Leverage and Deepening Business Cycle Skewness

Henrik Jensen[*] Ivan Petrella[†]

Søren H. Ravn[‡] Emiliano Santoro[§]

March 2019

Abstract

We document that the U.S. and other G7 economies have been characterized by an increasingly negative business cycle asymmetry over the last three decades. This finding can be explained by the concurrent increase in the financial leverage of households and firms. To support this view, we devise and estimate a dynamic general equilibrium model with collateralized borrowing and occasionally binding credit constraints. Improved access to credit increases the likelihood that financial constraints become non-binding in the face of expansionary shocks, allowing agents to freely substitute intertemporally. Contractionary shocks, on the other hand, are further amplified by drops in collateral values, since constraints remain binding. As a result, booms become progressively smoother and more prolonged than busts. Finally, in line with recent empirical evidence, financially-driven expansions lead to deeper contractions, as compared with equally-sized non-financial expansions.

Keywords: Credit constraints, business cycles, skewness, deleveraging.
JEL: E32, E44.

[*]University of Copenhagen and CEPR. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Henrik.Jensen@econ.ku.dk.

[†]University of Warwick and CEPR. Warwick Business School, University of Warwick, Scarman Rd, CV4 7AL Coventry, United Kingdom. *E-mail*: Ivan.Petrella@wbs.ac.uk.

[‡]University of Copenhagen. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Soren.Hove.Ravn@econ.ku.dk.

[§]University of Copenhagen. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Emiliano.Santoro@econ.ku.dk.

1 Introduction

Economic fluctuations across the industrialized world are typically characterized by asymmetries in the shape of expansions and contractions in aggregate activity. A prolific literature has extensively studied the statistical properties of this empirical regularity, reporting that the magnitude of contractions tends to be larger than that of expansions; see, among others, Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012) and Adrian *et al.* (2018). While these studies have generally indicated that business fluctuations are negatively skewed, the possibility that business cycle asymmetry has changed over time has been overlooked. Yet, the shape of the business cycle has evolved over the last three decades: For instance, since the mid-1980s the U.S. economy has displayed a marked decline in macroeconomic volatility, a phenomenon known as the Great Moderation (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000). This paper documents that, over the same period, the skewness of the U.S. business cycle has become increasingly negative. Our key contribution is to show that occasionally binding financial constraints, combined with a sustained increase in financial leverage, allow us to account for several facts associated with the evolution of business cycle asymmetry.

Figure 1 reports the post-WWII rate of growth of U.S. real GDP, together with the 68% and 90% confidence intervals from a Gaussian density fitted on pre- and post-1984 data. Three facts stand out: First, as discussed above, the U.S. business cycle has become less volatile in the second part of the sample, even if we take into account the major turmoil induced by the Great Recession. Second, real GDP growth displays large swings in both directions during the first part of the sample, while in the post-1984 period the large downswings associated with the three recessionary episodes are not matched by similar-sized upswings. In fact, if we examine the size of economic contractions in conjunction with the drop in volatility occurring since the mid-1980s, it appears that recessions have become relatively more 'violent', whereas the ensuing recoveries have become smoother, as recently pointed out by Fatás and Mihov (2013). Finally, recessionary episodes have become less frequent, thus implying more prolonged expansions.

[Insert Figure 1]

These properties, which are shared by all the G7 economies, translate into business cycles displaying increasingly negative asymmetry over the last three decades. Explaining this pattern represents a challenge for existing business cycle models. To meet this, a theory is needed that involves both non-linearities and a secular development of the underlying mechanism, so as to shape the evolution in business cycle skewness. As for the first prerequisite, the importance of borrowing constraints as a source of business cycle asymmetries has long been recognized in the literature; see, e.g., the survey by Brunnermeier *et al.* (2013). In expansions, households and firms may find it optimal to borrow less than their available credit limit. Instead, financial constraints tend to be binding during recessions, so that borrowing is tied to the value of collateral assets. The resulting non-linearity translates into a negatively skewed business cycle. As for the second prerequisite, the past decades have witnessed a major deregulation of financial markets, with one result being a substantial increase in the degree of leverage of advanced economies. To see this, Figure 2 reports the credit-to-GDP and the loan-to-asset (LTA) ratios of both households and the corporate sector in the US.[1] This leveraging process is also confirmed, e.g., by Jordà *et al.* (2017) in a large cross-section of countries.

[Insert Figure 2]

Based on these insights, the objective of this paper is to propose a structural explanation of deepening business cycle asymmetry. To this end, we devise and estimate a dynamic stochastic general equilibrium (DSGE) model that allows for the collateral constraints faced by the firms and a fraction of the households not to bind at all points in time. We examine the model in the presence of a realistic increase in the maximum loan-to-value (LTV) ratios facing both households and firms. Due to easier access to credit, the likelihood that collateral constraints become non-binding increases when expansionary shocks hit. The magnitude of the resulting boom is therefore attenuated as agents can freely smooth their actions over time. The model predicts this is typically the case for corporate borrowing. By contrast, financially constrained households never find themselves unconstrained, primarily because their debt contracts are mostly long-term, so that shocks tend to affect a relatively small part of the stock of debt being refinanced in each period.[2] In the face of contractionary shocks, instead, both types of borrowers tend to remain financially constrained. In light of these effects, business cycles become increasingly negatively skewed. Following an increase in household and corporate leverage, as observed in the post-1984 sample, the model accounts for up to 50% of the asymmetry in the growth of real GDP, as measured by both the skewness and the ratio between

[1] As we discuss in Appendix A, the aggregate LTA ratios reported in Figure 2 are likely to understate the actual loan-to-value (LTV) requirements faced by the marginal borrower. While alternative measures may yield higher LTV ratios, they point to the same behavior of leverage over time (see also Graham *et al.*, 2014, and Jordà *et al.*, 2017).

[2] On the other hand, the presence of long-term household borrowing proves to be important to reproduce other key features of changing business cycle asymmetry. Primarily, the substantial increase in the duration of expansions that has been observed since the mid-1980s.

the downside and the upside semivolatility of GDP growth. The model also predicts that the average duration of contractions does not change much as leverage increases, whereas that of expansions increases substantially in the post-1984 sample, closely in line with the changes observed in the data.

We then juxtapose the deepening in business cycle asymmetry with the Great Moderation in macroeconomic volatility. While increasing leverage cannot in itself account for the Great Moderation, our analysis shows that the deepening asymmetry of the business cycle is compatible with a drop in its volatility. Additionally, the decline in macroeconomic volatility mostly rests on the characteristics of the expansions, whose magnitude declines as an effect of collateral constraints becoming increasingly lax. This is in line with the empirical findings of Gadea-Rivas *et al.* (2014, 2015), who show that neither changes to the depth nor to the frequency of recessionary episodes account for the stabilization of macroeconomic activity in the US.[3]

Recently, increasing attention has been devoted to the connection between the driving factors behind business cycle expansions and the extent of the subsequent contractions. Jordà *et al.* (2013) report that more credit-intensive expansions tend to be followed by deeper recessions—irrespective of whether the latter are accompanied by a financial crisis. Our model accounts for this feature along two dimensions. First, we show that contractions become increasingly deeper as the average LTV ratio increases, even though the boom-bust cycle is generated by the same combination of expansionary and contractionary shocks. Second, financially-driven expansions lead to deeper contractions, when compared to similar-sized expansions generated by non-financial shocks. Both exercises emphasize that, following a contractionary shock, the aggregate repercussions of financially constrained agents' deleveraging increases in the size of their debt. As a result, increasing leverage makes it harder for savers to compensate for the drop in consumption and investment of constrained agents. This narrative of the boom-bust cycle characterized by a debt overhang is consistent with the results of Mian and Sufi (2010), who identify a close connection at the county level in the US between pre-crisis growth of household leverage and the severity of the Great Recession. Likewise, Giroud and Mueller (2017) document that, over the same period, counties with highly leveraged firms suffered larger employment losses.

[3] In this respect, downward wage rigidity has recently been pointed to as an alternative source of macroeconomic asymmetry (see Abbritti and Fahr, 2013). However, for this to act as a driver of deepening business cycle asymmetry, one would need to observe stronger rigidity over time. Most importantly, even if such a mechanism was at work, the resulting change in the skewness of the business cycle would primarily rest on the emergence of more dramatic recessionary episodes, without any major change in the key characteristics of expansions. However, this implication would stand in contrast with the evidence of Gadea-Rivas *et al.* (2014, 2015).

Our analysis stresses the functioning of occasionally binding financial constraints in combination with a sustained increase in financial leverage. This is consistent with existing accounts of the widespread financial liberalization that started in the US during the 1980s, which provide evidence of a relaxation of financial constraints over time (see, e.g., Justiniano and Primiceri, 2008). For households, Dynan *et al.* (2006) and Campbell and Hercowitz (2009) have discussed how the wave of financial deregulation taking place in the early 1980s paved the way for a substantial reduction in downpayment requirements and the rise of the subprime mortgage market. Combined with the boom in securitization some years later, this profoundly transformed household credit markets and gave rise to the leveraging process observed in Figure 2. As for the corporate sector, the period since around 1980 has witnessed the emergence of a market for high-risk, high-yield bonds (Gertler and Lown, 1999) along with enhanced access to both equity markets and bank credit for especially small- and medium-sized firms (Jermann and Quadrini, 2009). Over the same period the investment-cash flow sensitivity in the US has declined substantially, a fact interpreted by several authors as an alleviation of firms' financial frictions (see, e.g., Agca and Mozumdar, 2008, and Brown and Petersen, 2009).[4] Our findings point to these developments as an impetus of the deepening asymmetry of the U.S. business cycle observed during the same period.

The observation that occasionally binding credit constraints may give rise to macroeconomic asymmetries is not new. Mendoza (2010) explores this idea in the context of a small open economy facing a constraint on its access to foreign credit. As this constraint becomes binding, the economy enters a 'sudden stop' episode characterized by a sharp decline in consumption. Maffezzoli and Monacelli (2015) show that the aggregate implications of financial shocks are state-dependent, with the economy's response being greatly amplified in situations where agents switch from being financially unconstrained to being constrained. In a similar spirit, Guerrieri and Iacoviello (2017) report that house prices exerted a much larger effect on private consumption during the Great Recession—when credit constraints became binding—than in the preceding expansion. While all these studies focus on specific economic disturbances and/or historical episodes, a key insight of this paper is to show how different evolving traits of business cycle asymmetry may be accounted for by a secular process of financial liberalization, conditional on both financial and non-financial disturbances.

We document a strong connection between leverage and business cycle asymmetry across the G7 countries. This result is related to the findings of Jordà *et al.* (2017), who report a

[4]Specifically, Brown and Petersen (2009) show that the investment-cash flow sensitivity declined from around $0.3-0.4$ in the 1970-1981 period to around $0.1-0.2$ in the post-1982 period.

positive correlation between the skewness of real GDP growth and the credit-to-GDP ratio for a large cross-section of countries observed over a long time-span. Popov (2014) exclusively focuses on business cycle asymmetry in a large panel of developed and developing countries, documenting two main results. First, the average business cycle skewness across all countries became markedly negative after 1991, consistent with our findings for the US. Second, this pattern is particularly distinct in countries that liberalized their financial markets. Also Bekaert and Popov (2015) examine a large cross-section of countries, reporting that more financially developed economies have more negatively skewed business cycles. Finally, Rancière *et al.* (2008) establish a negative cross-country relationship between real GDP growth and the skewness of credit growth in financially liberalized countries. While we focus on the asymmetry of output, we observe a similar pattern for credit, making our results comparable with their findings. On a more general note, all of these studies focus on the connection between business cycle skewness and financial factors in the cross-country dimension, whereas we examine how financial leverage may have shaped various dimensions of business cycle asymmetry over time.

The rest of the paper is organized as follows. In Section 2 we report a host of evidence on the connection between leverage and changes in the shape of the business cycle. Section 3 inspects the key mechanisms at play in our narrative within a simple two-period model. Section 4 presents our DSGE model, and Section 5 discusses its solution and estimation. Section 6 reports the quantitative results based on the DSGE model. Section 7 shows that the model is capable of producing the type of debt-overhang recession emphasized in recent empirical studies. Section 8 concludes. The appendices contain supplementary material concerning the model solution and various empirical and computational details.

2 Empirical evidence

We first examine various aspects of business cycle asymmetry in the US, and how they have changed over the last three decades. We then enlarge our view to other G7 countries, and investigate how changes in business cycle asymmetry connect to the role of leverage. Finally, we take advantage of cross-sectional variation across U.S. States to document an empirical relationship between household leverage and the deepness of state-level contractions during the Great Recession.

2.1 Changing business cycle asymmetry

A number of empirical studies have documented a major reduction in the volatility of the U.S. business cycle since the mid-1980s. In this section we document changes in the asymmetry of the cycle that have occurred over the same time span. Table 1 reports the skewness of the rate of growth of different macroeconomic aggregates in the pre- and post-1984 period.

[Insert Table 1]

The skewness is typically negative and not too distant from zero in the first part of the sample, but becomes more negative thereafter.[5] To supplement this finding, we employ a battery of normality tests, which all reject normality in the second subsample, regardless of how the growth rate of real GDP is computed. For instance, we use the Kolmogorov-Smirnov test with estimated parameters (see Lilliefors, 1967), with the null hypothesis being that (year-on-year) GDP growth data in either of the two periods are drawn from a Normal distribution: This is strongly rejected for the second subsample (p-value=0.004), whereas it cannot be rejected in the first one (p-value=0.289).[6]

[Insert Table 2]

Another way to highlight changes in the shape of the business cycle is to compare the upside and the downside semivolatilities of real GDP growth over the two subsamples. The upside (downside) semivolatility is obtained as the average of the squared deviation from the mean of observations that are above (below) the mean. The overall volatility of the business cycle during the Great Moderation has dropped by more than 40%, compared to the pre-moderation period (from 3.07% to 1.75%, when calculated on year-on-year GDP growth). However, as indicated in Table 2, this drop has not been symmetric. In fact, whereas the upside and downside semivolatilities are roughly equal in the pre-moderation sample, in the post-1984 sample the (square root of the) downside semivolatility is around 35% larger than its upside

[5] Appendix B1 reports measures of time-varying volatility and skewness of real GDP growth, based on a non-parametric estimator. The downward pattern in business cycle asymmetry emerges as a robust feature of the data, along with the widely documented decline in macroeconomic volatility.

[6] Additional normality tests are reported in Appendix B2. We also check that the drop in the skewness does not result from a moderate asymmetry in the first part of the sample being magnified by a fall in the volatility, such as the Great Moderation. The skewness of a random variable is defined as m_3/σ^3, where m_3 is the third central moment of the distribution and σ denotes its standard deviation: Therefore, an increase in the absolute size of the skewness could merely reflect a fall in σ, with m_3 remaining close to invariant. However, this is not the case, as $m_3 = -2.817$ for the year-on-year growth rate of real GDP in the pre-1984 sample, while it equals -6.875 afterwards.

counterpart, when calculated on year-on-year GDP growth.[7] As highlighted in Figure 1, this implies an increase in the smoothness of the expansions, indicating that the emergence of the Great Moderation mostly rests on the characteristics of the upsides of the cycle, as recently argued by Gadea-Rivas *et al.* (2014, 2015). All in all, our evidence suggests that the U.S. business cycle has become more asymmetric in the last three decades.

The next step in the analysis consists of translating changes in the business cycle asymmetry into some explicit measure of the deepness of economic contractions, while accounting for time-variation in the dispersion of the growth rate process. In line with Jordà *et al.* (2017), the first column of Table 3 reports the fall of real GDP during a given recession, divided by the duration of the recession itself: this measure is labelled as 'violence'.[8]

[Insert Table 3]

Comparing the violence of the contractionary episodes before and after 1984, we notice that the 1991 and 2001 recessions have not been very different from earlier contractions. However, to compare the relative magnitude of different recessions over a period that displays major changes in the volatility of the business cycle, it is appropriate to control for the average variability of the cycle around a given recessionary episode. To this end, the second column of Table 3 reports standardized violence, which is obtained by normalizing violence by a measure of the variability of real GDP growth.[9] Using this metric we get a rather different picture. The three recessionary episodes occurred during the Great Moderation appear substantially deeper than the pre-1984 ones: averaging out the first seven recessionary episodes returns a standardized violence of 1.22%, against an average of 2.90% for the post-1984 period. Moreover, as highlighted in the last two columns of Table 3, the duration of business cycle contractions does not change much between the two samples, while the duration of the expansions doubles. This contributes to picturing the business cycle in the post-1984 sample as consisting of more smoothed and prolonged expansions, interrupted by short—yet, more dramatic—contractionary episodes.

[7] It is worth highlighting that the major drop in business cycle asymmetry does not uniquely depend on the Great Recession. If one looks at the asymmetry of real GDP growth over the 1984:III-2007:II sample, a sizable drop in the skewness can still be appreciated (from -0.09 to -0.70 for year-on-year growth, and from -0.11 to -0.39 for quarter-on-quarter growth). In addition, the ratio between the downside and the upside semivolatility of GDP growth goes from 1.06 to 1.24 in the case of year-on-year growth rates, and from 1.03 to 1.09 for quarter-on-quarter growth.

[8] For earlier analyses on the violence (and brevity) of economic contractions see Mitchell (1927) and, more recently, McKay and Reis (2008).

[9] The volatility is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year window. We exclude the period running up to the recession by calculating the standard deviation up to a year before the recession begins. Weighting violence by various alternative mesures of business cycle volatility returns a qualitatively similar picture: Appendix B3 reports additional robustness evidence on the standardized violence of the recessions in the US.

2.2 International evidence

This subsection brings further evidence on evolving asymmetries in the business cycle and connects this phenomenon to the role of leverage. To this end, we enlarge our view to the G7 countries, and investigate changes in the asymmetry of business fluctuations, and how they connect to the degree of leverage.

Table 4 reports the skewness and the ratio between the downside and the upside semivolatility of real GDP growth.[10] We use data over the 1961:II to 2016:II time window, and split the sample in the second quarter of 1984.[11] For all countries, we detect a more negative skewness, along with a relative increase in the downside semivolatility during the post-1984 sample, which implies that the volatility of expansions has declined relative to that of contractions.

[Insert Table 4]

The results so far highlight a more pronounced business cycle asymmetry in the post-1984 sample, both for the US and the remaining G7 countries. This period is also associated with an increase in leverage on a global scale, the so-called financial 'hockey stick' highlighted by Schularick and Taylor (2012). To gauge the connection between business cycle asymmetry and leverage, we follow Jordà *et al.* (2017), and summarize the correlation between some asymmetry statistics—namely, the skewness and the ratio between the downside and the upside semivolatility of real GDP growth—and the loan-to-GDP ratio. To this end, we take quarterly GDP growth rates for all the countries under investigation (Δy_{it}), and construct 8-year rolling windows of data. Thus, we compute country-window specific moments, $m(\Delta y_{it})$, and relate them to the average credit-to-GDP ratio calculated over the same sample.[12] Figure 3 reports binned scattered diagrams to summarize our results. The regression line is obtained by assuming a quadratic relationship between business cycle asymmetry and the loan-to-asset ratio (including country-level fixed effects). The evidence points to a close link between leverage and business cycle asymmetries: skewness decreases in leverage and, coherently, the magnitude of

[10] To calculate asymmetry statistics for these countries, we remove the underlying long-run growth appropriately, as the country-specific growth rates display large changes over the sample under investigation (see Antolin-Diaz *et al.*, 2017). This phenomenon is less evident for the US, though the results reported in Table 1 are robust to subtracting the underlying long-run growth rate. We estimate long-run growth as the first difference in the smooth trend of the real GDP series. The latter is retrieved through the modified HP filter of Rotemberg (1999). Using alternative filters delivers similar results.

[11] Stock and Watson (2005) have shown that, for these countries, the mid-1980s are associated with a sharp reduction in macroeconomic volatility. The results are robust to delaying the cut-off date.

[12] For the loan-to-GDP ratio we use the annual data provided by Jordà *et al.* (2017). In Appendix B4 we reproduce the same charts focusing on the loan-to-GDP ratio for the household and the corporate sector, respectively, confirming the results reported here for the aggregate. Using a short window to compute moments implicitly takes care of low-frequency variation in GDP growth. The results obtained by removing the trend in GDP growth (available upon request) are virtually unchanged.

the contractionary phases relative to that of the expansionary ones rises as countries display greater loan-to-GDP ratios.

[Insert Figure 3]

2.3 Leverage and the Great Recession: cross-state evidence

So far we have established that the post-1984 period is characterized by a smoother path of the expansionary periods and a stronger standardized violence of the recessionary episodes, as compared with the pre-1984 period. In addition, over the same time window the process of financial deregulation has been associated with a sizeable increase in leverage of households and firms, both in the US and other G7 countries. We now produce related evidence based on U.S. state-level data. Specifically, we take data on quarterly real Gross State Product (GSP) from the BEA Regional Economic Accounts and compute both the skewness of GSP growth and the violence of the Great Recession in the U.S. States.[13] Figure 4 correlates the resulting statistics to the average debt-to-income ratio prior to the recession. Notably, states where households were more leveraged not only have witnessed more severe GSP contractions during the last recession, but have also displayed a more negatively skewed GSP growth over the 2005-2016 time window.

[Insert Figure 4]

To gain further insights into the cross-sectional connection between the magnitude of the Great Recession and business cycle dynamics, we order the U.S. states according to households' average pre-crisis debt-to-income ratio. We then construct two synthetic series, computed as the growth rates of the median real GSP of the top and the bottom ten states in terms of leverage, respectively. According to Figure 5, there are no noticeable differences in the performance of the two groups before and after the Great Recession, with both of them growing at a roughly similar pace. However, the drop in real activity has been much deeper for relatively more leveraged states. Altogether, this evidence points to a close link between leverage and business cycle asymmetries.

[Insert Figure 5]

[13] To account for the possibility that the recession does not begin/end in the same period across the US, we define the start of the recession in a given state as the period with the highest level of real GSP in the window that goes from five quarters before the NBER peak date to one quarter after that. Similarly, the end of the recession is calculated as the period with the lowest real GSP in the window from one quarter before to five quarters after the NBER trough date.

3 A two-period model

Some preliminary insights about our main model can be offered through a simple model of collateralized debt. The model shares many of the central aspects of our DSGE economy, most notably entrepreneurs facing an asset-based credit constraint. The representative entrepreneur has utility $U = \log C_1 + \beta \mathrm{E}_1 \log C_2$, where $\beta \in (0,1)$ is the discount factor and C_t denotes the consumption of a non-durable good at time $t = 1, 2$. In both periods, production is carried out by a representative firm employing capital, K_{t-1}, and labor, L_t, as production inputs:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{1}$$

where A_1 is an exogenous stochastic variable with cumulative distribution function $\mathcal{F}$ (with $\mathcal{F}' > 0$), and $\alpha \in [0,1]$. As we will focus on the effects of technology shocks taking place in the first period, A_2 is set to a constant, A. We assume that entrepreneurs work as executives in the firms, and inelastically supply labor, $L_t = 1$, for $t = 1, 2$. The entrepreneurs' dynamic budget constraints in periods 1 and 2 are, respectively,

$$\begin{aligned} C_1 + I_1 - B_1 &= r_1^K K_0 + W_1 - RB_0, & (2)\\ C_2 &= \left(1 + r_2^K\right) K_1 + W_2 - RB_1, & (3)\\ \text{with } K_0 &> 0 \text{ and } B_0 \text{ given,} \end{aligned}$$

where W_t denotes the real labour income and r_t^K is the capital rental rate, for $t = 1, 2$, and $R > 1$ is the gross interest rate. The initial stock of debt is denoted by B_0. According to conventional arguments, we rule out debt in the second period, so that $B_2 = 0$. Investment is performed in period 1 only:

$$I_1 = K_1 - (1 - \delta) K_0, \tag{4}$$

where $\delta \in [0,1]$ is the depreciation rate. Finally, the stock of debt in period 1 cannot exceed a fraction of the present value of capital:

$$B_1 \leq s\frac{K_1}{R}, \qquad s \in (0,1), \tag{5}$$

where s is the LTV ratio.

Appendix C shows in detail the derivation of the model's competitive equilibrium. Here, it suffices to consider the capital stock, and thus investment, in period 1. When the constraint

(5) does not bind, we obtain

$$K_1 = \left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}}. \tag{6}$$

If (5) binds, instead, the solution for K_1 is characterized by

$$\Psi\left(K_1; A_1\right) = 0, \tag{7}$$

where

$$\begin{aligned}\Psi\left(K_1; A_1\right) \;&\equiv\; \beta\left(1 + \alpha A K_1^{\alpha-1} - s\right)\left[A_1 K_0^{\alpha} - RB_0 - \left(1 - \frac{s}{R}\right)K_1 + (1-\delta)\,K_0\right] \\ &\quad - \left[(1-s)\,K_1 + AK_1^{\alpha}\right]\left(1 - \frac{s}{R}\right).\end{aligned} \tag{8}$$

Differentiating $\Psi\left(K_1; A_1\right)$ with respect to K_1 shows that

$$\frac{\partial \Psi\left(K_1; A_1\right)}{\partial K_1} < 0. \tag{9}$$

Furthermore:

$$\lim_{K_1 \to 0} \Psi\left(K_1; A_1\right) = \infty, \tag{10}$$

$$\lim_{K_1 \to \Upsilon} \Psi\left(K_1; A_1\right) = -\frac{(1-s)\,\Upsilon + A\Upsilon^{\alpha}}{R-s}R, \tag{11}$$

where $\Upsilon \equiv R\left[A_1 K_0^{\alpha} - RB_0 + (1-\delta)\,K_0\right]/(R-s)$. $K_1 \to \Upsilon$ from below corresponds to moving towards zero period-1 consumption. Hence, as $\Psi\left(K_1; A_1\right)$ is a continuously decreasing function—which tends to infinity for $K_1 \to 0$, and moves downward towards a finite negative number as $K_1 \to \Upsilon$—a solution for $K_1 \in \mathcal{R}_+$ exists and is unique.

Crucially, in the face of an expansionary shock, $\Psi\left(K_1; A_1\right)$ moves up, for any K_1. Hence, when the credit constraint binds, investment responds to temporary shocks, which it does not in the unconstrained case, as implied by (6). In light of this, a negative shock will have adverse investment repercussions, while a similar-sized positive shock may have no impact at all, if it makes the entrepreneur unconstrained. This type of discontinuity—which extends to the behavior of production and consumption—is at the heart of negative asymmetry in the model. To refine our analysis, it is useful to denote with $\overline{A}_1$ the value of the technology shock that makes investment the same in the constrained and unconstrained regimes. The case of $A_1 > \overline{A}_1$ will then be one in which the credit constraint does not bind, and vice versa for $A_1 < \overline{A}_1$. It can be shown that

$$\overline{A}_1 = [1+\beta R - s(1+\beta)]\frac{1}{\beta R K_0^{\alpha}}\left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}} - \frac{A^{\frac{1}{1-\alpha}}}{\beta R K_0^{\alpha}}\left(\frac{\alpha}{R-1}\right)^{\frac{\alpha}{1-\alpha}} + \frac{RB_0}{K_0^{\alpha}} - (1-\delta)K_0^{1-\alpha}. \tag{12}$$

Coherently, $F\left(\overline{A}_1\right)$ is the probability that the economy is below the turning point between the constrained and the unconstrained regime, i.e., the probability that the entrepreneur is constrained. From (12), we can see that $\partial\overline{A}_1/\partial s < 0$. I.e., as the amount of debt that can be contracted for a given level of collateral increases, the minimum realization of the technology shock that makes the entrepreneur unconstrained decreases. In other words, a higher s implies that the entrepreneur has higher chances of becoming financially unconstrained when technology fluctuates.

Intuitively, when s is relatively high, the entrepreneur can obtain a debt level relatively close to what she would have desired in the absence of credit constraints. Therefore, it takes a relatively small positive technology shock to make the constraint slack by increasing income and reducing the need for debt. In contrast, had s been relatively low, the entrepreneur would be relatively far from obtaining the desired debt level, and it will require a much larger technology shock to increase income sufficiently to make the entrepreneur desire less debt and become unconstrained. In sum, the range of technology shocks that render the entrepreneur unconstrained increases with s. This explains why the probability of becoming unconstrained, $1-F\left(\overline{A}_1\right)$, increases with s.

The next section introduces a DSGE model where the mechanisms we have just described produce increasingly negative asymmetry in connection with a process of financial leveraging. Essentially, in such a model aggregate dynamics emerges as a mixture of the behavioral rules governing consumption and investment decisions under different regimes. A higher probability of non-binding financial constraints will be associated with a more marked business cycle asymmetry, as documented in Section 2.

4 A DSGE model

We adopt a standard real business cycle model augmented with collateral constraints, along the lines of Kiyotaki and Moore (1997), Iacoviello (2005), Liu *et al.* (2013), and Justiniano *et al.* (2015); *inter alia*. The economy is populated by three types of agents, each of mass one. These agents differ by their discount factors, with the so-called patient households displaying the highest degree of time preference, while impatient households and entrepreneurs have relatively lower discount factors. Patient and impatient households supply labor, consume nondurable goods and land services. Entrepreneurs only consume nondurable goods, and accumulate both land and physical capital, which they rent to firms. The latter are of unit mass and operate under perfect competition, taking labor inputs from both types of households, along with capital and land from the entrepreneurs. The resulting gross product may be used for investment and nondurable consumption.

4.1 Patient households

The utility function of patient households is given by:

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\log\left(C_t^P - \theta^P C_{t-1}^P\right) + \varepsilon_t \log\left(H_t^P\right) + \frac{\nu^P}{1-\varphi^P}\left(1 - N_t^P\right)^{1-\varphi^P} \right] \right\}, \tag{13}$$

$$0 < \beta^P < 1,\ \varphi^P \geq 0,\ \varphi^P \neq 1,\ \nu^P > 0,\ 0 \leq \theta^P < 1$$

where C_t^P denotes their nondurable consumption, H_t^P denotes land holdings, and N_t^P denotes the fraction of time devoted to labor. Moreover, β^P is the discount factor, θ^P measures the degree of habit formation in nondurable consumption and φ^P is the coefficient of relative risk aversion pertaining to leisure. Finally, ε_t is a land-preference shock satisfying

$$\log \varepsilon_t = \log \varepsilon + \rho_\varepsilon \left(\log \varepsilon_{t-1} - \log \varepsilon\right) + u_t, \qquad 0 < \rho_\varepsilon < 1, \tag{14}$$

where $\varepsilon > 0$ denotes the steady-state value and where $u_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$. Utility maximization is subject to the budget constraint

$$C_t^P + Q_t\left(H_t^P - H_{t-1}^P\right) + R_{t-1}B_{t-1}^P = B_t^P + W_t^P N_t^P, \tag{15}$$

where B_t^P denotes the stock of one-period debt held at the end of period t, R_t is the associated gross real interest rate, Q_t is the price of land in units of consumption goods, and W_t^P is the real wage.

4.2 Impatient households

The utility of impatient households takes the same form as that of patient households:

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\left(\beta^I\right)^t\left[\log\left(C_t^I-\theta^I C_t^I\right)+\varepsilon_t\log\left(H_t^I\right)+\frac{\nu^I}{1-\varphi^I}\left(1-N_t^I\right)^{1-\varphi^I}\right]\right\}, \tag{16}$$
$$0 \;<\; \beta^I<\beta^P,\ \varphi^I>0,\ \varphi^I\neq 1,\ \nu^I>0,\ 0\leq\theta^I<1$$

where, as for the patient households, C_t^I denotes nondurable consumption, H_t^I denotes land holdings, and N_t^I denotes the fraction of time devoted to labor. Households' difference in the degree of time preference is captured by imposing $\beta^P>\beta^I$. This ensures that, in the steady state, patient and impatient households act as lenders and borrowers, respectively. Impatient households are subject to the following budget constraint

$$C_t^I+Q_t\left(H_t^I-H_{t-1}^I\right)+R_{t-1}B_{t-1}^I=B_t^I+W_t^I N_t^I. \tag{17}$$

Impatient households are also subject to a collateral constraint. The nature of the constraint reflects the fact that the vast majority of household debt is effectively long-term. Specifically, building on the work of Kydland *et al.* (2016) and Gelain *et al.* (2017), we assume that impatient households' stock of debt, B_t^I, is constrained from above:

$$B_t^I\leq\vartheta^I s_t^I\frac{\mathrm{E}_t\left\{Q_{t+1}\right\}H_t^I}{R_t}+\left(1-\vartheta^I\right)\left(1-\xi^I\right)B_{t-1}^I,\qquad 0<\vartheta^I,\xi^I<1, \tag{18}$$

where we assume that impatient households refinance a fraction ϑ^I of their outstanding debt in each period. Their 'new' borrowing cannot exceed a fraction s_t^I of the expected present value of their land holdings at the beginning of period $t+1$. Of the remaining, non-refinanced, stock of debt, impatient households are assumed to amortize a constant fraction, ξ^I.[14] Finally, the LTV ratio (or credit limit) on new borrowing, s_t^I, is stochastic and aims at capturing financial shocks (as in, e.g., Jermann and Quadrini, 2012 and Liu *et al.*, 2013):

$$\log s_t^I \;=\; \log s^I+\log s_t \tag{19}$$
$$\log s_t \;=\; \rho_s\log s_{t-1}+v_t,\qquad 0<\rho_s<1, \tag{20}$$

where $v_t\sim\mathcal{N}\left(0,\sigma_s^2\right)$ and s^I, the steady-state LTV ratio, is a proxy for the *average* stance of

[14] Kydland *et al.* (2016) demonstrate that, with a time-varying amortization rate, the model-implied repayment profile mimics that of a standard annuity loan arbitrarily well. Given the different focus of our paper, we opt for a constant amortization rate, without loss of generality.

credit availability to the impatient households.

4.3 Entrepreneurs

Entrepreneurs have preferences over nondurables only (see Iacoviello, 2005; Liu *et al.*, 2013), and maximize

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_t^E - \theta^E C_{t-1}^E\right) \right\}, \quad 0 < \beta^E < \beta^P, \ 0 \leq \theta^E < 1, \tag{21}$$

where C_t^E denotes entrepreneurial nondurable consumption. Utility maximization is subject to the following budget constraint

$$C_t^E + I_t + Q_t\left(H_t^E - H_{t-1}^E\right) + R_{t-1}B_{t-1}^E = B_t^E + r_{t-1}^K K_{t-1} + r_{t-1}^H H_{t-1}^E, \tag{22}$$

where I_t denotes investment in physical capital, K_{t-1} is the physical capital stock rented to firms at the end of period $t-1$, and H_{t-1}^E is the stock of land rented to firms. Finally, r_{t-1}^K and r_{t-1}^H are the rental rates on capital and land, respectively. Capital depreciates at the rate δ, and its accumulation is subject to quadratic investment adjustment costs, so that its law of motion reads as

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \qquad 0 < \delta < 1, \quad \Omega > 0. \tag{23}$$

As the impatient households, entrepreneurs are subject to a credit constraint on their new borrowing, but are able to use both capital and their holdings of land as collateral assets:[15]

$$B_t^E \leq \vartheta^E s_t^E \mathrm{E}_t\left\{\frac{Q_{t+1}^K K_t + Q_{t+1}H_t^E}{R_t}\right\} + \left(1-\vartheta^E\right)\left(1-\xi^E\right)B_{t-1}^E, \qquad 0 < \vartheta^E, \xi^E < 1, \tag{24}$$

where Q_t^K denotes the price of installed capital in consumption units and s_t^E behaves in accordance with

$$\log s_t^E = \log s^E + \log s_t, \tag{25}$$

where s^E denotes entrepreneurs' steady-state LTV ratio.[16] Together with households' average LTV ratio, this parameter will assume a key role in the analysis of the evolving connection

[15] The importance of real estate as collateral for business loans has recently been emphasized by Chaney *et al.* (2012) and Liu *et al.* (2013).

[16] As we discuss in Appendix A1, once the low-frequency components of the LTA series are removed, their cyclical components strongly comove. In light of this, we opt for a common financial shock.

between macroeconomic asymmetries and financial leverage.

4.4 Firms

Firms operate under perfect competition, employing a constant-returns-to-scale technology. They rent capital and land from the entrepreneurs and hire labor from both types of households in order to maximize their profits. The production technology for output, Y_t, is given by:

$$Y_t = A_t \left[\left(N_t^P \right)^{\alpha} \left(N_t^I \right)^{1-\alpha} \right]^{\gamma} \left[\left(H_{t-1}^E \right)^{\phi} K_{t-1}^{1-\phi} \right]^{1-\gamma}, \qquad 0 < \alpha,\ \phi,\ \gamma < 1, \tag{26}$$

with total factor productivity A_t evolving according to

$$\log A_t = \log A + \rho_A \left(\log A_{t-1} - \log A \right) + z_t, \qquad 0 < \rho_A < 1, \tag{27}$$

where $A > 0$ is the steady-state value of A_t, and $z_t \sim \mathcal{N}\left(0, \sigma_A^2\right)$.

4.5 Market clearing

Aggregate supply of land is fixed at H, implying that land-market clearing is given by

$$H = H_t^P + H_t^I + H_t^E. \tag{28}$$

The economy-wide net financial position is zero, such that

$$B_t^P + B_t^I + B_t^E = 0. \tag{29}$$

The labor markets for each labor type clear, and the aggregate resource constraint reads as

$$Y_t = C_t^P + C_t^I + C_t^E + I_t. \tag{30}$$

5 Equilibrium, solution and estimation

An equilibrium is defined as a sequence of prices and quantities which, conditional on the sequence of shocks $\{A_t, \varepsilon_t, s_t\}_{t=0}^{\infty}$ and initial conditions, satisfy the agents' optimality conditions, the budget and credit constraints, as well as the technological constraints and the market-clearing conditions. The optimality conditions are reported in Appendix D. Due to the assumptions about the discount factors, $\beta^I < \beta^P$ and $\beta^E < \beta^P$, both collateral constraints are binding in the steady state. However, the optimal level of debt of one or both agents may fall short of the credit limit when the model is not at its steady state, in which case the collateral constraints will be non-binding.

To account for the occasionally binding nature of the credit constraints, our solution method follows Laséen and Svensson (2011) and Holden and Paetz (2012). The idea is to introduce a set of (anticipated) 'shadow value shocks' to ensure that the shadow values associated with each of the two collateral constraints remain non-negative at all times.[17] We present the technical details of the method in Appendix E.

5.1 Calibration and estimation

In the remainder we aim at assessing the extent to which a relaxation of the credit limits faced by the borrowers can account for the evolution of the asymmetry of the business cycle. With this in mind, we assign parameter values that allow us to match a set of characteristics of the U.S. business cycle in the pre-1984 sample. We do this by calibrating a subset of the parameters, while estimating the remaining ones using the simulated method of moments (SMM). Next, we simulate the model for progressively higher average LTV ratios faced by households and firms, and track the implied changes in the skewness of output and other macroeconomic variables, as well as other business cycle statistics.

5.1.1 Calibrated parameters

The calibrated parameters are summarized in Panel A of Table 5. We choose to calibrate a subset of the model parameters that can be pinned down using a combination of existing studies and first moments of U.S. data. We interpret one period as a quarter. We therefore set $\beta^P = 0.99$, implying an annualized steady-state rate of interest of about 4%. Moreover, we set $\beta^I = \beta^E = 0.96$, in the ballpark of the available estimates for relatively more impatient agents (see, e.g., Iacoviello, 2005 and references therein). The utility weight of leisure is set to ensure that both types of households work 1/4 of their time in the steady state. This implies a value of $\nu^i = 0.27$ for $i = \{P, I\}$. The Frisch elasticity of labor supply is given by the inverse of φ^i, multiplied by the steady-state ratio of leisure to labor hours. Having pinned down the latter to 3, we set $\varphi^i = 9$, $i = \{P, I\}$, implying a Frisch elasticity of 1/3, a value which is broadly in line with the available estimates (see, e.g., Herbst and Schorfheide, 2014). In line with Iacoviello (2005) and Iacoviello and Neri (2010), we set the share of labor income pertaining to patient

[17] For first-order perturbations, we have verified that our solution produces similar simulated moments as using the method of Guerrieri and Iacoviello (2015); see also Holden and Paetz (2012).

households, α, to 0.7. To pin down the labor income share we follow Elsby *et al.* (2013) and use the official estimate of the Bureau of Labor Statistics: The average value for the 1948-1983 time span implies $\gamma = 0.636$.

Regarding the parameters governing the refinancing and amortization of household debt, we build on Kydland *et al.* (2016), who employ a steady-state amortization rate, ξ^I, of 0.014. The implied maturity of a mortgage loan is close to 24 years, slightly longer than in Alpanda and Zubairy (2017), who report that the average remaining term of outstanding household mortgages in the US is close to 20 years.[18] In addition, Kydland *et al.* (2016) report a refinancing share of total mortgage lending of 39%. As we show in Appendix D1, matching this number implies a refinancing parameter $\vartheta^I = 0.009$. We then turn to the entrepreneurs. Most of the existing business cycle studies treat corporate debt as short-term (see, e.g., Jermann and Quadrini, 2012 and Liu *et al.*, 2013), with a duration of one quarter. Chodorow-Reich and Falato (2017) report an average maturity of long-term corporate debt of around 3 years, with long-term debt defined as bank loans with at least one year of residual maturity. This implies a weighted average maturity of total debt between 2 and 3 years. Based on Compustat data, Poeschl (2018) reports an average maturity of corporate debt of 2.3 years. Relying on these figures, we target a maturity of 2.5 years. To this end, we set $\xi^E = 0.125$.[19] Thus, we pin down the refinancing rate, ϑ^E, so as to ensure that the steady-state fraction of total lending that goes to refinance old debt equals 83%, as in the Thompson Reuters LPC Dealscan data (see Drechsel, 2018). This implies a value of $\vartheta^E = 0.698$ (see Appendix D1 for the details on this computation).

Given these parameters, we set $\delta, \varepsilon, \phi$, and s^I to jointly match the following four ratios (all at the annual frequency) for the period from World War II until 1984: A ratio of residential land to output of 1.098, a ratio of commercial land to output of 0.631, an average capital to output ratio of 1.109, and an average ratio of private nonresidential investment to output of 0.230.[20] The depreciation rate of capital consistent with these figures is 0.052, somewhat higher

[18] Following Alpanda and Zubairy (2017), we approximate maturity by two times the half-life of a loan.

[19] The amortization rates we employ are, if anything, on the conservative side. For households, Alpanda and Zubairy (2017) and Gelain *et al.* (2017) both use slightly higher values than we do. For firms, it is important to stress that business loans are frequently renegotiated. In a large sample of private credit agreements to publicly traded U.S. firms, Roberts and Sufi (2009) find that 90% of debt contracts with maturities longer than one year are renegotiated prior to maturity. Moreover, Chodorow-Reich and Falato (2017) emphasize the role of the 'loan covenant channel': While only 10% of bank loans to firms had a remaining maturity of less than a year at the onset of the financial crisis in 2008, a much larger share of firms breached a covenant associated with their loan during the crisis, thus allowing the lender to dictate new terms. We have verified that our results are robust to realistic changes in the amortization rates for both households and firms.

[20] Our computations of these ratios largely follow those of Liu *et al.* (2013). For residential land, we use owner-occupied real estate from the Flow of Funds tables. For commercial land, Liu *et al.* (2013) use Bureau of Labor Statistics data on land inputs in production, which are not available for the sample period we consider.

than standard values, as it reflects that our measure of capital excludes residential capital and structures, which feature lower depreciation rates than, e.g., intellectual properties. We obtain a value of $\phi = 0.134$, which, multiplied by $(1-\gamma)$, measures land's share of inputs, and a weight of land in the utility function of $\varepsilon = 0.084$. The implied value for impatient households' average LTV ratio is 0.673. The spread between the household and the entrepreneurial steady-state LTV ratios is calibrated to match the average difference in the low-frequency components over the entire sample, which is roughly equal to the average difference in the original series. As a result, the entrepreneurial average LTV ratio, s^E, is set to 0.763.[21]

5.1.2 Estimated parameters

We rely on the SMM to estimate the remaining model parameters, as this method is particularly well-suited for DSGE models involving non-binding constraints or other non-linearities. Ruge-Murcia (2012) studies the properties of SMM estimation of non-linear DSGE models, and finds that this method is computationally efficient and delivers accurate parameter estimates. Moreover, Ruge-Murcia (2007) performs a comparison of the SMM with other widely used estimation techniques applied to a basic RBC model, showing it fares quite well in terms of accuracy and computing efficiency, along with being less prone to misspecification issues than likelihood-based methods.

We estimate the following parameters: The investment adjustment cost parameter (Ω), the parameters measuring habit formation in consumption (θ^P, θ^I, and θ^E), and the parameters governing the persistence and volatility of the shocks $(\rho_A, \rho_s, \rho_\varepsilon, \sigma_A, \sigma_s, \sigma_\varepsilon)$.[22] In the estimation, we use five macroeconomic time series for the U.S. economy spanning the sample period 1952:I–1984:II: The growth rates of real GDP, real private consumption, real non-residential investment, real house prices, and the cyclical component of the LTA series in Figure 2, with the trend being computed as in Müller and Watson (2018).[23] The beginning of the sample is dictated by the availability of quarterly Flow of Funds data, while the end of the sample

Instead, we compute the sum of the real estate holdings of nonfinancial corporate and nonfinancial noncorporate businesses from the Flow of Funds, and then follow Liu *et al.* (2013) in multiplying this number by a factor of 0.5 to impute the value of land. For capital, we compute the sum of the annual stocks of equipment and intellectual property products of the private sector and consumer durables. We use the corresponding flow variables to measure investment. Finally, we measure output as the sum of investment (as just defined) and private consumption expenditures on nondurable goods and services.

[21] These values for the average LTV ratios are lower than those typically employed in models calibrated over the Great Moderation sample (see, e.g., Calza *et al.*, 2013, Liu *et al.*, 2013, and Justiniano *et al.*, 2014), as our calibration covers the period before the subsequent wave of financial liberalization.

[22] In the estimation we impose that $\theta^I = \theta^E$, as initial attempts to identify these two parameters separately proved unsuccessful.

[23] Since the cyclical components of the two LTA series are strongly correlated, we use the one obtained for the households. All results are robust to using the corporate one.

coincides with the onset of the Great Moderation.[24] In the estimation, we match the following empirical moments: The standard deviations and first-order autoregressive parameters of each of the five variables, the correlation of consumption, investment, and house prices with output, and the skewness of output, consumption, and investment. This gives a total of 16 moment conditions to estimate nine parameters. We provide more details about the data and our estimation strategy in Appendix F.

The estimated parameters are reported in Panel B of Table 5. The estimate of Ω is in line with existing results from estimated DSGE models (see, e.g., Christiano *et al.*, 2014). The degree of habit formation of patient households is close to the estimate of Liu *et al.* (2013), whereas the estimated habit parameter for impatient households and entrepreneurs is somewhat higher than most of the available estimates. The volatility and persistence parameters of the technology shock are in line with those typically found in the real business cycle literature; see, e.g., Mandelman *et al.*, 2011. The finding of rather large land-demand shocks is consistent with the results of Iacoviello and Neri (2010) and Liu *et al.* (2013). Finally, the financial shocks in our model are more volatile than found by Jermann and Quadrini (2012) and Liu *et al.* (2013), but less persistent.

The model-implied matched moments and their data counterparts are reported in Table F1 in Appendix F. Moreover, it is worth highlighting that we match quite closely a set of non-targeted moments of interest. For instance, the ratio between the downside and the upside semivolatility of real GDP growth and the standardized violence are 1.072 and 1.165, respectively, while the corresponding numbers in the data are 1.061 and 1.217. Moreover, applying the business cycle dating algorithm of Harding and Pagan (2002) to the simulated data, we obtain a duration of expansions and contractions of 22.156 and 3.765 quarters, respectively, as compared with 15.333 and 3.714 quarters in the data. While the duration of expansions is somewhat overestimated, these numbers confirm that the model generally produces a close match of key properties of the data over the pre-1984 sample.

[Insert Table 5]

6 Asymmetric business cycles and collateral constraints

We can now examine how our model generates stronger business cycle asymmetries as financial leverage increases. We do so in three steps. First, we inspect a set of impulse responses to

[24] In fact, house prices are only available starting in 1963:I. We choose not to delay the beginning of other data series to this date.

build some intuition around the non-linear transmission of different shocks. Next, we present various business cycle statistics obtained from simulating the model at different degrees of leverage. Finally, we examine the behavior of business cycle asymmetry in conjunction with the behavior of macroeconomic volatility. Our quantitative exercises primarily aim at assessing the model's ability to reproduce various dimensions of changing business cycle asymmetry, relying exclusively on an increase in financial leverage.[25]

6.1 Impulse-response analysis

To gain some preliminary insights into the nature of our framework, and how its properties evolve under different LTV ratios, we study the propagation of different shocks. Figure 6 displays the response of output to a set of positive shocks, as well as the mirror image of the response to equally-sized negative shocks, under different credit limits.[26] For purely illustrative purposes, we report impulse responses from a version of the model in which long-term debt is temporarily 'shut off', i.e. where both corporate and household debt have a duration of one period. This enhances the likelihood of observing episodes of non-binding collateral constraints in the face of a one-off expansionary shock.[27]

Looking at the first row of the figure, technology shocks of either sign produce symmetric responses under the calibrated LTV ratios for impatient households and entrepreneurs. By contrast, at higher credit limits a positive technology shock renders the borrowing constraint of the entrepreneurs slack for eight quarters, while impatient households remain constrained throughout. Entrepreneurs optimally choose to borrow less than they are able to. This attenuates the expansionary effect on their demand for land and capital, dampening and prolonging the boom in aggregate economic activity. On the contrary, following a similar-sized negative technology shock, the borrowing constraints remain binding throughout. As a result, both impatient households and entrepreneurs are forced to cut back on their borrowing in response to the drop in the value of their collateral assets. This produces a stronger and swifter output response.

[Insert Figure 6]

[25] The aim of the exercise is not to account for the process of financial innovation and liberalization lying behind the increase in leverage in the last decades—a task the model is not suitable for. Instead, we take this increase for granted and examine how it has affected the shape of the business cycle.

[26] Appendix G1 reports the corresponding impulse-responses for total consumption, investment, and total debt.

[27] In the dynamic simulations reported in the next subsections, instead, combinations of all the shocks at their baseline calibration have the potential to generate episodes of non-binding constraints, even in the presence of long-term debt.

As for the stochastic shifts in household preferences, the second row of Figure 6 indicates that entrepreneurs' collateral constraint becomes non-binding for three quarters after a positive land demand shock in the scenario with high LTV ratios, while impatient households remain constrained throughout. Therefore, entrepreneurs have no incentive to expand their borrowing capacity by increasing their stock of land. By contrast, there is no attenuation of negative shocks to the economy. In that case, both collateral constraints remain binding, giving rise to a large and immediate output drop.

Similar observations apply to the transmission of the financial shock. Under high average LTV ratios the entrepreneurs are unconstrained during the first three periods following a positive shock. For the reasons discussed above, this leads to a smooth response of output, as compared with what happens following a negative shock. In this case entrepreneurs are forced into a sizeable deleveraging, reducing the stock of land available for production. Also impatient households deleverage and bring down their stock of land, which further depresses the land price, and thus the borrowing capacity of both types of constrained agents. The result is a large drop in output.

The impulse-response analysis offers a clear message: As leverage increases, economic expansions tend to become smoother and more prolonged than contractions, paving the way to a negatively skewed business cycle. This is broadly consistent with the observation of lower volatility of the upside of the business cycle, as compared with its downside. Moreover, all the three types of shock we consider have the potential to generate episodes of non-binding constraints in response to positive innovations, thus contributing to business cycle asymmetries.

6.2 Leverage and asymmetries

To deepen our understanding of the model, we report a number of statistics from a rich set of dynamic simulations based on a gradual increase in the average LTV ratios of the financially constrained agents. Following the approach of Müller and Watson (2018), Appendix A1 documents that, at very low frequencies, the LTA series for the household and corporate sector display strong comovement. Therefore, starting from their calibrated values, both agents' steady-state LTV ratios are progressively raised by 23 basis points, in line with the increase in the low-frequency components of the LTA series reported in Figure 1 over the 1984-2016 time window.[28]

[28] In each simulation reported in this section, the entrepreneurial average LTV ratio is adjusted to be 9 basis points greater than any value we consider for impatient households' credit limit, in line with the baseline calibration of the model. In Section 6.3.1, instead, we feed in the estimated low-frequency components of the LTA ratios of households and firms. In both cases, we compute the statistics of interest as median values from

Figure 7 displays two key dimensions of business cycle asymmetry: the left panel reports the skewness of the growth rates of output, aggregate consumption and investment, while the right panel reports the ratio between the downside and the upside semivolatility of output growth. All the statistics in the left panel are negative at our calibrated average LTV ratios, and decline thereafter. Similarly, we observe a stable increase in the ratio between the downside and the upside semivolatility of output growth. Therefore, in connection with an increase in financial leverage, the model is capable of generating an increasingly negatively skewed business cycle.[29] In fact, relying exclusively on the role of occasionally binding financial constraints, the model accounts for up to 50% of the asymmetry in the (year-on-year) growth of real GDP in the post-1984 sample, as measured by both the skewness and the ratio between the downside and the upside semivolatility of GDP growth.

[Insert Figure 7]

These properties reflect into a marked transformation in the shape of the business cycle. As leverage increases, the model is capable of reproducing relative changes in the average duration of contractions and expansions that are broadly in line with those documented in Table 3: While the former invariantly last one year, the latter increase their duration from around four-five years to roughly eight years (see Figure 8). On the other hand, higher leverage is associated with relatively more severe contractionary episodes, as implied by the standardized violence reported in the last panel of the figure. These findings have a common root: An increase in their average LTV ratios allows financially constrained agents to be in a better position to smooth consumption and investment during expansions.

It is important to mention that changes in business cycle skewness are predominantly driven by the firm sector becoming increasingly unconstrained as the average LTV ratio increases. Impatient households, instead, never find themselves unconstrained, primarily because their debt contracts are long-term, implying that shocks tend to affect a relatively small part of the stock of debt being refinanced in each period.[30,31] On the other hand, household borrowing

501 simulations, all of which run for 2000 periods.

[29] Though the functioning of occasionally binding constraints does not primarily hinge on the prices of collateral assets, these play a quantitatively important role. To see this, the reader is referred to Figure G8 in Appendix G4, where we report a set of statistics obtained by simulating an alternative version of the model where the collateral assets are pledged at their steady-state prices.

[30] The frequency of non-binding constraints for both impatient households and the entrepreneurs is reported in Figure G4 in Appendix G2. Guerrieri and Iacoviello (2017) report that non-binding credit constraints were prevalent among U.S. households from the late 1990s until the onset of the Great Recession. The difference between our results and their evidence mainly lies in the fact that they calibrate a smaller degree of inertia in the borrowing limit.

[31] Moreover, households only feature one type of collateral asset in their borrowing constraint. This limits the amplification of shocks affecting their borrowing capacity.

helps to account for some desirable quantitative properties, primarily a sizeable increase in the duration of expansions (see Appendix G3).

[Insert Figure 8]

6.3 Skewness and volatility

Recent statistical evidence has demonstrated that the Great Moderation was never associated with smaller or less frequent downturns, but has been driven exclusively by the characteristics of the expansions, whose magnitude has declined over time (Gadea-Rivas *et al.*, 2014, 2015). We now examine this finding in conjunction with the change in the asymmetry of the business cycle, which has largely occurred over the same time span.

[Insert Figure 9]

The left panel of Figure 9 reports the standard deviation of output growth as a function of the average LTV ratios. As shown by Jensen *et al.* (2018) in a similar model, macroeconomic volatility displays a hump-shaped pattern: Starting from low credit limits, higher availability of credit allows financially constrained agents to engage in debt-financed consumption and investment, as dictated by their relative impatience, thus reinforcing the macroeconomic repercussions of shocks that affect their borrowing capacity. This pattern eventually reverts, as higher LTV ratios increase the likelihood that credit constraints become non-binding. In such cases, the consumption and investment decisions of households and entrepreneurs may delink from changes in the value of their collateral assets, dampening the volatility of aggregate economic activity. In fact, at the upper end of the range of average LTV ratios we consider, volatility drops below the value we match under the baseline calibration.

A key property of a model with occasionally binding constraints is that the volatility reversal is much stronger for positive than for negative shocks, in the face of which financial constraints tend to remain binding. This inherent property of our framework indicates that the drop in output volatility observed beyond $s^I \approx 0.8$ is mostly connected with expansionary periods. The right panel of Figure 9 confirms this view: Here, we compare the volatility of expansionary and contractionary episodes, respectively, as a function of the average LTV ratios. The volatility of expansions is always lower than that of contractions, and declines over the entire range of average credit limits. The volatility of contractions, on the other hand, increases steadily.

Notably, increasing leverage allows the model to account for different correlations between the volatility and the skewness of output growth. Based on the comparison between Figure

7 and the left panel of Figure 9, this correlation is increasingly negative until $s^I \approx 0.8$, thus becoming positive as financial deepening reaches very advanced stages. These results are reminiscent of the evidence reported by Bekaert and Popov (2015), who document a positive long-run correlation between the volatility and skewness of output growth in a large cross-section of countries, but also a negative short-run relationship: As financial leverage reaches a certain level across advanced economies, our results predict that skewness and volatility will eventually decline in conjunction.

A word of caution is in order at this stage. While our framework points to a hump-shaped relationship between credit limits and macroeconomic volatility, the key driver of business cycle asymmetry—endogenous shifts between binding and non-binding collateral constraints—in itself works as an impetus of lower macroeconomic volatility, *ceteris paribus*. Thus, despite our analysis not warranting the claim that the empirical developments in the volatility and skewness of the business cycle necessarily have the same origin, higher credit limits do eventually lead to a drop in the overall volatility of our model economy by making financial constraints increasingly slack.[32] A related question is whether our main finding of increasingly negative business cycle asymmetry would survive in the presence of a reduction in macroeconomic volatility of the magnitude observed during the Great Moderation. The next subsection tackles this point.

6.3.1 Counterfactual exercises: accounting for the Great Moderation

We now turn to a counterfactual exercise aimed at reconciling our main finding of increasingly negative business cycle asymmetry with a reduction in macroeconomic volatility of the magnitude observed during the Great Moderation.[33] To this end, we conduct two experiments: In the first experiment, we take a perspective similar to that of Section 6.2, but feed in the actual, estimated low-frequency components of the LTA ratios of households and firms, respectively, rather than relying on a linear and parallel increase in the leverage of the two agents.[34] In the second experiment, we repeat the exercise in combination with a gradual reduction of the

[32] In fact, several authors have pointed to financial liberalization and the associated easing of the financial constraints of both households and firms as a contributor to the Great Moderation (see, e.g., Justiniano and Primiceri, 2008 and, for a review of the literature, Den Haan and Sterk, 2010).

[33] In this respect, it is important to recognize that none of the factors to which the Great Moderation is typically ascribed are featured in our model. The most popular narratives about the Great Moderation are a drop in the volatility of economic shocks (see, e.g., Justiniano and Primiceri, 2008) and improvements in the conduct of monetary policy (see, e.g., Boivin and Giannoni, 2006).

[34] Specifically, for each year in the 1980-2016 time window, we feed in the annual average of the quarterly long-run LTV components, as depicted in Figure A.1 in Appendix A1. To make up for the difference in levels between the aggregated series and the (somewhat higher) calibrated LTV ratios, we add an (agent-specific) constant to the low-frequency components. If, at any time, the implied LTV ratio exceeds 0.99, we cap it at this value. The implied paths of the LTV series are reported in Figure G10 of Appendix G5.

standard deviations of all the shocks in the model. This reduction is reverse-engineered to obtain a decline in macroeconomic volatility similar to the one observed in the data during the Great Moderation. Specifically, we target a decline in the volatility of output growth of around 40%, in line with the evidence reported in Section 2.1. To this end, we assume that the reduction in the magnitude of the shocks starts in 1984, and is completed by 1989.[35]

[Insert Figure 10]

The results from our experiments are reported in Figure 10: here, the skewness, the ratio between the downside and the upside semivolatility, and the volatility of output growth are reported over the 1980-2016 time span. In both experiments, the skewness displays a decline of roughly the same magnitude as that observed in Figure 7, reaching a level of about -0.6 by the end of the sample; about half of the corresponding level reported in Table 1.[36] The ratio between the business cycle semivolatilities behaves coherently. As for the standard deviation, it drops by about 40% when we reduce the size of the shocks hitting the economy (by construction), while displaying a very small decline in the alternative scenario. Although the increase in LTV ratios in these experiments is not too different from the exercises in the previous subsection, it is important to stress that the coexistence of a large change in the asymmetry and the volatility of the business cycle is not trivial: All else equal, reducing the size of the shocks hitting the economy lowers the probability that collateral constraints become non-binding, thus potentially weakening the key driver of business cycle asymmetry in the model. However, we find that smaller shocks only slightly mitigate the potential to produce sizeable non-linearities.

7 Debt overhang and business cycle asymmetries

Several authors have recently pointed to the nature of the boom phase of the business cycle as a key determinant of the subsequent recession. For example, using data for 14 advanced economies for the period 1870–2008, Jordà *et al.* (2013) find that more credit-intensive expansions tend to be followed by deeper recessions, whether or not the recession is accompanied by a financial crisis.

[35] This exercise is thus consistent with the "Good Luck" narrative of the Great Moderation (Stock and Watson, 2003). Throughout both experiments, we keep the relative size of the shocks fixed and in accordance with the estimation reported in Section 5.1.2. The path of the scaling factor is illustrated in Figure G9 of Appendix G5. The results are generally robust to alternative choices of the transition window.

[36] The initial hike (drop) in the skewness (ratio between the downside and upside semivolatility) of output growth can be explained by the fact that, in the face of a reduction in the volatility of the shocks taking place over a rather limited time span, the LTV ratios of both agents rise over a much larger time window.

In this section we demonstrate that our model is also capable of reproducing these empirical facts. Figure 11 reports the results of the following experiment: Starting in the steady state, we generate a boom-bust cycle for different average LTV ratios. We first feed the economy with a series of positive shocks of all three types in the first five periods (up to period 0 in the figure). During the boom phase, we calibrate the size of the expansionary shocks hitting the economy so as to make sure that the boom in output is identical across all the experiments. Hereafter, starting in period 1 in the figure, we shock the economy with contractionary shocks of all three types for two periods, after which the negative shocks are 'phased out' over the next three periods. Crucially, the contractionary shocks are identical across calibrations. This ensures that the severity of the recession is solely determined by the endogenous response of the model at each different LTV ratio.[37] As the figure illustrates, the deepness of the contraction increases with the steady-state LTV ratios. A boom of a given size is followed by a more severe recession when debt is relatively high, as compared with the case of more scarce credit availability. At higher average LTV ratios, households and entrepreneurs are more leveraged during the boom, and they therefore need to face a more severe process of deleveraging when the recession hits. By contrast, when credit levels are relatively low, financially constrained agents face lower credit availability to shift consumption and investment forward in time during booms, and are therefore less vulnerable to contractionary shocks.

[Insert Figure 11]

We next focus on the nature of the boom and how this spills over to the ensuing contraction. The left panel of Figure 12 compares the path of output in two different boom-bust cycles, while the right panel shows the corresponding paths for aggregate debt. In each panel, the dashed line represents a non-financial boom generated by a combination of technology and land-demand shocks, while the solid line denotes a financial boom generated by credit limit shocks.[38] We calibrate the size of the expansionary shocks so as to deliver an identical increase in output during each type of boom (which lasts for five periods, up until period 0 in the figure). As in the previous experiment, we then subject the economy to identical sets of contractionary shocks of all three types, so as to isolate the role played by the specific type of boom in shaping

[37] During both the boom and the bust we keep the relative size of the three shocks fixed and equal to their estimated standard deviations. However, we set their persistence parameters to zero, in order to avoid that the shape of the recession may be determined by lagged values of the shocks during the boom. Finally, we make sure that impatient households and entrepreneurs remain constrained in all periods of each of the cases, so as to enhance comparability.

[38] In the non-financial boom we keep the relative size of the technology and land-demand shocks in line with the values estimated in Section 5.1.2. As in the previous experiment, we set the persistence parameters of all the shock processes to zero.

the subsequent recession. The contractionary shocks hit in periods 1 and 2 in the figure, and are then 'phased out' over the next three periods. While the size of the expansion in output is identical in each type of boom, the same is not the case for total debt, which increases by more than twice as much during the financial boom. The consequences of this build-up of credit show up during the subsequent contraction, which is much deeper following the financially fueled expansion, in line with the empirical findings of Jordà *et al.* (2013). As in Mian and Sufi (2010), this exercise confirms that the macroeconomic repercussions of constrained agents' deleveraging increases in the extent of leveraging.[39]

[Insert Figure 12]

8 Concluding comments

We have documented how different dimensions of business cycle asymmetry in the US and other G7 countries have changed over the last decades, and pointed to the concurrent increase in private debt as a potential driver of these phenomena. We have presented a dynamic general equilibrium model with credit-constrained households and firms, in which increasing leverage translates into a more negatively skewed business cycle. This finding relies on the occasionally binding nature of financial constraints: As their credit limits increase, financially constrained agents are more likely to become unconstrained during booms, while credit constraints tend to remain binding during downturns.

These insights shed new light on the analysis of the business cycle and its developments. The Great Moderation is widely regarded as the main development in the statistical properties of the U.S. business cycle since the 1980s. We point to a simultaneous change in the shape of the business cycle closely connected with financial factors. Enhanced credit access as observed over the last few decades implies both a prolonging and a smoothing of expansionary periods as well as less frequent—yet, relatively more dramatic—economic contractions, exacerbated by deeper deleveraging episodes. As for the first part of this story, several contributions have pointed to the attenuation of the upside of the business cycle as the main statistical trait of the Great Moderation. Nevertheless, insofar as financial liberalization and enhanced credit access

[39] Addressing the endogeneity of credit and business cycle dynamics, Gadea-Rivas and Perez-Quiros (2015) stress that growing credit is not a predictor of future contractions. Our model simulations are consistent with this view. In fact, as displayed by Figure 12, output and credit growth are strongly correlated, regardless of whether the boom is driven by financial shocks. At the same time, the model predicts that a boom driven by financial shocks is associated with a stronger increase in debt and a deeper contraction, as compared with an equally-sized non-financial boom.

can be pointed to as key drivers of an increasingly asymmetric business cycle, the second insight implies that large contractionary episodes, albeit less frequent, might represent a 'new normal'.

Our results are also of interest to macroprudential policymakers, as we complement a recent empirical literature emphasizing that the seeds of the recession are sown during the boom (see, e.g., Mian *et al.*, 2017). The nature of the expansionary phase, as much as its size, is an important determinant of the ensuing downturn, and policymakers should pay close attention to the build-up of credit during expansions in macroeconomic activity.

References

Abbritti, M., and S. Fahr, 2013, Downward Wage Rigidity and Business Cycle Asymmetries, Journal of Monetary Economics, 60, 871–886.

Adrian, T., N. Boyarchenko, and D. Giannone, 2018, Vulnerable Growth, American Economic Review, forthcoming.

Agca, S., and A. Mozumdar, 2008, The Impact of Capital Market Imperfections on Investment–Cash Flow Sensitivity, Journal of Banking and Finance, 32, 207–216.

Alpanda, S., and S. Zubairy, 2017, Addressing Household Indebtedness: Monetary, Fiscal or Macroprudential policy?, European Economic Review, 92, 47–73.

Antolin-Diaz, J., T. Drechsel, and I. Petrella, 2017, Tracking the Slowdown in Long-Run GDP Growth, Review of Economics and Statistics, 99, 343–356.

Bekaert, G., and A. Popov, 2015, On the Link between the Volatility and Skewness of Growth, mimeo, Columbia Business School and the European Central Bank.

Boivin, J., and M. Giannoni, 2006, Has Monetary Policy Become More Effective?, Review of Economics and Statistics, 88, 445–462.

Brown, J. R., and B. C. Petersen, 2009, Why Has the Investment-Cash Flow Sensitivity Declined so Sharply? Rising R&D and Equity Market Developments, Journal of Banking and Finance, 33, 971–984.

Brunnermeier, M. K., T. Eisenbach, and Y. Sannikov, 2013, Macroeconomics with Financial Frictions: A Survey, Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society. New York: Cambridge University Press.

Calza, A., T. Monacelli, and L. Stracca, 2013, Housing Finance and Monetary Policy, Journal of the European Economic Association, 11, 101–122.

Campbell, J. R., and Z. Hercowitz, 2009, Welfare Implications of the Transition to High Household Debt, Journal of Monetary Economics, 56, 1–16.

Chaney, T., D. Sraer, and D. Thesmar, 2012, The Collateral Channel: How Real Estate Shocks Affect Corporate Investment, American Economic Review, 102, 2381–2409.

Chodorow-Reich, G., and A. Falato, 2017, The Loan Covenant Channel: How Bank Health Transmits to the Real Economy, NBER Working Papers, No. 23879.

Christiano, L. J., R. Motto, and M. Rostagno, 2014, Risk Shocks, American Economic Review, 104, 27–65.

den Haan, W. and V. Sterk, 2010, The Myth of Financial Innovation and the Great Moderation, Economic Journal, 121, 707–739.

Drechsel, T., 2018, Flow-Based Borrowing Constraints and Macroeconomic Fluctuations, working paper, London School of Economics.

Dynan, K. E., D. W. Elmendorf, and D. E. Sichel, 2006, Can Financial Innovation Help to Explain the Reduced Volatility of Economic Activity?, Journal of Monetary Economics, 53, 123–150.

Elsby, M., B. Hobijn and A. Sahin, 2013, The Decline of the U.S. Labor Share, Brookings Papers on Economic Activity, 44, 1–63.

Fatás, A. and I. Mihov, 2013, Recoveries, CEPR Discussion Papers, No. 9551.

Gadea-Rivas, M. D., A. Gomez-Loscos, and G. Perez-Quiros, 2014, The Two Greatest: Great Recession vs. Great Moderation, CEPR Discussion Papers, No. 10092.

Gadea-Rivas, M. D., A. Gomez-Loscos, and G. Perez-Quiros, 2015, The Great Moderation in Historical Perspective: Is it that Great?, CEPR Discussion Papers, No. 10825.

Gadea-Rivas, M. D., and G. Perez-Quiros, 2015, The Failure To Predict The Great Recession—A View Through The Role Of Credit, Journal of the European Economic Association, 13, 534–559.

Gelain, P., K. J. Lansing, and G. J. Natvik, 2017, Leaning Against the Credit Cycle, Journal of the European Economic Association, forthcoming.

Gertler, M., and C. Lown, 1999, The Information in the High-Yield Bond Spread for the Business Cycle: Evidence and Some Implications, Oxford Review of Economic Policy, 15, 132–150.

Giroud, X., and H. M. Mueller, 2017, Firm Leverage, Consumer Demand, and Unemployment during the Great Recession, Quarterly Journal of Economics, 132, 271–316.

Graham, J. R., M. T. Leary, and M. R. Roberts, 2014, A Century of Capital Structure: The Leveraging of Corporate America, Journal of Financial Economics, 118, 658–683.

Guerrieri, L., and M. Iacoviello, 2015, OccBin: A Toolkit for Solving Dynamic Models With Occasionally Binding Constraints Easily, Journal of Monetary Economics, 70, 22–38.

Guerrieri, L., and M. Iacoviello, 2017, Collateral Constraints and Macroeconomic Asymmetries, Journal of Monetary Economics, 90, 28–49.

Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, Econometrica, 57, 357–384.

Harding, D. and A. Pagan, 2002, Dissecting the Cycle: A Methodological Investigation, Journal of Monetary Economics, 49, 365–381.

Herbst, E., and F. Schorfheide, 2014, Sequential Monte Carlo Sampling for DSGE Models, Journal of Applied Econometrics, 29, 1073–1098.

Holden, T., and M. Paetz, 2012, Efficient Simulation of DSGE Models with Inequality Constraints, School of Economics Discussion Papers 1612, University of Surrey.

Iacoviello, M., 2005, House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle, American Economic Review, 95, 739–764.

Iacoviello, M. and S. Neri, 2010, Housing Market Spillovers: Evidence from an Estimated DSGE Model, American Economic Journal: Macroeconomics, 2, 125–164.

Jensen, H., S. H. Ravn, and E. Santoro, 2018, Changing Credit Limits, Changing Business Cycles, European Economic Review, 102, 211–239.

Jermann, U. and V. Quadrini, 2009, Financial Innovations and Macroeconomic Volatility, working paper, Universities of Pennsylvania and Southern California.

Jermann, U. and V. Quadrini, 2012, Macroeconomic Effects of Financial Shocks, American Economic Review, 102, 238–271.

Jordà, O., M. Schularick, and A. M. Taylor, 2013, When Credit Bites Back, Journal of Money, Credit and Banking, 45, 3–28.

Jordà, O., M. Schularick, and A. M. Taylor, 2017, Macrofinancial History and the New Business Cycle Facts, NBER Macroeconomics Annual 2016, 213–263.

Justiniano, A., and G. E. Primiceri, 2008, The Time Varying Volatility of Macroeconomic Fluctuations, American Economic Review, 98, 604–641.

Justiniano, A., G. E. Primiceri, and A. Tambalotti, 2014, Credit Supply and the Housing Boom, forthcoming, Journal of Political Economy.

Justiniano, A., G. E. Primiceri, and A. Tambalotti, 2015, Household Leveraging and Deleveraging, Review of Economic Dynamics, 18, 3–20.

Kim, C.-J. and C. R. Nelson, 1999, Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle, The Review of Economics and Statistics, 81, 608–616.

Kiyotaki, N. and J. Moore, 1997, Credit Cycles, Journal of Political Economy, 105, 211–248.

Kydland, F. E., P. Rupert, and R. Sustek, 2016, Housing Dynamics over the Business Cycle, International Economic Review, 57, 1149–1177.

Laséen, S., and L. E. O. Svensson, 2011, Anticipated Alternative Policy Rate Paths in Policy Simulations, International Journal of Central Banking, 7, 1–35.

Lilliefors, H. W., 1967, On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown, Journal of the American Statistical Association, 62, 399–402.

Liu, Z., P. Wang, and T. Zha, 2013, Land-Price Dynamics and Macroeconomic Fluctuations, Econometrica, 81, 1147–1184.

Maffezzoli, M., and T. Monacelli, 2015, Deleverage and Financial Fragility, CEPR Discussion Papers, No. 10531.

Mandelman, F. S., P. Rabanal, J. F. Rubio-Ramírez, and D. Vilán, 2011, Investment-Specific Technology Shocks and International Business Cycles: An Empirical Assessment, Review of Economic Dynamics, 14, 136–155.

McConnell, M. M., and G. Perez-Quiros, 2000, Output Fluctuations in the United States: What Has Changed since the Early 1980's?, American Economic Review, 90, 1464–1476.

McKay, A., and R. Reis, 2008, The Brevity and Violence of Contractions and Expansions, Journal of Monetary Economics, 55, 738–751.

Mendoza, E. G., 2010, Sudden Stops, Financial Crises, and Leverage, American Economic Review, 100, 1941–1966.

Mian, A., and A. Sufi, 2010, Household Leverage and the Recession of 2007 to 2009, IMF Economic Review, 58, 74–117.

Mian, A., A. Sufi, and E. Verner, 2017, Household Debt and Business Cycles Worldwide, Quarterly Journal of Economics, 132, 1755–1817.

Mitchell, W. C., 1927, Business Cycles: The Problem and Its Setting, National Bureau of Economic Research: New York.

Morley, J., and J. Piger, 2012, The Asymmetric Business Cycle, Review of Economic and Statistics, 94, 208–221.

Müller, U., and M. Watson, 2018, Long-Run Covariability, Econometrica, 86, 775–804.

Neftci, S., 1984, Are Economic Time Series Asymmetric over the Business Cycle?, Journal of Political Economy, 92, 307–328.

Poeschl, J., 2018, Corporate Debt Maturity and Investment over the Business Cycle, Danmarks Nationalbank Working Paper 125.

Popov, A., 2014, Credit Constraints, Equity Market Liberalization, and Growth Rate Asymmetry, Journal of Development Economics, 107, 202–214.

Rancière, R., A. Tornell, and F. Westermann, 2008, Systemic Crises and Growth, Quarterly Journal of Economics, 123, 359–406.

Roberts, M. R., and A. Sufi, 2009, Renegotiation of Financial Contracts: Evidence from Private Credit Agreements, Journal of Financial Economics, 93, 159–184.

Rotemberg, J. J., 1999, A Heuristic Method for Extracting Smooth Trends from Economic Time Series, NBER Working Papers 7439, National Bureau of Economic Research.

Ruge-Murcia, F., 2007, Methods to Estimate Dynamic Stochastic General Equilibrium Models, Journal of Economic Dynamics and Control 31, 2599–2636.

Ruge-Murcia, F., 2012, Estimating Nonlinear DSGE Models by the Simulated Method of Moments: With an Application to Business Cycles, Journal of Economic Dynamics and Control 36, 914–938.

Schularick, M., and A. M. Taylor. 2012. Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises 1870-2008, American Economic Review, 102, 1029–61.

Sichel, D., 1993, Business Cycle Asymmetry: A Deeper Look, Economic Inquiry, 31, 224–236.

Stock, J., and M. Watson, 2003, Has the Business Cycle Changed and Why?, NBER Macroeconomics Annual 2002, 159–230.

Stock, J., and M. Watson, 2005, Understanding Changes In International Business Cycle Dynamics, Journal of the European Economic Association, 3, 968–1006.

White, H., 1980, A Heteroskedasticity-Consistent Covariance Matrix and a Direct Test for Heteroskedasticity, Econometrica, 48, 817–838.

Tables and Figures

Table 1. The skewness of the U.S. business cycle

	Quarter-on-Quarter Growth		Year-on-Year Growth	
	1947:I-1984:II	1984:III-2016:II	1947:I-1984:II	1984:III-2016:II
GDP	-0.118	-1.212	-0.098	-1.304
	[-0.325 ; 0.088]	[-1.559 ; -0.573]	[-0.285 ; 0.073]	[-1.516 ; -0.936]
Consumption	-0.506	-0.468	-0.202	-1.001
	[-1.134 ; 0.128]	[-0.725 ; -0.119]	[-0.368 ; -0.038]	[-1.181 ; -0.737]
Investment	-0.210	-0.827	-0.007	-1.399
	[-0.497 ; 0.096]	[-1.161 ; -0.277]	[-0.280 ; 0.229]	[-1.684 ; -0.983]

Notes: For different macroeconomic aggregates, we report the coefficient of skewness computed on the quarter-on-quarter and year-on-year growth rates, over the 1947:I-1984:II and 1984:III-2016:II samples. 68% confidence intervals (in brackets) are constructed by bootstrapping with 5000 replications. Data source: Federal Reserve Economic Data.

Table 2. U.S. business cycle volatility and semivolatilities

	GDP growth (quarter-on-quarter)		GDP growth (year-on-year)	
	1947:I-1984:II	1984:III-2016:II	1947:I-1984:II	1984:III-2016:II
σ	4.702	2.358	3.071	1.747
σ^-/σ^+	1.035	1.289	1.061	1.364
	[0.977; 1.0940]	[1.141; 1.409]	[1.007; 1.119]	[1.245; 1.467]

Notes: Table 2 reports the volatility of real GDP growth (both on a quarter-on-quarter and on a year-on-year basis) and the ratio between its downside and upside semivolatility. Specifically, $\sigma = \sqrt{\sum_{t=1}^{T}(x_t - \overline{x})^2/T}$, while the upside and downside semivolatility are defined as $\sigma^+ = \sqrt{\sum_{t=1}^{T}(x_t - \overline{x})^2 \mathbb{1}(x_t \geq \overline{x})/T}$ and $\sigma^- = \sqrt{\sum_{t=1}^{T}(x_t - \overline{x})^2 \mathbb{1}(x_t < \overline{x})/T}$, respectively, where $1(z)$ is an indicator function taking value 1 when condition z is true, and 0 otherwise. 68% confidence intervals (in brackets) are constructed by bootstrapping with 5000 replications. Data source: Federal Reserve Economic Data.

Table 3. The violence of recessions in the US

	Violence	Std. Violence	Duration (quarters)	
			Contractions	Expansions
1953:II – 1954:II	3.411	0.949	4	–
1957:III – 1958:II	7.309	2.542	3	13
1960:II – 1961:I	1.801	0.572	3	8
1969:IV – 1970:IV	0.471	0.267	4	35
1973:IV – 1975:I	2.529	1.228	5	12
1980:I – 1980:III	4.401	1.775	2	20
1981:III – 1982:IV	2.679	1.190	5	4
1990:III – 1991:I	2.651	3.624	2	31
2001:I – 2001:IV	1.267	1.785	3	40
2007:IV – 2009:II	2.891	3.297	6	24
Average				
Pre-1984	3.229	1.217	3.714	15.333
Post-1984	2.270	2.902	3.667	31.667

Notes: For every recession, we calculate 'Violence' as the annualized fall of real GDP from the peak to the trough of the contractionary episode, divided by the length of the recession; 'Std. Violence' standardizes the violence of the recession by the average business cycle volatility prior to the recession. The business cycle volatility is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year window. We exclude the period running up to the recession by calculating the standard deviation up to a year before the recession begins. Data source: NBER.

Table 4. Changing asymmetry in the G7 countries

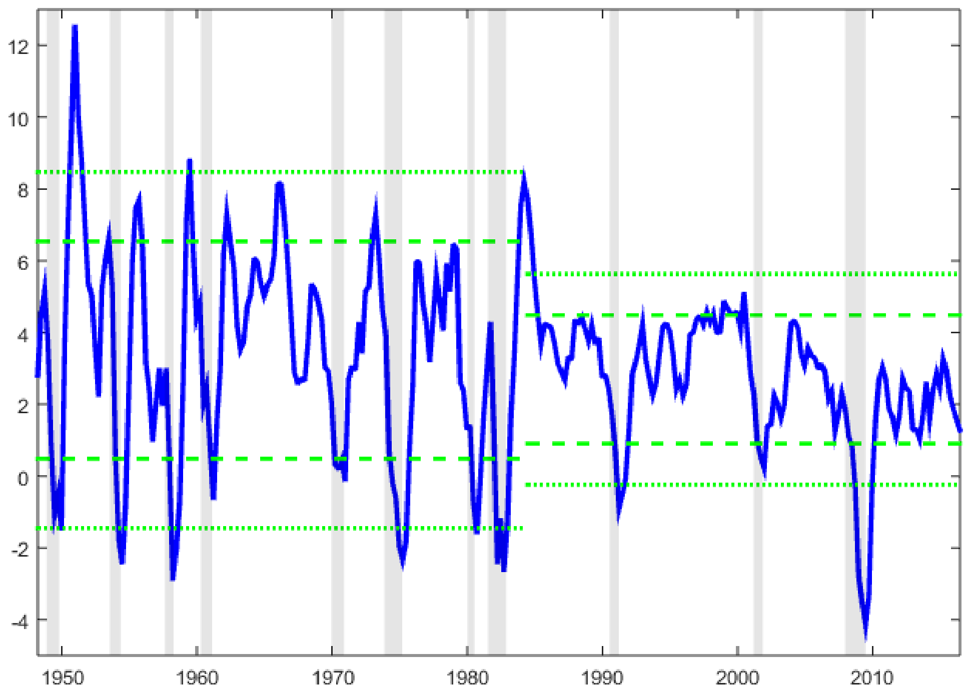
	Quarter-on-Quarter Growth				Year-on-Year Growth			
	Skewness		σ^-/σ^+		Skewness		σ^-/σ^+	
	Pre-84	Post-84	Pre-84	Post-84	Pre-84	Post-84	Pre-84	Post-84
Canada	-0.198	-0.946	1.040	1.227	-0.956	-1.023	1.284	1.313
	[-0.397; 0.036]	[-1.258; -0.492]	[0.976; 1.103]	[1.106; 1.326]	[-1.124; -0.692]	[-1.192; -0.768]	[1.187; 1.359]	[1.222; 1.394]
France	-0.045	-0.913	1.009	1.192	-0.711	-0.904	1.207	1.241
	[-0.882; 0.774]	[-1.217; -0.397]	[0.852; 1.196]	[1.067; 1.295]	[-1.2010 -0.2026]	[-1.101; -0.616]	[1.066 1.359]	[1.144; 1.318]
Germany	-0.356	-1.430	1.079	1.294	-0.508	-1.074	1.172	1.221
	[-0.739; 0.167]	[-1.919; -0.372]	[0.961; 1.195]	[1.084; 1.450]	[-0.710; -0.281]	[-1.353; -0.580]	[1.094; 1.246]	[1.083; 1.337]
Italy	0.247	-1.101	0.977	1.275	-0.431	-1.420	1.143	1.440
	[-0.448; 0.740]	[-1.454; -0.455]	[0.871; 1.117]	[1.135; 1.396]	[-0.611; -0.221]	[-1.648; -1.023]	[1.067; 1.212]	[1.318; 1.537]
Japan	-0.517	-0.982	1.085	1.205	-0.527	-1.021	1.166	1.247
	[-1.115; 0.392]	[-1.439; -0.149]	[0.928; 1.238]	[1.052; 1.330]	[-0.716; -0.311]	[-1.348; -0.451]	[1.088; 1.243]	[1.116; 1.361]
United Kingdom	0.633	-0.973	0.867	1.220	-0.436	-1.492	1.148	1.469
	[0.217; 0.951]	[-1.324; -0.443]	[0.788; 0.972]	[1.081; 1.350]	[-0.685; -0.175]	[-1.735; -1.105]	[1.064; 1.232]	[1.330; 1.582]

Notes: For each country, the table reports the skewness and the ratio between the downside and the upside semivolatility of detrended GDP growth (σ^-/σ^+) over the 1961:I-2016:II sample (both quarter-on-quarter and year-on-year). 68% confidence intervals (in brackets) are constructed by bootstrapping with 5000 replications. Data source: OECD database.

Table 5. Parameter values		
Panel A: Calibrated parameters		
Parameter	Description	Value
β^P	Discount factor, patient households	0.99
β^i, $i = \{I, E\}$	Discount factor, impatient households and entrepreneurs	0.96
φ^i, $i = \{P, I\}$	Curvature of utility of leisure	9
ν^i, $i = \{P, I\}$	Weight of labor disutility	0.27
ε	Weight of land utility	0.084
ϕ	Non-labor input share of land	0.134
γ	Labor share of production	0.636
δ	Capital depreciation rate	0.052
α	Income share of patient households	0.7
s^I	Initial loan-to-value ratio, impatient households	0.673
s^E	Initial loan-to-value ratio, entrepreneurs	0.763
ϑ^I	Refinancing rate, impatient households	0.009
ϑ^E	Refinancing rate, entrepreneurs	0.698
ξ^I	Amortization rate, impatient households	0.014
ξ^E	Amortization rate, entrepreneurs	0.125
Panel B: Estimated parameters		
Parameter	Description	Value
Ω	Investment adjustment cost parameter	8.933 (2.940)
θ^P	Habit formation, patient households	0.361 (0.112)
θ^I	Habit formation, impatient households + entrepreneurs	0.941 (0.045)
ρ_A	Persistence of technology shock	0.987 (0.044)
ρ_s	Persistence of credit-limit shock	0.853 (0.043)
ρ_ε	Persistence of land-demand shock	0.880 (0.398)
σ_A	Std. dev. of technology shock	0.009 (0.001)
σ_s	Std. dev. of credit-limit shock	0.033 (0.001)
σ_ε	Std. dev. of land-demand shock	0.072 (0.356)

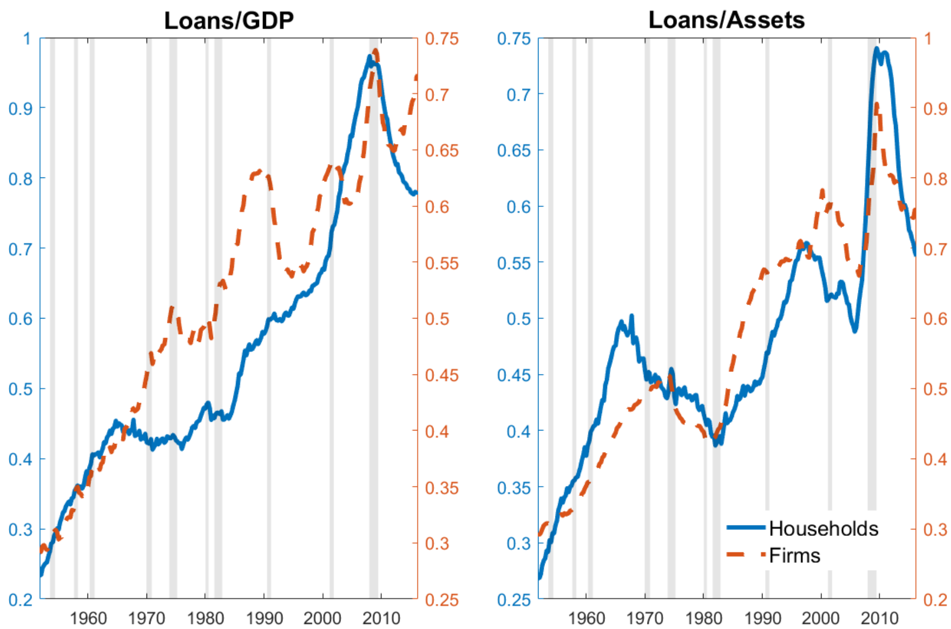
Note: The standard errors of the estimated parameters are reported in brackets.

Figure 1. Growth rates of U.S. real GDP



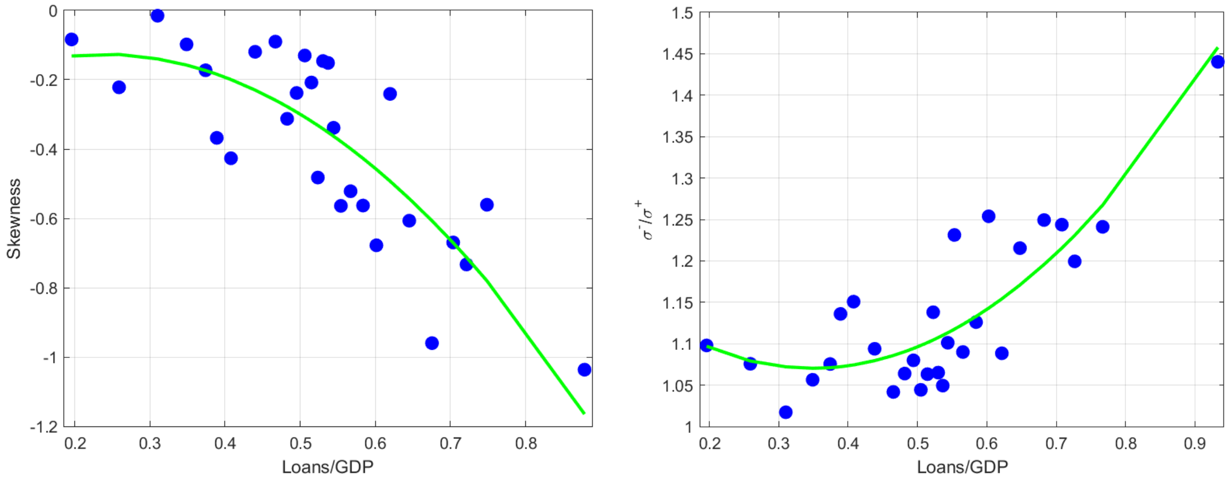
Notes: Year-on-year rate of growth of U.S. real GDP over the 1947:I-2016:II sample. The green bands correspond to the 68% and 90% confidence intervals from a Gaussian density fitted on the 1947:I-1984:II and 1984:III-2016:II samples. The vertical shadowed bands denote the NBER recession episodes. Data source: Federal Reserve Economic Data.

Figure 2. Household and corporate leverage in the US



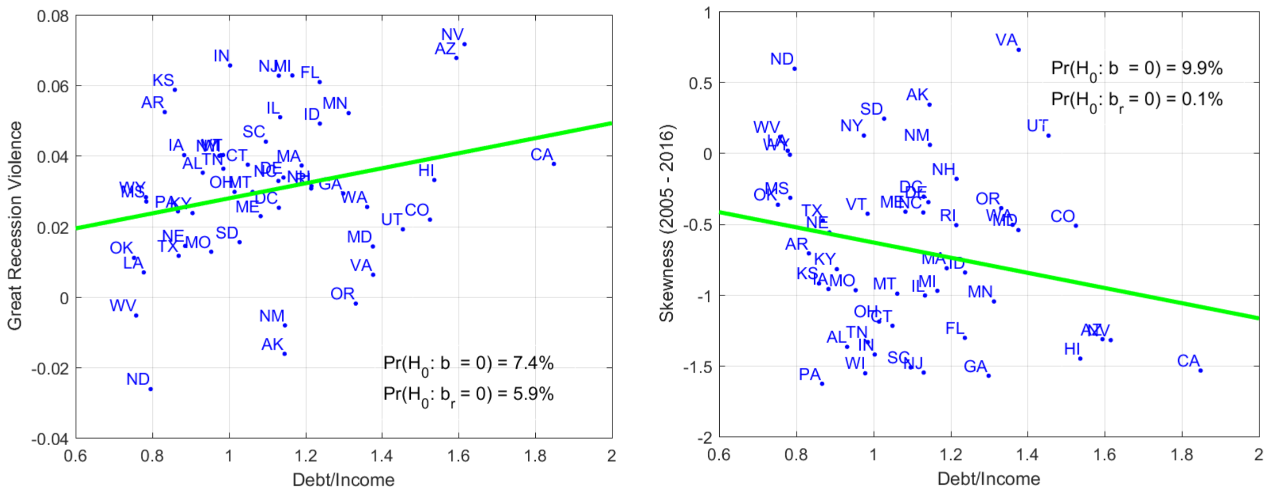
Notes: Left panel: the solid-blue line graphs the ratio between loans to households and GDP, while the dashed-red line reports the same variable at the corporate level. Right panel: the solid-blue line graphs the ratio between households' liabilities and assets, while the dashed-red line reports the same variable at the corporate level. The vertical shadowed bands denote the NBER recession episodes. Data source: Flow of Funds data, Financial Accounts of the US. See Appendix A for further details.

Figure 3. Leverage and asymmetry: G7 countries



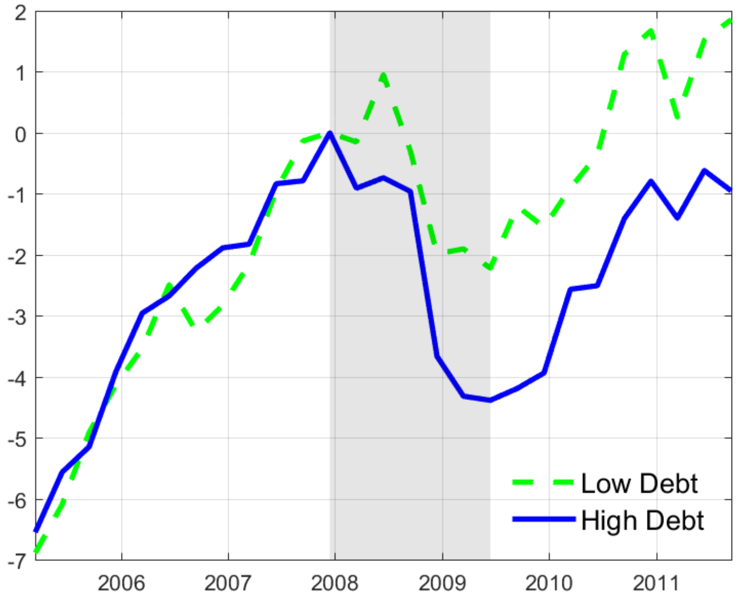
Notes: The left panel reports the skewness of quarter-on-quarter GDP growth, computed for the G7 countries, against their loan-to-GDP ratio. The right panel replaces the skewness of GDP growth with the ratio between its downside and upside semivolatilities (σ^-/σ^+). To construct the data points, we take quarterly GDP growth rates for all the countries under investigation, and construct 8-year rolling windows of data. Thus, we compute country-window specific moments, and relate them to the average credit-to-GDP ratio calculated over the same sample. The dots displayed in the figure are summary data for each moment, computed by grouping the credit-to-GDP ratio into 30 bins. The regression line is obtained by assuming a quadratic relationship between the two variables (accounting for country-level fixed effects). Data source: OECD and Jordà-Schularick-Taylor Macrohistory Database.

Figure 4. Leverage and asymmetry across U.S. States



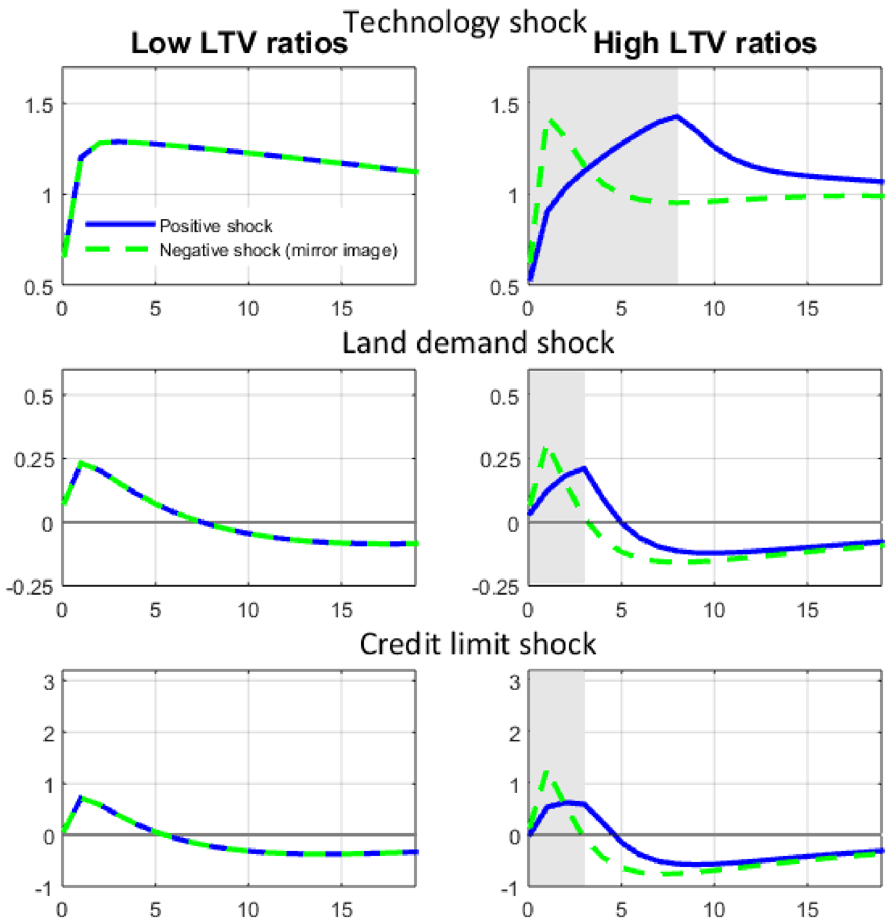
Notes: The left panel plots the violence of the Great Recession in each U.S. State against the average debt-to-income ratio at the household level over the period 2003:I-2007:I. To allow for the fact that the recession does not begin/end at the same time throughout the US, we calculate the start (end) of the recession in a given state as the period with the highest (lowest) level of real Gross State Product (GSP) in a window that goes from 5 quarters before (after) to one quarter after (before) the NBER dates. The right-hand panel plots the skewness of year-on-year real GSP growth over the 2005:I-2016:I period against the average debt-to-income ratio. In each panel we report the p-values associated with the slope coefficient: the first p-value is calculated on the slope coefficient estimated by OLS, while the second p-value refers to the slope estimated by excluding outliers (i.e., the observations whose standardized residuals from a first stage OLS regression are classified as being out of the 5/95% Gaussian confidence interval). In both cases we compute White (1980) heteroskedasticity-robust standard errors. Data sources: State Level Household Debt Statistics produced by the New York Fed and BEA Regional Economic Accounts.

Figure 5. GSP dynamics and household leverage



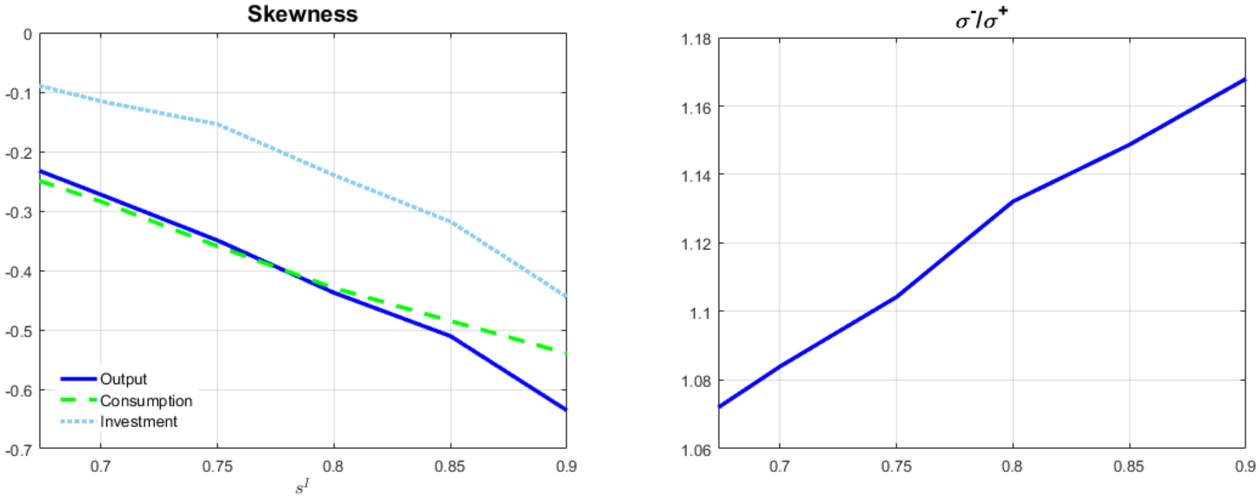
Notes: Growth rates of two synthetic GSP series obtained by ranking the U.S. States according to their average debt-to-income ratio in the 5 years before the Great Recession. The solid-blue line is calculated from the median real GSP of the top 10 states, while the dashed-green line is obtained from the median for the bottom 10 states. The resulting statistics have been normalized to zero at the beginning of the Great Recession (i.e., 2007:IV). The vertical shadowed band denotes the 2007:IV-2009:II recession episode. Data sources: State Level Household Debt Statistics produced by the New York Fed and BEA Regional Economic Accounts.

Figure 6. Impulse responses for different degrees of leverage



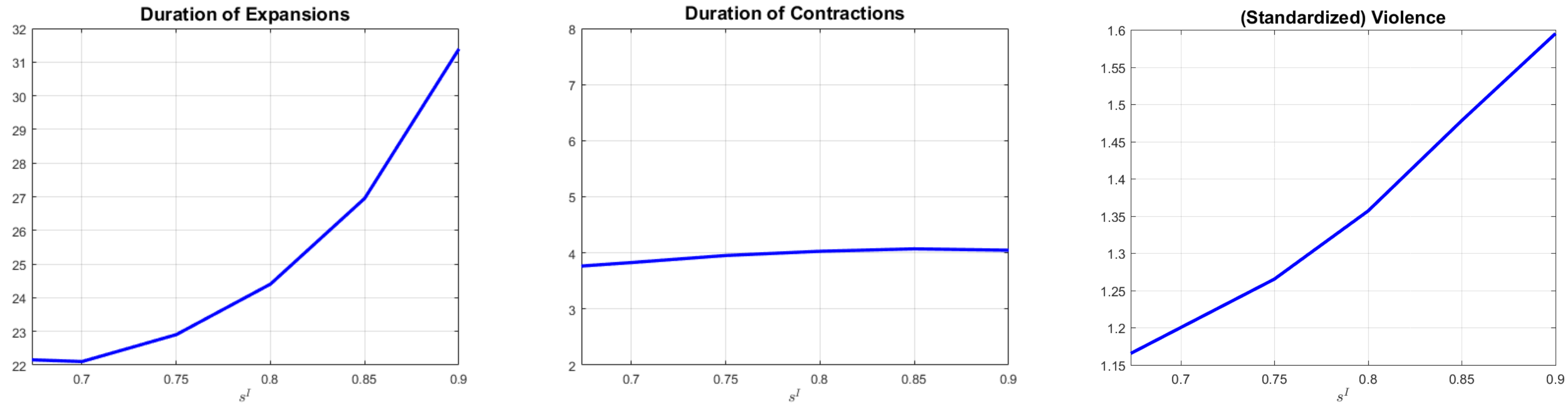
Notes: Impulse responses of output (in percentage deviation from the steady state) to a one-standard deviation shock to technology (row 1), a two-standard deviation shock to land demand (row 2), and a one-standard deviation shock to credit limits (row 3) in a model with debt duration of one quarter. Left column: $s^I = 0.67$, $s^E = 0.76$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Figure 7. Business cycle asymmetry



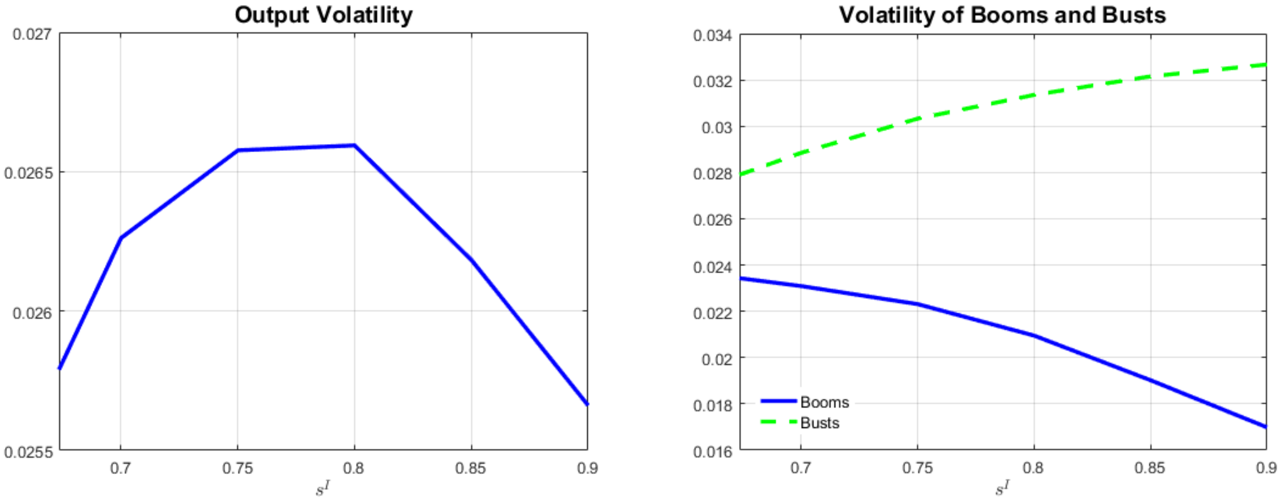
Notes: The left panel of the figure reports the skewness of the year-on-year growth rate of output, consumption and investment, while the right panel displays the ratio between the downside and the upside semivolatility of year-on-year output growth, for different average LTV ratios faced by the financially constrained agents. To identify the recessionary episodes in the simulated series, we use the Harding and Pagan (2002) algorithm. Across all the simulations the entrepreneurial average LTV ratio is adjusted to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 8. Expansions, contractions and leverage



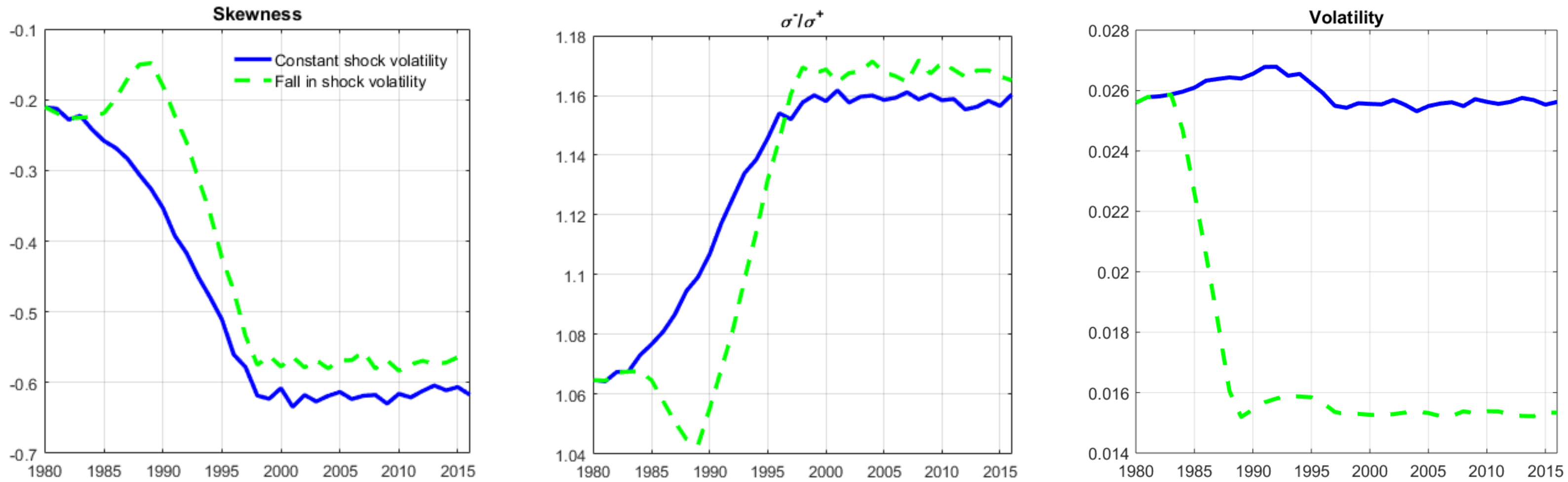
Notes: The first two panels report the duration (in quarters) of expansions and contractions, respectively. The last panel displays the standardized violence of recessionary episodes. To identify expansions and contractions in the simulated gross output series, we use the Harding and Pagan (2002) algorithm. We compute the violence as the average fall of output over a given recession, divided by the length of the recession itself. Finally, we standardize the violence by means of the volatility of year-on-year output growth over the five years prior to the recession. All statistics are conditional on different average LTV ratios faced by the financially constrained agents. Across all the simulations the entrepreneurial average LTV ratio is adjusted to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 9. Leverage and volatility



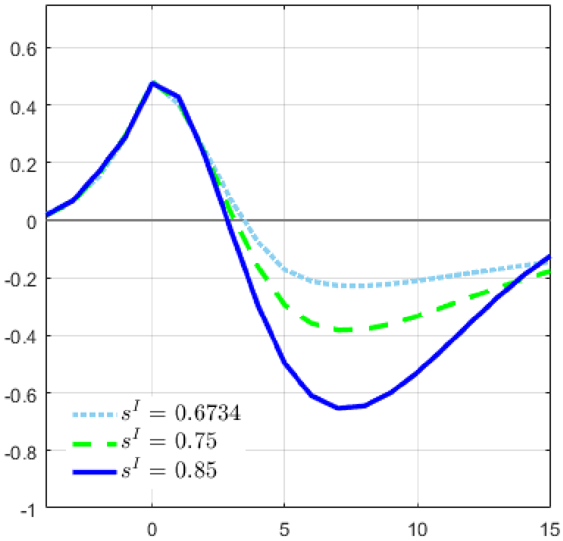
Notes: The left panel reports the standard deviation of year-on-year output growth, while the right panel reports the standard deviation of expansions (solid-blue line) and contractions (dashed-green line) in economic activity. These are determined based on whether output is above or below its steady-state level. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 10. Business cycle asymmetry and the Great Moderation



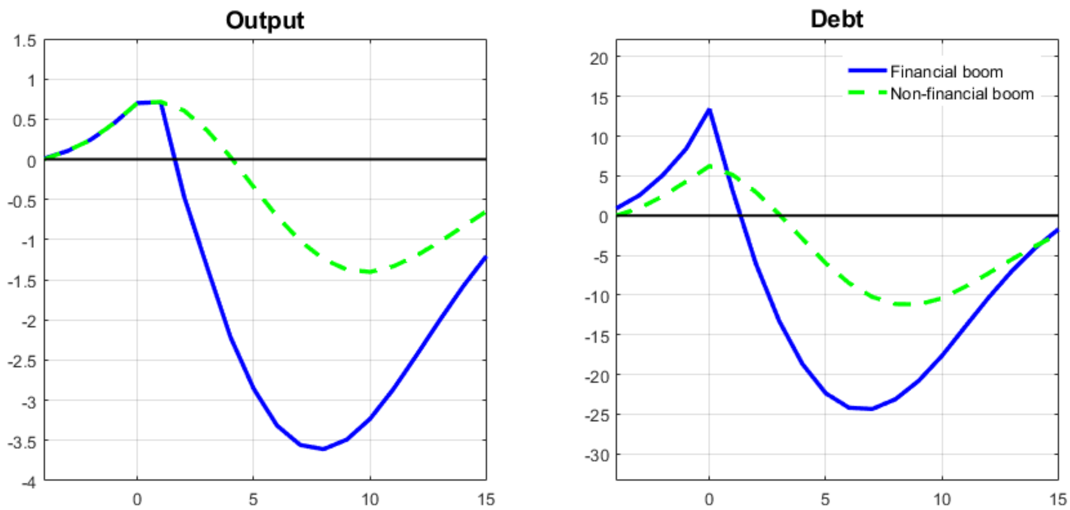
Notes. The figure displays changes in the skewness, the ratio between the downside and the upside semivolatility and the standard deviation of (year-on-year) output growth. Two counterfactual experiments are performed. In both of them, we feed the estimated trends of the LTV ratios of households and firms into the model (see Appendix A1), from 1980 to 2016. The solid-blue line refers to an experiment in which we keep the magnitude of the shocks to the economy constant and equal to their estimated values. The dashed-green line, instead, refers to an experiment in which the size of the shocks is gradually reduced so as to obtain a 40% reduction in the volatility of output growth over the 1984-1989 time span, and kept constant thereafter.

Figure 11. Boom-bust cycles and leverage



Notes: The figure shows the path of output (in percentage deviation from the steady state). Starting in steady state, we generate a boom-bust cycle for different steady-state debt levels, as implied by different average LTV ratios. We first feed the economy with a series of positive shocks during the first five periods, up until period 0. The size of the expansionary shocks is set so as to make sure that the boom is identical across all the calibrations. Thus, we shock the economy with identical contractionary shocks for two periods, after which the negative shocks are 'phased out' over the next three periods, i.e., their size is reduced successively and linearly. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 12. Financial vs. non-financial recessions



Notes: The figure shows the path of output (left panel) and aggregate debt (right panel), both in percentage deviations from the steady state. The solid-blue line represents a financial boom, while the dashed-green line represents a non-financial boom. Impatient households and entrepreneurs remain constrained throughout both types of booms. In this experiment, we set the average LTV ratios to $s^I = 0.85$ and $s^E = 0.94$. We calibrate the size of the expansionary shocks so as to deliver an identical increase in output during each type of boom (which lasts for five periods, up until period 0). We then subject the economy to identical sets of contractionary shocks of all three types. The contractionary shocks hit in periods 1 and 2, and are then 'phased out' over the next three periods, i.e., their size is reduced linearly.

Appendix A. Assets and liabilities in the US

Figure 2 shows the ratio of liabilities to assets for households and firms in the United States, respectively. All data are taken from FRED (Federal Reserve Economic Data), Federal Reserve Bank of St. Louis. The primary source is Flow of Funds data from the Board of Governors of the Federal Reserve System. For business liabilities we use the sum of debt securities and loans of nonfinancial corporate and noncorporate businesses. For assets we follow Liu *et al.* (2013) and use data on both sectors' equipment and software as well as real estate at market value. For households and nonprofit organizations, we again use the sum of debt securities and loans as data for liabilities and use as assets both groups' real estate at market value and equipment and software of nonprofit organizations.

The ratios reported in Figure 2 are aggregate measures, and may therefore not reflect actual loan-to-value (LTV) requirements for the marginal borrower. Nonetheless, we report these figures since the flow of funds data deliver a continuous measure of LTV ratios covering the entire period 1952–2016. For households, the aggregate ratio of credit to assets in the economy is likely to understate the actual downpayment requirements faced by households applying for a mortgage loan, since loans and assets are not evenly distributed across households. In our model we distinguish between patient and impatient households, and we assume that only the latter group is faced with a collateral constraint. In the data we do not make such a distinction, so that the LTV ratio for households reported in Figure 2 represents an average of the LTV of patient households (savers), who are likely to have many assets and small loans, and that of impatient households (borrowers), who on average have larger loans and fewer assets. Justiniano *et al.* (2014) use the Survey of Consumer Finances and identify borrowers as households with liquid assets of a value less than two months of their income. Based on the surveys from 1992, 1995, and 1998, they arrive at an average LTV ratio for this group of around 0.8, while our measure fluctuates around 0.5 during the 1990s. Following Duca *et al.* (2011), an alternative approach is to focus on first-time home-buyers, who are likely to fully exploit their borrowing capacity. Using data from the American Housing Survey, these authors report LTV ratios approaching 0.9 towards the end of the 1990s; reaching a peak of almost 0.95 before the onset of the recent crisis. While these alternative approaches are likely to result in higher *levels* of LTV ratios, we are especially interested in the development of these ratios over a rather long time span. While we believe the Flow of Funds data provide the most comprehensive and consistent time series evidence in this respect, substantial increases over time in the LTV ratios faced by households have been extensively documented; see, e.g., Campbell and Hercowitz (2009), Duca *et al.* (2011), Favilukis *et al.* (2017), and Boz and Mendoza (2014). It should be noted that for households, various government-sponsored programs directed at lowering the down-payment requirements faced by low-income or first-time home buyers have been enacted by different administrations (Chambers *et al.*, 2009). These are likely to have contributed to the increase in the ratio of loans to assets illustrated in the left panel of Figure 2.

Likewise, the aggregate ratio of business loans to assets in the data may cover for a disparate distribution of credit and assets across firms. In general, the borrowing patterns and conditions of firms are more difficult to characterize than those of households, as their credit demand is more volatile, and their assets are less uniform and often more difficult to assess. Liu *et al.* (2013) also use Flow of Funds data to calibrate the LTV ratio of the entrepreneurs, and arrive at a value of 0.75. This ratio is based on the assumption that commercial real estate enters with a weight of 0.5 in the asset composition of firms. The secular increase in firm leverage over the second half of the 20th century has also been documented by Graham *et al.* (2014)

using data from the Compustat database.[40,41] These authors report loan-to-asset ratios that are broadly in line with those we present. More generally, an enhanced access of firms to credit markets over time has been extensively documented in the literature, as also discussed in the main text.

A1. Long-run properties of the LTV ratios

In this subsection we investigate the low-frequency properties of the LTV ratios of households and the corporate sector over the 1952:I-2016:II time window. We follow the approach of Müller and Watson (2018), who develop methods to investigate the long-run comovement of two time series.

Since it has been argued that the amplitude of the financial cycle can potentially be much longer than the business cycle (Borio, 2014), we focus on the very low-frequency movements in the LTV ratios. In our baseline specification, we focus on fluctuations over periods longer than 30 years. Table A1 reports the long-run correlation coefficients, as well as the slope coefficient of a linear regression relating household to corporate debt, together with the 68% confidence interval. Figure A1 reports the two LTV ratios, together with their low-frequency components. The two series display strong comovement at the very low frequency, with the slope coefficient containing 1 in the confidence interval. Between 1984 and 2016, this component increased by 20 and 23 basis points for households and firms, respectively. It is also worth emphasizing that, once we remove low-frequency variation in the LTV ratios, their 'cyclical' variations are strongly correlated (about 65%). This evidence supports our modelling choice for the behavior of the LTV ratios, with the trend components for the household and the corporate sector rising in tandem by 23 basis points in Section 6.2, and a common cyclical component. The spread between the household and entrepreneurial steady-state LTV ratios is set to match the average difference in the low-frequency components over the entire sample, which is roughly equal to the average difference in the original series.

Table A1 also reports additional robustness results for different choices of the minimum-length period of the low-frequency component. The results of the baseline specification are quite robust for reasonable variations of the cut-off choice.

[40] It should be mentioned that they also show a Flow of Funds-based measure of debt to total assets at historical cost (or book value) for firms. The increase over time in this measure is smaller. However, we believe that the ratio of debt to *pledgeable* assets at market values (as shown in Figure 2) is the relevant measure for firms' access to collateralized loans, and hence more appropriate for our purposes.

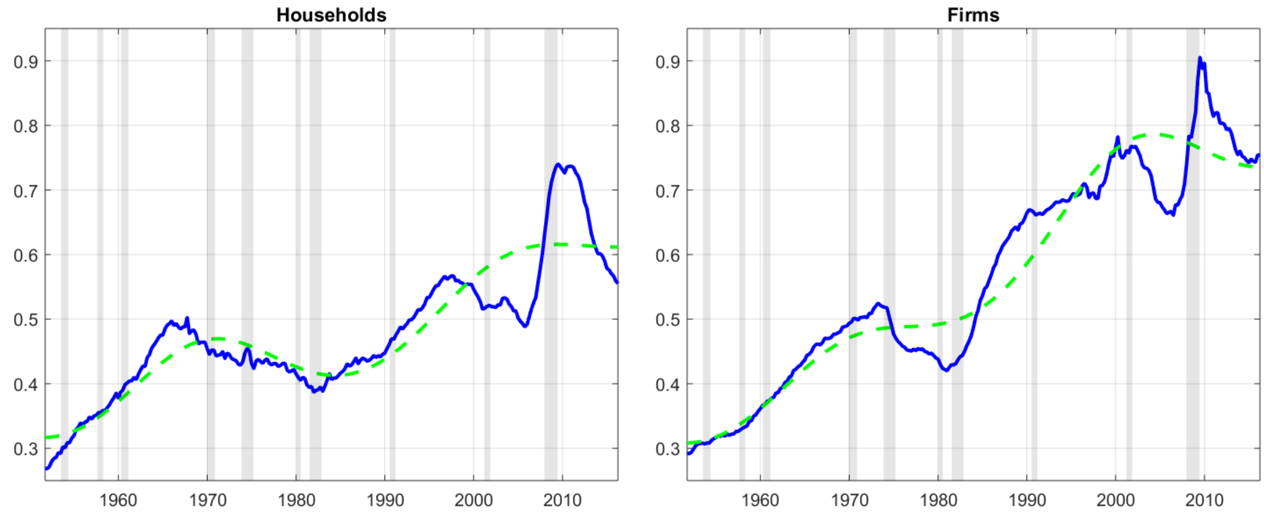
[41] We emphasize that Figure 2 reports a *gross* measure of firm leverage. Bates *et al.* (2009) report that firm leverage *net of* cash holdings has been declining since 1980, but that this decline is entirely due to a large increase in cash holdings.

Table A1. Household and corporate LTV ratios in the long run

Periods longer than 30 years	
$\widehat{\rho}$	$\widehat{\beta}$
0.847	1.224
[0.511 - 0.947]	[0.680 - 1.618]
Periods longer than 35 years	
$\widehat{\rho}$	$\widehat{\beta}$
0.776	1.317
[0.250 - 0.950]	[0.439 - 1.952]
Periods longer than 25 years	
$\widehat{\rho}$	$\widehat{\beta}$
0.892	1.160
[0.703 - 0.957]	[0.769 - 1.605]

Notes: Table A1 summarizes the long-run covariance ($\hat{\rho}$ denotes the correlation coefficient and $\hat{\beta}$ denotes the slope coefficient of the linear relationship) and the 68% confidence set (in brackets) for the household and the corporate LTV ratios.

Figure A1. Low-frequency components of the LTV ratios



Notes: Each plot reports the LTV ratio (solid-blue line) and its low-frequency component (dashed-green line). The left panel reports data for the household sector, whereas the right panel refers to the corporate sector.

Appendix B. Additional empirical evidence

B1. Time-varying volatility and skewness

In the main text we report evidence on the skewness of real GDP growth being different before and during the Great Moderation. The choice of a cut-off date is inspired by a large literature that has documented a drop in the volatility over the two samples. This exercise entails a possible drawback: The estimates of the skewness can be biased by the first and second moment of the business cycle changing over time. In particular: i) There is now ample evidence that the volatility of the business cycle displays a cyclical behavior (see, e.g., Kim and Nelson, 1999; and McConnell and Perez-Quiros, 2000) and ii) the long-run growth rate of the economy since around 2000 is substantially lower than the average for the entire sample (see, e.g., Antolin-Diaz *et al.*, 2017). To account for these issues we report a measure of time-varying skewness of real GDP growth for the entire sample, relying on a nonparametric estimator. To this end, take a generic time series, y_t, so that its variance and skewness can be respectively calculated as

$$\begin{aligned}
\sigma^2 &= Var\left(y_t\right) = \frac{1}{T}\sum_{t=1}^{T}\left(y_t-\mu\right)^2, \\
\varrho &= Skew\left(y_t\right) = \left\{\frac{1}{T}\sum_{t=1}^{T}\left(y_t-\mu\right)^2\right\}^{-3/2}\left\{\frac{1}{T}\sum_{t=1}^{T}\left(y_t-\mu\right)^3\right\},
\end{aligned}$$

where T denotes the number of observations in the sample and $\mu = E\left(y_t\right) = T^{-1}\sum_{t=1}^{T} y_t$ is the sample average. Define the sample autocovariance and autocorrelation as

$$\begin{aligned}
\gamma_\tau &= \frac{1}{T}\sum_{t=1}^{T-|\tau|}\left(y_{t-|\tau|}-\mu\right)\left(y_t-\mu\right), \\
\rho_\tau &= \frac{\gamma_\tau}{\sigma^2}.
\end{aligned}$$

When y_t is a Gaussian process with absolutely summable autocovariances, it can be shown that the standard errors associated with the two measures are:[42]

$$\begin{aligned}
Var\left(\sigma^2\right) &= \frac{2}{T}\left(\sum_{\tau=-\infty}^{\infty}\gamma_\tau\right)^2, \\
Var\left(\varrho\right) &= \frac{6}{T}\sum_{\tau=-\infty}^{\infty}\rho_\tau^3.
\end{aligned}$$

In practice the two summations are truncated at some appropriate (finite) lag k.

The framework we follow in order to account for time-variation in the variance and skewness has a long pedigree in statistics, starting with the work of Priestley (1965), who introduced the concept of slowly varying process. This work suggests that time series may have time-varying spectral densities which change slowly over time, and proposed to describe those changes as the result of a non-parametric process. This work has more recently been followed up by Dahlhaus (1996), as well as Kapetanios (2007) and Giraitis *et al.* (2014) in the context of time-varying regression models and economic forecasting, respectively. Specifically, the time-varying variance and skewness are calculated as

$$\begin{aligned}
\sigma_t^2 &= Var_t\left(y_t\right) = \sum_{j=1}^{t}\omega_{j,t}\left(y_j-\mu_t\right)^2, \\
\varrho_t &= Skew_t\left(y_t\right) = \left\{\sum_{j=1}^{t}\omega_{j,t}\left(y_j-\mu_t\right)^2\right\}^{-3/2}\left\{\sum_{j=1}^{t}\omega_{j,t}\left(y_j-\mu_t\right)^3\right\},
\end{aligned}$$

[42] The first expression computes the variance as the Newey-West variance of the squared residuals, in order to account for the autocorrelation of the errors. The second equality follows from Gasser (1975) and Psaradakis and Sola (2003).

where $\mu_t = \sum_{j=1}^{t} \omega_{j,t} y_j$. Thus, the sample moments are discounted by the function $\omega_{t,T}$:

$$\omega_{j,t} = cK\left(\frac{t-j}{H}\right),$$

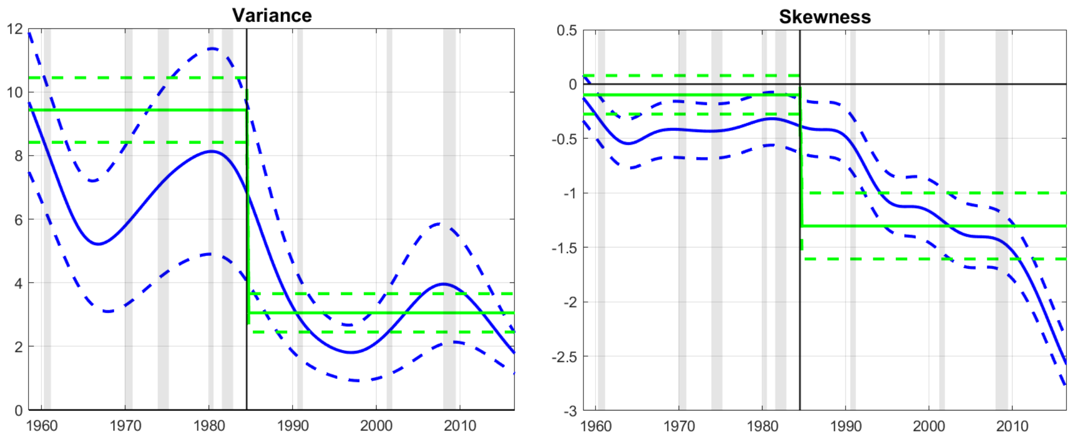
where c is an integration constant and $K\left(\frac{T-t}{H}\right)$ is the kernel function determining the weight of each observation j in the estimation at time t. This weight depends on the distance to t normalized by the bandwidth H. Giraitis *et al.* (2014) show that the estimator has desirable frequentist properties. They suggest using Gaussian kernels with the optimal bandwidth value $H = T^{1/2}$.

Similarly, we can compute the time-varying standard deviation of variance and skewness estimates using time-varying estimates of the sample autocovariance and autocorrelations:

$$\begin{aligned}\gamma_{\tau,t} &= \sum_{j=1}^{t-|\tau|} \omega_{j,t}\left(y_{j-|\tau|} - \mu_t\right)\left(y_j - \mu_t\right), \\ \rho_{\tau,t} &= \frac{\gamma_{\tau,t}}{\sigma_t^2}.\end{aligned}$$

Based on this, Figure B1 reports time-varying measures of volatility and skewness of GDP growth. The left panel confirms the widely documented decline in volatility. From the right panel, it is clear that skewness drops in the second subsample, with a first drop being identified after the 1991 recession and a further one after the Great Recession.

Figure B1. Time-varying volatility and skewness



Notes. Figure B1 reports the time-varying variance and skewness of year-on-year growth of real GDP (solid-blue lines)—obtained by using a nonparametric estimator in the spirit of Giraitis *et al.* (2014)—as well as the associated 68% confidence interval (dashed-blue lines). We also report the variance and skewness of real GDP growth computed over the pre- and post-Great Moderation sample (solid-green lines), as well as the associated 68% confidence interval (dashed-green lines). The vertical shadowed bands denote the NBER recession episodes. Sample: 1947:I-2016:II. The first 10 years of data are dropped to initialize the algorithm. Data source: FRED.

B2. Normality tests

Table B1. Normality tests

	GDP growth (QoQ)	
	1947:I-1984:II	1984:III-2016:II
KS	0.638	0.002
AD	0.534	0.000
SW	0.507	0.000
JB	$\gg$0.50	$\ll$0.001
	GDP growth (YoY)	
	1947:I-1984:II	1984:III-2016:II
KS	0.289	0.004
AD	0.060	0.000
SW	0.091	0.000
JB	$\gg$0.50	$\ll$0.001

Notes. Table B1 reports the p-values of a battery of tests assuming the null hypothesis that real GDP growth is normally distributed in a given sample. KS refers to Kolmogorov-Smirnov test with estimated parameters (see Liliefors, 1967); AD refers to the test of Anderson and Darling (1954); SW refers to the Shapiro-Wilk test (Shapiro and Wilk, 1965) with p-values calculated as outlined by Royston (1992); JB refers to the Jarque-Bera test for normality (Jarque and Bera, 1987). Data source: FRED.

B3. Additional evidence on the standardized violence of the US business cycle

Table B2. Standardized violence of U.S. recessions (Robustness)

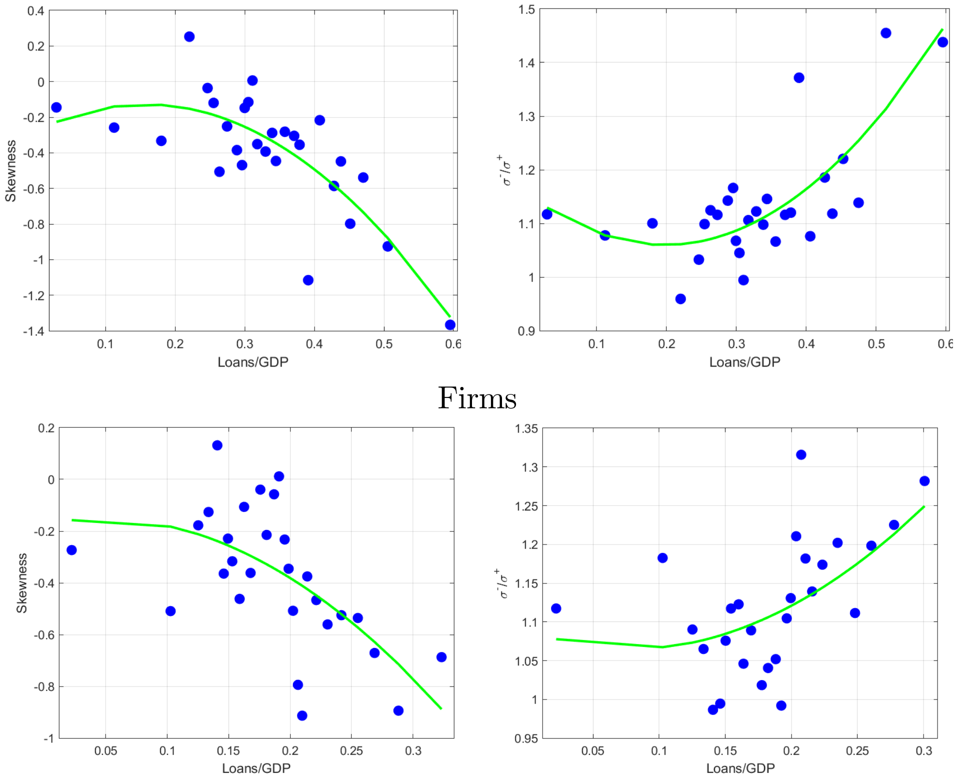
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1953:II – 1954:II	0.635	1.127	0.734	0.731	0.599	0.664	0.584
1957:III – 1958:II	1.629	2.415	1.572	1.767	1.716	1.688	1.785
1960:II – 1961:I	0.351	0.595	0.387	0.429	0.307	0.464	0.304
1969:IV – 1970:IV	0.163	0.156	0.101	0.248	0.155	0.240	0.158
1973:IV – 1975:I	0.662	0.836	0.544	0.847	0.542	0.833	0.509
1980:I – 1980:III	0.999	1.454	0.947	1.239	0.972	1.234	0.863
1981:III – 1982:IV	0.598	0.885	0.576	0.645	0.448	0.621	0.400
1990:III – 1991:I	1.910	1.527	1.132	1.363	1.255	1.229	1.301
2001:I – 2001:IV	0.730	0.730	0.541	0.755	0.463	0.701	0.419
2007:IV – 2009:II	1.847	1.665	1.234	2.020	1.607	1.915	1.571
Average							
Pre-84	0.720	1.067	0.695	0.844	0.677	0.821	0.657
Post-84	1.495	1.307	0.969	1.380	1.108	1.282	1.0967

Notes: Table B2 reports different measures of standardized violence that change depending on the business cycle volatility employed in the denominator. Column (1) follows the same procedure employed to obtain standardized violence in Table 3, though the volatility measure is retrieved from quarter-on-quarter growth rates of real GDP. In the remaining computations, even column numbers report violence statistics that are standardized by volatility measures retrieved from quarter-on-quarter growth rates or real GDP, while in odd column numbers the standardization is operated through volatility measures obtained from year-on-year growth rates. Columns (2) and (3) calculate the volatility by splitting the data between pre- and post-Great Moderation. In columns (4) and (5) the standardization is operated by considering the following stochastic volatility model for real GDP growth: $y_t = \rho_0 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \sigma_t \varepsilon_t$, where $\sigma_t^2 = \sigma_{t-1}^2 + \kappa \sigma_t^2 \left(\varepsilon_t^2 - 1\right)$ and $\varepsilon_t \sim N\left(0, 1\right)$. In columns (6) and (7) the standardization is operated by considering a time-varying AR model for real GDP growth with stochastic volatility similar to that of Stock and Watson (2005), where all the time-varying parameters follow random walk laws of motion (as in Delle Monache and Petrella, 2017). Data source: NBER.

B4. Leverage and asymmetry (G7 countries)

Figure B2. Leverage and asymmetry: G7 countries

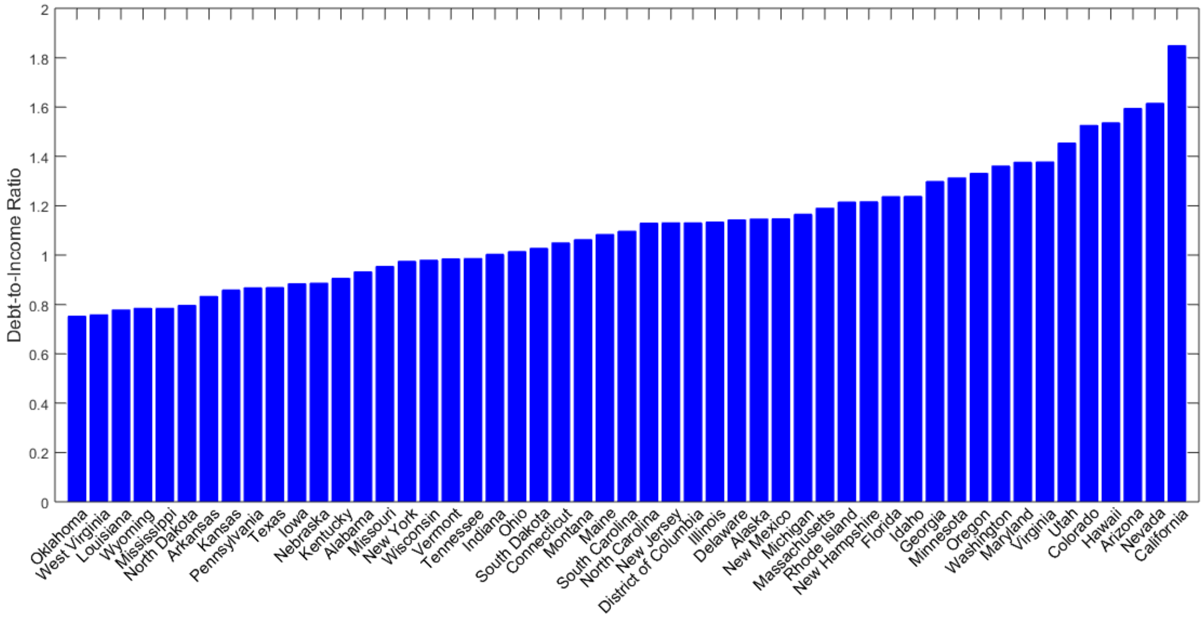
Households



Notes: The top panels refer to the household sector, while the bottom panels refer to the corporate sector. The left panel of each line reports the skewness of GDP growth, computed for each G7 country, against the loan-to-GDP ratio of a specific sector. In the right panels we replace the skewness with the ratio between the downside and the upside semivolatility of business fluctuations. The regression line is obtained by assuming a quadratic relationship between the two variables (accounting for sector-specific fixed effects). Data source: OECD and Jordà-Schularick-Taylor Macrohistory Database.

B5. Household leverage in the US

Figure B3. U.S. States ordered by households' average debt-to-income ratio



Notes. U.S. States ordered by the average debt-to-income ratio in the household sector, over the period 2003-2007. Data source: State Level Household Debt Statistics produced by the New York Fed.

Appendix C. Details on the solution of the two-period model

Here, we provide details on the computation of the competitive equilibrium of the two-period model discussed in Section 3. The notation is explained in the main text.

Optimality

We first derive the optimality conditions. Rewrite the maximand with the budget constraints and the definition of capital accumulation to get

$$\begin{aligned}\widetilde{U} \;=\;& \log\left[r_1^K K_0 + W_1 - RB_0 + B_1 - K_1 + (1-\delta) K_0\right] \\ &+\beta \log\left[\left(1+r_2^K\right) K_1 + W_2 - RB_1\right].\end{aligned}$$

We maximize $\widetilde{U}$ w.r.t. K_1 and B_1, subject to (5). Factor payments are taken as given, as these are co-determined by the demands of all firms in the economy.

We get the first-order conditions

$$-\frac{1}{r_1^K K_0 + W_1 - RB_0 + B_1 - K_1 + (1-\delta) K_0} + \beta \frac{1+r_2^K}{\left(1+r_2^K\right) K_1 + W_2 - RB_1} + \mu \frac{s}{R} = 0, \quad (31)$$

$$\frac{1}{r_1^K K_0 + W_1 - RB_0 + B_1 - K_1 + (1-\delta) K_0} - \beta \frac{R}{\left(1+r_2^K\right) K_1 + W_2 - RB_1} - \mu = 0, \quad (32)$$

$$\mu\left(B_1 - s\frac{K_1}{R}\right) = 0, \qquad \mu \geq 0, \quad (33)$$

where μ is the multiplier applying to the collateral constraint.

Otherwise, the production factors are remunerated at their marginal product:

$$r_t^K = \alpha A_t K_{t-1}^{\alpha-1} L_t^{1-\alpha}, \tag{34}$$

$$W_t = (1-\alpha) A_t K_{t-1}^{\alpha} L_t^{-\alpha}, \qquad t = 1, 2. \tag{35}$$

The case of an equilibrium with a non-binding constraint

In this case, $\mu = 0$, so that (31) and (32) become

$$\begin{aligned} \frac{1}{C_1} &= \beta \frac{1 + r_2^K}{C_2}, \\ \frac{1}{C_1} &= \beta \frac{R}{C_2}, \end{aligned}$$

and no-arbitrage implies

$$1 + r_2^K = R. \tag{36}$$

This pins down K_1 from (34):

$$K_1 = \left[\frac{\alpha A}{R-1}\right]^{\frac{1}{1-\alpha}}. \tag{37}$$

From (35) we can also recover the wage rate in period 2:

$$W_2 = (1-\alpha) A \left[\frac{\alpha A}{R-1}\right]^{\frac{\alpha}{1-\alpha}}. \tag{38}$$

Thus, total income amounts to

$$\left(1 + r_2^K\right) K_1 + W_2 = \frac{1}{\alpha} \left[\frac{\alpha A}{R-1}\right]^{\frac{1}{1-\alpha}} (\alpha + R - 1),$$

so that we retrieve

$$C_2 = \Gamma - RB_1, \tag{39}$$

where $\Gamma \equiv \frac{1}{\alpha}\left[\frac{\alpha A}{R-1}\right]^{\frac{1}{1-\alpha}} (\alpha + R - 1)$. Plugging (39) into (32), together with (2), returns

$$\frac{1}{\underbrace{r_1^K K_0 + W_1 - RB_0 + B_1 - K_1 + (1-\delta) K_0}_{= C_1}} = \beta \frac{R}{\Gamma - RB_1},$$

and, therefore:

$$\Gamma - RB_1 = \beta R \left[r_1^K K_0 + W_1 - RB_0 + B_1 - K_1 + (1-\delta) K_0\right].$$

Plugging in the solutions for W_1, r_1^K and K_1 results into

$$\Gamma - RB_1 = \beta R \left[A_1 K_0^{\alpha} - RB_0 + B_1 - \left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}} + (1-\delta) K_0\right],$$

from which we can characterize B_1 as

$$B_1 = \frac{\Gamma}{R(1+\beta)} - \frac{\beta}{1+\beta}\left[A_1 K_0^{\alpha} - RB_0 - \left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}} + (1-\delta) K_0\right]. \tag{40}$$

We can then derive I_1, C_1 and C_2. We have, by solution of K_1, that

$$I_1 = \left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}} - (1-\delta) K_0. \tag{41}$$

We find C_2 by combining (40) with (39):

$$C_2 = \frac{\beta\Gamma}{1+\beta} + \frac{\beta R}{1+\beta}\left[A_1 K_0^{\alpha} - RB_0 - \left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}} + (1-\delta) K_0\right].$$

Finally, using $1/C_1 = \beta R/C_2$ in the unconstrained case, we get

$$C_1 = \frac{\Gamma}{R(1+\beta)} + \frac{1}{1+\beta}\left[A_1 K_0^{\alpha} - RB_0 - \left(\frac{\alpha A}{R-1}\right)^{\frac{1}{1-\alpha}} + (1-\delta) K_0\right].$$

The case of a binding constraint

In this case, $\mu > 0$. We first use (31) and (32):

$$-\frac{1}{C_1} + \beta\frac{1+r_2^K}{C_2} + \mu\frac{s}{R} = 0, \tag{42}$$

$$\frac{1}{C_1} - \beta\frac{R}{C_2} - \mu = 0. \tag{43}$$

Adding the left- and the right-hand side terms gives

$$\beta\frac{1+r_2^K - R}{C_2} = \mu\left(1 - \frac{s}{R}\right) > 0. \tag{44}$$

This shows how a binding borrowing constraint induces a wedge between the return on borrowing and capital; i.e., (36) ceases to hold. Specifically, investment is depressed, which drives the gross marginal return of capital above R.

C_2 depends on K_1 and W_2 as before, but B_1 and K_1 are now linked by the credit constraint. However, r_2^K does not pin down K_1 as in the unconstrained case, as $\mu > 0$; cf. (44). Using (42) and (43) eliminate μ:

$$\frac{1}{C_1} = \beta\frac{1+r_2^K - s}{C_2\left(1-\frac{s}{R}\right)}.$$

Thus, using (2) and (3):

$$\frac{1}{r_1^K K_0 + W_1 - RB_0 + B_1 - K_1 + (1-\delta) K_0} = \beta\frac{1+r_2^K - s}{\left[\left(1+r_2^K\right) K_1 + W_2 - RB_1\right]\left(1-\frac{s}{R}\right)}.$$

We can now use the expressions for r_1^K, r_2^K, W_1 and W_2 to get

$$\frac{1}{A_1 K_0^{\alpha} - RB_0 + B_1 - K_1 + (1-\delta) K_0} = \beta\frac{1+\alpha A K_1^{\alpha-1} - s}{\left[K_1 + A K_1^{\alpha} - RB_1\right]\left(1-\frac{s}{R}\right)}.$$

Finally, we use the binding credit constraint,

$$B_1 = s\frac{K_1}{R},$$

to eliminate B_1:

$$\frac{1}{A_1K_0^{\alpha} - RB_0 - \left(1 - \frac{s}{R}\right)K_1 + (1-\delta)K_0} = \beta\frac{1 + \alpha AK_1^{\alpha-1} - s}{\left[(1-s)K_1 + AK_1^{\alpha}\right]\left(1 - \frac{s}{R}\right)}. \tag{45}$$

This provides a non-linear characterization of K_1 (and, thus, investment). The expression above can be reshuffled to get

$$\Psi\left(K_1; A_1\right) = 0,$$

where

$$\begin{aligned}\Psi\left(K_1; A_1\right) \equiv\ & \beta\left(1 + \alpha AK_1^{\alpha-1} - s\right)\left[A_1K_0^{\alpha} - RB_0 - \left(1 - \frac{s}{R}\right)K_1 + (1-\delta)K_0\right] \\ & - \left[(1-s)K_1 + AK_1^{\alpha}\right]\left(1 - \frac{s}{R}\right).\end{aligned}$$

Appendix D. Details on the design and solution of the DSGE model

This appendix reports further information on the design and solution of the DSGE model. We first provide some details on the modeling and calibration of the debt contracts. We then proceed to state the first-order conditions, the steady state, and the log-linearization of the model.

D1. Debt contracts

Impatient households and entrepreneurs take up debt with maturity greater than one period. The borrowing constraints presented in the main text, (18) and (24), are rationalized in line with Kydland *et al.* (2016). Let L_t^i denote the flow of lending to agent $i = \{I, E\}$ in period t. This consists of two elements: Agent i's share of existing, non-amortized debt that is refinanced in period t, $\vartheta^i(1-\xi^i)B_{t-1}^i$, and new 'net' lending, $L_t^{i,net}$. Thus:

$$L_t^i = L_t^{i,net} + \vartheta^i(1-\xi^i)B_{t-1}^i. \tag{46}$$

The flow of lending is related to the stock of debt via the following law of motion:

$$B_t^i = \left(1 - \vartheta^i\right)\left(1 - \xi^i\right)B_{t-1}^i + L_t^i, \tag{47}$$

or, using (46):

$$B_t^i = \left(1 - \xi^i\right)B_{t-1}^i + L_t^{i,net}.$$

When taking on new debt, borrowers can pledge as collateral only the fraction of their assets not already used to secure the existing stock of debt. Since ϑ^i denotes the fraction of existing debt that is refinanced, the remaining share $1 - \vartheta^i$ of existing debt is collateralized by the same fraction of the borrower's assets. This implies the upper bounds on new lending, for each agent:

$$L_t^I \leq \vartheta^I s_t^I \frac{\mathrm{E}_t\left\{Q_{t+1}\right\}H_t^I}{R_t},$$

$$L_t^E \leq \vartheta^E s_t^E \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\}.$$

Combining these two expressions with the law of motion for debt, (47), we obtain the borrowing constraints presented in the main text:

$$B_t^I \leq \vartheta^I s_t^I \frac{\mathrm{E}_t \left\{Q_{t+1}\right\} H_t^I}{R_t} + \left(1-\vartheta^I\right)\left(1-\xi^I\right) B_{t-1}^I,$$

$$B_t^E \leq \vartheta^E s_t^E \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} + \left(1-\vartheta^E\right)\left(1-\xi^E\right) B_{t-1}^E.$$

Steady state and calibration of the debt contracts

It is useful to introduce Λ_t^i (for $i = \{I, E\}$) to denote the fraction of total lending that goes into the refinancing of old debt. From (46), it follows that:

$$\Lambda_t^i \equiv \frac{L_t^i - L_t^{i,net}}{L_t^i} = \vartheta^i(1-\xi^i)\frac{B_{t-1}^i}{L_t^i}.$$

In the steady state, this becomes:

$$\Lambda^i = \vartheta^i(1-\xi^i)\frac{B^i}{L^i}.$$

We can obtain an expression for $\frac{B^i}{L^i}$ from the steady-state version of the debt-accumulation equation (47):

$$\frac{B^i}{L^i} = \frac{1}{1-\left(1-\vartheta^i\right)\left(1-\xi^i\right)},$$

which can be inserted into the previous expression to obtain:

$$\Lambda^i = \frac{\vartheta^i(1-\xi^i)}{1-\left(1-\vartheta^i\right)\left(1-\xi^i\right)}.$$

This pins down the steady-state value of the refinancing parameter, ϑ^i, for given values of the amortization rate, ξ^i, and the share of refinancing to total loans, Λ^i. Solving for ϑ^i, we obtain:

$$\vartheta^i = \frac{\Lambda^i \xi^i}{\left(1-\Lambda^i\right)\left(1-\xi^i\right)}. \tag{48}$$

As discussed in Section 5.1.1, this expression is employed in our calibration strategy: For both household and corporate debt, we set empirical values of Λ^i and ξ^i. We then use (48) to calibrate the refinancing parameter for each of the two agents. For households, we set $\xi^I = 0.014$ and $\Lambda^I = 0.39$, thus obtaining $\vartheta^I = 0.009$. For firms, we set $\xi^I = 0.125$ and $\Lambda^I = 0.83$, so that $\vartheta^I = 0.698$.

D2. First-order conditions

Here we report the first-order conditions from the optimization problems faced by the three types of agents in the model.

Patient households

Patient households' optimal behavior is described by the following first-order conditions:

$$
\begin{aligned}
\frac{1}{C_t^P - \theta^P C_{t-1}^P} - \frac{\beta\theta^P}{\mathrm{E}_t\left\{C_{t+1}^P\right\} - \theta^P C_t^P} &= \lambda_t^P, & (49)\\
\nu^P\left(1 - N_t^P\right)^{-\varphi^P} &= \lambda_t^P W_t^P, & (50)\\
\lambda_t^P &= \beta^P R_t \mathrm{E}_t\left\{\lambda_{t+1}^P\right\}, & (51)
\end{aligned}
$$

$$
Q_t = \frac{\varepsilon_t}{\lambda_t^P H_t^P} + \beta^P \mathrm{E}_t\left\{\frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1}\right\}, \tag{52}
$$

where λ_t^P is the multiplier associated with (15).

Impatient households

The first-order conditions of the impatient households are given by:

$$
\begin{aligned}
\frac{1}{C_t^I - \theta^I C_{t-1}^I} - \frac{\beta\theta^I}{\mathrm{E}_t\left\{C_{t+1}^I\right\} - \theta^I C_t^I} &= \lambda_t^I, & (53)\\
\nu^I\left(1 - N_t^I\right)^{-\varphi^I} &= \lambda_t^I W_t^I, & (54)
\end{aligned}
$$

$$
\lambda_t^I - \mu_t^I = \beta^I R_t \mathrm{E}_t\left\{\lambda_{t+1}^I\right\} - \beta^I\left(1 - \vartheta^I\right)\left(1 - \xi^I\right)\mathrm{E}_t\left\{\mu_{t+1}^I\right\}, \tag{55}
$$

$$
Q_t = \frac{\varepsilon_t}{\lambda_t^I H_t^I} + \beta^I \mathrm{E}_t\left\{\frac{\lambda_{t+1}^I}{\lambda_t^I} Q_{t+1}\right\} + \vartheta^I s_t^I \frac{\mu_t^I}{\lambda_t^I}\frac{\mathrm{E}_t\left\{Q_{t+1}\right\}}{R_t}, \tag{56}
$$

where λ_t^I is the multiplier associated with (17), and μ_t^I is the multiplier associated with (18). Additionally, the complementary slackness condition

$$
\mu_t^I\left(B_t^I - \vartheta^I s_t^I \frac{\mathrm{E}_t\left\{Q_{t+1}\right\} H_t^I}{R_t} - \left(1 - \vartheta^I\right)\left(1 - \xi^I\right) B_{t-1}^I\right) = 0, \tag{57}
$$

must hold along with $\mu_t^I \geq 0$ and (18).

Entrepreneurs

The optimal behavior of the entrepreneurs is characterized by:

$$
\frac{1}{C_t^E - \theta^E C_{t-1}^E} - \frac{\beta\theta^E}{\mathrm{E}_t\left\{C_{t+1}^E\right\} - \theta^E C_t^E} = \lambda_t^E, \tag{58}
$$

$$
\lambda_t^E - \mu_t^E = \beta^E R_t \mathrm{E}_t\left\{\lambda_{t+1}^E\right\} - \beta^E\left(1 - \vartheta^E\right)\left(1 - \xi^E\right)\mathrm{E}_t\left\{\mu_{t+1}^E\right\}, \tag{59}
$$

$$
\lambda_t^E = \psi_t^E\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E\Omega\mathrm{E}_t\left\{\psi_{t+1}^E\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right\}, \tag{60}
$$

$$\psi_t^E = \beta^E r_t^K \mathrm{E}_t\left\{\lambda_{t+1}^E\right\} + \beta^E (1-\delta)\mathrm{E}_t\left\{\psi_{t+1}^E\right\} + \vartheta^E \mu_t^E s_t^E \frac{\mathrm{E}_t\left\{Q_{t+1}^K\right\}}{R_t}, \tag{61}$$

$$Q_t = \beta^E r_t^H \mathrm{E}_t\left\{\frac{\lambda_{t+1}^E}{\lambda_t^E}\right\} + \beta^E \mathrm{E}_t\left\{\frac{\lambda_{t+1}^E}{\lambda_t^E} Q_{t+1}\right\} + \vartheta^E s_t^E \frac{\mu_t^E}{\lambda_t^E} \frac{\mathrm{E}_t\left\{Q_{t+1}\right\}}{R_t}, \tag{62}$$

where λ_t^E, ψ_t^E, and μ_t^E are the multipliers associated with (22), (23), and (24), respectively. Moreover,

$$\mu_t^E \left(B_t^E - \vartheta^E s_t^E \mathrm{E}_t\left\{\frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t}\right\} - \left(1-\vartheta^E\right)\left(1-\xi^E\right) B_{t-1}^E \right) = 0, \tag{63}$$

holds along with $\mu_t^E \geq 0$ and (24). Finally, the definition of Q_t^K implies that

$$Q_t^K = \psi_t^E / \lambda_t^E. \tag{64}$$

Firms

Firms' first-order conditions determine the optimal demand for the input factors:

$$\alpha\gamma Y_t / N_t^P = W_t^P, \tag{65}$$

$$(1-\alpha)\gamma Y_t / N_t^I = W_t^I, \tag{66}$$

$$(1-\gamma)(1-\phi)\mathrm{E}_t\left\{Y_{t+1}\right\}/K_t = r_t^K, \tag{67}$$

$$(1-\gamma)\phi \mathrm{E}_t\left\{Y_{t+1}\right\}/H_t^E = r_t^H. \tag{68}$$

D3. Steady state

The deterministic steady state of the model is described in the following. Variables without time subscripts indicate their steady-state values. We first consider the implications of the patient households' optimality conditions. From (49) and (50), we get

$$\frac{1-\beta^P\theta^P}{\left(1-\theta^P\right) C^P} = \lambda^P \tag{69}$$

and

$$\nu^P \left(1-N^P\right)^{-\varphi^P} = \lambda^P W^P, \tag{70}$$

respectively. The steady-state gross interest rate on loans is recovered from (51):

$$R = \frac{1}{\beta^P}, \tag{71}$$

emphasizing that it is the time preference of the most patient individual that determines the steady-state rate of interest. From (52) we find

$$H^P = \frac{\varepsilon}{Q\lambda^P\left(1-\beta^P\right)}. \tag{72}$$

Turning to impatient households, (53) and (54) lead to

$$\frac{1-\beta^I\theta^I}{\left(1-\theta^I\right) C^I} = \lambda^I, \tag{73}$$

and

$$\nu^I \left(1 - N^I\right)^{-\varphi^I} = \lambda^I W^I, \tag{74}$$

respectively. From (55) we obtain the steady-state value of the multiplier on the credit constraint:

$$\mu^I = \frac{\lambda^I \left(1 - \beta^I R\right)}{1 - \beta^I \left(1 - \vartheta^I\right)\left(1 - \xi^I\right)},$$

which, by use of (71), yields

$$\mu^I = \frac{\lambda^I \left(1 - \frac{\beta^I}{\beta^P}\right)}{1 - \beta^I \left(1 - \vartheta^I\right)\left(1 - \xi^I\right)}. \tag{75}$$

From (75) we see that, in the steady state, $\mu^I > 0$ provided that $\beta^P > \beta^I$, which implies that the credit constraint (18) is binding. In a similar fashion, from (59) we get

$$\mu^E = \frac{\lambda^E \left(1 - \frac{\beta^E}{\beta^P}\right)}{1 - \beta^E \left(1 - \vartheta^E\right)\left(1 - \xi^E\right)}. \tag{76}$$

Hence, $\mu^E > 0$ provided that $\beta^P > \beta^E$, implying that the entrepreneurs' credit constraint, (24), is also binding in the steady state. From (56) we get

$$H^I = \frac{\varepsilon}{Q\lambda^I \left[1 - \beta^I - \frac{\left(1 - \frac{\beta^I}{\beta^P}\right)}{1 - \beta^I \left(1 - \vartheta^I\right)\left(1 - \xi^I\right)} \vartheta^I s^I \beta^P\right]}, \tag{77}$$

where the last line makes use of (71) and (75).

Turning to the remaining optimality conditions of the entrepreneurs, (58) gives

$$\frac{1 - \beta^E \theta^E}{\left(1 - \theta^E\right) C^E} = \lambda^E, \tag{78}$$

and (60) implies

$$\psi^E \left[1 - \frac{\Omega}{2}\left(\frac{I}{I} - 1\right)^2\right] - \psi^E \Omega \frac{I}{I}\left(\frac{I}{I} - 1\right) + \beta^E \psi^E \Omega \left(\frac{I}{I}\right)^2 \left(\frac{I}{I} - 1\right) = \lambda^E,$$

leading to

$$\psi^E = \lambda^E. \tag{79}$$

This reflects that there are no investment adjustment costs in the steady state. Therefore, the shadow value of a unit of capital equals the shadow value of wealth. Combining (79) with (64), we obtain

$$Q^K = 1. \tag{80}$$

After imposing (71), (76), and (79), (61) returns

$$r^K = \frac{\left[1 - \beta^E \left(1 - \vartheta^E\right)\left(1 - \xi^E\right)\right]\left[1 - \beta^E (1 - \delta)\right] - \left(\beta^P - \beta^E\right) \vartheta^E s^E Q^K}{\beta^E \left[1 - \beta^E \left(1 - \vartheta^E\right)\left(1 - \xi^E\right)\right]}. \tag{81}$$

From (62), instead, we find

$$r^H = \frac{\left(1-\beta^E\right)Q}{\beta^E} - \frac{\mu^E \vartheta^E s^E}{\lambda^E \beta^E}\frac{Q}{R}. \tag{82}$$

We then turn to the remaining equilibrium conditions in the steady state. As we saw above, the two credit constraints are binding in the steady state. Hence,

$$B^I = \frac{\vartheta^I s^I}{1-\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\frac{QH^I}{R}, \tag{83}$$

$$B^E = \frac{\vartheta^E s^E}{1-\left(1-\vartheta^E\right)\left(1-\xi^E\right)}\frac{Q^K K + QH^E}{R}. \tag{84}$$

The production function is

$$Y = \left[\left(N^P\right)^\alpha \left(N^I\right)^{1-\alpha}\right]^\gamma \left[\left(H^E\right)^\phi K^{1-\phi}\right]^{1-\gamma}. \tag{85}$$

The steady-state counterparts of firms' first-order conditions, (65)–(68), are:

$$\begin{aligned}
\alpha\gamma\frac{Y}{N^P} &= W^P, && (86)\\
(1-\alpha)\gamma\frac{Y}{N^I} &= W^I, && (87)\\
(1-\gamma)(1-\phi)\frac{Y}{K} &= r^K, && (88)\\
(1-\gamma)\phi\frac{Y}{H^E} &= r^H. && (89)
\end{aligned}$$

In the steady state, the law of motion for capital implies

$$I = \delta K. \tag{90}$$

We have the following steady-state resource constraints:

$$Y = C^P + C^I + C^E + I, \tag{91}$$

$$H = H^P + H^I + H^E, \tag{92}$$

$$B^P + B^I + B^E = 0. \tag{93}$$

Also, we have the steady-state versions of the agents' budget constraints:

$$\begin{aligned}
C^P &= W^P N^P - (R-1)B^P, && (94)\\
C^I &= W^I N^I - (R-1)B^I, && (95)\\
C^E + I &= r^K K + r^H H^E - (R-1)B^E && (96)
\end{aligned}$$

We therefore have that the steady state is characterized by the vector

$$\left[\begin{array}{c} Y, C^P, C^I, C^E, I, H^P, H^I, H^E, K, N^P, N^I, B^P, B^I, B^E, \\ Q, Q^K, R, r^K, r^H, W^P, W^I, \lambda^P, \lambda^I, \lambda^E, \mu^I, \mu^E, \psi^E \end{array}\right].$$

These 27 variables are determined by the 27 equations: (69), (70), (71), (72), (73), (74), (75), (76), (77), (78), (79), (80), (81), (82), (83), (84), (85), (86), (87), (88), (89), (90), (91), (92), (93), (94), and (95).

We now briefly proceed with the characterization of the steady state, finding some variables' equilibrium in a closed form. To this end, we define these variables as a ratio of total output. The resulting system, which comprises seven equations, is then solved numerically. The remaining variables then follow from the characterizations above.

First, combine (81) and (88) to get an expression for capital-output ratio:

$$\frac{K}{Y}=\frac{(1-\gamma)(1-\phi)\beta^E\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]}{\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]\left[1-\beta^E(1-\delta)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E Q^K}, \tag{97}$$

where we have used $Q^K=1$ from (80). Thus, we combine (82) and (89) to get an expression for entrepreneurs' land-output ratio:

$$\frac{QH^E}{Y}=\frac{(1-\gamma)\phi\beta^E\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]}{\left(1-\beta^E\right)\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E}, \tag{98}$$

where we have made use of (76). Again, based on $Q^K=1$, the entrepreneurial borrowing constraint can be rewritten as

$$\frac{B^E}{Y}=\frac{\vartheta^E}{1-\left(1-\vartheta^E\right)\left(1-\xi^E\right)}\frac{s^E}{R}\left(\frac{K}{Y}+\frac{QH^E}{Y}\right), \tag{99}$$

where we can insert from (71), (97), and (98). The resulting closed-form solution of the entrepreneurial steady-state loan-to-output ratio is central in setting up a sub-system of seven central variables. First, it can be plugged into the entrepreneurs' budget constraint, (96), so as to obtain:

$$\frac{C^E}{Y}+\frac{I}{Y}=r^K\frac{K}{Y}+r^H\frac{H^E}{Y}-(R-1)\frac{B^E}{Y},$$

which, by use of (90), becomes

$$\frac{C^E}{Y}=\left(r^K-\delta\right)\frac{K}{Y}+r^H\frac{H^E}{Y}-(R-1)\frac{B^E}{Y}.$$

Using (81) and (89), we get

$$\frac{C^E}{Y}=\left(\frac{\left(1-\beta^E\right)\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E Q^K}{\beta^E\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]}\right)\frac{K}{Y}+(1-\gamma)\phi-(R-1)\frac{B^E}{Y},$$

which, by use of (97), returns the entrepreneurs' consumption-to-output ratio:

$$\begin{aligned}\frac{C^E}{Y} &= \frac{(1-\gamma)(1-\phi)\left[\left(1-\beta^E\right)\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E\right]}{\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]\left[1-\beta^E(1-\delta)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E} \\ &\quad +(1-\gamma)\phi-\frac{1-\beta^P}{\beta^P}\frac{B^E}{Y}.\end{aligned} \tag{100}$$

We then turn to the impatient households. Their budget constraint can be written as

$$\frac{C^I}{Y}=\frac{W^I N^I}{Y}-(R-1)\frac{B^I}{Y},$$

which, by use of (71) and (87), becomes

$$\frac{C^I}{Y} = (1-\alpha)\,\gamma - \frac{1-\beta^P}{\beta^P}\frac{B^I}{Y}.$$

Likewise, patient households' budget constraint can be written as

$$\frac{C^P}{Y} = \frac{W^P N^P}{Y} - (R-1)\,\frac{B^P}{Y},$$

which, by use of (71) and (86), becomes

$$\frac{C^P}{Y} = \alpha\gamma - \frac{1-\beta^P}{\beta^P}\frac{B^P}{Y}.$$

Adding up these constraints gives

$$\frac{C^I + C^P}{Y} = \gamma + \frac{1-\beta^P}{\beta^P}\frac{B^E}{Y}, \tag{101}$$

where (93) has been invoked. Note that the right-hand-side of (101) is known, by virtue of (99).

Combining (69), (70) and (86) gives the steady-state equilibrium condition for patient households' labor:

$$\nu^P\left(1-N^P\right)^{-\varphi^P} C^P \frac{1-\theta^P}{1-\beta^P\theta^P} = \alpha\gamma\frac{Y}{N^P}. \tag{102}$$

Similarly, (73), (74) and (87) characterize impatient households' equilibrium labor:

$$\nu^I\left(1-N^I\right)^{-\varphi^I} C^I \frac{1-\theta^I}{1-\beta^I\theta^I} = (1-\alpha)\,\gamma\frac{Y}{N^I}. \tag{103}$$

Combining the two households' land-demand expressions, (72) and (77), gives

$$\frac{H^I}{H^P} = \frac{\lambda^P\left(1-\beta^P\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]}{\lambda^I\left\{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I\right\}}.$$

Eliminating the multipliers by (69) and (73), and eliminating H^P through (92), we obtain the following land-market equilibrium characterization:

$$\frac{H^I}{H-H^I-H^E}\frac{C^P}{C^I} = \frac{\left(1-\beta^P\theta^P\right)\left(1-\theta^I\right)\left(1-\beta^P\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]}{\left(1-\beta^I\theta^I\right)\left(1-\theta^P\right)\left\{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I\right\}}. \tag{104}$$

We also take the impatient households' borrowing constraint into consideration. Using (83) to eliminate B^I in the budget constraint,

$$\frac{C^I}{Y} = (1-\alpha)\,\gamma - \left(1-\beta^P\right)\frac{\vartheta^I}{1-\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\frac{s^I Q H^I}{Y}. \tag{105}$$

Relying on (77),

$$QH^I = \frac{\varepsilon\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]}{\lambda^I\left\{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I\right\}},$$

and, again, (73), we obtain

$$\begin{aligned} QH^I &= \frac{\varepsilon\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]\frac{\left(1-\theta^I\right)}{1-\beta^I\theta^I}C^I}{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I}, \quad (106)\\ Q &= \frac{\varepsilon\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]\frac{\left(1-\theta^I\right)}{1-\beta^I\theta^I}C^I}{H^I\left\{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I\right\}}. \quad (107)\end{aligned}$$

We then use (106) to rewrite the consumption-output ratio for impatient households, (105), as

$$\begin{aligned}\frac{C^I}{Y} &= (1-\alpha)\gamma \\ &-\left(1-\beta^P\right)\frac{\vartheta^I}{1-\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\frac{s^I}{Y}\frac{\varepsilon\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]\frac{\left(1-\theta^I\right)}{1-\beta^I\theta^I}C^I}{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I}.\end{aligned} \quad (108)$$

Likewise, we can use (107) to eliminate Q from (98), and obtain:

$$\begin{aligned}\frac{H^E}{Y} &= \frac{(1-\gamma)\phi\beta^E\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]}{\left(1-\beta^E\right)\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E}\cdot \\ &\cdot\frac{\left\{\left(1-\beta^I\right)\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]-\left(\beta^P-\beta^I\right)\vartheta^I s^I\right\}}{\varepsilon\left[1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\right]\frac{\left(1-\theta^I\right)}{1-\beta^I\theta^I}}\frac{H^I}{C^I}.\end{aligned} \quad (109)$$

Thus, the production function (85) is rewritten as a function of the derived ratios:

$$Y^\gamma = A\left[\left(N^P\right)^\alpha\left(N^I\right)^{1-\alpha}\right]^\gamma\left[\left(\frac{H^E}{Y}\right)^\phi\left(\frac{K}{Y}\right)^{1-\phi}\right]^{1-\gamma},$$

Using (97), we finally obtain

$$\begin{aligned} Y &= A^{\frac{1}{\gamma}}\left(N^P\right)^\alpha\left(N^I\right)^{1-\alpha}\cdot \\ &\cdot\left[\left(\frac{H^E}{Y}\right)^\phi\left(\frac{(1-\gamma)(1-\phi)\beta^E\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]}{\left[1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\right]\left[1-\beta^E(1-\delta)\right]-\left(\beta^P-\beta^E\right)\vartheta^E s^E Q^K}\right)^{1-\phi}\right]^{\frac{1-\gamma}{\gamma}}.\end{aligned} \quad (110)$$

We have now reduced the steady state to a matter of finding the vector

$$\left[Y, C^P, C^I, H^I, H^E, N^P, N^I\right],$$

which satisfies the equations (101), (102), (103), (104), (108), (109) and (110), *given* the solution for B^E/Y, (99), and given all the parameters and exogenous variables of the model. We compute the vector numerically using `fsolve` in MATLAB. The remaining variables then follow analytically from the steady-state equations presented above.

D4. Log-linearization

We log-linearize the model around the steady state found in the previous section. In the following, we let $\widehat{X_t}$ denote the log-deviation of a generic variable X_t from its steady state value X, except for the following variables: For the interest rates, $\widehat{R}_t \equiv R_t - R$, $\widehat{r}_t^H \equiv r_t^H - r^H$ and $\widehat{r}_t^K \equiv r_t^K - r^K$; for debt, $\widehat{B}_t^i \equiv (B_t^i - B^i)/Y$, $i = \{P, I, E\}$. We first derive the log-linear versions of the agents' optimality conditions and conclude with the expressions for market clearing.

Optimality Conditions of Patient Households

Once log-linearized, equations (49), (50) and (51) become

$$\beta^P \theta^P \mathrm{E}_t \left\{ \widehat{C}_{t+1}^P \right\} - \left(1 + \beta^P \left(\theta^P \right)^2 \right) \widehat{C}_t^P + \theta^P \widehat{C}_{t-1}^P = \left(1 - \theta^P \right) \left(1 - \beta^P \theta^P \right) \widehat{\lambda}_t^P , \tag{111}$$

$$\varphi^P \frac{N^P}{1 - N^P} \widehat{N}_t^P = \widehat{\lambda}_t^P + \widehat{W}_t^P , \tag{112}$$

$$\beta^P \widehat{R}_t + \mathrm{E}_t \left\{ \widehat{\lambda}_{t+1}^P \right\} = \widehat{\lambda}_t^P , \tag{113}$$

Log-linearizing (52) yields

$$\frac{\varepsilon}{H^P} \left(\widehat{\varepsilon}_t - \widehat{H}_t^P \right) + \beta^P \lambda^P Q \mathrm{E}_t \left\{ \widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1} \right\} = \lambda^P Q \left(\widehat{\lambda}_t^P + \widehat{Q}_t \right) .$$

Now use steady-state equation (72) to get

$$-Q\lambda^P \left(1 - \beta^P \right) \widehat{H}_t^P + Q\lambda^P \left(1 - \beta^P \right) \widehat{\varepsilon}_t + \beta^P \lambda^P Q \mathrm{E}_t \left\{ \widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1} \right\} = \lambda^P Q \left(\widehat{\lambda}_t^P + \widehat{Q}_t \right) ,$$

and thereby

$$\beta^P \mathrm{E}_t \left\{ \widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1} \right\} - \left(1 - \beta^P \right) \widehat{H}_t^P + \left(1 - \beta^P \right) \widehat{\varepsilon}_t = \widehat{\lambda}_t^P + \widehat{Q}_t . \tag{114}$$

Moreover, the log-linearized budget constraint reads as

$$\begin{aligned} & \frac{C^P}{Y} \widehat{C}_t^P + \frac{QH^P}{Y} \left(\widehat{H}_t^P - \widehat{H}_{t-1}^P \right) + \frac{B^P}{Y} \widehat{R}_{t-1} + \frac{1}{\beta^P} \widehat{B}_{t-1}^P \\ = \; & \widehat{B}_t^P + \alpha\gamma \left(\widehat{W}_t^P + \widehat{N}_t^P \right) . \end{aligned}$$

where we have used (86).

Optimality Conditions of Impatient Households

From (53), (54) and (55) we obtain

$$\beta^I \theta^I \mathrm{E}_t \left\{ \widehat{C}_{t+1}^I \right\} - \left(1 + \beta^I \left(\theta^I \right)^2 \right) \widehat{C}_t^I + \theta^I \widehat{C}_{t-1}^I = \left(1 - \theta^I \right) \left(1 - \beta^I \theta^I \right) \widehat{\lambda}_t^I , \tag{115}$$

$$\varphi^I \frac{N^I}{1 - N^I} \widehat{N}_t^I = \widehat{\lambda}_t^I + \widehat{W}_t^I , \tag{116}$$

and

$$\lambda^I\widehat{\lambda}_t^I - \mu^I\widehat{\mu}_t^I = \beta^I\lambda^I\widehat{R}_t + \beta^I R\lambda^I \mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^I\right\} - \beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\mu^I\mathrm{E}_t\left\{\widehat{\mu}_{t+1}^I\right\},$$

respectively. The last expression is rewritten, by means of (75), as

$$\begin{aligned}\widehat{\lambda}_t^I &= \beta^I\widehat{R}_t + \beta^I R\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^I\right\} + \frac{\left(1-\frac{\beta^I}{\beta^P}\right)}{1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\widehat{\mu}_t^I \\ &\quad -\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)\frac{\left(1-\frac{\beta^I}{\beta^P}\right)}{1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\mathrm{E}_t\left\{\widehat{\mu}_{t+1}^I\right\}.\end{aligned} \tag{117}$$

Furthermore, (56) becomes

$$\begin{aligned}Q\widehat{Q}_t &= \frac{\varepsilon}{H^I\lambda^I}\left(\widehat{\varepsilon}_t - \widehat{\lambda}_t^I - \widehat{H}_t^I\right) + \beta^I Q\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^I + \widehat{Q}_{t+1} - \widehat{\lambda}_t^I\right\} \\ &\quad + \frac{\mu^I}{\lambda^I}\frac{\vartheta^I s^I Q}{R}\left[\widehat{\mu}_t^I - \widehat{\lambda}_t^I + \widehat{s}_t + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} - \beta^P\widehat{R}_t\right],\end{aligned}$$

which, by use of (75) and (77), becomes

$$\begin{aligned}\widehat{Q}_t &= \left[1-\beta^I - \frac{\left(\beta^P-\beta^I\right)}{1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\vartheta^I s^I\right]\left(\widehat{\varepsilon}_t - \widehat{\lambda}_t^I - \widehat{H}_t^I\right) + \beta^I\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^I + \widehat{Q}_{t+1} - \widehat{\lambda}_t^I\right\} \\ &\quad + \frac{\left(\beta^P-\beta^I\right)}{1-\beta^I\left(1-\vartheta^I\right)\left(1-\xi^I\right)}\vartheta^I s^I\left[\widehat{\mu}_t^I - \widehat{\lambda}_t^I + \widehat{s}_t + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} - \beta^P\widehat{R}_t\right],\end{aligned} \tag{118}$$

where, again, we have used (71). The budget constraint becomes

$$\frac{C^I}{Y}\widehat{C}_t^I + \frac{QH^I}{Y}\left(\widehat{H}_t^I - \widehat{H}_{t-1}^I\right) + \frac{B^I}{Y}\widehat{R}_{t-1}^{M,I} + \frac{1}{\beta^P}\widehat{B}_{t-1}^I = \widehat{B}_t^I + (1-\alpha)\gamma\left(\widehat{W}_t^I + \widehat{N}_t^I\right), \tag{119}$$

where we have used (87). Finally, the log-linearized version of the collateral constraint is:

$$Y\widehat{B}_t^I \leq \frac{\vartheta^I s^I QH^I}{R}\left(\widehat{s}_t^I + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^I - \beta^P\widehat{R}_t\right) + \left(1-\vartheta^I\right)\left(1-\xi^I\right)Y\widehat{B}_{t-1}^I. \tag{120}$$

Optimality Conditions of the Entrepreneurs

From (58) and (59) we get

$$\beta^E\theta^E\mathrm{E}_t\left\{\widehat{C}_{t+1}^E\right\} - \left(1+\beta^E\left(\theta^E\right)^2\right)\widehat{C}_t^E + \theta^E\widehat{C}_{t-1}^E = \left(1-\theta^E\right)\left(1-\beta^E\theta^E\right)\widehat{\lambda}_t^E, \tag{121}$$

$$\lambda^E\widehat{\lambda}_t^E - \mu^E\widehat{\mu}_t^E = \beta^E\lambda^E\widehat{R}_t + \beta^E R\lambda^E\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} - \beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\mu^E\mathrm{E}_t\left\{\widehat{\mu}_{t+1}^E\right\},$$

respectively. The latter we can be rewritten using (76):

$$\begin{aligned}\widehat{\lambda}_t^E &= \beta^E\widehat{R}_t + \beta^E R\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} + \frac{\left(1-\frac{\beta^E}{\beta^P}\right)}{1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)}\widehat{\mu}_t^E \\ &\quad -\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)\frac{\left(1-\frac{\beta^E}{\beta^P}\right)}{1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)}\mathrm{E}_t\left\{\widehat{\mu}_{t+1}^E\right\}.\end{aligned} \tag{122}$$

From (60) we get

$$\widehat{\psi}_t^E - \Omega\left(1+\beta^E\right)\widehat{I}_t + \Omega\widehat{I}_{t-1} + \beta^E\Omega \mathrm{E}_t\left\{\widehat{I}_{t+1}\right\} = \widehat{\lambda}_t^E, \tag{123}$$

where we have made use of (79). Equation (61) becomes

$$\begin{aligned}\widehat{\psi}_t^E &= \beta^E r^K \mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} + \beta^E \widehat{r}_t^K + \beta^E(1-\delta)\mathrm{E}_t\left\{\widehat{\psi}_{t+1}^E\right\} \\ &+\vartheta^E \frac{\left(\beta^P-\beta^E\right)}{1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)} s^E Q^K\left[\widehat{\mu}_t^E + \widehat{s}_t^E + \mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} - \beta^P\widehat{R}_t\right],\end{aligned} \tag{124}$$

where we have used (71), (76), and (79). Moreover, (64) becomes

$$\widehat{\psi}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K. \tag{125}$$

Finally, (62) is approximated as

$$\begin{aligned}Q\widehat{Q}_t &= \beta^E r^H\left(\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} - \widehat{\lambda}_t^E + \frac{1}{r^H}\widehat{r}_t^H\right) + \beta^E Q\left(\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} - \widehat{\lambda}_t^E\right) \\ &+\vartheta^E s^E \frac{\mu^E}{\lambda^E}\frac{Q}{R}\left[\widehat{s}_t^E + \widehat{\mu}_t^E - \widehat{\lambda}_t^E + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} - \beta^P\widehat{R}_t\right],\end{aligned}$$

which we can rewrite, using (71) and (76), as

$$\begin{aligned}Q\widehat{Q}_t &= \beta^E r^H\left(\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} - \widehat{\lambda}_t^E + \frac{1}{r^H}\widehat{r}_t^H\right) + \beta^E Q \mathrm{E}_t\left(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1} - \widehat{\lambda}_t^E\right) \\ &+\frac{\left(\beta^P-\beta^E\right)}{1-\beta^E\left(1-\vartheta^E\right)\left(1-\xi^E\right)}\vartheta^E s^E Q\left[\widehat{s}_t^E + \widehat{\mu}_t^E - \widehat{\lambda}_t^E + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} - \beta^P\widehat{R}_t\right].\end{aligned} \tag{126}$$

Furthermore, the budget constraint becomes

$$\begin{aligned}&\frac{C^E}{Y}\widehat{C}_t^E + \frac{I}{Y}\widehat{I}_t + \frac{QH^E}{Y}\left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + \frac{B^E}{Y}\widehat{R}_{t-1}^{M,E} + \frac{1}{\beta^P}\widehat{B}_{t-1}^E \\ &= \widehat{B}_t^E + \frac{K}{Y}\widehat{r}_{t-1}^K + \frac{H^E}{Y}\widehat{r}_{t-1}^H + (1-\gamma)\phi\widehat{H}_{t-1}^E + (1-\gamma)(1-\phi)\widehat{K}_{t-1},\end{aligned} \tag{127}$$

where we have used (88) and (89). Finally, the borrowing constraint reads as

$$\begin{aligned}Y\widehat{B}_t^E &\leq \vartheta^E s^E \frac{\left(K+QH^E\right)}{R}\left(\widehat{s}_t^E - \beta^P\widehat{R}_t\right) + \vartheta^E s^E \frac{K}{R}\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K + \widehat{K}_t\right\} \\ &+\vartheta^E s^E \frac{QH^E}{R}\mathrm{E}_t\left\{\widehat{Q}_{t+1} + \widehat{H}_t^E\right\} + \left(1-\vartheta^E\right)\left(1-\xi^E\right)Y\widehat{B}_{t-1}^E.\end{aligned} \tag{128}$$

Firms' Optimality Conditions

Firms' first-order conditions, (65), (66), (67) and (68), are log-linearized as

$$\widehat{Y}_t - \widehat{N}_t^P = \widehat{W}_t^P, \tag{129}$$

$$\widehat{Y}_t - \widehat{N}_t^I = \widehat{W}_t^I, \tag{130}$$

$$\mathrm{E}_t\left\{\widehat{Y}_{t+1}\right\} - \widehat{K}_t = \left(r^K\right)^{-1}\widehat{r}_t^K, \tag{131}$$

$$\mathrm{E}_t\left\{\widehat{Y}_{t+1}\right\}-\widehat{H}_t^E=\left(r^H\right)^{-1}\widehat{r}_t^H, \tag{132}$$

respectively.

Market Clearing and Resource Constraints

From the law of motion for capital, (23), we get

$$\widehat{K}_t=(1-\delta)\,\widehat{K}_{t-1}+\delta\widehat{I}_t, \tag{133}$$

where we have used (90). Moreover, from the resource constraint, (30), we have

$$\widehat{Y}_t=\frac{C^P}{Y}\widehat{C}_t^P+\frac{C^I}{Y}\widehat{C}_t^I+\frac{C^E}{Y}\widehat{C}_t^E+\delta\frac{K}{Y}\widehat{I}_t. \tag{134}$$

We also have the log-linearized versions of (26), (28) and (29):

$$\widehat{Y}_t=\widehat{A}_t+\alpha\gamma\widehat{N}_t^P+(1-\alpha)\,\gamma\widehat{N}_t^I+(1-\gamma)\,(1-\phi)\,\widehat{K}_{t-1}+(1-\gamma)\,\phi\widehat{H}_{t-1}^E, \tag{135}$$

$$0=H^P\widehat{H}_t^P+H^I\widehat{H}_t^I+H^E\widehat{H}_t^E, \tag{136}$$

$$0=\widehat{B}_t^P+\widehat{B}_t^I+\widehat{B}_t^E. \tag{137}$$

As for the shocks processes, (27), (14) and (19) imply

$$\widehat{A}_t=\rho_A\widehat{A}_{t-1}+z_t, \tag{138}$$

$$\widehat{\varepsilon}_t=\rho_\varepsilon\widehat{\varepsilon}_{t-1}+u_t, \tag{139}$$

$$\widehat{s}_t=\rho_s\widehat{s}_{t-1}+v_t, \tag{140}$$

respectively. This completes our list of log-linearized equations.

The log-linearized system consists of 30 equations: 18 first-order conditions, 2 budget constraints, 2 credit constraints, 1 production function, 3 market clearing conditions, 1 capital accumulation equation, and 3 shock processes. The 30 variables of the system are given by the vector

$$\left[\begin{array}{c}\widehat{C}_t^P,\widehat{C}_t^I,\widehat{C}_t^E,\widehat{\lambda}_t^P,\widehat{\lambda}_t^I,\widehat{\lambda}_t^E,\widehat{\psi}_t^E,\widehat{\mu}_t^I,\widehat{\mu}_t^E,\widehat{R}_t,\widehat{N}_t^P,\widehat{N}_t^I,\widehat{W}_t^P,\widehat{W}_t^I,\\ \widehat{H}_t^P,\widehat{H}_t^I,\widehat{H}_t^E,\widehat{Q}_t,\widehat{Q}_t^K,\widehat{r}_t^H,\widehat{r}_t^K,\widehat{K}_t,\widehat{I}_t,\widehat{Y}_t,\widehat{B}_t^P,\widehat{B}_t^I,\widehat{B}_t^E,\widehat{A}_t,\widehat{\varepsilon}_t,\widehat{s}_t\end{array}\right],$$

and are determined by equations (111)-(140).

Appendix E. Solution method

We solve the model numerically, as described in the following. When solving the model, we treat the collateral constraints as inequalities, accounting for two complementary slackness conditions (57) and (63). We then adopt the solution method of Holden and Paetz (2012), on which this appendix builds. In turn, Holden and Paetz (2012) expand on previous work by Laséen and Svensson (2011). With first-order perturbations, this solution method is equivalent to the piecewise linear approach discussed by Guerrieri and Iacoviello (2015). We have verified that their proposed solution method does indeed produce identical results. Furthermore, Holden and Paetz (2012) and Guerrieri and Iacoviello (2015) evaluate the accuracy of their respective methods against a global solution based on projection methods. This is done for a very simple model with a borrowing constraint, for which a highly accurate global solution can be obtained and used as a benchmark. They find that the local approximations are very accurate. For the model used in this paper, the large number of state variables (14 endogenous state variables and three shocks) renders the use of global solution methods impractical due to the curse of dimensionality typically associated with such methods.

The collateral constraints put an upper bound on the borrowing of each of the two constrained agents. While the constraints are binding in the steady state, this may not be the case outside the steady state, where the constraints may not bind. Observe that we can reformulate the collateral constraints in terms of restrictions on each agent's shadow value of borrowing; μ_t^j, $j = \{I, E\}$: We know that $\mu_t^j \geq 0$ if and only if the optimal debt level of agent j is exactly at or above the collateral value. In other words, we need to ensure that $\mu_t^j \geq 0$. If this restriction is satisfied with inequality, the constraint is binding, so the slackness condition is satisfied. If it holds with equality, the collateral constraint becomes non-binding, but the slackness condition is still satisfied. If instead $\mu_t^j < 0$, agent j's optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow 'too much'. In this case, the slackness condition is violated. We then need to add shadow price shocks so as to 'push' μ_t^j back up until it exactly equals its lower limit of zero and the slackness condition is satisfied. To ensure compatibility with rational expectations, these shocks are added to the model as 'news shocks'. The idea of adding such shocks to the model derives from Laséen and Svensson (2011), who use such an approach to deal with pre-announced paths for the interest rate setting of a central bank. The contribution of Holden and Paetz (2012) is to develop a numerical method to compute the size of these shocks that are required to obtain the desired level for a given variable in each period, and to make this method applicable to a general class of potentially more complicated problems than the relatively simple experiments conducted by Laséen and Svensson (2011).

We first describe how to compute impulse responses to a single generic shock, e.g., a technology shock. The first step is to add independent sets of shadow price shocks to each of the two log-linearized collateral constraints. To this end, we need to determine the number of periods T for which we conjecture that the collateral constraints may be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses; $T \leq T^{IRF}$. For each period $t \leq T$, we then add shadow price shocks which hit the economy in period t but become known at period 0, that is, at the same time the economy is hit by the technology shock.

Let $\widehat{X_t}$ denote the log-deviation of a generic variable X_t from its steady-state value X, except for the following variables: For the interest rates, $\widehat{R}_t \equiv R_t - R$, $\widehat{r}_t^H \equiv r_t^H - r^H$ and $\widehat{r}_t^K \equiv r_t^K - r^K$, and for debt, $\widehat{B}_t^i \equiv (B_t^i - B^i)/Y$, $i = P, I, E$. We can then write the log-linearized collateral constraints, augmented with the shadow price shocks, as follows:

$$Y\widehat{B}_t^I \leq \frac{\vartheta^I s^I Q H^I}{R}\left(\widehat{s}_t^I + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^I - \beta^P \widehat{R}_t\right) + \left(1-\vartheta^I\right)\left(1-\xi^I\right) Y\widehat{B}_{t-1}^I - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,I},$$

$$\begin{aligned} Y\widehat{B}_t^E \ \leq\ & \vartheta^E s^E \frac{\left(K+QH^E\right)}{R}\left(\widehat{s}_t^E - \beta^P \widehat{R}_t\right) + \vartheta^E s^E \frac{K}{R}\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K + \widehat{K}_t\right\} \\ & + \vartheta^E s^E \frac{QH^E}{R}\mathrm{E}_t\left\{\widehat{Q}_{t+1} + \widehat{H}_t^E\right\} + \left(1-\vartheta^E\right)\left(1-\xi^E\right) Y\widehat{B}_{t-1}^E - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,E}, \end{aligned}$$

where $\varepsilon_{s,t-s}^{SP,j}$ is the shadow price shock that hits agent j in period $t = s$, and is anticipated by all agents in period $t = t - s = 0$ ensuring consistency with rational expectations. We let all shadow price shocks be of unit magnitude. We then need to compute two sets of weights α_{μ_I} and α_{μ_E} to control the impact of each shock on μ_t^I and μ_t^E. The 'optimal' sets of weights ensure that μ_t^I and μ_t^E are bounded below at exactly zero. The weights are computed by solving the following quadratic programming problem:

$$\begin{aligned} \alpha^* \ &\equiv\ \begin{bmatrix} \alpha_{\mu_I}^{*\prime} & \alpha_{\mu_E}^{*\prime} \end{bmatrix}' \\ &=\ \arg\min \begin{bmatrix} \alpha_{\mu_I}' & \alpha_{\mu_E}' \end{bmatrix} \left[\begin{bmatrix} \mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A} \end{bmatrix} + \begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}\begin{bmatrix} \alpha_{\mu_I} \\ \alpha_{\mu_E} \end{bmatrix}\right], \end{aligned}$$

subject to

$$\alpha_{\mu_j}' \geq 0,$$
$$\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}}\alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}}\alpha_{\mu_k} \geq 0,$$

$j = \{I, E\}$. Here, μ^j and $\widetilde{\mu}^{j,A}$ denote, respectively, the steady-state value and the unrestricted relative impulse response of μ^j to a technology shock, that is, the impulse-response of μ^j when the collateral constraints are assumed to always bind. In this respect, the vector $\begin{bmatrix} \mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A} \end{bmatrix}$ contains the absolute, unrestricted impulse responses of the two shadow values stacked. Further, each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ contains the relative impulse responses of μ^j to shadow price shocks to agent k's constraint for $j, k = \{I, E\}$, in the sense that column s in $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ represents the response of the shadow value to a shock $\varepsilon_{s,t-s}^{SP,j}$, i.e. to a shadow price shock that hits in period s but is anticipated at time 0, as described above.[43] The off-diagonal elements of the matrix $\begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}$ take into account that the impatient household may be affected if the collateral constraint of the entrepreneur becomes non-binding, and *vice versa*. Following the discussion in Holden and Paetz (2012), a sufficient condition for the existence of a unique solution to the optimization problem is that the matrix $\begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix} + \begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}'$ is positive definite. We have checked and verified that this condition is in fact always satisfied.

We can explain the nature of the optimization problem as follows. First, note that $\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}}\alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}}\alpha_{\mu_k}$ denotes the combined response of μ_t^j to a given shock (here, a

[43] Each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ needs to be a square matrix, so if the number of periods in which we guess the constraints may be non-binding is smaller than the number of periods for which we compute impulse responses, $T < T^{IRF}$, we use only the first T rows of the matrix, i.e., the upper square matrix.

technology shock) *and* a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow-price shocks is exactly large enough to make sure that the response of μ_t^j is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, positive shadow price shocks occur if and only if at least one of the two constrained variables, μ_t^I and μ_t^E, are at their lower bound of zero in that period. As pointed out by Holden and Paetz (2012), this can be thought of as a complementary slackness condition on the two inequality constraints of the optimization problem. Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. In this case, however, we need to allow for more than one type of shock. For each period t, we first generate the shocks hitting the economy. We then compute the unrestricted path of the endogenous variables given those shocks and given the simulated values in $t-1$. The unrestricted paths of the bounded variables (μ_t^I and μ_t^E) then take the place of the impulse responses in the optimization problem. If the unrestricted paths of μ_t^I and μ_t^E never hit the bounds in future periods, our simulation for period t is fine. If the bounds are hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period t. Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and push the restricted variables away from their bounds.

Appendix F. Data description and estimation strategy

As described in the main text, we use data for the following five macroeconomic variables of the U.S. economy spanning the period 1952:I–1984:II: The year-on-year growth rates (in log-differences) of real GDP, real private consumption, real non-residential investment, and real house prices, and the cyclical component of the LTA series in Figure 2, with the trend being computed as in Müller and Watson (2018). Since the cyclical components of the two LTA series are strongly correlated, we use the one obtained for the households.[44] All data series are taken from the Federal Reserve's FRED database, with the exception of the house price, which is provided by the US Census Bureau. The series are the following:

- Growth rate of *Real Gross Domestic Product*, billions of chained 2009 dollars, seasonally adjusted, annual rate (FRED series name: GDPC1).
- Growth rate of *Real Personal Consumption Expenditures*, billions of chained 2009 dollars, seasonally adjusted, annual rate (FRED series name: PCECC96).
- Growth rate of *Real private fixed investment: Nonresidential* (chain-type quantity index), index 2009=100, seasonally adjusted (FRED series name: B008RA3Q086SBEA).
- Growth rate of *Price Index of New Single-Family Houses Sold Including Lot Value*, index 2005=100, not seasonally adjusted. This series is available only from 1963:Q1 onwards.

[44] All results are robust to using the corporate one.

- To obtain the house price in real terms, this series is deflated using the GDP deflator (*Gross Domestic Product: Implicit Price Deflator*, index 2009=100, seasonally adjusted, FRED series name: GDPDEF).

- LTA data: We employ the series in the right panel of Figure 2 for the period up until 1984:II. As described in Appendix A1, we extract the trend from these series using the method of Müller and Watson (2018). We then use the cyclical component in the estimation of the model. Since the cyclical components of the two series are strongly correlated, we use the series for households, but all results are robust to using the series for firms instead.

Estimation

We use 16 empirical moments in the SMM estimation: The standard deviations and first-order autoregressive parameters of each of the five variables described above, the correlation of consumption, investment, and house prices with output, and the skewness of output, consumption, and investment. These moments are matched to their simulated counterparts from the theoretical model. Our estimation procedure seeks to minimize the sum of squared deviations between empirical and simulated moments. As some of the moments are measured in different units (e.g., standard deviations vs. correlations), we use the percentage deviation from the empirical moment in each case. In order for the minimization procedure to converge, it is crucial to use the same set of shocks repeatedly, making sure that the only change in the simulated moments from one iteration to the next is that arising from updating the parameter values. In practice, since the list of parameter values to be estimated includes the variance of the shocks in the model, we draw from the standard normal distribution with zero mean and unit variance, and then scale the shocks by the variance of each of the three shock processes, allowing us to estimate the latter. We use a draw of 2000 realizations of each of the three shocks in the model, thus obtaining simulated moments for 2000 periods.[45] To make sure that the draw of shocks used is representative of the underlying distribution, we make 501 draws of potential shock matrices, rank these in terms of the standard deviations of each of the three shocks, and select the shock matrix closest to the median along all three dimensions. This matrix of shocks is then used in the estimation. In the estimation, we impose only very general bounds on parameter values: All parameters are bounded below at zero, and the habit formation parameters along with all AR(1)-coefficients are bounded above at 0.99—a bound that is never attained.

To initiate the estimation procedure a set of initial values for the estimated parameters are needed. These are chosen based on values reported in the existing literature. The estimation results proved robust to changes in the set of initial values, as long as these remain within the range of available estimates. In line with the existing literature, we set the initial values of the investment adjustment cost parameter (Ω) and the parameters governing habit formation in consumption for the three agents to 4 and 0.5, respectively.[46] For the technology shock, we choose values similar to those used in most of the real business cycle literature, $\rho_A = 0.97$ and $\sigma_A = 0.005$ (see, e.g., Mandelman *et al.*, 2011). For the land-demand shock, we set $\rho_\varepsilon = 0.99$

[45] Our simulated sample is thus more than 15 times longer than the actual dataset (which spans 130 quarters). Ruge-Murcia (2012) finds that SMM is already quite accurate when the simulated sample is five or ten times longer than the actual data.

[46] Unlike the other estimated parameters, θ^P and θ^I also affect the steady state of the model. To account for this, we rely on the following iterative procedure: We first calibrate the model based on the starting value for θ^P and θ^I. Upon estimation, but before simulating the model, we recalibrate it for the estimated values of the habit parameters. This leads only to a very small change in the values of ε, ϕ, and s^I, while the remaining parameters are unaffected.

and $\sigma_\varepsilon = 0.03$, in line with Liu *et al.* (2013). Finally, for the credit limit shock, we set the persistence parameter $\rho_s = 0.95$, while the standard deviation is set to $\sigma_s = 0.04$.

We abstain from using an optimal weighting matrix in the estimation. This choice is based on the findings of Altonji and Segal (1996), who show that when GMM is used to estimate covariance structures and, potentially, higher-order moments such as variances, as in our case, the use of an optimal weighting matrix causes a severe downward bias in estimated parameter values. Similar concerns apply to SMM as to GMM. The bias arises because the moments used to fit the model itself are correlated with the weighting matrix, and may thus be avoided by the use of fixed weights in the minimization. Altonji and Segal (1996) demonstrate that minimization schemes with fixed weights clearly dominate optimally weighted ones in such circumstances. Ruge-Murcia (2012) points out that parameter estimates remain consistent for any positive-definite weighting matrix, and finds that the accuracy and efficiency gains associated with an optimal weighting matrix are not overwhelming. The empirical moments and their model counterparts upon estimation are reported in Table F1.

When computing standard errors, we rely on a version of the delta method, as described, e.g., in Hamilton (1994). We approximate the numerical derivative of the moments with respect to the estimated parameters using the secant that can be computed by adding and subtracting ϵ to/from the estimates, where ϵ is a very small number. The covariance (or spectral density) matrix is estimated using the Newey-West estimator.

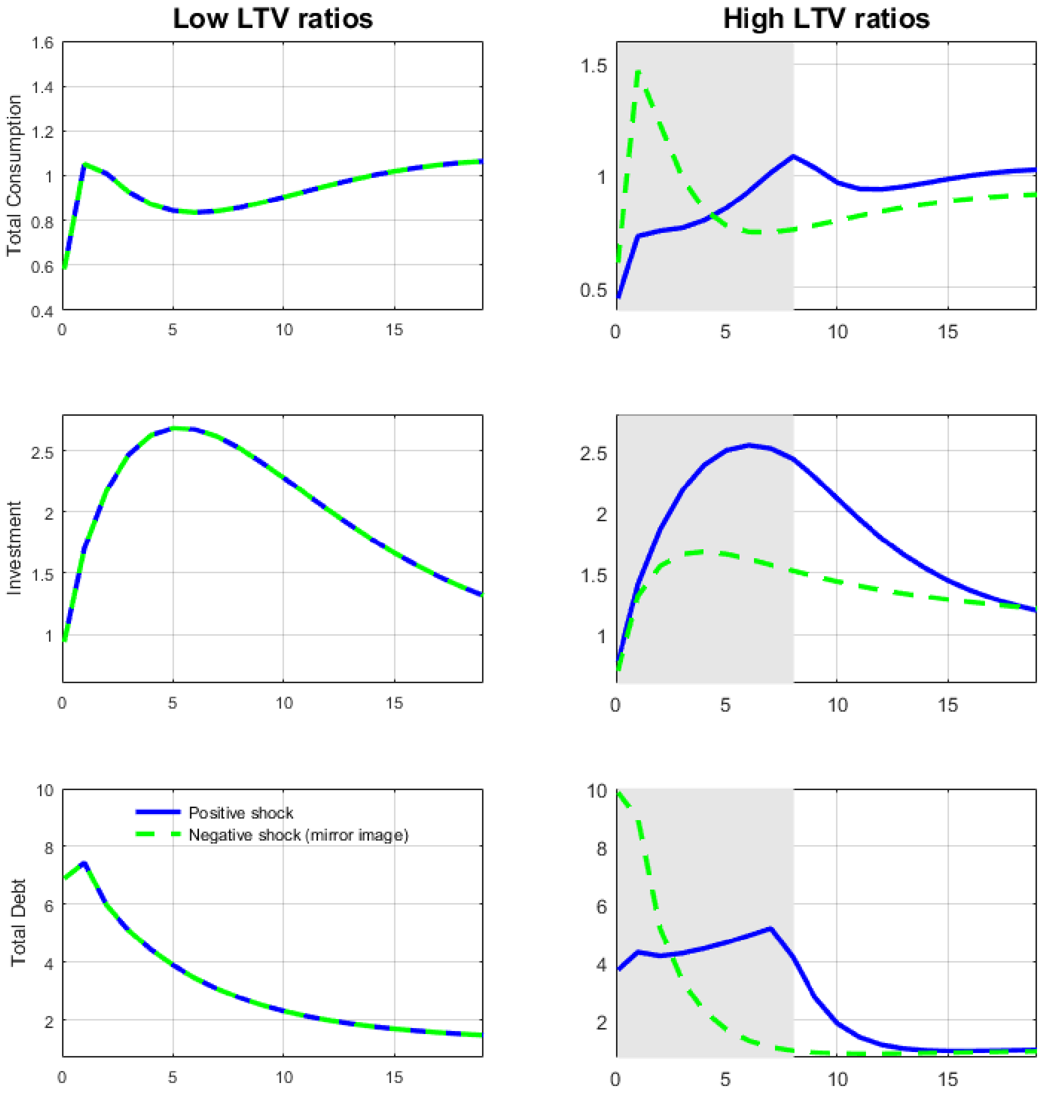
Table F1. Empirical and simulated moments

	Model simulations	U.S. data (1952:I–1984:II)
	Standard deviations (percent)	
Output	2.72	2.84
Consumption	1.87	2.23
Investment	6.52	6.91
House price	4.26	3.05
LTV ratio	6.32	5.68
	Skewness	
Output	−0.14	−0.38
Consumption	−0.21	−0.31
Investment	−0.02	−0.41
	Autocorrelations	
Output	0.90	0.82
Consumption	0.85	0.81
Investment	0.94	0.84
House price	0.63	0.79
LTV ratio	0.85	0.94
	Correlations with output	
Consumption	0.92	0.85
Investment	0.93	0.75
House price	0.73	0.38

Appendix G. Additional numerical evidence

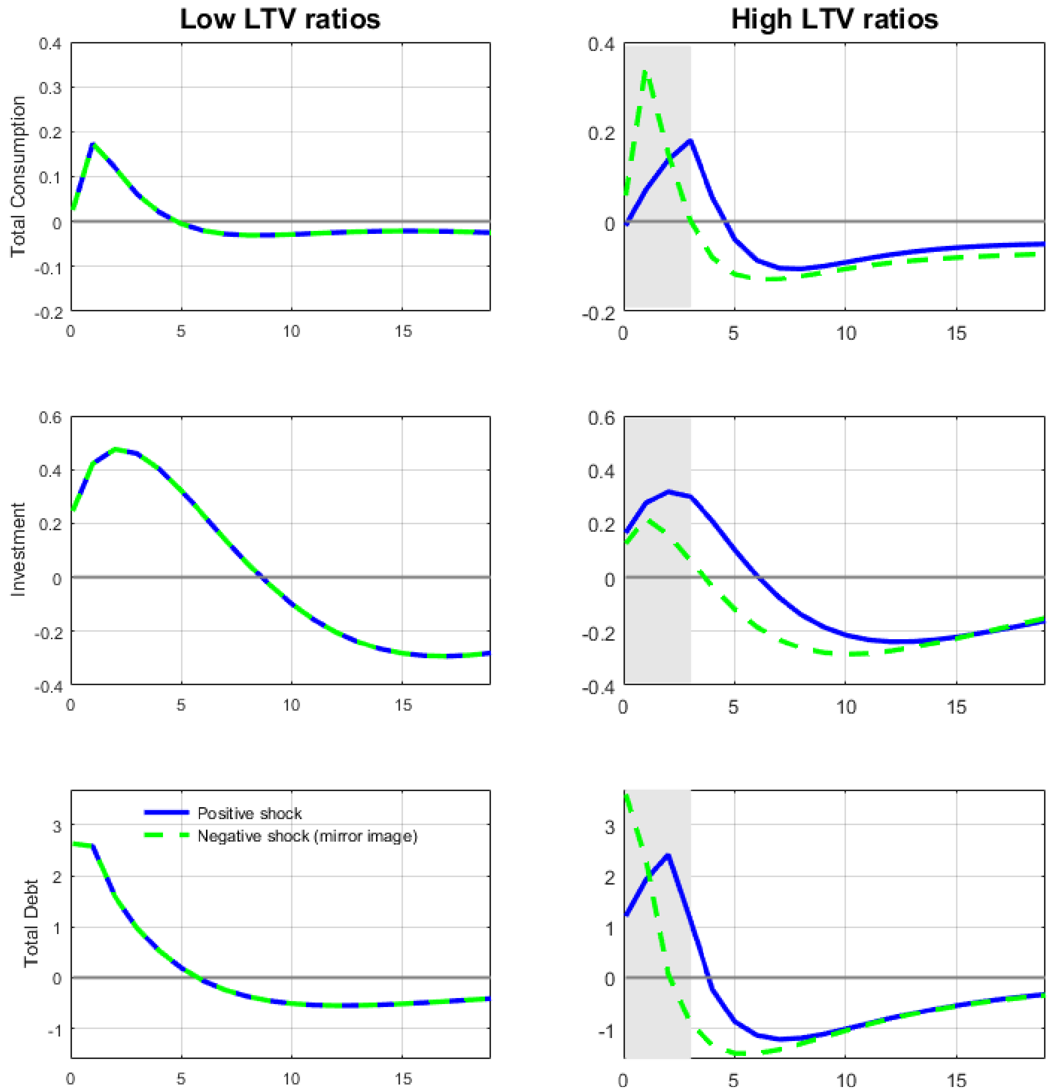
G1. Impulse responses

Figure G1. Impulse responses to a technology shock



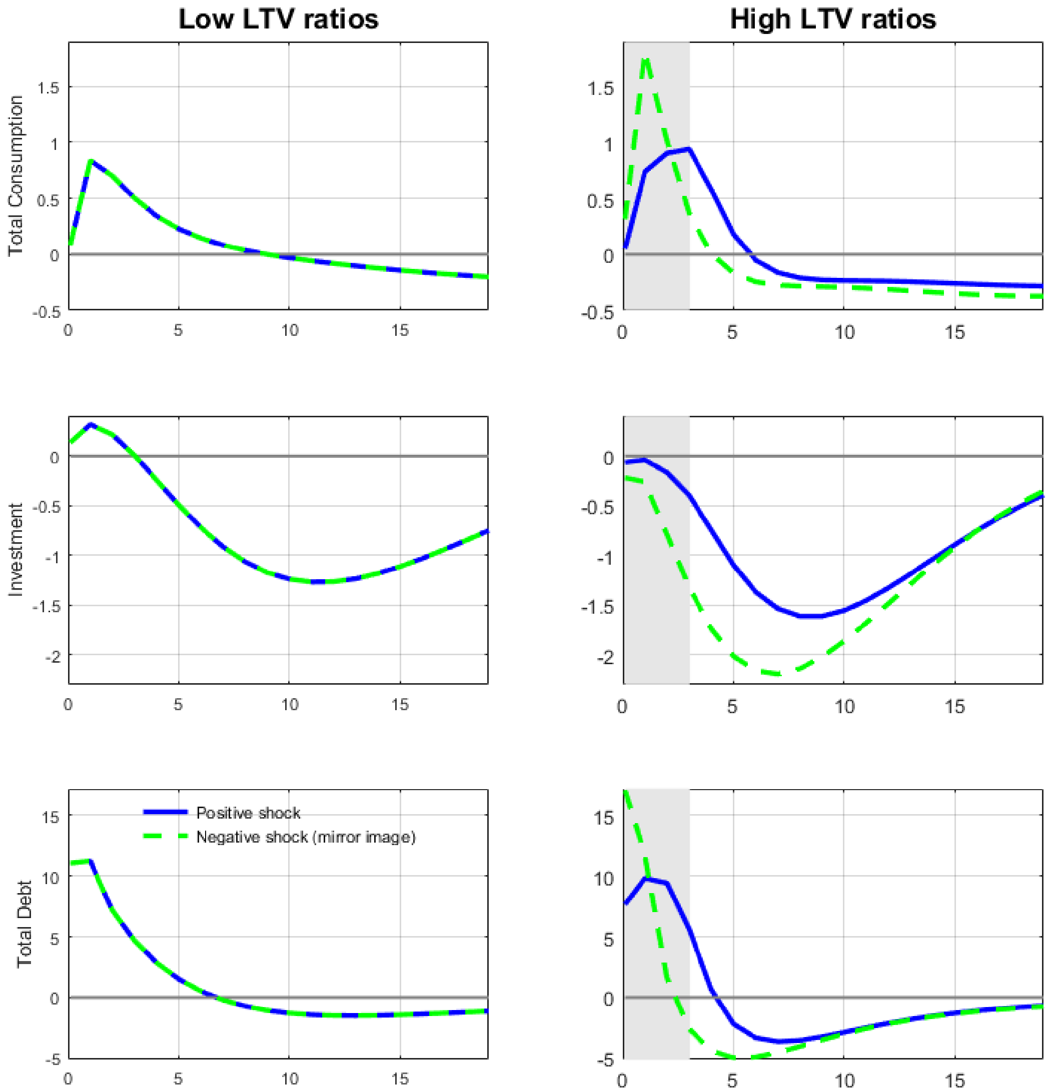
Notes: Impulse responses of key macroeconomic variables (in percentage deviation from the steady state) to a one-standard deviation shock to technology. Left column: $s^I = 0.67$, $s^E = 0.76$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Figure G2. Impulse responses to a land demand shock



Notes: Impulse responses of key macroeconomic variables (in percentage deviation from the steady state) to a two-standard deviations shock to land demand. Left column: $s^I = 0.67$, $s^E = 0.76$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

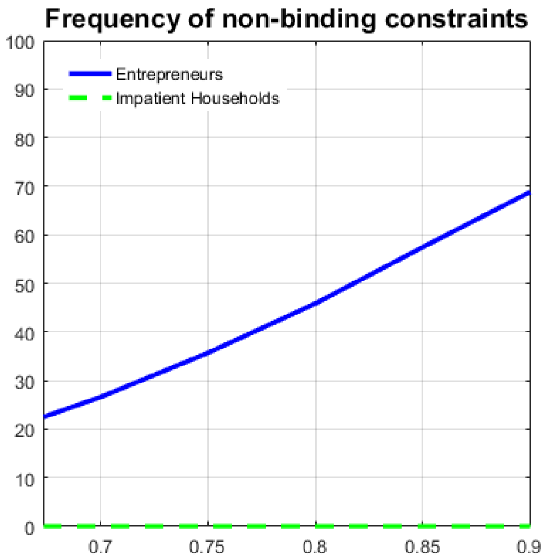
Figure G3. Impulse responses to a credit limit shock



Notes: Impulse responses of key macroeconomic variables (in percentage deviation from the steady state) to a one-standard deviation shock to credit limits. Left column: $s^I = 0.67$, $s^E = 0.76$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

G2 On the occurrence of non-binding collateral constraints

Figure G4. Leverage and non-binding collateral constraints



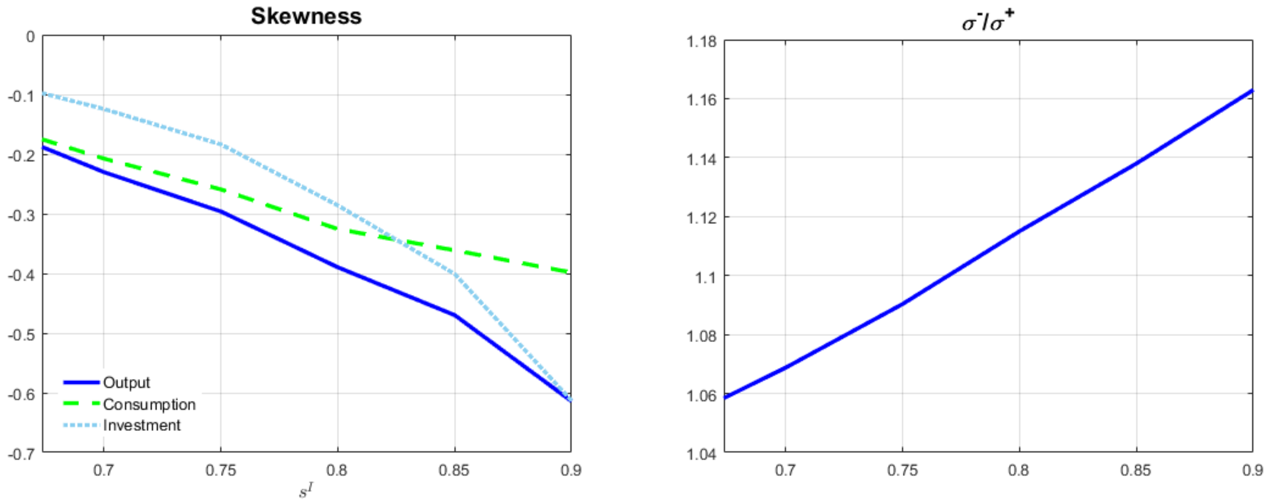
Notes: Frequency of non-binding constraints for the entrepreneurs (solid-blue line) and the impatient households (dashed-green line). Both statistics are graphed for different average LTV ratios faced by the impatient household. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

G3. The model with no household debt

In this appendix we report numerical evidence from an alternative model with no role for collateralized household debt. We effectively exclude impatient households from the model by setting their income share to a very low number (i.e., $1 - \alpha = 0.01$). All other parameters are as described in Section 5.1. We then perform the same simulation exercise as that reported in Section 6.2. The results are reported below.[47] As displayed by Figure G5, the alternative model generates an amount of skewness similar to that of the baseline framework. However, as illustrated in the left panel of Figure G6, the model's ability to reproduce the increase in the duration of expansions observed in the data is impaired substantially. This can be explained based on the fact that impatient households contract long-term debt, which induces a certain smoothness in the consumption/investment profiles of all agents in the model. In addition, the left panel of Figure G7 indicates that the alternative model implies a much larger increase in output volatility when leverage increases, and a much smaller reversal. This pattern represents a further challenge to a model with no household borrowing, as it makes our findings harder to reconcile with the Great Moderation in output volatility. In fact, attaining such a fall in volatility would entail a rather large scaling of the structural shocks (recall the analysis in Section 6.3.1).

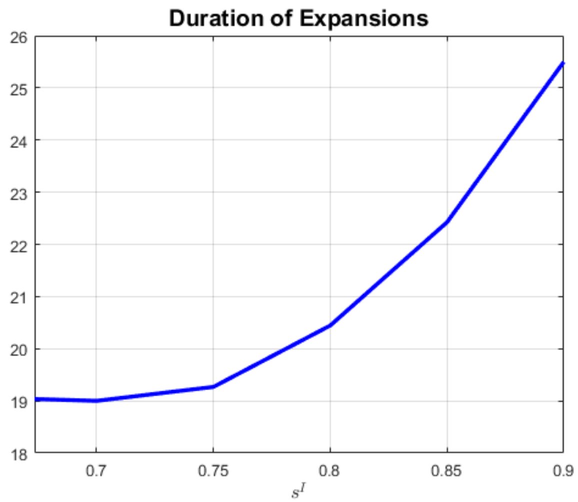
[47] Note that the impatient household is still present in the model, albeit playing a very small role. Thus, when reporting the results from this model, we choose to keep s^I on the horizontal axis, so as to facilitate comparison with the results in the main text.

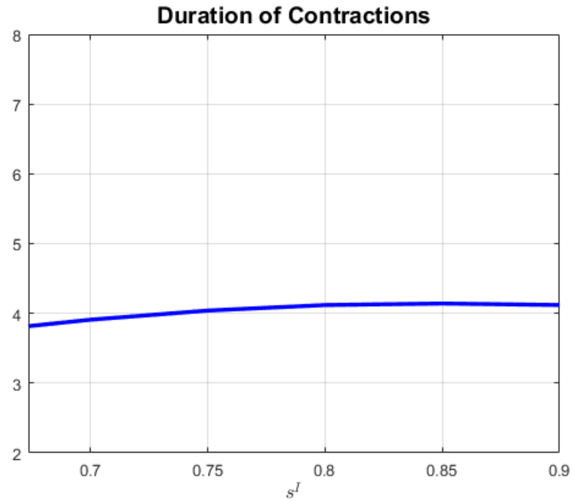
Figure G5. Business cycle asymmetry

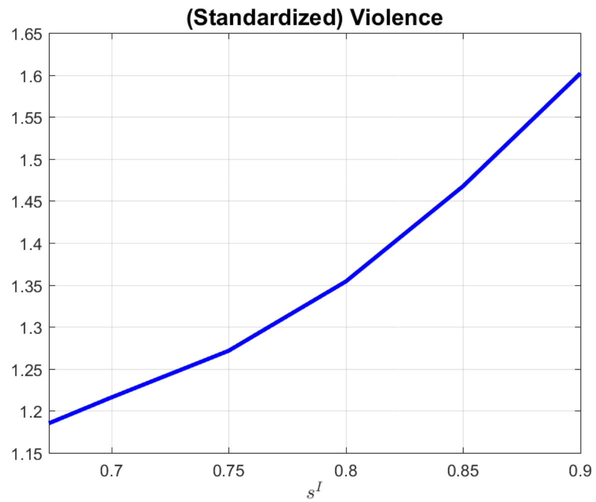


Notes: The left panel of the figure reports the skewness of the year-on-year growth rate of output, consumption and investment, while the right panel displays the ratio between the downside and the upside semivolatility of year-on-year output growth, for different average LTV ratios faced by the financially constrained agents. To identify the recessionary episodes in the simulated series, we use the Harding and Pagan (2002) algorithm. Across all the simulations the entrepreneurial average LTV ratio is adjusted to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure G6. Expansions, contractions and leverage

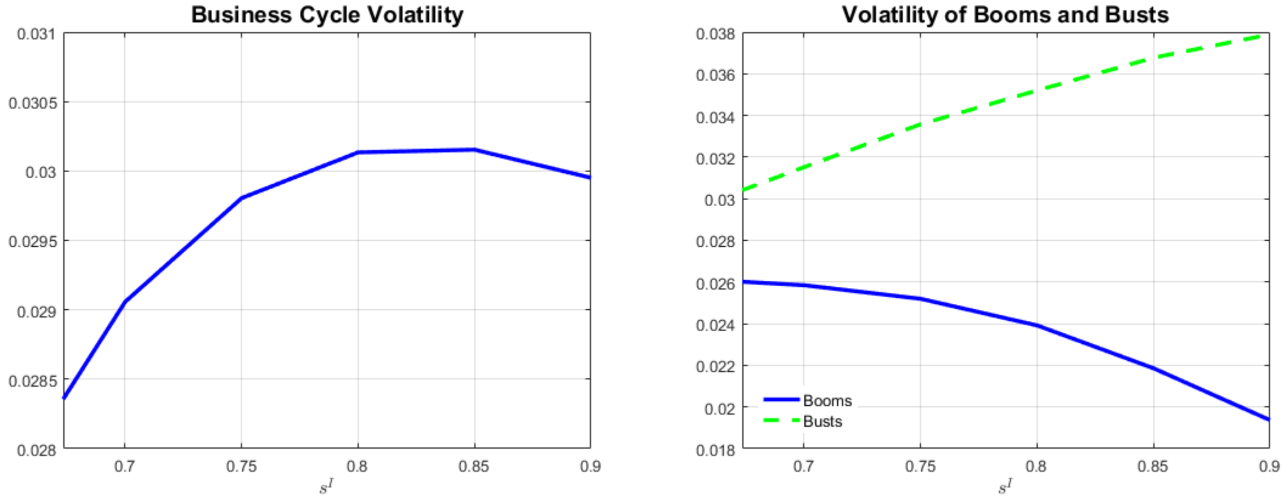






Notes: The first two panels report the duration (in quarters) of expansions and contractions, respectively. The last panel displays the standardized violence of recessionary episodes. To identify expansions and contractions in the simulated gross output series, we use the Harding and Pagan (2002) algorithm. We compute the violence as the average fall of output over a given recession, divided by the length of the recession itself. Finally, we standardize the violence by means of the volatility of year-on-year output growth over the five years prior to the recession. All statistics are conditional on different average LTV ratios faced by the financially constrained agents. Across all the simulations the entrepreneurial average LTV ratio is adjusted to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure G7. Leverage and volatility

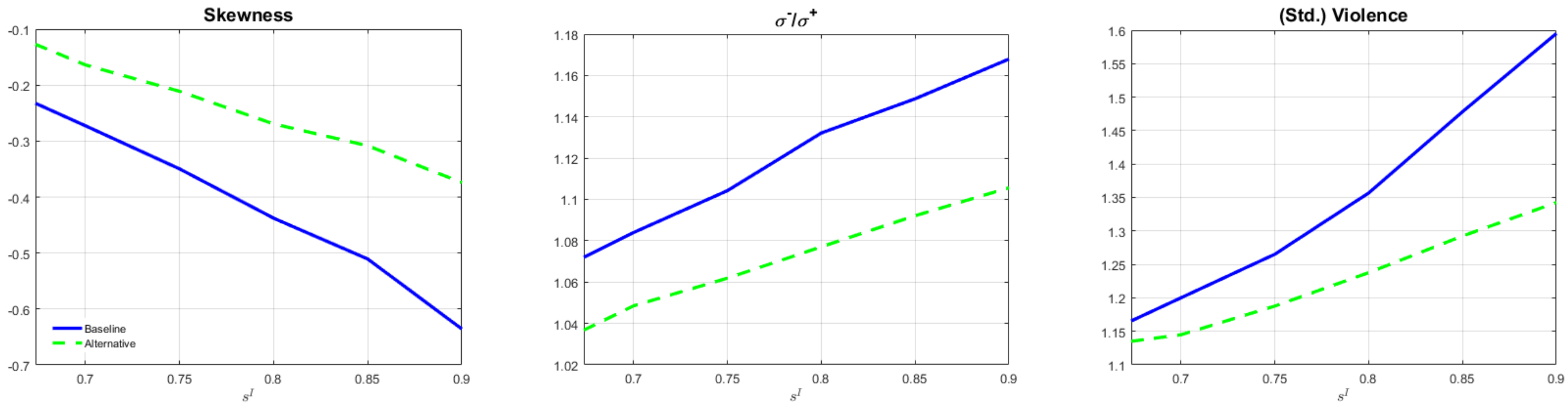


Notes: The left panel reports the standard deviation of year-on-year output growth, while the right panel reports the standard deviation of expansions (solid-blue line) and contractions (dashed-green line) in economic activity. These are determined based on whether output is above or below its steady-state level. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

G4. Asymmetry and collateral prices

In this appendix we report results obtained by simulating an alternative version of the model where the collateral assets are pledged at their steady-state prices. We also report results from our baseline model for comparison.

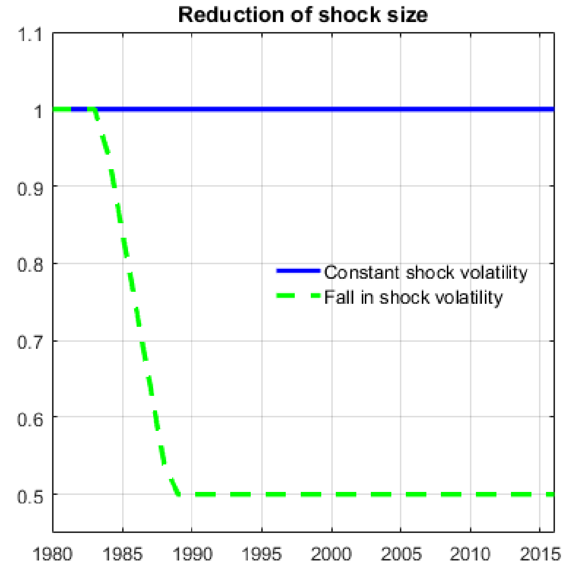
Figure G8. Asymmetry and collateral prices



Notes: Solid-blue lines: baseline model. Dashed-green lines: alternative model with asset prices fixed at their steady-state values in the collateral constraints.

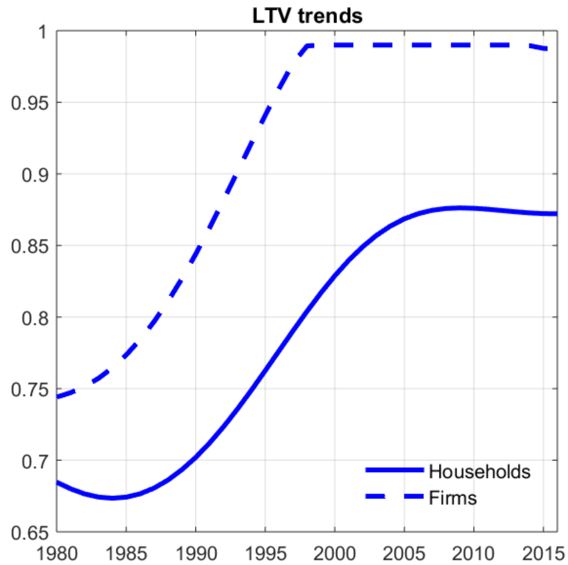
G5. Counterfactual exercise

Figure G9. Scaling factor



Notes: Scaling factor applied to the shocks to attain a 40% reduction in the volatility of output growth over the 1984-1989 time window.

Figure G10. Sequences of LTV ratios



Notes: Sequence of LTV ratios used in each of the two counterfactual scenarios reported in Figure 10. We use the long-term components reported in Figure A1, to which we add a constant in order to match the calibrated LTV ratios from Section 5.1.1 in 1984. If the resulting LTV ratio exceeds 0.99, we cap it at this value.

References

Altonji, J. G., and L. M. Segal, 1996, Small-Sample Bias in GMM Estimation of Covariance Structures, Journal of Business and Economic Statistics 14, 353–366.

Anderson, T. W. and D. A. Darling, 1954, A Test of Goodness-of-Fit, Journal of the American Statistical Association, 49, 765–769.

Bates, T. W., K. M. Kahle, and R. M. Stulz, 2009, Why Do U.S. Firms Hold So Much More Cash than They Used To?, Journal of Finance, 64, 1985–2021.

Borio, C., 2014, The Financial Cycle and Macroeconomics: What Have We Learnt?, Journal of Banking and Finance, 45, 182–198.

Boz, E., and E. Mendoza, 2014, Financial Innovation, the Discovery of Risk, and the U.S. Credit Crisis, Journal of Monetary Economics, 62, 1–22.

Chambers, M., C. Garriga, and D. Schlagenhauf, 2009, Accounting for Changes in the Homeownership Rate, International Economic Review, 50, 677–726.

Dahlhaus, R., 1996, Fitting Time Series Model to Nonstationary Processes, Annals of Statistics, 25, 1–37.

Delle Monache, D. and I. Petrella, 2017, Adaptive Models and Heavy Tails with an Application to Inflation Forecasting, International Journal of Forecasting, 33, 482–501.

Duca, J. V., J. Muellbauer, and A. Murphy, 2011, House Prices and Credit Constraints: Making Sense of the U.S. Experience, Economic Journal, 121, 533–551.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh, 2017, The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium, Journal of Political Economy, 125, 140–223.

Gasser, T., 1975, Goodness-of-fit Tests for Correlated Data, Biometrika, 62, 563–570.

Giraitis, L., Kapetanios, G., and Yates, T., 2014, Inference on Stochastic Time-Varying Coefficient Models, Journal of Econometrics, 179, 46–65.

Hamilton, J. D., 1994, Time Series Analysis, Princeton University Press, Princeton, NJ.

Jarque, C., and A. Bera, 1987, A Test for Normality of Observations and Regression Residuals, International Statistical Review, 55, 163–172.

Kapetanios, G., 2007, Estimating Deterministically Time-varying Variances in Regression Models, Economics Letters, 97, 97–104.

Priestley, M., 1965, Evolutionary Spectra and Nonstationary Processes, Journal of the Royal Statistical Society, Series B, 27, 204–37.

Psaradakis, Z., and M. Sola, 2003, On Detrending and Cyclical Asymmetry, Journal of Applied Econometrics, 18, 271–289.

Royston, P., 1992, Approximating the Shapiro-Wilk W-test for Non-normality, Statistics and Computing, 2, 117–119.

Shapiro, S., and M. Wilk, 1965, An Analysis of Variance Test for Normality (Complete Samples), Biometrika, 52, 591–611.