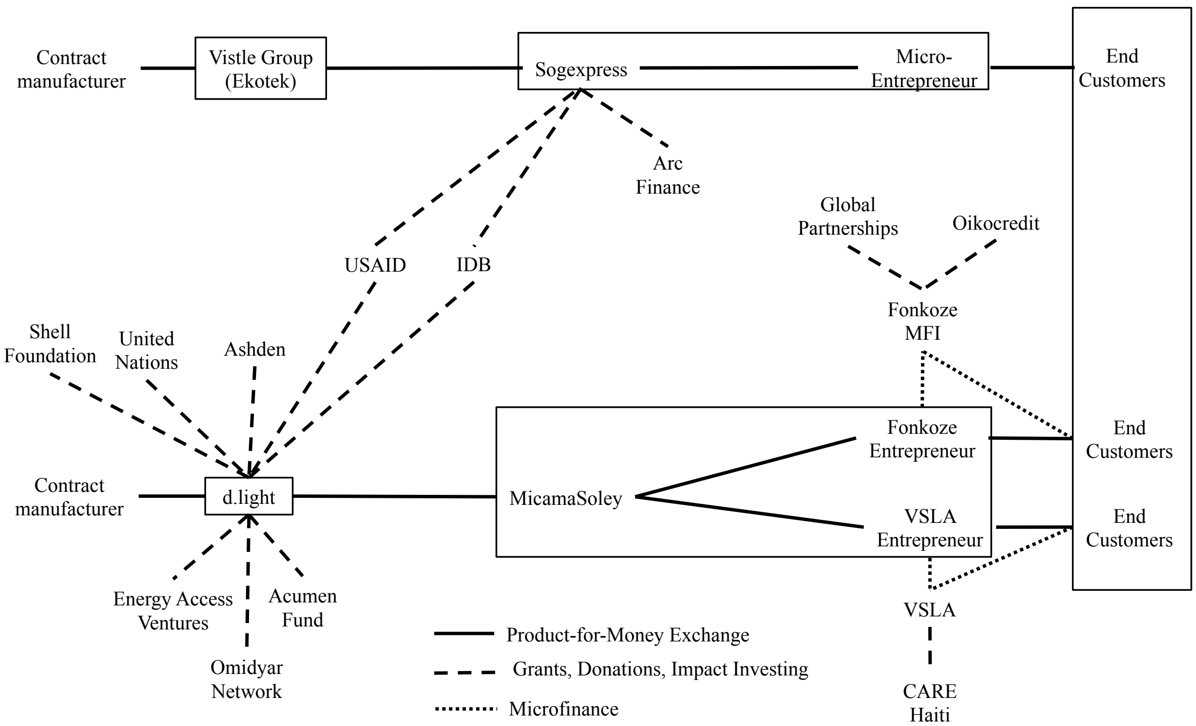

Figure 3 Two retailers, MicamaSoley and Sogexpress, selling substitutable products from d.light and Ekotek

In the next section, we develop three separate models to analyze subsidies in these three settings. Our goal is to determine the optimal subsidy strategy for donors to maximize the impact of their planned budget when the wholesale price is exogenous. (We shall extend this to endogenous wholesale price in Section 4.)

3. Base Model: Exogenous wholesale price

We present a stylized three-echelon model of those three supply chain settings as described in Section 2. In our model, we "aggregate" importers, distributors, and retailers all as "retailer" into a single echelon, and model "manufacturer" and "beneficiaries" in two separate echelons. The wholesale price w is exogenous (as discussed in finding 1 of our field study). This occurs when the manufacturer has established a common wholesale price w for different countries, and will not change its wholesale price for the market in question due to "parallel imports arbitrage" concerns. Import cost and distribution cost are taken to be zero; however, our analysis can be easily extended to including various costs: import tax, logistics and distribution costs. To capture findings 2 and 3 of our field study in our base model, the three supply-chain entities and the donor make the following decisions:

1. Given a planned budget K, the donor selects the *retailer subsidy* s_r and the *beneficiary subsidy* s_b to maximize product adoption. (Because the wholesale price w is exogenous in the base model, there is no incentive for the donor to offer subsidy s_m to the manufacturer because such subsidy will not support the donor's cause to increase product adoption. However, when the wholesale price is endogenous (Section 4), the donor may offer subsidy s_m to the manufacturer.

2. When the wholesale price w is exogenous, the manufacturer is passive in the base model, but becomes active in choosing the endogenous wholesale price w in an extension of the base model in Section 4.

3. Given the subsidies (s_r, s_b) and the wholesale price w, the retailer sets its retail price p to maximize its profit.

4. Given the subsidy s_b and the retail price, each beneficiary decides whether to purchase the product or not. To capture heterogeneity among customers (cf., Lilien et al. 2010), we assume beneficiaries (consumers) value the product (e.g., d.light's solar lantern) as v, where $v \sim U[0,1]$ so that beneficiaries with $v \geq (p - s_b)$ will purchase the product.

We model the above interactions as a three-stage Stackelberg game when the wholesale price is exogenous. (The analysis in Section 4 for endogenous wholesale price will involve a four-stage Stackelberg game.) In this three-stage game, the donor is the leader who determines the subsidies (s_r, s_b) to be given to the retailer and the beneficiary; respectively. Given these subsidies, the retailer is the follower who sets the retail price p, and, finally, the beneficiaries decide whether to purchase the product or not. (In

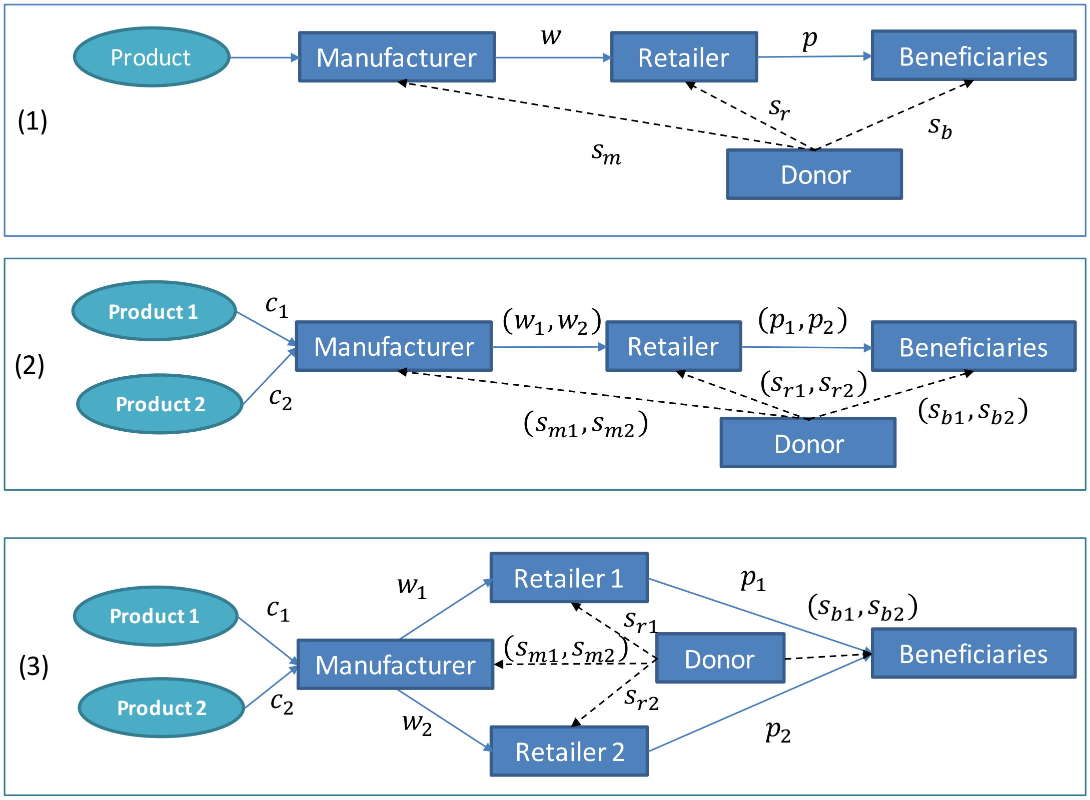

Figure 4 **Three supply chain settings: (1) single product, one retailer, (2) two substitutable products, one retailer, (3) two substitutable products, each with separate retailer.**

the base model, we scale the size of the beneficiary market M to 1, however, in Section 5, we extend this analysis to the case where the market size M is uncertain with $E(M) = 1$.) In the remainder of this section, we use backward induction to solve different variants of this 3-stage Stackelberg game by considering three different supply chain structures (Figure 4) in which a single manufacturer sells: (1) a single product through one retailer, (2) two substitutable products through one retailer, or (3) two substitutable products separately through two competing retailers. For each setting, we determine the optimal subsidy and the corresponding maximum product adoption. Then we compare these equilibrium outcomes across these three settings.

Before we solve the Stackelberg game for each setting (Figure 4) with $s_m = 0$ as the wholesale w is exogenous, let us first formulate the donor's problem involving two products (taking the single product case in setting 1 as a special case). Let D_1 and D_2 be the demand for product 1 and 2 so that the donor's total subsidy cost is equal to $(s_{r1}+s_{b1})D_1+(s_{r2}+s_{b2})D_2$. Given a budget K, the donor's problem is:

$$\max_{(s_{r1},s_{r2};s_{b1},s_{b2})} \quad (D_1+D_2) \tag{1}$$

$$\textit{subject to} \qquad (s_{r1}+s_{b1})D_1+(s_{r2}+s_{b1})D_2 \le K. \tag{2}$$

Next, we analyze the donor's problem for each setting as depicted in Figure 4.

3.1. Setting 1: Selling a single product through a single retailer

In setting 1 (Figure 4-1), the manufacturer sells one product through a single retailer. For any given retail price p and subsidy s_b, the product demand $D = 1-(p-s_b)$ because the beneficiary valuation $v \sim U[0,1]$. Anticipating this demand function along with any given subsidy s_r, the retailer solves:

$$\pi_r(s_b,s_r) = \max_p\{[p-(w-s_r)]\cdot[1-(p-s_b)]\}, \tag{3}$$

which yields optimal retail price $p^*(s_b,s_r)$ satisfying:

$$p^*(s_b,s_r) = \frac{1+w}{2}+\frac{s_b-s_r}{2}. \tag{4}$$

The first term $\frac{1+w}{2}$ corresponds to the base retail price, and the second term represents the upward (downward) adjustment in retail price in response to the subsidy (s_r,s_b).

It follows from (4) and $D=1-(p-s_b)$, the corresponding $D=\frac{1-w}{2}+\frac{s_b+s_r}{2}$ so that the beneficiary demand is increasing in the subsidies (s_b,s_r). To ensure that demand is always greater than 0 even when there is no subsidy, we should assume $w<1$. Similarly, we get $\pi_r(s_b,s_r)=\frac{(1-w+(s_b+s_r))^2}{4}$ via substitution so that the retailer's profit is non-negative and increasing in the donor's subsidies (s_b,s_r). By denoting $s \equiv s_b+s_r$ as the "total subsidy level", we can express $s=2D-1+w$. We assume the budget is reasonably low so that the donor cannot use K to set subsidy $s \ge w$ so that the effect cost of the lantern is essentially free. To ensure $s<w$, we assume that $K<\frac{1}{2}\cdot w$. By using $s=2D-1+w$, the donor's problem (1) can be reformulated as:

$$\max_D \quad D \quad \text{s.t.} \quad (2D-1+w)\cdot D \le K \tag{5}$$

By noting that the adoption D and total subsidy cost (left hand side of the budget constraint) are increasing in D, we can show that the budget constraint is binding and then obtain:

PROPOSITION 1. *When selling a single product through a single retailer, the optimal demand* $D^* = \frac{1-w+\sqrt{(1-w)^2+8K}}{4}$ *and the optimal subsidy* $s_b^* + s_r^* \equiv s^* = \frac{-(1-w)+\sqrt{(1-w)^2+8K}}{2}$*, where* s^* *is increasing in* w *and* K*.*

Proposition 1 implies that, although the optimal subsidies (s_b^*, s_r^*) are not unique to the supply chain entities, the total subsidy value s^* across the supply chain is uniquely defined. Also, when the exogenous wholesale price w is higher, the donor will also increase its subsidy s^* to compensate the retailer/beneficiary accordingly. However, due to the binding budget constraint, a higher subsidy would yield a lower demand D^* as w increases.

The single product case yields two key results: (1) the budget constraint is binding, and (2) it does not matter whether the donor subsidize the retailer or the beneficiary as long as the total subsidy s^* is maintained at the optimal level. We now examine whether these two results will hold in settings 2 and 3 (Figure 4), and examine which configuration would yield a higher product adoption.

3.2. Setting 2: Selling two substitutable products through a single retailer

When selling two substitutable products (Figure 4-2), we assume that these two products have different beneficiary valuations, where $v_1 \sim U[0,1]$, and $v_2 = \delta \cdot v_1$ with $\delta > 1$. (We assume that the wholesale price $w_2 > w_1$ to rule out product 2 dominating product 1 completely as a trivial case. Analogous to setting 1, we also assume $w_1 < 1$ and $w_2 < \delta$ to ensure the demand of each product will not always be 0.) Because the wholesale price is product-specific, we consider the case where retail price p_i and the subsidies (s_{r_i}, s_{b_i}) offered by the donor are product-specific, $i = 1, 2$.

Consider the beneficiary purchasing decision for any given retail price p_i and subsidy s_{b_i} for product $i = 1, 2$. By using the effective price $(p_i - s_{b_i})$ and valuations of different products, a beneficiary will buy product 1 if $v_1 > (p_1 - s_{b_1})$ and $v_1 - (p_1 - s_{b_1}) > v_2 - (p_2 - s_{b_2})$; buy product 2 if $v_2 > (p_2 - s_{b_2})$ and $v_2 - (p_2 - s_{b_2}) > v_1 - (p_1 - s_{b_1})$; and buy nothing, otherwise. By using $v_1 \sim U[0,1]$, $v_2 = \delta \cdot v_1$, demand D_i for either product i satisfies:

$$D_1 = \frac{(p_2 - s_{b_2}) - \delta(p_1 - s_{b_1})}{\delta - 1}, \quad D_2 = 1 - \frac{(p_2 - s_{b_2}) - (p_1 - s_{b_1})}{\delta - 1} \tag{6}$$

Anticipating the demand D_i in (6) along with any given subsidy s_{r_i}, the retailer solves: $\pi_r(s_{b_i}, s_{r_i}; i = 1,2) = \max_{p_1,p_2} \sum_{i=1}^{2} \{(p_i - (w_i - s_{r_i})) \cdot D_i\}$. It is easy to check then that the optimal retail price $p_i^*(s_{b_i}, s_{r_i})$ for $i = 1,2$ satisfies:

$$p_1^*(s_{b_i}, s_{r_i}; i = 1,2) = \frac{1+w_1}{2} + \frac{s_{b_1} - s_{r_1}}{2}, \quad p_2^*(s_{b_i}, s_{r_i}; i = 1,2) = \frac{\delta + w_2}{2} + \frac{s_{b_2} - s_{r_2}}{2} \tag{7}$$

The optimal retail price possesses the same properties as the optimal price p^* as stated in (4); we omit details. Substituting p_i^* into D_1 and D_2 and by denoting $s_1 \equiv s_{b_1} + s_{r_1}$ and $s_2 \equiv s_{b_2} + s_{r_2}$, we get:

$$D_1 = \frac{\delta(s_1 - w_1) - (s_2 - w_2)}{2(\delta - 1)}, \; D_2 = \frac{(\delta - 1) - (s_1 - w_1) + (s_2 - w_2)}{2(\delta - 1)}. \tag{8}$$

From (8), $s_1 = 2(D_1 + D_2) + (w_1 - 1)$ and $s_2 = 2(D_1 + \delta D_2) + (w_2 - \delta)$. By considering donor's budget constraint $s_1 D_1 + s_2 D_2 \leq K$, the donor's problem (1) can be reformulated as:

$$\max_{D_1, D_2} \; D_1 + D_2 \quad \text{s.t.} \quad [2(D_1 + D_2) + (w_1 - 1)] \cdot D_1 + [2(D_1 + \delta D_2) + (w_2 - \delta)] \cdot D_2 \leq K. \tag{9}$$

By noting that the objective function of (9) is increasing in both D_1 and D_2, the budget constraint will be binding if the subsidy cost $s_1 D_1 + s_2 D_2$ (i.e., the left hand side of the budget constraint of problem (9)) is also increasing in D_1 and D_2. By differentiating the subsidy cost function $f_1(D_1, D_2) = 2D_1^2 + 2\delta D_2^2 + 4D_1 D_2 + (w_1 - 1)D_1 + (w_2 - \delta)D_2$ with respect to D_1 and D_2, we find that the function is indeed increasing in D_1 and D_2. In this case, we can conclude that the donor's budget constraint is binding so that the budget constraint will satisfy as an equation. By treating the budget constraint (9)) as a quadratic equation, we can determine the optimal D_1^* as a function of D_2, getting $D_1^* = \frac{1}{4} \cdot [1 - w_1 - 4D_2 + \sqrt{(4D_2 - 1 + w_1)^2 - 8[D_2(w_2 - \delta) + 2D_2^2\delta - K}]$. Through substitution, the donor's problem (9) simplifies to a single-variable problem:

$$\max_{D_2 \geq 0} \; \frac{1}{4} \cdot [1 - w_1 + \sqrt{(4D_2 - 1 + w_1)^2 - 8[D_2(w_2 - \delta) + 2D_2^2\delta - K}]. \tag{10}$$

By solving this, we determine the optimal demand D_2^* and then retrieve D_1^*, getting:

Proposition 2. *When selling two substitutable products through a single retailer, the donor's optimal subsidy s_i^* and the corresponding optimal (D_1^*, D_2^*) satisfy:*

1. *When* $\delta - w_2 > 1 - w_1$, $(D_1^*, D_2^*) = (\frac{w_2-\delta w_1}{4(\delta-1)} + \frac{1}{4}\sqrt{\frac{(w_1-w_2)^2}{\delta-1} + w_1^2 - 2w_2 + \delta + 8K}, \frac{(\delta-w_2)-(1-w_1)}{4(\delta-1)})$, $(s_1^*, s_2^*) = (\frac{1}{2}(w_1 - 1 + \sqrt{8K + w_1^2 - 2w_2 + \frac{(w_1-w_2)^2}{\delta-1} + \delta}), \frac{1}{2}(w_2 - \delta + \sqrt{8K + w_1^2 - 2w_2 + \frac{(w_1-w_2)^2}{\delta-1} + \delta}))$;
2. *When* $\delta - w_2 \leq 1 - w_1$, $(D_1^*, D_2^*) = (\frac{1}{4}(1 - w_1 + \sqrt{8K + (1-w_1)^2}), 0)$, *and* $(s_1^*, s_2^*) = (\frac{1}{2}(w_1 - 1 + \sqrt{8K + (1-w_1)^2}), \frac{1}{2}(1 - w_1 + \sqrt{8K + (1-w_1)^2}) + w_2 - \delta)$.

Also, the total demand under setting 2: $D_1^* + D_2^* \geq \frac{1}{4}(1 - w_1 + \sqrt{8K + (1-w_1)^2})$.

Proposition 2 possesses structure similar to Proposition 1: the optimal subsidies $(s_{b_i}^*, s_{r_i}^*)$ are not unique but the total subsidy per unit s_i^* for product i is uniquely defined. Next, observe that the valuations of products 1 and 2 are bounded above by 1 and δ and that the retail prices are bounded below by w_1 and w_2 (without subsidies), we can interpret $(1 - w_1)$ and $(\delta - w_2)$ as the maximum consumer surplus for products 1 and 2; respectively. In this case, when $(\delta - w_2) \leq (1 - w_1)$, the second statement reveals that when product 2 offers a lower maximum surplus than product 1. Hence, there is no demand for product 2 in equilibrium so that $D_2^* = 0$, and the problem reduces to the single product case as in setting 1. However, when $(\delta - w_2) > (1 - w_1)$, the first statement implies that both products have positive demands in equilibrium and the last statement reveals that the total demand $D_1^* + D_2^*$ is higher than the demand obtained in the single product case as in Proposition 1 for setting 1. Proposition 2 *implies* the two key results in setting 1, and even though the products are substitutable, offering product choice to beneficiaries can increase the *total* demand in line with the donor's goals.

Uniform subsidies. Instead of product-specific subsidies, let us examine a special case in which the subsidy is uniform so that $s_1 = s_2 = s$. This special setting occurs in various situations as explained in Taylor and Xiao (2014). By substituting $s_1 = s_2 = s$ into (8), we can solve the simplified donor's problem (below) and obtain the following result:

$$\max_s \quad \frac{s - w_1 + 1}{2} \quad \text{s.t.} \quad s \cdot \frac{s - w_1 + 1}{2} \leq K \tag{11}$$

Corollary 1. *When the donor offers uniform subsidy, the optimal per unit subsidy* $s^* = \frac{-(1-w_1)+\sqrt{(1-w_1)^2+8K}}{2}$ *and the total demand* $D_1^* + D_2^* = \frac{1-w_1+\sqrt{8K+(1-w_1)^2}}{4}$.

Corollary 1 reveals that, relative to Proposition 1 in setting 1, the total demand for selling two products ($D_1^* + D_2^*$) is the same when the donor offers uniform subsidies. Combining this observation with the above discussion about Proposition 2, we gain a new insight: When selling two substitutable products through a single retailer, the increase in the total demand is not driven by offering more products to the beneficiaries. Instead, the increase is driven by the *product-specific* subsidies.

3.3. Setting 3: Selling two products through two competing retailers

Consider the case when two substitutable products are sold separately through two competing retailers (Figure 4-3), where $v_1 \sim U[0,1]$ and $v_2 = \delta \cdot v_1$ with $\delta > 1$. Noting that the beneficiary purchasing decision is driven purely by the retail price p_i and the subsidy s_{b_i} for product $i = 1, 2$, it is easy to check that the demand for each product is as shown in (6).

Anticipating the demand function D_i as in (6) along with any subsidy s_{r_i}, the retailer i's problem can be formulated as $\pi_i(s_{b_i}, s_{r_i}; i = 1, 2) = \max_{p_i}\{(p_i - (w_i - s_{r_i})) \cdot D_i\}$. In this case, it is easy to check that the "best response" functions are: $p_1^*(p_2) = \frac{(p_2 - s_{b_2}) + \delta w_1}{2\delta} + \frac{s_{b_1} - s_{r_1}}{2}$ and $p_2^*(p_1) = \frac{(\delta-1)+(p_1 - s_{b_1}) + w_2}{2} + \frac{s_{b_2} - s_{r_2}}{2}$. By considering these two equations simultaneously, we obtain retailer i's equilibrium retail price p_i^e:

$$p_1^e = \frac{(\delta - 1) - (s_2 - w_2) - 2\delta(s_1 - w_1)}{4\delta - 1} + s_{b_1},\ p_2^e = \frac{2\delta(\delta - 1) - 2\delta(s_2 - w_2) - \delta(s_1 - w_1)}{4\delta - 1} + s_{b_2} \tag{12}$$

where $s_i \equiv s_{b_i} + s_{r_i}$. By substituting p_i^e into D_1 and D_2 given in (6), it can be shown that the equilibrium demand for each product i can be expressed as:

$$D_1 = \frac{\delta(\delta - 1) - \delta(s_2 - w_2) + \delta(2\delta - 1)(s_1 - w_1)}{(4\delta - 1)(\delta - 1)},\ D_2 = \frac{2\delta(\delta - 1) + (2\delta - 1)(s_2 - w_2) - \delta(s_1 - w_1)}{(4\delta - 1)(\delta - 1)} \tag{13}$$

By using (D_1, D_2) given above, we can express subsidy $s_1 = \frac{2\delta - 1}{\delta} D_1 + D_2 + (w_1 - 1)$ and $s_2 = D_1 + (2\delta - 1)D_2 + (w_2 - \delta)$. By using the same approach as before, the donor's problem can be simplified as:

$$\max_{D_1, D_2} \quad D_1 + D_2 \quad \text{s.t.} \quad \frac{2\delta - 1}{\delta} {D_1}^2 + 2D_1D_2 + (2\delta - 1){D_2}^2 + (w_1 - 1)D_1 + (w_2 - \delta)D_2 \le K \tag{14}$$

Like before, we show that the subsidy cost (i.e., the left hand side of (14)) is increasing in D_1 and D_2 so that the budget constraint is binding. By using this result, we get:

PROPOSITION 3. *When selling two substitutable products through two competing retailers, the optimal demand* (D_1^*, D_2^*) *satisfies* $\frac{2\delta-1}{\delta}{D_1^*}^2 + 2D_1^*D_2^* + (2\delta-1){D_2^*}^2 + (w_1-1)D_1^* + (w_2-\delta)D_2^* = K$ *so that the corresponding optimal subsidy* s_i^* *satisfies the binding budget constraint. Moreover, the optimal subsidy* $(s_{b_i}^*, s_{r_i}^*)$ *are not unique, but the total subsidy per unit* s_i^* *for product* i *is uniquely determined.*

For setting 3, Proposition 3 implies that the two key results obtained from the single product case in setting 1 continue to hold: it does not matter whether the donor subsidizes the retailer or the beneficiaries as long as the optimal subsidy is maintained at the optimal level.

By comparing the outcome of setting 2 and setting 3, we can obtain the following corollary:

COROLLARY 2. *Selling two substitutable products through two competing retailers instead of a single retailer can achieve a higher total demand (i.e.,* $D_1^* + D_2^*$*).*

Corollary 2 implies that the total demand reach can be increased further by when the manufacturer sells its substitutable products through two competing retailers. Also, Corollary 2 and Proposition2 show that both retailer competition and having substitutable products can help to increase total demand. As such, the donor should (1) offer different (optimal) subsidies to different products; (2) encourage more retailers to distribute the products to increase adoption of essential products.

Uniform Subsidies. When the donor offers the same subsidy across products with $s_1 = s_2 = s$, using (13), the donor's problem becomes:

$$\max_s \quad \frac{3\delta + (s-w_2) + 2\delta(s-w_1)}{4\delta-1} \quad \text{s.t.} \quad s \cdot \frac{3\delta + (s-w_2) + 2\delta(s-w_1)}{4\delta-1} \le K \tag{15}$$

and we obtain:

COROLLARY 3. *When the donor offers uniform subsidies, the optimal subsidy* s^* *is lower in setting 3 with two competing retailers than in setting 2 with one retailer. However, the total demand* $D_1^* + D_2^*$ *is higher in setting 3 than in setting 2.*

Recall from Corollary 1 that the donor cannot increase the total demand relative to setting 1 if it offers uniform subsidies in setting 2. In contrast, Corollary 3 above states that, even when the donor offers uniform subsidies in Setting 3, the donor can still increase the total demand, relative to settings 1 and 2, by selling two products through two competing retailers. Therefore, while substitutable products create product-competition, selling them separately through two competing retailers in setting 3 intensifies retail-competition and each retailer reduces its retail price. As such, the donor can afford to reduce its uniform subsidy s^* in setting 3. However, because the budget constraint is binding with $s^* \cdot (D_1^* + D_2^*) = K$, total demand will increase even as the unit subsidy s^* decreases.

In summary, our base model offers the following insights: (1) the optimal per unit subsidy is not unique: the donor can offer per unit subsidy to the retailers and/or to the beneficiaries, as long as the total subsidy per unit is set at a certain optimal level; (2) by introducing substitutable products, the donor can increase the total reach of the products; however, product-specific subsidies (not uniform subsidies) is effective for boosting the total demand; (3) the donor should support a supply chain structure embedded with retail-competition to further boost total demand. Recognizing the fact that these insights rely on the assumptions: (a) exogenous wholesale price; and (b) fixed market size $M = 1$. We consider two extensions in the following two sections by relaxing these assumptions to see if these results are robust.

4. Extension 1: Endogenous wholesale price

We now extend our base model to the case when the manufacturer sets its wholesale price to maximize its profit, thus extending the 3-stage Stackelberg game earlier to a four-stage Stackelberg game involving beneficiaries, the retailer, the manufacturer, and the donor as the leader and analyze the same three supply chain settings (Figure 4).

4.1. Setting 1: Selling one product through a single retailer

For setting 1 of the model thus extended from setting 1 for the base model (Figure 4-1), we can show that product demand D is equal to $1 - (p - s_b)$ for any given wholesale price w, subsidy (s_m, s_r, s_b) and retail price p. The retailer's problem is the same as (3) so that the optimal price $p^* = \frac{1+w}{2} + \frac{s_b - s_r}{2}$ and the corresponding demand $D = \frac{1-w}{2} + \frac{s_b+s_r}{2}$ remains the same as before. Anticipating this demand function for any wholesale price w and unit cost c, and subsidy s_m, the manufacturer solves:

$$\pi_m = \max_{w} \quad (w + s_m - c) \cdot \left(\frac{1-w}{2} + \frac{s_b + s_r}{2} \right) \tag{16}$$

By showing that optimal wholesale price $w^* = \frac{1+c}{2} + \frac{s-s_m}{2}$, the corresponding demand $D = \frac{1-c}{4} + \frac{s_b+s_r+s_m}{4}$. By letting the total subsidy $s' \equiv s_b + s_r + s_m$, the donor solves:

$$\max_{s'} \quad D \equiv \frac{1-c}{4} + \frac{s'}{4} \quad \text{s.t. } s' \cdot (\frac{1-c}{4} + \frac{s'}{4}) \leq K \tag{17}$$

Because the objective function and the subsidy cost (i.e., left hand side of (17)) are increasing in s', the budget constraint is binding and we get:

PROPOSITION 4. *When selling one product through a single retailer and when the wholesale price is endogenously determined by the manufacturer, the optimal total subsidy* $s'^* = \frac{-(1-c)+\sqrt{(1-c)^2+16K}}{2}$ *and the optimal demand* $D^* = \frac{(1-c)+\sqrt{(1-c)^2+16K}}{8}$.

Proposition 4 is analogous to Proposition 1: the total subsidy per unit s'^* is uniquely determined but the optimal subsidies (s_b^*, s_r^*, s_m^*) are not. Also, the optimal subsidy s'^* is increasing in the production cost c and the budget K, while the optimal total demand is decreasing in production cost c and increasing in K. Moreover, "double marginalization" persists: the manufacturer's profit $\pi_m^* = 2(D^*)^2$, which is twice that of the retailer $\pi_r^* = (D^*)^2 > 0$.

4.2. Setting 2: Selling two products through a single retailer

We now consider Setting 2 as depicted in Figure 4-2. For any given w_i and subsidy (s_{mi}, s_{ri}, s_{bi}) for product i, the retailer's pricing problem is the same as Setting 2 as presented in Section 3.2: the retail price p_i^* is given in (7) and the corresponding demand function D_i is given in (8). Hence, for any given subsidy (s_{m1}, s_{m2}), the manufacturer solves:

$$\max_{w_1, w_2} \quad \sum_{i=1,2} (w_i + s_{m_i} - c_i) \cdot D_i, \tag{18}$$

where D_i is given by (8) and c_i is the per unit production cost. The optimal wholesale price w_i^* satisfies:

$$w_1^* = \frac{1}{2}(1 + c_1 + s_1 - s_{m_1}), \quad w_2^* = \frac{1}{2}(c_2 + s_2 - s_{m_2} + \delta) \tag{19}$$

By substituting w_1^* and w_2^* into (8) and by denoting the total subsidy $s_1' \equiv s_1 + s_{m_1}$ and $s_2' \equiv s_2 + s_{m_2}$, where s_1 and s_2 represent the total retailer and beneficiary subsidies as in the base case, we get:

$$D_1 = \frac{(c_2 - s_2') + (s_1' - c_1)\delta}{4(\delta - 1)}, \quad D_2 = \frac{\delta - 1 + (c_1 - s_1') - (c_2 - s_2')}{4(\delta - 1)} \tag{20}$$

It follows from (20), we get: $s_1' = -1 + c_1 + 4(D_1 + D_2)$, $s_2' = -\delta + c_2 + 4(D_1 + \delta D_2)$ so that we can express the budget constraint $s_1' D_1 + s_2' D_2 \leq K$ in terms of D_i. Hence, the donor's problem becomes:

$$\max_{D_1, D_2} \quad D_1 + D_2 \quad \text{s.t. } [-1 + c_1 + 4(D_1 + D_2)] \cdot D_1 + [-\delta + c_2 + 4(D1 + \delta D_2)] \cdot D_2 \leq K \tag{21}$$

By using the same argument as presented in Section 3.2, we can show that the budget constraint is binding so that the optimal demand (D_1^*, D_2^*) satisfies $4(D_1^*)^2 + 4\delta(D_2^*)^2 + 8D_1^* D_2^* + (c_1 - 1)D_1^* + (c_2 - \delta)D_2^* = K$. The binding budget constraint yields $D_1^* = -D_1 + \frac{1}{8}[1 - c_1 + \sqrt{(-1 + c_1 + 8D_2)^2 - 16(4\delta D_2^2 - K + D_2(c_2 - \delta)}]$. Through substitution, the donor's problem can be further simplified:

$$\max_{D_2 \geq 0} \quad \frac{1}{8}[1 - c_1 + \sqrt{(-1 + c_1 + 8D_2)^2 - 16(4\delta D_2^2 - K + D_2(c_2 - \delta)}] \tag{22}$$

Proposition 5. *When selling two substitutable products through a single retailer and when the wholesale price is endogenous, we get:*

1. *When* $\delta - c_2 > 1 - c_1$, $D_1^* = \frac{c_2 - c_1\delta}{8(\delta - 1)} + \frac{1}{8}\sqrt{c_1^2 - 2c_2 + 16K + \frac{(c_1 - c_2)^2}{\delta - 1} + \delta}$, $D_2^* = \frac{\delta - c_2 - (1 - c_1)}{8(\delta - 1)}$*; and*
2. *When* $\delta - c_2 \leq 1 - c_1$, $D_1^* = \frac{1}{8}[(1 - c_1) + \sqrt{(1 - c_1)^2 + 16K}]$, $D_2^* = 0$.

Also, the optimal total subsidy level $(s_1'^*, s_2'^*) = (-1 + c_1 + 4(D_1^* + D_2^*), -\delta + c_2 + 4(D_1^* + \delta D_2^*))$.

Analogous to Proposition 2, Proposition 5 suggests that the structural results remain the same even when the wholesale price is endogenous; i.e., (a) the optimal subsidies $(s_{b_i}^*, s_{r_i}^*, s_{m_i}^*)$ are not unique but the total subsidy per unit $s_i'^*$ for product i is uniquely defined; and (2) the total demand under setting 2 (i.e., $D_1^* + D_2^*$) will always be greater than the total demand under setting 1 with one product (i.e., $D = \frac{1}{4}(1 - c_1 + \sqrt{8K + (1 - c_1)^2})$.

Uniform subsidies. When the donor offers uniform subsidies so that $s_1' = s_2' = s'$, we can use the same approach as before to get:

Corollary 4. *When selling two substitutable products through a single retailer and when the donor offers uniform subsidy across both products, the optimal per unit subsidy* $s'^* = \frac{-(1 - c_1) + \sqrt{(1 - c_1)^2 + 16K}}{2}$*, and the total demand* $D_1^* + D_2^* = \frac{(1 - c_1) + \sqrt{(1 - c_1)^2 + 16K}}{8}$.

Observe that Corollary 4 is analogous to Corollary 1: when selling two products through one retailer by using uniform subsidy, the donor cannot increase the total demand. Hence, the result obtained from Setting 2 in the based model continued to hold: the increase in the total demand is driven by the product-specific subsidy, not by having more products.

4.3. Setting 3: Selling two products separately through two competing retailers

By noting that this setting is akin to Setting 3 as presented in Section 3.3, one can check that the retailer's pricing problem is the same as in Section 3.3. So the retail price p_i^* is given by (12) and the corresponding demand function D_i is given by (13), where $s_1 \equiv s_{b_1} + s_{r_1}$ and $s_2 \equiv s_{b_2} + s_{r_2}$. In this case, the manufacturer's problem is $\max_{w_1,w_2} \sum_{i=1,2}(w_i + s_{m_i} - c_i) \cdot D_i$. Also, the optimal wholesale price and the the corresponding demand satisfy:

$$w_1^* = \frac{1}{2}(1 + c_1 + s_1 - s_{m_1}), \quad w_2^* = \frac{1}{2}(\delta + c_2 + s_2 - s_{m_2}), \tag{23}$$

$$D_1 = \frac{\delta[\delta - 1 - (s_2' - c_2) + (2\delta - 1)(s_1' - c_1)]}{2(\delta - 1)(4\delta - 1)}, \quad D_2 = \frac{2\delta(\delta - 1) + (2\delta - 1)(s_2' - c_2) + \delta(s_1' - c_1)}{2(\delta - 1)(4\delta - 1)}, \tag{24}$$

where $s_1' \equiv s_1 + s_{m_1}$ and $s_2' \equiv s_2 + s_{m_2}$. Through (24), we can express s_1' and s_2' as: $s_1' = c_1 - 1 + 2[(2 - \frac{1}{\delta})D_1 + D_2]$ and $s_2' = c_2 - \delta + 2[D_1 + (2\delta - 1)D_2]$ so that the donor's problem can be written as:

$$\max_{D_1,D_2} \quad D_1 + D_2 \quad \text{s.t. } (c_1 - 1 + 2[(2 - \frac{1}{\delta})D_1 + D_2]) \cdot D_1 + (c_2 - \delta + 2[D_1 + (2\delta - 1)D_2]) \cdot D_2 \leq K \tag{25}$$

As before, by showing that the subsidy cost (i.e., left hand side (25)) is increasing in D_1 and D_2, we get:

PROPOSITION 6. *When selling two substitutable products separately through two competing retailers and when the wholesale price is endogenous,*

1. *the donor's budget constraint (25) is binding;*
2. *The optimal subsidy* $(s_{b_i}^*, s_{r_i}^*, s_{m_i}^*)$ *are not unique but the total subsidy per unit* $s_i'^*$ *for product* i *is uniquely determined; and*
3. *Selling through two competing retailers will generate a higher total demand than selling through a single retailer.*

Furthermore, when the donor offers uniform subsidies, the optimal subsidy (i.e., $s_1 = s_2 = s$*), the optimal subsidy* $s'^* = \frac{-(3\delta - c_2 - 2\delta c_1) + \sqrt{(3\delta - c_2 - 2\delta c_1)^2 + 4(4\delta - 1)(2\delta + 1)K}}{2(2\delta + 1)}$ *and the corresponding total demand* $D_1^* + D_2^* = \frac{3\delta - c_2 - 2\delta c_1 + \sqrt{(3\delta - c_2 - 2\delta c_1)^2 + 4(4\delta - 1)(2\delta + 1)K}}{2(4\delta - 1)}$.

Proposition 6 shows that the donor can achieve a higher product adoption in Setting 3 than in Setting 2. In summary, when the wholesale price is endogenous, the results from the base model continued to hold: (1) the budget constraint is binding; (2) it does not matter who to subsidize as long as the total subsidy $s_i^{'}$ is maintained at the optimal level; (3) product-specific subsidies can enable the donor achieve a higher demand; and (4) retail competition can enable the donor to generate a higher demand.

5. Extension 2: Market Uncertainty

Instead of assuming that the market size $M = 1$ in the base model, we now extend our base model to the case when M follows a probability density function $f(m)$ with $m \in (0, \infty)$. Following the modeling approach of Taylor and Xiao (2014), there are five steps:

1. The donor determines and announces the subsidies s_k for entity $k = r, b$.
2. The retailer knows the density function $f(m)$ and selects the order quantity z.
3. The retailer observes the realized market size $M = m$.
4. The retailer decides on the retail price p by taking the order quantity z and the realized market size m into consideration.
5. The beneficiary demand D is realized.

This extension is more complex because, as noted in Steps 2 that the retailer selects the order quantity "before", but he decides on the retail price "after" the market size is realized as in Step 4. To solve the 3-stage Stackelberg game for each of the three settings, we use backward induction beginning with Step 5 and ending with Step 1. In the remainder of this section, we first characterize the optimal total subsidy level for each setting for the case when the probability density function $f(m)$ of the market size M is continuous. Then, by considering the case when the market size M follows the uniform or the normal distribution, we show numerically that the key results obtained from the base model continued to hold.

5.1. Setting 1: Selling one product through one retailer

Consider Setting 1 as depicted in Figure 4-1 (base case with $s_m = 0$). For any given subsidy s_b, any realized market size m and any retail price p, the beneficiary demand (in step 5) is given by $D = (1 - p + s_b) \cdot m$.

Retailer's pricing problem. Observe from step 4 that the retailer's pricing problem takes place "after" the order z is placed and the market size m is realized. Therefore, the ordering cost $w \cdot z$ is sunk, the actual sales $S = \min\{D, z\}$, where $D = (1 - p + s_b) \cdot m$, and the retailer's pricing problem for any given subsidy s_r is: $\max_p \quad (p + s_r) \cdot \min\{(1 - p + s_b) \cdot m,\ z\}$, which can be reformulated as:

$$\max_p \quad (p + s_r) \cdot (1 - p + s_b) \cdot m \quad \text{s.t.} \quad (1 - p + s_b) \cdot m \le z. \tag{26}$$

By solving the above problem, the optimal price satisfies:

$$p^* = \begin{cases} \frac{1+s_b-s_r}{2} & \text{if } m \le \frac{2z}{1+s_b+s_r} \\ 1 + s_b - \frac{z}{m} & \text{if } m \ge \frac{2z}{1+s_b+s_r} \end{cases} \tag{27}$$

By denoting the total subsidy $s \equiv s_b + s_r$, the corresponding sale $S = \min\{D, z\}$ and the retailer's per unit revenue $p^* + s_r$ are:

$$S = \begin{cases} \frac{1+s}{2} \cdot m & \text{if } m \le \frac{2z}{1+s} \\ z & \text{if } m \ge \frac{2z}{1+s} \end{cases}, \qquad p^* + s_r = \begin{cases} \frac{1+s}{2} & \text{if } m \le \frac{2z}{1+s} \\ 1 + s - \frac{z}{m} & \text{if } m \ge \frac{2z}{1+s} \end{cases} \tag{28}$$

Retailer's ordering problem. Observe that the sales S and the retailer's revenue $(p^* + s_r\ )$ depends only on the total subsidy s. Hence, it suffices to focus on s only when we examine the retailer's ordering that takes place "before" the market size is realized as in step 2. For any given per unit total subsidy $s = s_r + s_b$, the retailer's (ex-post) profit is:

$$\Pi_r(m) = (p^* + s_r) \cdot S - w \cdot z = \begin{cases} (\frac{1+s}{2})^2 \cdot m - w \cdot z & \text{if } m \le \frac{2z}{1+s} \\ (1 + s - \frac{z}{m} - w) \cdot z & \text{if } m \ge \frac{2z}{1+s} \end{cases}. \tag{29}$$

To maximize the retailer's (ex-ante) expected profit, the retailer's ordering problem is:

$$\max_z \ E_m[\Pi_r(m)] = \int_0^{\frac{2z}{1+s}} [(\frac{1+s}{2})^2 m - wz] f(m) \mathrm{d}m + \int_{\frac{2z}{1+s}}^{\infty} (1 + s - \frac{z}{m} - w) z f(m) \mathrm{d}m. \tag{30}$$

By differentiating $E_m[\Pi_r(m)]$ with respect to z and by applying the Leibniz rule, we get:

$$\frac{\partial E_m[\Pi_r(m)]}{\partial z} = \int_{\frac{2z}{1+s}}^{\infty} (1 + s - \frac{2z}{m}) \cdot f(m) \mathrm{d}m - w$$

By considering the first order condition and by using the implicit function theorem, we get:

Proposition 7. *When selling one product through one retailer, the retailer's optimal ordering decision* z^* *satisfies* $\int_{\frac{2z^*}{1+s}}^{\infty}(1+s-\frac{2z^*}{m})\cdot f(m)\mathrm{d}m - w = 0$. *Also, the optimal order quantity* z^* *is increasing in the donor's subsidy* s *and decreasing in the wholesale price* w.

Donor's problem. By substituting the optimal retailer's ordering quantity $z = z^*$ given in Proposition 7 into the sales function S as given above, the expected sale S is:

$$E_M[S] = \int_0^{\frac{2z^*}{1+s}}(\frac{1+s}{2}m)f(m)\mathrm{d}m + \int_{\frac{2z^*}{1+s}}^{\infty} z^* f(m)\mathrm{d}m = \frac{1+s}{2}E[M] - \frac{1}{2}\int_{\frac{2z^*}{1+s}}^{\infty}[((1+s)m - 2z^*)f(m)]\mathrm{d}m, \tag{31}$$

where $\int_{\frac{2z^*}{1+s}}^{\infty}[(1+s-\frac{2z^*}{m})\cdot f(m)]\mathrm{d}m = w$. As such, the donor's problem in step 1 can be written as:

$$\max_s \quad E_M[S] \ \text{ s.t. } s\cdot E_M[S] \leq K. \tag{32}$$

Proposition 8. *When selling one product through one retailer, the donor's budget constraint is binding so that the optimal subsidy* s^* *satisfies* $s^*\cdot E_M[S] = K$. *Also, the donor's optimal subsidy* s^* *is increasing in the budget* K *and the wholesale price* w.

Proposition 8 reveals that even when market size is uncertain, the key results obtained in the base case in Section 3.1 continue to hold.

5.2. Setting 2: Selling two substitutable products through one retailer

We now consider setting 2 (Figure 4-2) corresponding to the base case with $s_{m1} = s_{m2} = 0$. To obtain tractable results, we focus on the following scenario: (a) the donor offers uniform subsidies so that the subsidy is product-independent; (b) product 1 has a long replenishment lead time so that the retailer needs to place the order z_1 "before" the market size is realized. However, product 2 has a short lead time so that the retailer can place the order z_2 "after" the market size is realized. (Observe that this scenario can occur when product 1 is sourced from afar and product 2 is sourced nearby.)

In the remainder of this section, we shall analyze settings 2 and 3 as depicted in Figure 4-2 and Figure 4-3 by focusing on this scenario. We begin with step 5. For any given wholesale price, per unit subsidy, market size, and retail price, the demand function is equal to (6) multiplied by m: $D_1 = m\cdot\frac{(p_2-s_{b_2})-\delta(p_1-s_{b_1})}{\delta-1}$, $D_2 = m\cdot[1-\frac{(p_2-s_{b_2})-(p_1-s_{b_1})}{\delta-1}]$.

Retailer's pricing problem. Recall that the retailer's pricing problem in step 4 occurs after the order z_1 is placed and the market size m is realized. Hence, the ordering cost for product 1; i.e., $w_1 \cdot z_1$ is sunk, and the actual sales for product 1 is $S_1 = \min\{D_1, z_1\}$, where $D_1 = m \cdot \frac{(p_2 - s_{b_2}) - \delta(p_1 - s_{b_1})}{\delta - 1}$. However, because product 2 is ordered after the market size is realized, $z_2^* = D_2 = S_2$. Hence, we only need to determine the optimal order quantity for product 1. For any given subsidy s_r for the retailer, we can use the same approach as in setting 1 to show that the retailer's pricing problem is $\max_{p_1, p_2} \quad (p_1 + s_{r_1}) \cdot S_1 + (p_2 + s_{r_2} - w_2) \cdot S_2$, which can be reformulated as:

$$\max_{p_1, p_2} \quad (p_1 + s_{r_1}) \cdot D_1 + (p_2 + s_{r_2} - w_2) \cdot D_2 \quad \text{s.t.} \quad D_1 \leq z_1. \tag{33}$$

By considering the first order condition and by defining a threshold $M_1 = \frac{2z_1(\delta-1)}{\delta s_1 + w_2 - s_2}$, the optimal retail price (p_1^*, p_2^*) and the corresponding sale (S_1, S_2) satisfy:

$$p_1^* = \begin{cases} \frac{1}{2}(1 + s_{b_1} - s_{r_1}) & \text{if } m \leq M_1 \\ \frac{1}{2m\delta}[-2z_1(\delta-1) + m(\delta + w_2 - s_2 + 2s_{b_1}\delta)] & \text{if } m > M_1 \end{cases}, \quad p_2^* = \frac{1}{2}(s_{b_2} - s_{r_2} + w_2 + \delta) \tag{34}$$

$$S_1 = \begin{cases} m \cdot \frac{\delta \cdot s_1 + w_2 - s_2}{2(\delta-1)} & \text{if } m \leq M_1 \\ z_1 & \text{if } m > M_1 \end{cases}, \quad S_2 = \begin{cases} m \cdot \frac{\delta - 1 - s_1 + s_2 - w_2}{2(\delta-1)} & \text{if } m \leq M_1 \\ \frac{1}{2\delta} \cdot [-2z_1 + m(s_2 - w_2 + \delta)] & \text{if } m > M_1 \end{cases} \tag{35}$$

Retailer's ordering problem. From (34) and (35), the retailer's profit in step 2 can be written as:

$$\begin{aligned} \Pi_r(m) &= (p_1^* + s_{r_1}) \cdot S_1 - w_1 z_1 + (p_2^* + s_{r_2} - w_2) \cdot S_2 \\ &= \begin{cases} \frac{m}{4(\delta-1)}[(s_2 - w_2)(s_2 - w_2 - 2(s_1+1) + (s_1^2 - 1 + 2s_2 - 2w_2)\delta + \delta^2] - w_1 z_1 & \text{if } m \leq M_1 \\ \frac{1}{4m\delta}[-4z_1^2(\delta-1) + m^2(s_2 - w_2 + \delta)^2 - 4mz_1(s_2 - w_2 - \delta(s_1 - w_1))] & \text{if } m > M_1. \end{cases} \end{aligned}$$

By letting $\Pi_{r,1}(m)$ and $\Pi_{r,2}(m)$ be $\Pi_r(m)$ when $m \leq M_1$ and $m > M_1$; respectively, the retailer's (ex-ante) expected profit is:

$$E_M[\Pi_r(m)] = \int_0^{M_1} \Pi_{r,1}(m) \cdot f(m)\mathrm{d}m + \int_{M_1}^{\infty} \Pi_{r,1}(m) \cdot f(m)\mathrm{d}m. \tag{36}$$

Hence, the retailer's ordering problem is: $\max_{z_1} \quad E_M[\Pi_r(m)]$, and

$$\frac{\partial E_M[\Pi_r(m)]}{\partial z_1} = \int_0^{M_1} (-w_1) \cdot f(m)\mathrm{d}m + \int_{M_1}^{\infty} [\frac{-2z_1(\delta-1)}{m\delta} - \frac{1}{\delta}((s_2 - w_2) - (s_1 - w_1)\delta)] \cdot f(m)\mathrm{d}m.$$

By considering the first order condition and by applying the implicit function theorem, we get:

PROPOSITION 9. *When selling two substitutable products through one retailer, the retailer's optimal order quantity for product 1* z_1^* *satisfies* $\int_{\frac{2z_1^*(\delta-1)}{\delta s_1+w_2-s_2}}^{\infty}[\frac{-2(\delta-1)z_1^*}{m\delta}+\frac{\delta s_1-s_2+w_2}{\delta}]\cdot f(m)\mathrm{d}m-w_1=0$*, where* z_1^* *is increasing in* s_1 *and* w_2 *and decreasing in* s_2 *and* w_1.

Donor's problem. When the donor offers uniform subsidy so that $s_1=s_2=s$, we can use the optimal order quantity z_1^* given in Proposition 9 along with the sales function given in (35) to show that the expected total sales is:

$$E_M[S_1+S_2]=\int_0^{M_1}\frac{s+1}{2}\cdot m\cdot f(m)\mathrm{d}m+\int_{M_1}^{\infty}(\frac{\delta-1}{\delta}z_1^*+\frac{m(s-w_2+\delta)}{2\delta})\cdot f(m)\mathrm{d}m. \tag{37}$$

Given the budget K, the donor's problem in step 1 is:

$$\max_s \quad E_M[S_1+S_2] \text{ s.t. } E_M[s\cdot(S_1+S_2)]\le K \tag{38}$$

PROPOSITION 10. *When selling two substitutable products through one retailer, the donor's budget constraint is binding: the optimal subsidy* s^* *satisfies* $s^*\cdot[\int_0^{\frac{2z_1^*(\delta-1)}{s^*(\delta-1)+w_2}}\frac{s^*+1}{2}\cdot m\cdot f(m)\mathrm{d}m+\int_{\frac{2z_1^*(\delta-1)}{s^*(\delta-1)+w_2}}^{\infty}(\frac{\delta-1}{\delta}z_1^*+\frac{m(s^*-w_2+\delta)}{2\delta})\cdot f(m)\mathrm{d}m]=K$.

When the market size is uncertain, the key results obtained in the base case as presented in Section 3.2 continued to hold.

5.3. Setting 3: Selling two substitutable products separately through two retailers

Consider Setting 3 as depicted in Figure 4-3 (base case with $s_{m1}=s_{m2}=0$). By considering the same scenario as described in the last subsection, we can show that for any given wholesale price, per unit subsidy, market size, and retail price, the demand function in step 5 is given in (13) multiplied by m. Because the order for product 1 (i.e., z_1) is placed by retailer "before" the market size is realized, the sales for product 1 is given by $S_1=\min\{D_1,z_1\}$. However, retailer 2 can postpone it ordering decision of product 2 "after" the market size is realized so that $z_2^*=D_2$ and the sales for product 2 is equal to $S_2=D_2$. It remains to focus on retailer 1's order quantity z_1.

Retailers' pricing problem. By using the same arguments as presented in the last subsection for setting 2, retailers' pricing problem in step 4 can be formulated as follows:

$$\max_{p_1} \quad (p_1+s_{r_1})\cdot D_1 \text{ s.t. } D_1=m\cdot\frac{(p_2-s_{b_2})-\delta(p_1-s_{b_1})}{\delta-1}\le z_1,\text{and}$$
$$\max_{p_2} \quad (p_2+s_{r_2}-w_2)\cdot m\cdot[1-\frac{(p_2-s_{b_2})-(p_1-s_{b_1})}{\delta-1}]. \tag{39}$$

By solving the above two pricing problems simultaneously and by defining a threshold for m as $M_2 = \frac{z_1\cdot(\delta-1)\cdot(4\delta-1)}{(1+2s_1)\delta^2-\delta(1+s_1+s_2-w_2)}$, the equilibrium retail price and the equilibrium sales satisfy:

$$p_1^* = \begin{cases} \frac{1+s_{b_1}+s_2-w_2-\delta-2\delta\cdot s_{b_1}+2\delta\cdot s_{r_1}}{1-4\delta} & \text{if } m \le M_2 \\ \frac{m(-1-s_{b_1}-s_2+w_2+\delta+2\delta s_{b_1})-2z_1(\delta-1)}{m(2\delta-1)} & \text{if } m > M_2 \end{cases}, \quad p_2^* = \begin{cases} \frac{s_{b_2}(1-2\delta)+\delta(2+s_1+2s_{r_2}-2w_2-2\delta)}{1-4\delta} & \text{if } m \le M_2 \\ \frac{z_1(1-\delta)+m(s_{b_2}(\delta-1)+\delta(\delta-1-s_{r_2}+w_2))}{m(2\delta-1)} & \text{if } m > M_2, \end{cases} \tag{40}$$

$$S_1 = \begin{cases} m\cdot\frac{-\delta(1-w_2+s_1+s_2)+(1+2s_1)\delta^2}{(\delta-1)(4\delta-1)} & \text{if } m \le M_2 \\ z_1 & \text{if } m > M_2 \end{cases}, \quad S_2 = \begin{cases} m\cdot\frac{w_2-(2+s_1+2w_2)\cdot\delta+2\delta^2+(2\delta-1)\cdot s_2}{(\delta-1)(4\delta-1)} & \text{if } m \le M_2 \\ \frac{-z_1+m(s_2-w_2+\delta)}{2\delta-1} & \text{if } m > M_2 \end{cases} \tag{41}$$

Retailer's ordering problem. Because retailer 2 can postpone its ordering decision of product 2 until after the market size is realized, the order quantity $z_2^* = D_2 = S_2$. It remains to solve retailer 1's ordering problem for product 1 in step 2. For any order quantity z_1, we can use the retail price p_1^* and the sales of product 1 S_1 as stated above to compute retailer 1's (ex-post) profit for selling product 1, where:

$$\Pi_{r_1}(m) = (p_1^*+s_{r_1})\cdot S_1 - w_1z_1 = \begin{cases} \frac{m\delta(1+s_1+s_2-w_2-\delta-2s_1\delta)^2}{(1-4\delta)^2(\delta-1)} - w_1z_1 & \text{if } m \le M_2 \\ (s_1+\frac{w_2-1+\delta-s_2}{2\delta-1}+\frac{2z_1(1-\delta)}{m(2\delta-1)}-w_1)\cdot z_1 & \text{if } m > M_2 \end{cases} \tag{42}$$

As before, we use $\Pi_{r_1,1}(m)$ and $\Pi_{r_1,2}(m)$ to represent the profit of retailer 1 under the cases when $m \le M_2$ and $m \ge M_2$; respectively. Hence, retailer 1's (ex-ante) expected profit is:

$$E_M[\Pi_{r_1}(m)] = \int_0^{M_2}\Pi_{r_1,1}(m)\cdot f(m)\mathrm{d}m + \int_{M_2}^{\infty}\Pi_{r_1,2}(m)\cdot f(m)\mathrm{d}m, \tag{43}$$

and retailer 1's ordering problem is: $\max_{z_1}\ E_M[\Pi_{r_1}(m)]$.

PROPOSITION 11. *When selling two substitutable products through two separate retailers, retailer 1's optimal ordering quantity z_1^* satisfies* $\int_{\frac{z_1^*\cdot(\delta-1)\cdot(4\delta-1)}{(1+2s_1)\delta^2-\delta(1+s_1+s_2-w_2)}}^{\infty}(s_1+\frac{w_2-1+\delta-s_2}{2\delta-1}+\frac{4z_1^*(1-\delta)}{m(2\delta-1)})\cdot f(m)\mathrm{d}m - w_1 = 0.$

Donor's problem. When the donor offers uniform subsidy with $s_1 = s_2 = s$, we can use sales functions given above to determine the expected total sale (i.e., $E_M[S_1+S_2]$). From (41) and formulate the donor's problem in step 1 as:

$$\max_s \quad E_M[S_1+S_2] \quad \text{s.t. } s\cdot E_M[S_1+S_2] \le K, \text{ where} \tag{44}$$

$$E_M[S_1+S_2]=\int_0^{M_2}\frac{3\delta+(s-w_2)+2\delta s}{4\delta-1}\cdot m\cdot f(m)\mathrm{d}m+\int_{M_2}^{\infty}\frac{2(\delta-1)z_1^*+m(s-w_2+\delta)}{2\delta-1}\cdot f(m)\mathrm{d}m \quad (45)$$

By using the same approach as in Setting 2, we get:

PROPOSITION 12. *When selling two products through two separate retailers, the budge constraint is binding: the donor's optimal subsidy s^* satisfies* $s^*\cdot[\int_0^{\frac{z_1^*\cdot(\delta-1)\cdot(4\delta-1)}{(1+2s^*)\delta^2-\delta(1+2s^*-w_2)}}\frac{3\delta+(s^*-w_2)+2\delta s^*}{4\delta-1}\cdot m\cdot f(m)\mathrm{d}m+\int_{\frac{z_1^*\cdot(\delta-1)\cdot(4\delta-1)}{(1+2s^*)\delta^2-\delta(1+2s^*-w_2)}}^{\infty}\frac{2(\delta-1)z_1^*+m(s^*-w_2+\delta)}{2\delta-1}\cdot f(m)\mathrm{d}m]=K.$

By reviewing the results presented in Propositions 8, 10, 12 in this section, we can conclude that, when the market size is uncertain, our two key results obtained from the base model continued to hold; namely, the donor can offer per unit subsidy to the retailers and/or to the beneficiaries, as long as the total subsidy per unit is set at a certain optimal level; and the donor's budget constraint is binding. To complete our analysis, it remains to examine whether it is still true that the donor can increase the total demand by introducing substituting products (as in Settings 2 and 3) and by supporting a supply chain structure that entails retail competition (as in Setting 3).

Comparisons to examine these issues analytically are intractable as no closed-form expressions are available for general probability distribution $f(m)$ so we investigate numerically.

Numerical comparison. We consider four cases: (1) market size M is deterministic with $m=1$; (2) market size $M\sim N(1,0.04)$; (3) market size $M\sim N(1,0.09)$; (4) market size $M\sim U[0,2]$, noting that $E[M]=1$ in all cases. In our numerical analysis, we set the exogenously given wholesale price as $w_1=0.5$ (for the single product in Setting 1, and for product 1 in Settings 2 and 3 when there are two products), $w_2=0.8$ (for product 2 in Settings 2 and 3), and set the valuation multiplier for product 2 $\delta=1.2$.

For each of these 4 cases, we compute the optimal uniform subsidy s^* and the optimal expected total sales S^* by varying the budget K from 0 to 0.18. Our numerical results are summarized in Figures 5, 6, 7, and 8. In each figure, we depict the optimal subsidy s^* on the left panel and the total expected sales S^* on the right panel.

We now observe:

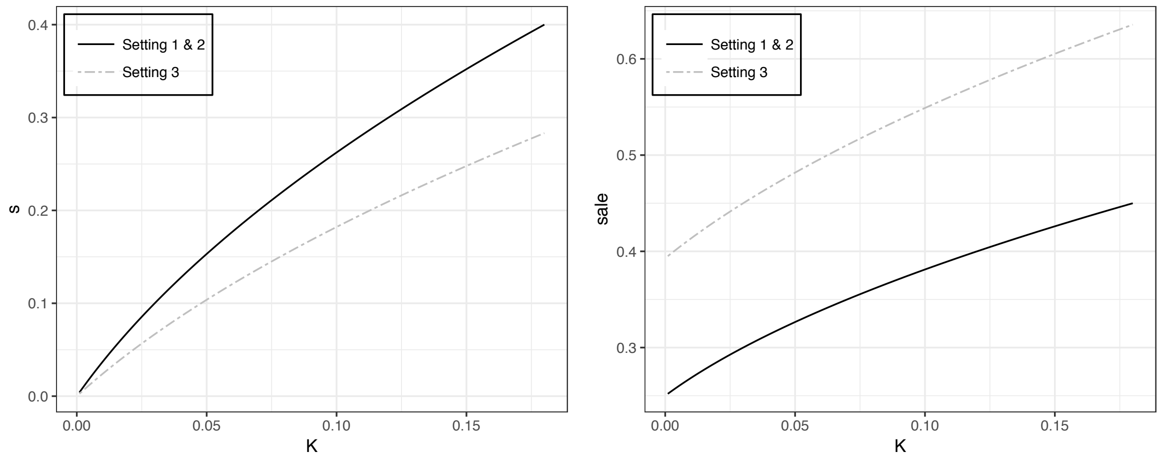

Figure 5 Optimal uniform subsidy (left) and the corresponding total sale (right) when the market size M is deterministic with $m = 1$.

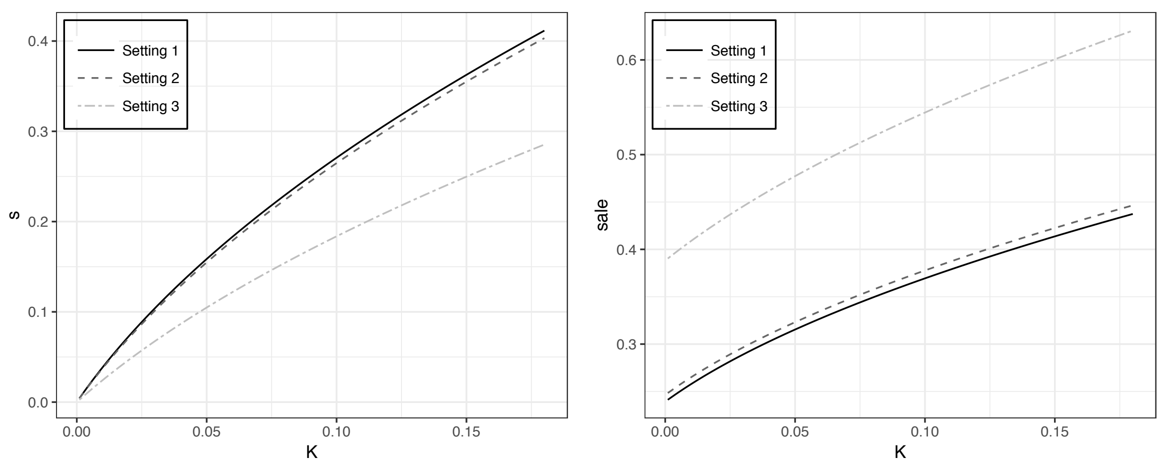

Figure 6 Optimal uniform subsidy (left) and the corresponding total sale (right) when the market size $M \sim N(1, 0.04)$

1. The optimal per unit subsidy s^* is the lowest in setting 3, followed by that in setting 2. This implies that the donor can afford to reduce its per unit subsidy s^* when there are more product available in the market (as in Setting 2), and can reduce the subsidy even further when there is retail competition (as in setting 3).

2. Combining observation 1 above with our analytical observation that the budget constraint is binding in all three settings, we see that the optimal total sales is the highest in Setting 3, followed by Setting 2 for any given budget K.

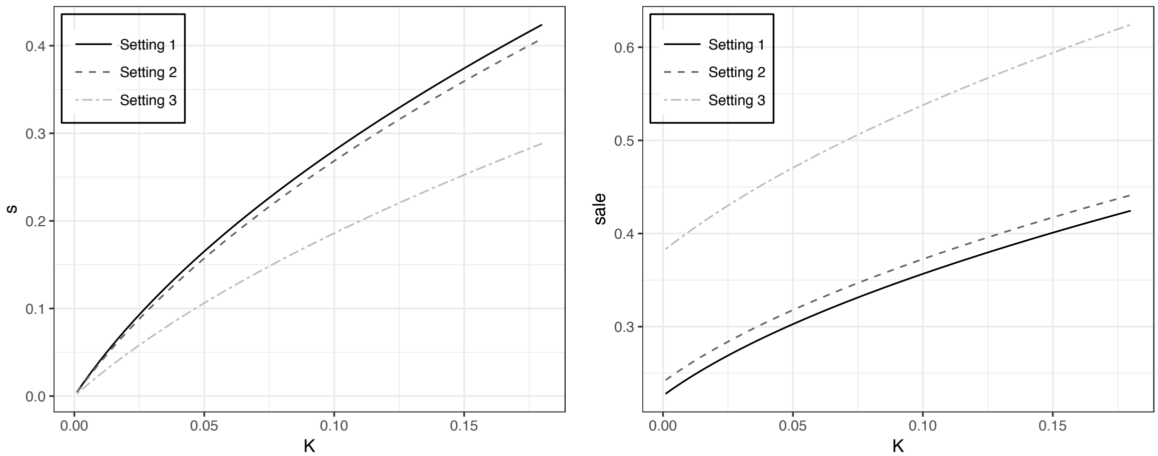

Figure 7 **Optimal uniform subsidy (left) and the corresponding total sale (right) when the market condition** $M \sim N(1, 0.09)$

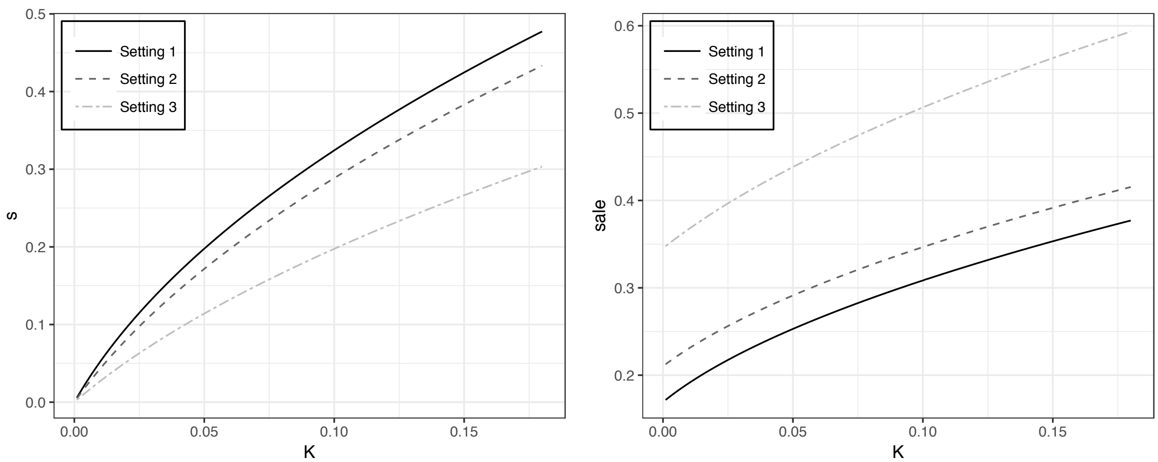

Figure 8 **Optimal uniform subsidy (left) and the corresponding total sale (right) when the market condition** $M \sim U[0, 2]$

3. Both the optimal subsidy s^* and the total sale are increasing in the budget K under all three supply chain structures.

4. When market size is deterministic, Figure 5 illustrates Corollary 1: when the donor offers uniform subsidy, the donor cannot increase the total demand in setting 2. As discussed in Section 3, the increase in the total demand in setting 2 is driven by product-specific subsidies, not by offering one more products. This explains why the total demand are the same for both settings 1 and 2 (when the donor offers uniform subsidy). However, as shown in Corollary 3, despite the "uniform subsidy" effect, Figure 5

verifies that retail competition in Setting 3 can enable the donor to obtain a higher total demand even when the donor offers uniform subsidy.

5. The variance of the market size M increases as we progress from Figure 6 to Figure 7, and from Figure 7 to Figure 8. By comparing the total demand across Figures 5, 6, 7, and 8, we notice that the result obtained in Corollary 1 for the deterministic case (Figure 5) is no longer valid when the market size is uncertain. Specifically, s the variance of the market size M increases, the donor can obtain a higher total demand by selling two products through one retailer in setting 2 compared to selling one product through one retailer in setting 1 even when the donor offers uniform subsidy. Hence, product choice can increase total demand further when the underlying market size is uncertain. The result in Corollary 3 continues to hold when the market size is uncertain: retail competition can enable the donor to obtain a higher total demand even when the market size is uncertain.

We conclude from these observations that the results obtained in the base model are robust in that they continue to hold even when the market size is uncertain, More importantly, we find the demand increase – in setting 3 over setting 2 and the in setting 2 over setting 1 – becomes more pronounced when the market size becomes more uncertain.

6. Conclusion

We introduced development supply chains as a 'hybrid of commercial and humanitarian supply chains. Using three settings of three-echelon stylized supply-chain model, we modeled competition among donor, manufacturers, retailers and consumers as a 4-stage Stackelberg game. These settings incorporated product substitution, retail competition, and demand uncertainty. Results from different variations of this game obtained using backward induction indicated that the donor can subsidize any echelon as long as the total subsidy per unit is maintained at the optimal level. Having more product choice (especially when the subsidies are product-specific) and more channel choice can increase the number of beneficiaries adopting the products, and this increase is more pronounced as the market size becomes more uncertain.

Scholars have studied the question of subsidizing manufacturers or consumers in a variety of contexts with specific policy goals such as consumer welfare with: private and public firms (cf. Myles 2002;

Poyago-Theotoky 2001; Scrimitore 2014); promotion of investment in a particular sector, say telecommunication (cf. Jeanjean 2010); or promotion of environmental sustainability (cf. Bansal and Gangopadhyay 2003). Part of environmental sustainability (and energy security) is adoption of solar technology among consumers (cf. Lobel and Perakis 2011; Cohen, Lobel, and Perakis 2016). There is also the rationale for health of the poor with subsidies for malaria medication in many developing countries (cf. Levi et al. 2017; Taylor and Xiao 2014). Other contexts for subsidies include *education* (Schultz, 2004); *electricity* (Goodarzi et al., 2015); *food* (Peeters and Albers, 2013); *housing* (Gilbert, 2004); and *smallholders farmers* (Tang et al., 2015). As regards energy or lighting specifically, there is the question of empirically establishing *willingness to pay* (Yoon et al. 2016), consumer adoption of *alternative lighting products* (Uppari et al. 2017), and *supply chain coordination* for photovoltaic modules (Chen and Su, 2014).

Our work complements this literature by specifically analyzing product-specific subsidies in different supply-chain settings with substitutable products and retail competition. For instance, Levi et al. (2017) investigate uniform subsidies to competing manufacturers of drugs against malaria. Like us, the donor's goal is maximizing adoption but, we show that *product-specific subsidies* are better than uniform subsidies in our context for the donor's objective. Also, our scope is the entire supply chain and apart from incorporating demand uncertainty. Rather than provide only upper bounds, we provide closed-form solutions for optimal subsidy in the deterministic demand case and characterize the solution in the uncertain demand case.

Shen et al. (2016) analyze how the Chinese government should subsidize home appliances for residents in rural areas. We use a three-echelon model in contrast to their two-echelon subsidy model. Their subsidies are a fixed percentage of the retail price across all products whereas we have product-specific subsidies and in three different supply chain settings. Moreover, Shen et al. (2016) determine the optimal percentage that maximizes consumer welfare, whereas we focus on total beneficiary demand, a common measure for the effectiveness of a subsidy program. Furthermore, they assume deterministic demand whereas we include the case of uncertain demand.

Goodarzi et al. (2015) study the interaction among the regulator, manufacturer and customers, focusing on the optimal feed-in tariff policies of the regulator to minimize the grid operator's total cost. In contrast, we seek to maximize product adoption under different supply-chain structures using unit subsidies. Our paper also complements Taylor and Xiao (2014) by considering three different supply chain settings with two substitutable products and two competing retailers in contrast to their one setting with one product and one retailer. Doing so affords us a broader set of results.

A secondary contribution is to the humanitarian operations literature. Our work is motivated by the example of Haiti following the 2010 earthquake. Supply chains such as one we presented help alleviate poverty, and poverty alleviation in turn reduces vulnerability to future disasters and crises (Sodhi 2016, Sodhi and Tang 2014, Van Wassenhove and Pedraza Martinez 2012). Post-disaster development as well as poverty alleviation in general require supply chains to incorporate local communities (Hall and Matos 2010) to provide jobs and investment, and address institutional voids in low-income markets (Parmigiani and Rivera-Santos 2015).

Our work has implications for donors to maximize adoption by beneficiaries:

1. *Supply chain considerations*: Donors must map out the supply chains of the products they want to subsidize – echelons, substitutable products, and competition between retailers – and coordinate donor action along these supply chain as well as those of intersecting supply chains.

2. *Product choice and retailer competition*: Donors must encourage competition in the distribution channels and ensure there is a range of substitutable products across the quality-price range. Supporting retailers who compete in offering these different products will further the donors' objective .

3. *Choice of subsidy target*: While an important question for the donor is where to apply subsidies in the supply chain for maximum effect, our work shows that the echelon does not matter so the decision can be made only on the basis of echelon-specific transaction costs, e.g., in places with mobile money, subsidizing end customers via their mobile money accounts may be an option.

4. *Optimum subsidy*: The donor must recognize there is an optimal level of subsidy – greater levels may increase the number of potential consumers, but would only increase the donor's investment and possibly lower the profitability for the micro-entrepreneur retailer. Likewise, donors must take care not to over-subsidize certain products to price out substitutes, something that has been observed in practice.

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Appendix A: Proof

Proof of Proposition 1 By considering the budget constraint, we can obtain that $D \leq \frac{1-w+\sqrt{(1-w)^2+8K}}{4}$. As the objective function is increasing in D, we know that the optimal $D^* = \frac{1-w+\sqrt{(1-w)^2+8K}}{4}$. And we can then calculate the optimal s^* via substitution.

Proof of Proposition 2 By taking the first order derivative of $f_1(D_1, D_2)$ with respect to D_1, D_2, we get:

$$\frac{\partial f_1}{\partial D_1} = 4(D_1 + D_2) + (w_1 - 1) = 2s_1 + (1 - w_1) = 2(D_1 + D_2) + s_1 > 0,$$
$$\frac{\partial f_1}{\partial D_2} = 4(D_1 + \delta D_2) + (w_2 - \delta) = 2s_2 + (\delta - w_2) = 2(D_1 + \delta D_2) + s_2 > 0,$$

from which we know that $f_1(D_1, D_2)$ is increasing in both D_1 and D_2. As the objective function $D_1 + k \cdot D_2$ is also increasing in both D_1 and D_2, we know that the optimal D_1^* and D_2^* should satisfy the binding budget constraint (i.e., $f_1(D_1^*, D_2^*) = K$). Next, by considering the first order condition of the objective function of the donor's problem given by (10), we obtain $D_2^* = \frac{(\delta-w_2)-(1-w_1)}{4(\delta-1)}$. When $\delta - w_2 \geq 1 - w_1$, then D_2^* is feasible, else when $\delta - w_2 < 1 - w_1$, we can find that the objective function is always decreasing in D_2 when $D_2 > 0$, thus we can obtain $D_2^* = 0$. As such, we can get the corresponding D_1^* and optimal subsidy $(s_{b_i}^*, s_{r_i}^*)$ via substitution. Moreover, as $(D_1^*, D_2^*) = (\frac{1}{4}(1 - w_1 + \sqrt{8K + (1-w_1)^2}), 0)$ is always a feasible solution of donor's problem in setting 2, we know that total demand in setting 2 $D_1^* + D_2^* \geq \frac{1}{4}(1 - w_1 + \sqrt{8K + (1-w_1)^2})$.

Proof of Corollary 1 As the donor's objective function and the expense are both increasing in s, it is easy to check that the budget constraint should be binding at the optimal solution. By solving the binding budget constraint, we obtain $s^* = \frac{-(1-w_1)+\sqrt{(1-w_1)^2+8K}}{2}$. And we can obtain the corresponding D_1^* and D_2^* via substitution.

Proof of Proposition 3 By denoting the subsidy cost (i.e., the left hand side of (14)) as $f_2(D_1, D_2)$ and by taking the first order derivative of $f_2(\cdot)$ with respect to D_1, D_2, we get:

$$\frac{\partial f_2}{\partial D_1} = 2D_1 \cdot \frac{2\delta - 1}{\delta} + 2D_2 + (w_1 - 1) = 2s_1 + (1 - w_1) = \frac{2\delta - 1}{2\delta} \cdot D_1 + D_2 + s_1 > 0,$$
$$\frac{\partial f_2}{\partial D_2} = 2(2\delta - 1)D_2 + 2D_1 + (w_2 - \delta) = 2s_2 + (\delta - w_2) = (2\delta - 1)D_2 + D_1 + s_2 > 0,$$

from which we know that $f_2(D_1, D_2)$ is increasing in both D_1 and D_2. As the objective function $D_1 + D_2$ is also increasing in both D_1 and D_2, we know that the optimal D_1^* and D_2^* should satisfy the binding budget constraint (i.e., $f_2(D_1^*, D_2^*) = K$). Also, from (13), we know that D_i only depends on the total subsidy s_i for each product so that we can solve out the unique s_i based on the binding budget constraint, while the optimal $s_{b_i}^*$ and $s_{r_i}^*$ are not uniquely determined.

Proof of Corollary 2 To achieve the same demand (D_1, D_2), the donor should spend $f_1(D_1, D_2) = 2D_1^2 + 2\delta D_2^2 + 4D_1D_2 + (w_1 - 1)D_1 + (w_2 - \delta)D_2$ in setting 2 and spend $f_2(D_1, D_2) = \frac{2\delta-1}{\delta}D_1^2 + 2D_1D_2 + (2\delta - 1)D_2^2 + (w_1 - 1)D_1 + (w_2 - \delta)D_2$ in setting 3. By comparing $f_1(D_1, D_2)$ and $f_2(D_1, D_2)$, we obtain:

$$f_1(D_1, D_2) - f_2(D_1, D_2) = (2 - \frac{2\delta - 1}{\delta}) \cdot (D_1^2 + \delta D_2^2) + 2D_1D_2 > 0.$$

Hence we know that to get the same (D_1, D_2), the donor needs to spend more money in a single retailer case (i.e., setting 2) than two competing retailers case (i.e., setting 3). Recall Proposition2 and 3, the optimal solutions of the donor's problem all satisfy the binding constraint. Therefore, we know that the optimal solution $(D_{1,1}^*, D_{1,2}^*)$ of setting 2 with a single retailer satisfies $f_1(D_{1,1}^*, D_{1,2}^*) = K$. Meanwhile, we also know that $f_2(D_{1,1}^*, D_{1,2}^*) < K$, which means $(D_{1,1}^*, D_{1,2}^*)$ is not the optimal solution of setting 3 with two competing retailers. As such, we know that the optimal solution of setting 3 yields a greater total demand (i.e., the objective function $D_1 + D_2$) than the optimal solution of setting 2.

Proof of Corollary 3 We use $s^{(A)}$ and $s^{(B)}$ to denote the optimal per uniform subsidy under setting 2 and setting 3. It is easy to check that both the total demand and the donor's expense given by (15) are both increasing in the per unit subsidy $s^{(B)}$. Hence the budget constraint should be binding at the optimal solution. By solving the binding budget constraint, we obtain the optimal $s^{(B)} = \frac{-(3\delta - w_2 - 2\delta w_1) + \sqrt{(3\delta - w_2 - 2\delta w_1)^2 + 4(4\delta - 1)(2\delta + 1)K}}{2(2\delta + 1)}$ and the corresponding total demand. We define the function $s(a, b) = -a + \sqrt{a^2 + b}$, from which we can easily check that $s(a, b)$ is decreasing in a and increasing in b. And we obtain $s^{(A)} = s(\frac{1-w_1}{2}, 2K)$ and $s^{(B)} = s(\frac{3\delta - w_2 - 2\delta w_1}{2(2\delta+1)}, \frac{4\delta-1}{2\delta+1}K)$. As $\delta - w_2 > 1 - w_1$, we can obtain that $\frac{1-w_1}{2} - \frac{3\delta - w_2 - 2\delta w_1}{2(2\delta+1)} = \frac{-(\delta - w_2) + 1 - w_1}{2(2\delta+1)} < 0$. Also, as $2K > \frac{4\delta-1}{2\delta+1}K$, we can further obtain $s^{(A)} > s^{(B)}$. As the total budget K is fixed and the budget constraint is binding, we can easily get the total demand under setting 2 is lower than the setting 3.

Proof of Proposition 4 It is easy to check that the objective function $D = \frac{1-c}{4} + \frac{s'}{4}$ and the donor's subsidy cost $s' \cdot (\frac{1-c}{4} + \frac{s'}{4})$ are both increasing in s'. Hence we know that the budget constraint is binding at the optimal solution. By solving the binding budget constraint, we obtain $s' = \frac{-(1-c) + \sqrt{(1-c)^2 + 16K}}{2}$ and we then get $D^* = \frac{(1-c) + \sqrt{(1-c)^2 + 16K}}{8}$ via substitution.

Proof of Corollary 4 When the donor offers the uniform subsidy across both products, then the donor's problem can be rewritten as:

$$\begin{aligned} \max_{s'} \quad & \frac{1 - (c_1 - s')}{4} \\ s.t. \quad & s' \cdot \frac{1 - (c_1 - s')}{4} \leq K \end{aligned}$$

It is obvious that both the objective function and the total cost are increasing in s' so that we can obtain the budget constraint should be binding at the optimal solution. By solving the binding budget constraint, we get $s'^* = \frac{-(1-c_1)+\sqrt{(1-c_1)^2+16K}}{2}$ and $D_1^* + D_2^* = \frac{(1-c_1)+\sqrt{(1-c_1)^2+16K}}{8}$.

Proof of Proposition 5 By denoting $f_1(D_1, D_2)$ as the subsidy cost (i.e., the left hand side of (21)) and taking the first order derivative, we obtain:

$$\frac{\partial f_1}{\partial D_1} = [-1 + c_1 + 4(D_1 + D_2)] + 4D_1 + 4D_2 = s_1' + 4(D_1 + D_2) > 0,$$
$$\frac{\partial f_2}{\partial D_2} = [-\delta + c_2 + 4(\delta D_2 + D_1)] + 4(D_1 + \delta D_2) = s_2' + 4(D_1 + \delta D_2) > 0.$$

Hence we know that for feasible s_1', s_2', D_1, D_2, the donor's expense $f_1(D_1, D_2)$ is increasing in D_1 and D_2. As the objective function $D_1 + D_2$ is also increasing in D_1 and D_2, we know the optimal (D_1^*, D_2^*) satisfies the binding budget constraint (i.e., $[-1 + c_1 + 4(D_1^* + D_2^*)] \cdot D_1^* + [-\delta + c_2 + 4(D_1^* + \delta D_2^*)] \cdot D_2^* = K$). Next, by considering the first order condition of donor's objective function given by (22), we obtain $D_2^* = \frac{\delta - c_2 - (1-c_1)}{\delta(\delta-1)}$. When $\delta - c_2 \geq 1 - c_2$, $D_2^* > 0$ so that we can further compute $D_1^* = \frac{c_2 - c_1\delta}{8(\delta-1)} + \frac{1}{8}\sqrt{c_1^2 - 2c_2 + 16K + \frac{(c_1-c_2)^2}{\delta-1} + \delta}$ via substitution. When $\delta - c_2 < 1 - c_2$, $\frac{\delta - c_2 - (1-c_1)}{\delta(\delta-1)} < 0$ so that the objective function is always increasing in D_2 when $D_2 > 0$. Hence we get the optimal $D_2^* = 0$ and $D_1^* = \frac{1}{8}(1 - c_1) + \sqrt{(1-c_1)^2 + 16K}$.

Proof of Proposition 6 By denoting $f_2(D_1, D_2)$ as the subsidy cost (i.e., the left hand side of (25)) and taking the first order derivative of $f_2(D_1, D_2)$ with respect to D_1 and D_2, we get:

$$\frac{\partial f_2}{\partial D_1} = c_1 - 1 + 4D_2 + (8 - \frac{4}{\delta})D_1 = s_1' + 2D_2 + (4 - \frac{2}{\delta})D_1 > 0$$
$$\frac{\partial f_2}{\partial D_2} = c_2 - \delta + 4D_1 + (4\delta - 2)D_2 = s_2' + 2D_1 + (4\delta - 2)D_2 > 0$$

Hence we know for feasible s_1', s_2', D_1, D_2, the donor's expense $f_2(D_1, D_2)$ is increasing in D_1 and D_2. As the objective function $D_1 + D_2$ is also increasing in D_1 and D_2, we obtain that the optimal (D_1^*, D_2^*) should satisfy the binding budget constraint (i.e., $(c_1 - 1 + 2[(2 - \frac{1}{\delta})D_1^* + D_2^*]) \cdot D_1 + (c_2 - \delta + 2[D_1^* + (2\delta - 1)D_2^*]) \cdot D_2^* = K$), which is stated as the first statement of Proposition 6. Next, we know from (24) that D_i only depends on s_i', which also implies that the total subsidy per unit s_i' for product i is uniquely determined but the optimal subsidy $(s_{b_i}^*, s_{r_i}^*, s_{m_i}^*)$ are not unique. Finally, we show the third statement by the following. To achieve the same demand (D_1, D_2), the donor should spend $f_1(D_1, D_2)$ in the setting 2 and spend $f_2(D_1, D_2)$ in the setting 3. By comparing $f_1(D_1, D_2)$ and $f_2(D_1, D_2)$, we obtain:

$$f_1(D_1, D_2) - f_2(D_1, D_2) = \frac{2}{\delta}D_1^2 + 2D_2^2 + 4D_1D_2 > 0.$$

Hence we know that to get the same (D_1, D_2), the donor needs to spend more money in setting 2 than setting 3. As the optimal solutions of the donor's problem all satisfy the binding constraint, we know that the optimal solution $(D^*_{1,1}, D^*_{1,2})$ of setting 2 satisfies $f_1(D^*_{1,1}, D^*_{1,2}) = K$. Meanwhile, we also know that $f_2(D^*_{1,1}, D^*_{1,2}) < K$, which means $(D^*_{1,1}, D^*_{1,2})$ is not the optimal solution of setting 3. As such, we know that the optimal solution of setting 3 yields a greater total demand (i.e., $D_1 + D_2$) than setting 2.

When the donor offers the uniform subsidy across both products, the donor's problem can be rewritten as:

$$\max_{s'} \quad \frac{3\delta + (s' - c_2) + 2\delta(s' - c_1)}{4\delta - 1} \quad s.t. \quad s' \cdot \frac{3\delta + (s' - c_2) + 2\delta(s' - c_1)}{4\delta - 1} \leq K$$

It is obvious that both the objective function and the total cost are increasing in s' so that we can obtain the budget constraint should be binding at the optimal solution. By solving the binding budget constraint, we get $s'^* = \frac{-(3\delta - c_2 - 2\delta c_1) + \sqrt{(3\delta - c_2 - 2\delta c_1)^2 + 4(4\delta-1)(2\delta+1)K}}{2(2\delta+1)}$ and $D_1^* + D_2^* = \frac{3\delta - c_2 - 2\delta c_1 + \sqrt{(3\delta - c_2 - 2\delta c_1)^2 + 4(4\delta-1)(2\delta+1)K}}{2(4\delta-1)}$.

Proof of Proposition7 Then by taking the second order derivative of $E_m[\Pi_r(m)]$ with respect to z and using the Leibniz integral rule, we obtain

$$\frac{\partial^2 E_m[\Pi_r(m)]}{\partial z^2} = -\int_{\frac{2z}{1+s}}^{\infty} \frac{2}{m} \cdot f(m)\mathrm{d}m < 0$$

Hence we know the expected profit function of the retailer is concave. Hence the optimal z^* satisfies the first order condition (i.e., $\int_{\frac{2z^*}{1+s}}^{\infty} (1 + s - \frac{2z^*}{m}) \cdot f(m)\mathrm{d}m - w = 0$). We use $g(z, s, w)$ to represent the function $\int_{\frac{2z^*}{1+s}}^{\infty} (1 + s - \frac{2z^*}{m}) \cdot f(m)\mathrm{d}m - w$, and we have $g(z^*, s, w) = 0$. By taking the first order derivative of $g(z, s, w)$ with respect to z, s and w, we get:

$$\frac{\partial g}{\partial z} = -\int_{\frac{2z}{1+s}}^{\infty} \frac{2}{m} \cdot f(m)\mathrm{d}m < 0, \quad \frac{\partial g}{\partial s} = \int_{\frac{2z}{1+s}}^{\infty} f(m)\mathrm{d}m > 0, \quad \frac{\partial g}{\partial w} = -1 < 0$$

From the above, we know that $g(z, s, w)$ is increasing in s and decreasing in z and w. Hence to ensure $g(z^*, s, w) = 0$, we can easily know that z^* is increasing in s and decreasing in w.

Proof of Proposition 8 By taking the first order derivative of $E_M[S]$ with respect to s, we get:

$$\begin{aligned}\frac{\partial E_M[S]}{\partial s} =& \frac{1+s}{2} \cdot \frac{2z^*}{1+s} \cdot f(\frac{2z^*}{1+s}) \cdot \partial(\frac{2z^*}{1+s})/\partial s + \int_0^{\frac{2z^*}{1+s}} \frac{m}{2} \cdot f(m)\mathrm{d}m \\ & - z^* f(\frac{2z^*}{1+s}) \cdot \partial(\frac{2z^*}{1+s})/\partial s + \int_{\frac{2z^*}{1+s}}^{\infty} \frac{\partial z^*}{\partial s} \cdot f(m)\mathrm{d}m \\ =& \int_0^{\frac{2z^*}{1+s}} \frac{m}{2} \cdot f(m)\mathrm{d}m + \int_{\frac{2z^*}{1+s}}^{\infty} \frac{\partial z^*}{\partial s} \cdot f(m)\mathrm{d}m\end{aligned}$$

From Proposition 7 we know that z^* is increasing in s. Hence we obtain that $\frac{\partial E_M[S]}{\partial s} > 0$, which indicates that the total sale is increasing in the donor's subsidy s. With the objective function $E_M[S]$ and the total subsidy cost $s \cdot E_M[S]$ both increasing in s, we know that the optimal solution will be achieved at the binding budget constraint. With the binding budget constraint, we know that when the budget K increase, the optimal s^* will increase.

By taking the first order derivative of the subsidy cost $s \cdot E_M[S]$ with respect to z^*, we get $\frac{\partial(s \cdot E_M[S])}{\partial z^*} = s \cdot (\int_{\frac{2z}{1+s}}^{\infty} f(m)\mathrm{d}m) > 0$, from which we know the cost is increasing in z^*. As we have shown in Proposition 7 that z^* is decreasing in the wholesale price w, we obtain that the cost is decreasing in w. To ensure budget constraint is binding, we get that when w increases, the optimal s^* will increase.

Proof of Proposition 9 By taking the second order derivative of $E_M[\Pi_r(m)]$, we get:

$$\frac{\partial E_M^2[\Pi_r(m)]}{\partial z_1^2} = \frac{\partial M_1}{\partial z_1} \cdot 0 + \int_{M_1}^{\infty} (\frac{-2(\delta-1)}{m\delta}) \cdot f(m)\mathrm{d}m < 0,$$

from which we know the retailer's expected profit by selling product 1 is a concave function of z_1. By considering the first order condition, we obtain that the optimal ordering decision for product 1 (i.e., z_1^*) satisfies $\int_{\frac{2z_1^*(\delta-1)}{\delta s_1+w_2-s_2}}^{\infty} [\frac{-2(\delta-1)z_1^*}{m\delta} + \frac{\delta s_1 - s_2 + w_2}{\delta}] \cdot f(m)\mathrm{d}m - w_1 = 0$.

We use $g(z_1, s_1, s_2, w_1, w_2)$ to represent $\int_{\frac{2z_1(\delta-1)}{\delta s_1+w_2-s_2}}^{\infty} [\frac{-2(\delta-1)z_1}{m\delta} + \frac{\delta s_1 - s_2 + w_2}{\delta}] \cdot f(m)\mathrm{d}m - w_1$, and we have shown that $g(z_1^*, s_1, s_2, w_1, w_2) = 0$. By taking the first order derivative of $g(z_1, s_1, s_2, w_1, w_2)$ with respect to z_1, s_1, s_2, w_1, w_2, we get:

$$\frac{\partial g}{\partial z_1} = \int_{M_1}^{\infty} (\frac{-2(\delta-1)}{m\delta}) \cdot f(m)\mathrm{d}m < 0, \quad \frac{\partial g}{\partial s_1} = \int_{M_1}^{\infty} f(m)\mathrm{d}m > 0,$$
$$\frac{\partial g}{\partial s_2} = \int_{M_1}^{\infty} -\frac{1}{\delta} f(m)\mathrm{d}m < 0, \quad \frac{\partial g}{\partial w_1} = -1 < 0, \quad \frac{\partial g}{\partial w_2} = \int_{M_1}^{\infty} \frac{1}{\delta} f(m)\mathrm{d}m > 0$$

To ensure $g(z_1^*, s_1, s_2, w_1, w_2) = 0$, we can easily obtain that z_1^* is increasing s_1 and w_2, while is decreasing in s_2 and w_1.

Proof of Proposition 10 We use $SS_1(m)$ and $SS_2(m)$ to represent the total sales (i.e., $S_1 + S_2$) under cases when $m \le M_1$ and $m \ge M_1$, respectively; and we have $SS_1(M_1) = SS_2(M_1)$. By taking the first order derivative of $E_M[S_1 + S_2]$ with respect to s, we obtain:

$$\begin{aligned}\frac{\partial E_M[S_1+S_2]}{\partial s} =& \frac{\partial M_1}{\partial s} \cdot SS_1(M_1) \cdot f(M_1) + \int_0^{M_1} \frac{m}{2} \cdot f(m)\mathrm{d}m \\ & - \frac{\partial M_1}{\partial s} \cdot SS_2(M_1) \cdot f(M_1) + \int_{M_1}^{\infty} (\frac{\partial z_1^*}{\partial s} + \frac{m}{2\delta}) \cdot f(m)\mathrm{d}m \\ =& \int_0^{M_1} \frac{m}{2} \cdot f(m)\mathrm{d}m + \int_{M_1}^{\infty} (\frac{\partial z_1^*}{\partial s} + \frac{m}{2\delta}) \cdot f(m)\mathrm{d}m\end{aligned}$$

When $s_1 = s_2 = s$, we know that the optimal order quantity z_1^* satisfies $g(z_1^*, s, w_1, w_2) = \int_{\frac{2z_1^*(\delta-1)}{\delta s+w_2-s}}^{\infty}[\frac{-2(\delta-1)z_1}{m\delta} + \frac{\delta s-s+w_2}{\delta}] \cdot f(m)\mathrm{d}m - w_1 = 0$. By taking the first order derivative of $g(\cdot)$, we find that $\frac{\partial g}{\partial z_1} < 0$ and $\frac{\partial g}{\partial s} > 0$, from which we can further know z_1^* is increasing in s so as to ensure $g(z_1^*, s, w_1, w_2) = 0$. As z_1^* is increasing in s, we can obtain that the total expected sales is increasing in s (i.e.,$\frac{\partial E_M[S_1+S_2]}{\partial s} > 0$). Moreover, it is obvious that the total expense $E_M[s \cdot (S_1 + S_2)] = s \cdot E_M[S_1 + S_2]$ is also increasing in s. Hence we know that the optimal per unit subsidy s^* should satisfy the binding budget constraint.

Proof of Proposition 11 By taking the first order derivative of $E_M[\Pi_{r_1}(M)]$ with respect to z_1, we get:

$$\begin{aligned}\frac{\partial E_M[\Pi_{r_1}(m)]}{\partial z_1} =& \frac{\partial M_2}{\partial z_1} \cdot \Pi_{r_1,1}(M_2) \cdot f(M_2) + \int_0^{M_2} (-w_1) \cdot f(m)\mathrm{d}m \\ &- \frac{\partial M_2}{\partial z_1} \cdot \Pi_{r_1,2}(M_2) \cdot f(M_2) + \int_{M_2}^{\infty} [-\frac{4(\delta-1)}{m(2\delta-1)} \cdot z_1 + \frac{\delta - 1 - (s_2 - w_2)}{2\delta - 1} + s_1 - w_1] \cdot f(m)\mathrm{d}m \\ =& - w_1 + \int_{M_2}^{\infty} [-\frac{4(\delta-1)}{m(2\delta-1)} \cdot z_1 + \frac{\delta - 1 - (s_2 - w_2)}{2\delta - 1} + s_1] \cdot f(m)\mathrm{d}m\end{aligned}$$

By checking the second order derivative of $E_M[\Pi_{r_1}(m)]$, we obtain: $\frac{\partial^2 E_M[\Pi_{r_1}(m)]}{\partial z_1^2} = \frac{\delta-1}{2\delta-1} \cdot [\frac{1}{\delta} \cdot f(M_2) - 4\int_{M_2}^{\infty} \frac{1}{m} f(m)\mathrm{d}m] < 0$ when $\frac{1}{\delta} \cdot f(M_2) < 4\int_{M_2}^{\infty} \frac{1}{m} f(m)\mathrm{d}m$. Hence we know that $E_M[\Pi_{r_1}(M)]$ is a concave function of z_1; and we can obtain Proposition 11 by considering the first order condition.

Proof of Proposition 12 By taking the first order derivative of $E_M[S_1 + S_2]$ with respect to s, we get:

$$\frac{\partial E_M[S_1 + S_2]}{\partial s} = \int_0^{M_2} \frac{1 + 2\delta}{4\delta - 1} \cdot m \cdot f(m)\mathrm{d}m + \int_{M_2}^{\infty} [\frac{2(\delta-1)}{2\delta - 1} \cdot \frac{\partial z_1^*}{\partial s} + \frac{m}{2\delta - 1}] \cdot f(m)\mathrm{d}m$$

From Proposition 11, we know that $-w_1 + \int_{M_2}^{\infty}[-\frac{4(\delta-1)}{m(2\delta-1)} \cdot z_1^* + \frac{\delta-1-(s_2-w_2)}{2\delta-1} + s_1] \cdot f(m)\mathrm{d}m = 0$. Hence when $s_1 = s_2 = s$, we denote $g(s, z_1) = -w_1 + \int_{M_2}^{\infty}[-\frac{4(\delta-1)}{m(2\delta-1)} \cdot z_1^* + \frac{\delta-1-(s-w_2)}{2\delta-1} + s] \cdot f(m)\mathrm{d}$ and we know $g(s, z_1^*) = 0$. It is easy to check that $\frac{\partial g}{\partial z} < 0$ and $\frac{\partial g}{\partial s} > 0$, from which we can obtain that z_1^* is increasing in s so as to ensure $g(s, z_1^*) = 0$. With $\frac{\partial z_1^*}{\partial s} > 0$, we can show $\frac{\partial E_M[S_1+S_2]}{\partial s} > 0$. Therefore, we obtain that both the objective function and the subsidy cost shown in the donor's problem (44) is increasing in s, from which we know that the budget constraint should be binding at the optimal solution.