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Research Article

Viscoelastic Effects on Free Convective Three-Dimensional Flow with Heat and Mass Transfer

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A theoretical study of free convective three-dimensional heat and mass transfer flow of a viscoelastic fluid along a steadily moving porous vertical plate in presence of transverse sinusoidal suction velocity distribution, and uniform free stream velocity has been considered. The flow becomes three dimensional due to this suction velocity. The governing equations of the flow field are solved by using series expansion method, and the expressions for velocity field, temperature field, skin friction, heat flux in terms of Nusselt number, and mass flux in terms of Sherwood number are obtained. The effects of the viscoelastic parameter on velocity profiles and shear stress with the combination of the other flow parameters are discussed graphically.

1. Introduction

The study of combined heat and mass transfer problems with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Such phenomenon is observed in buoyancy-induced motions in the atmosphere, in bodies of water, quasisolid bodies such as earth, and so on. In nature and industrial applications, many transport processes exist where the heat and mass transfer takes place simultaneously as a result of combined effects of thermal diffusion and diffusion of chemical species. In addition, the phenomenon of heat and mass transfer is also encountered in chemical processes industries such as food processing and polymer production. Soundalgekar and Warve [1] have analyzed two-dimensional unsteady free convection flow, past an infinite vertical plate with oscillating wall temperature and constant suction. Lin and Wu [2] have analyzed the problem of simultaneous heat and mass transfer with entire range of buoyancy ratio for most practical chemical species in dilute and aqueous solutions. Muthucumaraswamy et al. [3] studied the heat and mass transfer effects on flow past an impulsively started infinite vertical plate.

Many research workers are doing investigation of the problem of laminar flow control due to its importance in the field of aeronautical engineering, in view of its applications to reduce drag and enhance the vehicle power requirement by a substantial amount. Initially this subject has been developed by Lachmann [4]. Theoretical and experimental investigations have shown that the transition from laminar to the turbulent flow, which causes the drag coefficient to increase, may be prevented by suction of the fluid and heat and mass transfer from boundary to the wall.

Singh et al. [5] studied the effect on wall shear stress and heat transfer of the flow caused by the periodic suction velocity perpendicular to the flow direction when the difference between the wall temperature and the free stream temperature gives rise to buoyancy force in the direction of the free stream. The effect of the porous medium on the three-dimensional Couette flow and heat transfer was presented by Singh and Sharma [6]. Chaudhary and Sharma [7] studied the three-dimensional convection flow through a porous medium and estimated the effect in heat and mass transfer. Ahmed [8] has studied the effects of heat and mass transfer on the steady three-dimensional flow of a viscous incompressible fluid along a steadily moving porous vertical plate subjected to a transverse sinusoidal velocity.

The present paper is concerned with the free convective three-dimensional heat and mass transfer flow of visco-elastic incompressible fluid characterized by second-order fluid along a steadily moving porous vertical plate in presence of transverse sinusoidal suction velocity distribution and uniform free stream velocity.

The constitutive equation for the incompressible second-order fluid is

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2, \tag{1}$$

where σ is the stress tensor, p is hydrostatic pressure, I is unit tensor, A_n $(n = 1, 2)$ are the kinematic Rivlin-Ericksen tensors, and $\mu_1, \mu_2,$ and μ_3 are the material coefficients describing the viscosity, elasticity, and cross-viscosity, respectively. The material coefficients $\mu_1, \mu_2,$ and μ_3 are taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Markovitz and Coleman [9]). Equation (1) was derived by Coleman and Noll [10] from that of simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

The expression for $A_{(1)ij}$ and $A_{(2)ij}$ is given by

$$\begin{aligned} A_{(1)ij} &= v_{i,j} + v_{j,i}, \\ A_{(2)ij} &= a_{i,j} + a_{j,i} + 2v^m{}_{,i} v_{m,j}, \end{aligned} \tag{2}$$

where v_i and a_i are the ith component of the velocity and acceleration vectors, respectively, and a comma denotes covariant differentiation with respect to the symbol following it.

2. Basic Equations

A rectangular Cartesian co-ordinate system is introduced such that the plate lies in xz-plane, x-axis being vertically upwards in the direction of buoyancy force which arises out of a difference in temperature of the plate and the free stream. The y-axis is taken perpendicular to the plate and directed into the fluid which is flowing laminarly with free stream velocity U. The transverse sinusoidal suction velocity distribution is assumed to be of the form

$$\overline{v}_w(\overline{z}) = -v_0 \left(1 + \varepsilon \cos \frac{\pi\, \overline{z}}{L}\right), \tag{3}$$

where $\varepsilon \ll 1$ and L is the wave length of the periodic suction. All physical quantities are independent of $\overline{x}$ for this problem of fully developed laminar flow.

The boundary conditions relevant to the problem are

$$\begin{aligned} &\overline{y} = 0 : \ \overline{u} = \overline{V}, \quad \overline{v} = \overline{v}_w, \\ &\overline{w} = 0, \quad \overline{T} = \overline{T}_w, \quad \overline{C} = \overline{C}_w, \\ &\overline{y} \longrightarrow \infty : \ \overline{u} \longrightarrow \overline{U}, \quad \overline{v} \longrightarrow -v_0, \quad \overline{w} \longrightarrow 0, \\ &\overline{T} \longrightarrow \overline{T}_\infty, \quad \overline{p} \longrightarrow \overline{p}_\infty, \quad \overline{C} \longrightarrow \overline{C}_\infty. \end{aligned} \tag{4}$$

We introduce the following nondimensional quantities:

$$\begin{aligned} &y = \frac{\overline{y}}{L}, \quad z = \frac{\overline{z}}{L}, \quad u = \frac{\overline{u}}{v_0}, \\ &v = \frac{\overline{v}}{v_0}, \quad w = \frac{\overline{w}}{v_0}, \quad U = \frac{\overline{U}}{v_0}, \\ &\theta = \frac{\overline{T} - \overline{T}_\infty}{T_w - T_\infty}, \quad C = \frac{\overline{C} - \overline{C}_\infty}{\overline{C}_w - \overline{C}_\infty}, \quad V = \frac{\overline{V}}{v_0}, \\ &\mathrm{Pr} = \frac{v_1}{\alpha}, \quad \mathrm{S} = \frac{v_1}{D}, \quad p = \frac{\overline{p}}{\rho (v_1/L)^2}, \\ &p_\infty = \frac{\overline{p}_\infty}{\rho (v_1/L)^2}, \quad \mathrm{Gr} = \frac{Lg\beta \left(\overline{T}_w - \overline{T}_\infty\right)}{v_0^2}, \\ &\mathrm{Gm} = \frac{Lg\overline{\beta} \left(\overline{T}_w - \overline{T}_\infty\right)}{v_0^2}, \quad \mathrm{Re} = \frac{v_0 L}{v_1}. \end{aligned} \tag{5}$$

Here, $(\overline{u}, \overline{v}, \overline{w})$ are the velocity components along the $(\overline{x}, \overline{y}, \overline{z})$ directions, respectively, g the acceleration due to gravity, $v_i = (\mu_i/\rho)$ $(i = 1, 2, 3)$, ρ is the density, β is the coefficient of volume expansion for heat transfer, $\overline{\beta}$ is the co-efficient of volume expansion for mass transfer, $\overline{p}$ is the pressure, $\overline{T}$ is the fluid temperature, $\overline{C}$ is the species concentration, α is the thermal diffusivity, D is the chemical molecular diffusivity, v_0 is the dimensionless suction velocity, Pr is the Prandtl number, S is the Schmidt number, Gr is the Grashof number for heat transfer, Gm is the Grashof number for mass transfer, Re is the Reynolds number, $\overline{T}_w$ is the fluid temperature on the wall, and $\overline{T}_\infty$ is the fluid temperature in the free stream.

In view of the above nondimensional quantities, the governing equations for heat and mass transfer flow are

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{6}$$

$$\begin{aligned} &v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \\ &= \mathrm{Gr}\ \theta + \mathrm{Gm}\ C + \frac{1}{\mathrm{Re}} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\ &\quad + \alpha_1 \left(2 \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial^2 v}{\partial y^2} \frac{\partial u}{\partial y}\right. \\ &\qquad + \frac{\partial^2 w}{\partial y^2} \frac{\partial u}{\partial z} + 2 \frac{\partial w}{\partial y} \frac{\partial^2 u}{\partial y \partial z} + w \frac{\partial^3 u}{\partial y^2 \partial z} \\ &\qquad + \frac{\partial^2 v}{\partial z^2} \frac{\partial u}{\partial y} + 2 \frac{\partial v}{\partial z} \frac{\partial^2 u}{\partial y \partial z} + v \frac{\partial^3 u}{\partial y \partial z^2} \\ &\qquad \left. + \frac{\partial^2 w}{\partial z^2} \frac{\partial u}{\partial z} + 2 \frac{\partial w}{\partial z} \frac{\partial^2 u}{\partial z^2} + w \frac{\partial^3 u}{\partial z^3}\right) \\ &\quad + \alpha_2 \left(\frac{\partial^2 v}{\partial y^2} \frac{\partial u}{\partial y} + 2 \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y^2} + 2 \frac{\partial^2 u}{\partial y \partial z} \frac{\partial w}{\partial y}\right. \end{aligned}$$

$$+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial^2 u}{\partial y\partial z}\frac{\partial v}{\partial z}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial z^2}+2\frac{\partial^2 u}{\partial z^2}\frac{\partial w}{\partial z}\Bigg), \tag{7}$$

$$\begin{aligned}
&v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\\
&=-\frac{1}{\mathrm{Re}^2}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\\
&\quad+\alpha_1\Bigg(v\frac{\partial^3 v}{\partial y^3}+w\frac{\partial^3 v}{\partial y^2\partial z}+v\frac{\partial^3 v}{\partial y\partial z^2}\\
&\qquad+w\frac{\partial^3 v}{\partial z^3}+2\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial y\partial z}+3\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}\\
&\qquad+2\frac{\partial^2 u}{\partial y\partial z}\frac{\partial u}{\partial z}+13\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\\
&\qquad+3\frac{\partial^2 w}{\partial y^2}\frac{\partial v}{\partial z}+4\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\\
&\qquad+4\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z^2}+\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2}\Bigg)\\
&\quad+\alpha_2\Bigg(2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+8\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\\
&\qquad+2\frac{\partial^2 v}{\partial y\partial z}\frac{\partial w}{\partial y}+2\frac{\partial^2 w}{\partial y^2}\frac{\partial v}{\partial z}+2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}\\
&\qquad+2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 u}{\partial y\partial z}\frac{\partial u}{\partial z}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z^2}\Bigg),
\end{aligned} \tag{8}$$

$$\begin{aligned}
&v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\\
&=-\frac{1}{\mathrm{Re}^2}\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\\
&\quad+\alpha_1\Bigg(w\frac{\partial^3 v}{\partial y^3}+v\frac{\partial^3 w}{\partial y\partial z^2}+w\frac{\partial^3 w}{\partial z^3}\\
&\qquad+v\frac{\partial^3 w}{\partial y^3}+2\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial y\partial z}+3\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y\partial z}\\
&\qquad+2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial z}+13\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}\\
&\qquad+3\frac{\partial^2 v}{\partial z^2}\frac{\partial w}{\partial y}+4\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\\
&\qquad+4\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}+2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y^2}+\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial y^2}\Bigg)\\
&\quad+\alpha_2\Bigg(2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}+8\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial y\partial z}\\
&\qquad+2\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial z^2}+2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y\partial z}\\
&\qquad+2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 u}{\partial y\partial z}\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y^2}\Bigg),
\end{aligned} \tag{9}$$

$$v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=\frac{1}{\mathrm{Re\ Pr}}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right), \tag{10}$$

$$v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{1}{\mathrm{Re\ S}}\left(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right), \tag{11}$$

where $\alpha_1 = v_2/L^2$ and $\alpha_2 = v_3/L^2$.

The corresponding boundary conditions are

$$\begin{gathered}
y=0:\ u=V,\quad v(z)=-(1+\varepsilon\cos\pi z),\\
w=0,\quad \theta=1,\quad C=1,\\
y\longrightarrow\infty:\ u\longrightarrow U,\quad v\longrightarrow -1,\\
w\longrightarrow 0,\ \theta\longrightarrow 0,\quad C\longrightarrow 0,\quad p\longrightarrow p_\infty.
\end{gathered} \tag{12}$$

3. Method of Solution

When the amplitude $\varepsilon(\ll 1)$ of the suction velocity is small, we assume the solutions of the nonlinear partial differential equations (7) to (11) of the form

$$\begin{aligned}
u(y,z)&=u_0(y)+\varepsilon u_1(y,z)+o(\varepsilon^2),\\
v(y,z)&=v_0(y)+\varepsilon v_1(y,z)+o(\varepsilon^2),\\
w(y,z)&=w_0(y)+\varepsilon w_1(y,z)+o(\varepsilon^2),\\
p(y,z)&=p_0(y)+\varepsilon p_1(y,z)+o(\varepsilon^2),\\
\theta(y,z)&=\theta_0(y)+\varepsilon\theta_1(y,z)+o(\varepsilon^2),\\
C(y,z)&=C_0(y)+\varepsilon C_1(y,z)+o(\varepsilon^2).
\end{aligned} \tag{13}$$

When $\varepsilon = 0$, the problem reduces to the two-dimensional case with $w = w_0 = 0$ and $p = p_0 = p_\infty$. Substituting (13) into (6) to (11), the terms free from ε of both sides are

$$v_0' = 0, \tag{14}$$

$$\alpha_1 v_0 u_0'' + \frac{1}{\mathrm{Re}}u_0'' - v_0 u_0' + \mathrm{Gr}\theta_0 + \mathrm{Gm}\ C_0 = 0, \tag{15}$$

$$-\frac{1}{\mathrm{Re}^2}\frac{\partial p_0}{\partial y}+(4\alpha_1+2\alpha_2)\frac{\partial u_0}{\partial y}\frac{\partial^2 u_0}{\partial y^2}=0, \tag{16}$$

$$-\frac{\partial\theta_0}{\partial y}=\frac{1}{\mathrm{Re\ Pr}}\frac{\partial^2\theta_0}{\partial y^2}, \tag{17}$$

$$-\frac{\partial C_0}{\partial y}=\frac{1}{\mathrm{Re\ S}}\frac{\partial^2 C_0}{\partial y^2}, \tag{18}$$

where prime denotes differentiation with respect to y.

The corresponding boundary conditions are

$$\begin{aligned} y = 0 &: \; u_0 = V, \quad v_0 = -1, \\ & \; w_0 = 0, \quad \theta_0 = 1, \quad C_0 = 1, \\ y \longrightarrow \infty &: \; u_0 \longrightarrow U, \quad v_0 \longrightarrow -1, \quad w_0 \longrightarrow 0, \\ & \; \theta_0 \longrightarrow 0, \quad C_0 \longrightarrow 0, \quad p_0 \longrightarrow p_\infty. \end{aligned} \tag{19}$$

Solving (14), (17), and (18) under boundary conditions (19), we get

$$\begin{aligned} v_0 &= -1, \\ \theta_0 &= \exp(-\mathrm{Re}\ \mathrm{Pr}\ y), \\ C_0 &= \exp(-\mathrm{Re}\ \mathrm{S}\ y). \end{aligned} \tag{20}$$

As $|\alpha_1| \ll 1$ (due to small shear rate) therefore substituting

$$u_0(y) = u_{00}(y) + \alpha_1 u_{01}(y) + o(\alpha_1^2), \tag{21}$$

into (15) and boundary conditions (19) up to the first order of α_1, and equating the coefficients of like powers of α_1, we obtain the following sets of ordinary differential equations and corresponding boundary conditions

$$\begin{aligned} u_{00}'' + \mathrm{Re}\ u_{00}' + \mathrm{Re}\{&\mathrm{Grexp}(-\mathrm{Re}\ \mathrm{Pr}\ y) \\ &+\mathrm{Gm}\ \exp(-\mathrm{Re}\ \mathrm{S}\ y)\} = 0, \end{aligned} \tag{22}$$

$$\mathrm{Re}\ u_{00}''' - u_{01}'' - \mathrm{Re}\ u_{01}' = 0, \tag{23}$$

$$\begin{aligned} y = 0 &: \; u_{00} = V, \quad u_{01} = 0, \\ y \longrightarrow \infty &: \; u_{00} \longrightarrow U, \quad u_{01} \longrightarrow 0. \end{aligned} \tag{24}$$

Solving (22) and (23) under boundary conditions (24) and then substituting into (21) (neglecting α_1^2), we get

$$\begin{aligned} u_0 = {} & U + (V - U)\exp(-\mathrm{Re}\ y) \\ & + \frac{\mathrm{Gr}}{\mathrm{Re}\ \mathrm{Pr}(\mathrm{Pr}-1)}\{\exp(-\mathrm{Re}\ y) - \exp(-\mathrm{Re}\ \mathrm{Pr}\ y)\} \\ & + \frac{\mathrm{Gm}}{\mathrm{Re}\ \mathrm{S}(\mathrm{S}-1)}\{\exp(-\mathrm{Re}\ y) - \exp(-\mathrm{Re}\ \mathrm{S}\ y)\} \\ & - \frac{\alpha_1}{\mathrm{Re}}\left[\left\{\frac{A_2}{\mathrm{Pr}(\mathrm{Pr}-1)} + \frac{A_3}{\mathrm{S}(\mathrm{S}-1)}\right\}\exp(-\mathrm{Re}\ y)\right. \\ & \quad + A_1\,\mathrm{Re}\, y \exp(-\mathrm{Re}\ y) \\ & \quad \left. - \frac{A_2\ \exp(-\mathrm{Re}\ \mathrm{Pr}\ y)}{\mathrm{Pr}(\mathrm{Pr}-1)} - \frac{A_3 \exp(-\mathrm{Re}\ \mathrm{S}\ y)}{\mathrm{S}(\mathrm{S}-1)}\right], \end{aligned} \tag{25}$$

where $A_1 = \mathrm{Re}^3\{U - V - (\mathrm{Gr}/\mathrm{Pr}\ \mathrm{Re}\ (\mathrm{Pr}-1)) - (\mathrm{Gm}/\mathrm{Re}\ \mathrm{S}(\mathrm{S}-1))\}$, $A_2 = (\mathrm{Gr}\,\mathrm{Re}^2\,\mathrm{Pr}^2/(\mathrm{Pr}-1))$, $A_3 = \mathrm{Gm}\ \mathrm{Re}^2\mathrm{S}^2/(\mathrm{S}-1)$ with $\mathrm{Pr} \neq 1$, $\mathrm{S} \neq 1$.

Equating the coefficients of ε from both sides after substitution of (13) in (6) to (11), neglecting those of ε^2, using (16), and assuming $\partial p_0/\partial y = 0$, we get

$$\frac{\partial v_1}{\partial u} + \frac{\partial w_1}{\partial z} = 0, \tag{26}$$

$$\begin{aligned} \nu_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = {} & \mathrm{Gr}\,\theta_1 + \mathrm{Gm}\,C_1 + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) \\ & + \alpha_1\left(-2\frac{\partial \nu_1}{\partial y}\frac{\partial^2 u_0}{\partial y^2} - \frac{\partial^3 u_1}{\partial y^3} + \nu_1 \frac{\partial^3 u_0}{\partial y^3}\right. \\ & \left. - \frac{\partial^2 \nu_1}{\partial y^2}\frac{\partial u_0}{\partial y} - \frac{\partial u_0}{\partial y}\frac{\partial^2 \nu_1}{\partial z^2} - \frac{\partial^3 u_1}{\partial y \partial z^2}\right), \end{aligned} \tag{27}$$

$$\begin{aligned} -\frac{\partial \nu_1}{\partial y} = {} & -\frac{1}{\mathrm{Re}^2}\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \nu_1}{\partial y^2} + \frac{\partial^2 \nu_1}{\partial z^2}\right) \\ & - \alpha_1\left(\frac{\partial^3 \nu_1}{\partial y^3} + \frac{\partial^3 \nu_1}{\partial y \partial z^2}\right), \end{aligned} \tag{28}$$

$$\begin{aligned} -\frac{\partial w_1}{\partial y} = {} & -\frac{1}{\mathrm{Re}^2}\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) \\ & - \alpha_1\left(\frac{\partial^3 w_1}{\partial y \partial z^2} + \frac{\partial^3 w_1}{\partial y^3}\right), \end{aligned} \tag{29}$$

$$-\frac{\partial \theta_1}{\partial y} + \nu_1 \frac{\partial \theta_0}{\partial y} = \frac{1}{\mathrm{Re}\ \mathrm{Pr}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right), \tag{30}$$

$$-\frac{\partial C_1}{\partial y} + \nu_1 \frac{\partial C_0}{\partial y} = \frac{1}{\mathrm{Re}\ \mathrm{S}}\left(\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2}\right), \tag{31}$$

with relevant boundary conditions

$$\begin{aligned} y = 0 &: \; u_1 = 0, \quad v_1 = -\cos \pi z, \\ & \; w_1 = 0, \quad \theta_1 = 0, \quad p_1 = 0, \quad C_1 = 0, \\ y \longrightarrow \infty &: \; u_1 \longrightarrow 0, \quad v_1 \longrightarrow 0, \quad w_1 \longrightarrow 0, \\ & \; \theta_1 \longrightarrow 0, \quad p_1 \longrightarrow 0, \quad C_1 \longrightarrow 0. \end{aligned} \tag{32}$$

Equations (26), (28), and (29) govern the cross-flow, and (27), (30), and (31) govern the main flow, the temperature and the species concentration, respectively.

4. Cross-Flow Solution

In order to solve (26), (28), and (29), being independent of the main flow component u_1 and the temperature field θ_1, we assume that

$$v_1(y,z) = -\pi\ v_{11}(y)\cos \pi z, \tag{33}$$

$$w_1(y,z) = v_{11}'(y)\sin \pi z, \tag{34}$$

$$p_1(y,z) = \mathrm{Re}^2 p_{11}(y)\cos \pi z. \tag{35}$$

The prime in v_{11}' denotes differentiation with respect to y. Equations (33) and (34) have been chosen so that the continuity equation (26) is satisfied. Substituting these equations into (28) and (29), two ordinary differential

equations for v_{11} and p_{11} are obtained:

$$v_{11}'' + \mathrm{Re}\, v_{11}' - \pi^2 v_{11} + \alpha_1 \mathrm{Re}(\pi^2 v_{11}' - v_{11}''') = -\frac{p_{11}'}{\pi} \mathrm{Re}, \tag{36}$$

$$v_{11}''' + \mathrm{Re}\, v_{11}'' - \pi^2 v_{11}' + \alpha_1 \mathrm{Re}\left(\pi^2 v_{11}'' - v_{11}^{\mathrm{IV}}\right) = -\pi\ p_{11} \mathrm{Re}, \tag{37}$$

with the boundary conditions

$$\begin{aligned} &y = 0 :\ v_{11} = \frac{1}{\pi}, \quad v_{11}' = 0, \\ &y \longrightarrow 0 :\ v_{11} \longrightarrow 0, \quad v_{11}' \longrightarrow 0. \end{aligned} \tag{38}$$

On eliminating the pressure p_{11}' from (36) and (37), we get the following differential equation in v_{11} as

$$\begin{aligned} &v_{11}^{\mathrm{IV}} + \mathrm{Re}\, v_{11}''' - 2\pi^2 v_{11}' - \pi^2 \mathrm{Re}\, v_{11}' + \pi^4 v_{11} \\ &\quad + \alpha_1 \mathrm{Re}\left(2\pi^2 v_{11}''' - v_{11}^{\mathrm{V}} - \pi^4 v_{11}'\right) = 0. \end{aligned} \tag{39}$$

To solve (39), we note that $\alpha_1 < 1$ for small shear, and so we can assume that

$$v_{11}(y) = v_{110}(y) + \alpha_1 v_{111}(y) + o(\alpha_1^2). \tag{40}$$

Substituting from (40) into (39) then equating the like powers of α_1 and neglecting the higher powers of α_1,we get

$$v_{110}^{\mathrm{IV}} + \mathrm{Re}\, v_{110}''' - 2\pi^2 v_{110}'' - \pi^2 \mathrm{Re}\, v_{110}' + \pi^4 v_{110} = 0, \tag{41}$$

$$\begin{aligned} &v_{111}^{\mathrm{IV}} + \mathrm{Re}\, v_{111}''' - 2\pi^2 v_{111}'' - \pi^2 \mathrm{Re}\, v_{111}' + \pi^4 v_{111} + 2\,\mathrm{Re}\, \pi^2 v_{110}'' \\ &\quad - \mathrm{Re}\, v_{110}^{\mathrm{V}} - \pi^4 \mathrm{Re}\, v_{110}' = 0. \end{aligned} \tag{42}$$

The corresponding boundary conditions are:

$$\begin{aligned} &y = 0 :\ v_{110} = \frac{1}{\pi}, \quad v_{111} = 0, \\ &v_{110}' = 0, \quad v_{111}' = 0, \\ &y \longrightarrow \infty :\ v_{110} \longrightarrow 0, \quad v_{111} \longrightarrow 0, \\ &v_{110}' \longrightarrow 0, \quad v_{111}' \longrightarrow 0. \end{aligned} \tag{43}$$

Solving (41) and (42) under boundary conditions (43), we get the expression for v_{11} and hence the solutions for velocity components v_1, w_1 and the pressure p_1.

5. Solution for Main Flow, Temperature, and Molar Concentration Fields

To solve (27), (30), and (31), we assume the following form:

$$\begin{aligned} u_1(y,z) &= u_{11}(y)\cos \pi z, \\ \theta_1(y,z) &= \theta_{11}(y)\cos \pi z, \\ C_1(y,z) &= C_{11}(y)\cos \pi z. \end{aligned} \tag{44}$$

On using (44) into (27), (30), and (31), we get

$$\begin{aligned} &u_{11}'' - \pi^2 u_{11} + \mathrm{Re}(\pi v_{11} u_0' + u_{11}') + \mathrm{Gr}\theta_{11} + \mathrm{Gm}\ C_{11} \\ &\quad + \alpha_1 (2\pi v_{11}' u_0'' - u_{11}''' - \pi v_{11} u_0''' + \pi v_{11}'' u_0' \\ &\qquad - u_0' \pi^3 v_{11} + \pi^2 u_{11}') = 0, \end{aligned} \tag{45}$$

$$\theta_{11}'' - \pi^2 \theta_{11} + \mathrm{Re}\ \mathrm{Pr}(\theta_{11}' + \pi v_{11}\theta_0') = 0, \tag{46}$$

$$C_{11}'' - \pi^2 C_{11} + \mathrm{Re}\ \mathrm{S}(C_{11}' + \pi v_{11} C_0') = 0, \tag{47}$$

subject to boundary conditions

$$\begin{aligned} &y = 0 :\ u_{11} = 0, \quad \theta_{11} = 0, \quad C_{11} = 0 \\ &y \longrightarrow \infty :\ u_{11} \longrightarrow 0, \\ &\theta_{11} \longrightarrow 0, \quad C_{11} \longrightarrow 0. \end{aligned} \tag{48}$$

Solving (46) and (47) under boundary conditions (48), we get the expressions for θ_{11} and C_{11}, and hence the solutions for θ_1 and C_1 have been obtained.

Again, substituting

$$u_{11}(y) = u_{110}(y) + \alpha_1 u_{111}(y) + o(\alpha_1^2) \tag{49}$$

into (45) and boundary conditions (48) up to first order of α_1 and comparing the coefficients of like powers of α_1, we obtain

$$u_{110}'' + \mathrm{Re}\, u_{110}' - \pi^2 u_{110} = -\mathrm{Re}(\mathrm{Gr}\,\theta_{11} + \mathrm{Gm}\, C_{11} + \pi v_{11} u_0'), \tag{50}$$

$$\begin{aligned} u_{111}'' + \mathrm{Re}\, u_{111}' - \pi^2 u_{111} = \mathrm{Re}(&-2\pi v_{11}' u_0'' + u_{110}''' + \pi v_{11} u_0''', \\ &-\pi v_{11}'' u_0' + u_0' \pi^3 v_{11} - \pi^2 u_{110}'), \end{aligned} \tag{51}$$

subject to boundary conditions

$$\begin{aligned} &y = 0 :\ u_{110} = 0, \quad u_{111} = 0, \\ &y \longrightarrow \infty :\ u_{110} \longrightarrow 0, \quad u_{111} \longrightarrow 0. \end{aligned} \tag{52}$$

Solving (50) and (51) under boundary conditions (52), we get the expression for u_{110} and u_{111}, and hence the solutions for u_{11} and u_1 have been obtained. The solutions of the differential equations are not presented here for the sake of brevity.

6. Results and Discussion

The nondimensional skin friction coefficient σ_{xy} at the plate $y = 0$ in the main flow direction is

$$\begin{aligned} \sigma_{xy} = \frac{\overline{\sigma}_{xy}}{\rho v_0^2} &= \left(\frac{1}{\mathrm{Re}}\frac{\partial u}{\partial y}\right)_{y=0} + \alpha_1 \\ &\times \left(\nu \frac{\partial^2 u}{\partial y^2} + w\frac{\partial^2 u}{\partial y \partial z} - 3\frac{\partial u}{\partial y}\frac{\partial v}{\partial y} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - 2\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}\right)_{y=0}. \end{aligned} \tag{53}$$

Table 1: Various combinations of flow parameters.

Cases	Gm	Pr	Re	S
I	2	3	3.5	0.1
II	5	3	3.5	0.1
III	5	5	3.5	0.1
IV	5	5	4.5	0.1
V	5	5	4.5	0.6

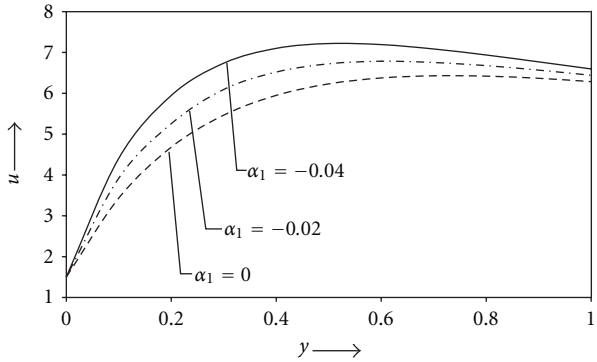

Figure 1: Variation of u against y for case I.

The nondimensional heat flux at the plate $y = 0$ in terms of Nusselt number Nu is

$$\begin{aligned} \mathrm{Nu} &= \frac{k}{\rho \nu_0 C_p \left(\overline{T}_w - \overline{T}_\infty\right)} \left(\frac{\partial \overline{T}}{\partial \overline{y}}\right) \\ &= \frac{1}{\mathrm{Pr\ Re}} \left[\theta_0'(0) + \varepsilon \theta_{11}'(0) \sin \pi z\right]. \end{aligned} \tag{54}$$

The nondimensional mass flux at the plate $y = 0$ in terms of Sherwood number Sh_0 is

$$\begin{aligned} \mathrm{Sh}_0 &= \frac{D}{\nu_0 \left(\overline{C}_w - \overline{C}_\infty\right)} \left(\frac{\partial \overline{C}}{\partial \overline{y}}\right)_{y=0} \\ &= \frac{1}{\mathrm{S\ Re}} \left[C_0'(0) + \varepsilon C_{11}'(0) \cos \pi z\right]. \end{aligned} \tag{55}$$

The purpose of this study is to bring out the effects of the non-Newtonian parameter on the governing flow with the combinations of the other flow parameters as the effects of the other parameters discussed by Ahmed. The non-Newtonian effect is exhibited through the nondimensional visco-elastic parameter α_1. The corresponding results for Newtonian fluid are obtained by setting $\alpha_1 = 0$.

Figures 1 to 5 represent the velocity profiles u against y to observe the visco-elastic effects for various sets of values (Table 1) of the Grashof number Gm for mass transfer, Prandtl number Pr, Reynolds number Re and Schmidt number S. It is evident from Figures 1 to 5 that the values of the velocity u increase with the increasing values of the non-Newtonian parameter $|\alpha_1|$, ($\alpha_1 = 0, -0.02, -0.04$) in comparison with the Newtonian fluid ($\alpha_1 = 0$) for all the cases in Table 1.

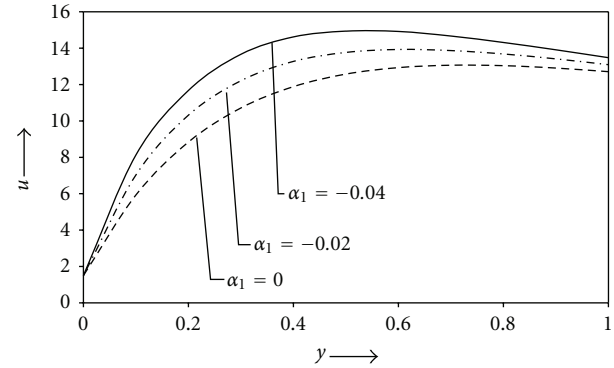

Figure 2: Variation of u against y for case II.

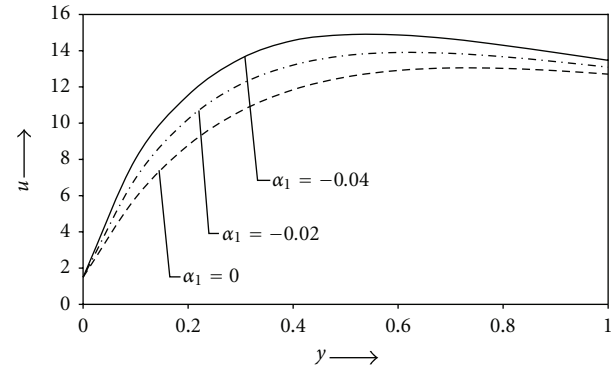

Figure 3: Variation of u against y for case III.

Figures 6 to 8 exhibit the effects of $|\alpha_1|$ on shear stress σ against Gm for various sets of values of S and Pr. It is evident from the Figures 6 to 8 that the values of σ decrease with the increasing values of the non-Newtonian parameter $|\alpha_1|$ in comparison with the Newtonian fluid for increasing values of the Schmidt number S (Figures 6 and 7) or Prandtl number Pr (Figures 7 and 8).

Figures 9 to 11 depict the shear stress σ against Schmidt number S for various sets of values of the Grashof number Gm for mass transfer and Prandtl number Pr. It is observed from the figures that the shear stress decreases with the increasing values of the non-Newtonian parameter $|\alpha_1|$, in comparison to the Newtonian fluid for increasing values of Grashof number (Figures 9 and 10) or Prandtl number (Figures 10 and 11).

It has also been observed that the heat and mass flux at the plate $y = 0$ are not significantly affected by the non-Newtonian parameter.

7. Conclusions

The present work is an attempt to study the viscoelastic effects on free convective three-dimensional flow along a steadily moving porous vertical plate in presence of transverse sinusoidal suction velocity and uniform free stream velocity. The second-order fluid model for a viscoelastic fluid flow is assumed. The effects of viscoelastic parameter

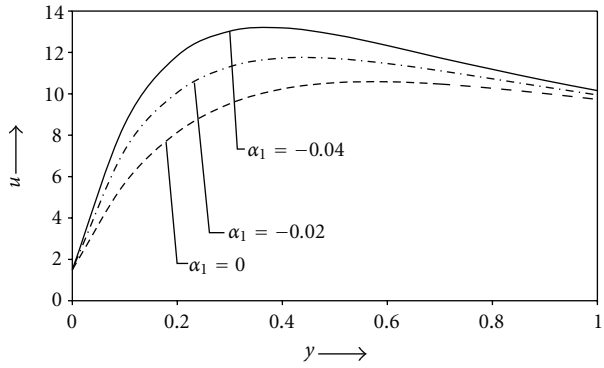

Figure 4: Variation of u against y for case IV.

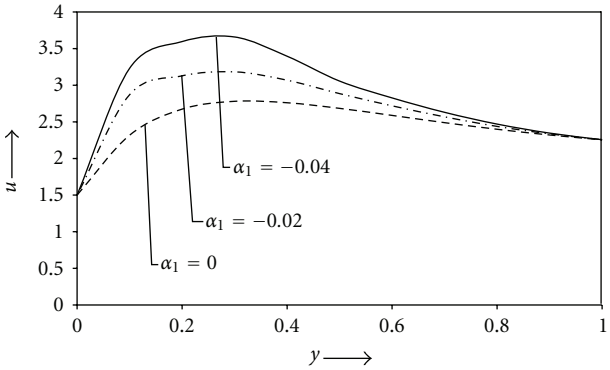

Figure 5: Variation of u against y for case V.

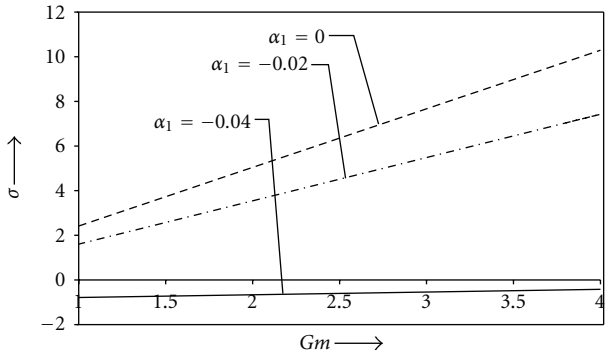

Figure 6: Variation of σ against Gm for $s = 0.1$, Pr = 3.

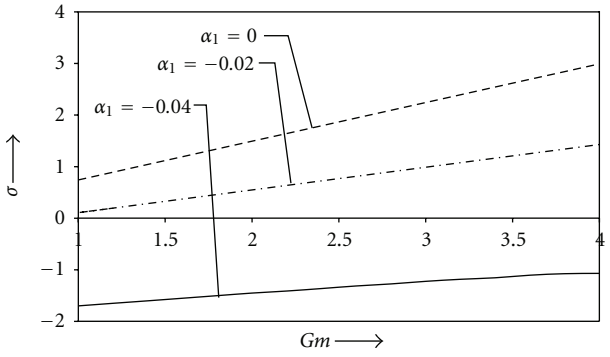

Figure 7: Variation of σ against Gm for $s = 0.4$, Pr = 3.

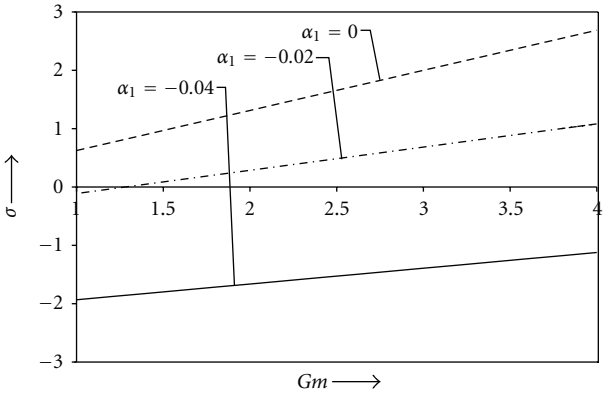

Figure 8: Variation of σ against Gm for $s = 0.4$, Pr = 5.

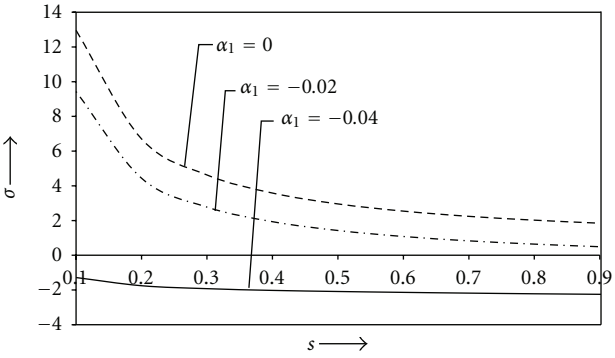

Figure 9: Variation of σ against s for Gm = 2, Pr = 3.

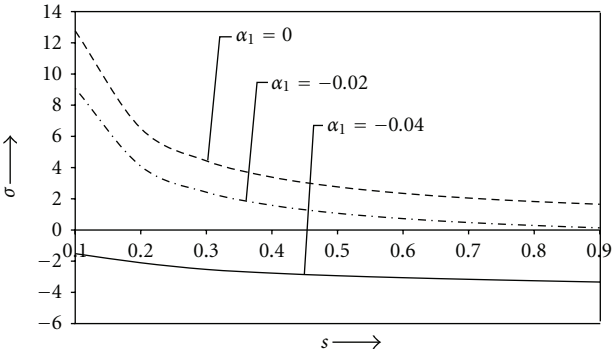

Figure 10: Variation of σ against s for Gm = 5, Pr = 3.

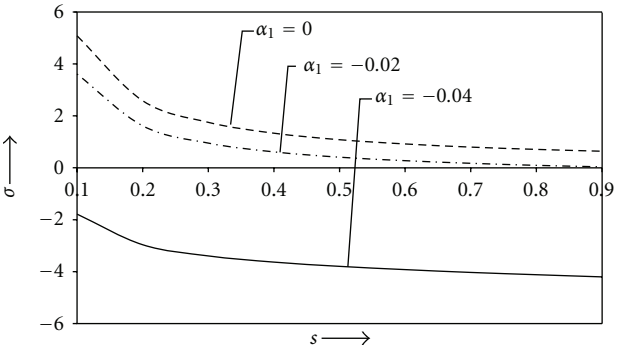

Figure 11: Variation of σ against s for Gm = 5, Pr = 5.

on velocity profile for different Gm, Pr, Re, and S are studied in detail. It was found that the velocity increases with the increasing values of the visco-elastic parameter in comparison to the Newtonian fluid. Also, in all the cases studied, it was found that the shear stress decreases with the increasing values of the viscoelastic parameter in comparison with the Newtonian fluid. Further, it was observed that the shear stress increases with the increasing values of Grashof number Gm for both Newtonian and non-Newtonian cases. But shear stress decreases with the increasing values of the Schmidt number for both Newtonian and non-Newtonian cases.

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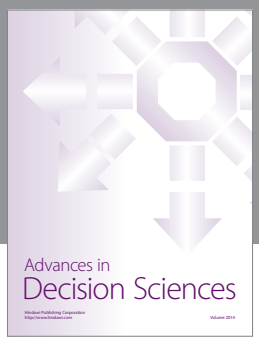

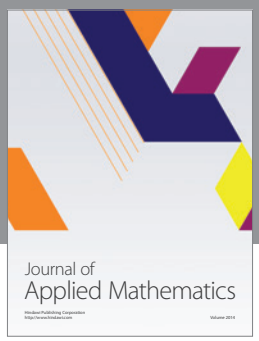

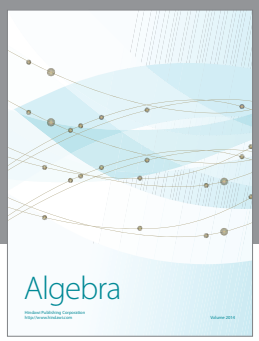

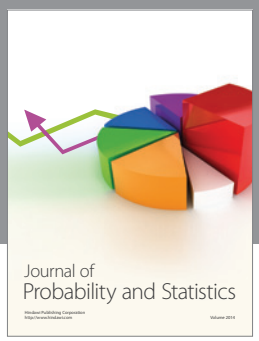

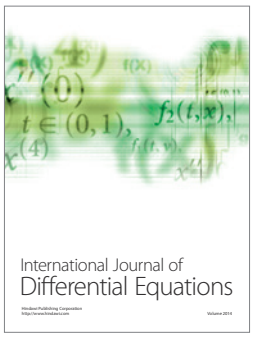

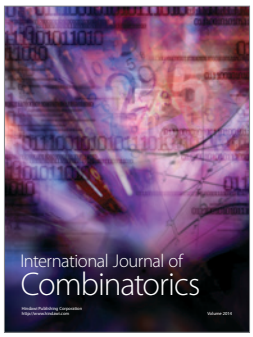

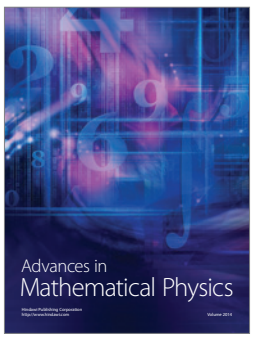

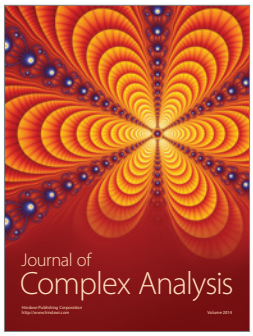

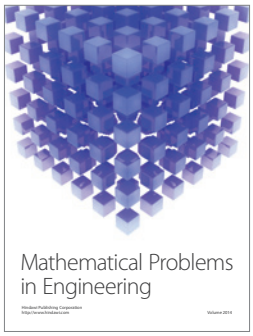

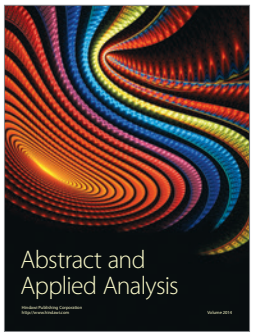

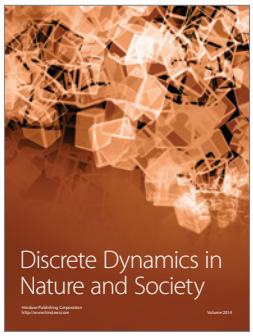

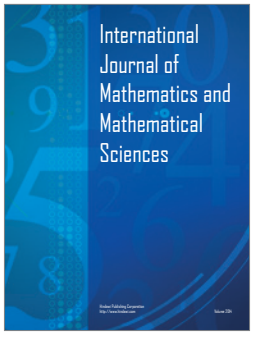

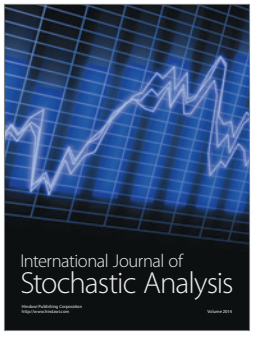

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