Zhang et al. *Advances in Difference Equations* (2015) 2015:371
DOI 10.1186/s13662-015-0705-2

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Positive periodic solution for Nicholson-type delay systems with impulsive effects

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Abstract
In this paper, a class of Nicholson-type delay systems with impulsive effects is considered. First, an equivalence relation between the solution (or positive periodic solution) of a Nicholson-type delay system with impulsive effects and that of the corresponding Nicholson-type delay system without impulsive effects is established. Then, by applying the cone fixed point theorem, some criteria are established for the existence and uniqueness of positive periodic solutions of the given systems. Finally, an example and its simulation are provided to illustrate the main results. Our results extend and improve greatly some earlier works reported in the literature.

Keywords: Nicholson-type systems; positive periodic solutions; delay; impulsive effect; cone fixed point theorem

1 Introduction

To describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained in [1], Gurney *et al.* [2] proposed the following Nicholson blowflies model:

$$N'(t) = -\delta N(t) + PN(t-\tau)e^{-aN(t-\tau)}, \tag{1.1}$$

where $N(t)$ is the size of the population at time t, P is the maximum *per capita* daily egg production, $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate, δ is the *per capita* daily adult death rate, and τ is the generation time. Nicholson's blowflies model and many generalized Nicholson's blowflies models have attracted more attention because of their extensively realistic significance; see [3–9]. Recently, in order to describe the models of marine protected areas and B-cell chronic lymphocytic leukemia dynamics, which are examples of Nicholson-type delay differential systems, Berezansky *et al.* [10], Wang *et al.* [11], and Liu [12] studied the following Nicholson-type delay systems:

$$\begin{cases} N_1'(t) = -\alpha_1(t)N_1(t) + \beta_1(t)N_2(t) + \sum_{j=1}^{m} c_{1j}(t)N_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)N_1(t-\tau_{1j}(t))}, \\ N_2'(t) = -\alpha_2(t)N_2(t) + \beta_2(t)N_1(t) + \sum_{j=1}^{m} c_{2j}(t)N_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)N_2(t-\tau_{2j}(t))}, \end{cases} \tag{1.2}$$

where $\alpha_i(t), \beta_i(t), c_{ij}(t), \gamma_{ij}(t), \tau_{ij}(t) \in C(R, (0,\infty))$, $i = 1, 2$, $j = 1, 2, \ldots, m$. For constant coefficients and delays, Berezansky *et al.* [10] presented several results for the permanence and globally asymptotic stability of system (1.2). Supposing that $\alpha_i(t)$, $\beta_i(t)$, $c_{ij}(t)$, $\gamma_{ij}(t)$, and $\tau_{ij}(t)$ are almost periodic functions, Wang *et al.* [11] obtained some criteria to ensure that the solutions of system (1.2) converge locally exponentially to a positive almost periodic solution. Furthermore, Liu [12] established some criteria for the existence and uniqueness of a positive periodic solution of system (1.2) by applying the method of the Lyapunov function.

However, species living in certain medium might undergo abrupt change of state at certain moments, and this occurs due to some seasonal effects such as weather change, food supply, and mating habits. That is to say, besides delays, impulsive effects likewise exist widely in many evolution processes. In the last two decades, the theory of impulsive differential equations has been extensively investigated due to its widespread applications [13–16].

Therefore, it is more realistic to investigate Nicholson-type delay systems with impulsive effects. However, to the best of our knowledge, few authors [17] have considered the conditions for existence and uniqueness of positive periodic solution for system (1.2) with impulsive effects. Thus, techniques and methods on the existence and uniqueness of a positive periodic solution for system (1.2) with impulsive effects should be developed and explored.

In this paper, we consider the following class of Nicholson-type delay systems with impulsive effects:

$$\begin{cases} y_1'(t) = -\alpha_1(t)y_1(t) + \beta_1(t)y_2(t) + \sum_{j=1}^m c_{1j}(t)y_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)y_1(t-\tau_{1j}(t))}, \\ y_2'(t) = -\alpha_2(t)y_2(t) + \beta_2(t)y_1(t) + \sum_{j=1}^m c_{2j}(t)y_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)y_2(t-\tau_{2j}(t))}, \\ \quad t \geq t_0 > 0, t \neq t_k, \\ y_i(t_k^+) = (1+b_k)y_i(t_k), \quad t \geq t_0, t = t_k i = 1,2, k = 1,2,\ldots, \end{cases} \tag{1.3}$$

where $\alpha_i(t), \beta_i(t), c_{ij}(t), \gamma_{ij}(t), \tau_{ij}(t) \in C([0,\infty),(0,\infty))$, $i = 1,2$, $j = 1,2,\ldots,m$. $\triangle y_i(t_k) = y_i(t_k^+) - y_i(t_k^-)$ are the impulses at moments t_k.

In Equation (1.3), we shall use the following hypotheses:

(H_1) $0 < t_0 < t_1 < t_2 < \cdots$, t_i, $i = 1,2,\ldots$ are fixed impulsive points with $\lim_{k\to\infty} t_k = \infty$;
(H_2) $\{b_k\}$ is a real sequence, and $b_k > -1$, $k = 1,2,\ldots$;
(H_3) $\alpha_i(t)$, $\beta_i(t)$, $c_{ij}(t)$, $\gamma_{ij}(t)$, $\tau_{ij}(t)$, and $\prod_{0<t_k<t}(1+b_k)$ are periodic functions with common period $\omega > 0$, $i = 1,2$, $j = 1,2,\ldots,m$, $k = 1,2,\ldots$.

Here and in the sequel, we assume that a product equals unit if the number of factors is equal to zero.

Let $\tau = \max\{\tau_{ij}^+\}$, $\tau_{ij}^+ = \max_{0\leq t\leq\omega} \tau_{ij}(t)$, $i = 1,2$, $j = 1,2,\ldots,m$. If $y_i(t)$ is defined on $[t_0-\tau,\sigma]$ with $t_0,\sigma \in R$, then we define $y_t \in C([-\tau,0],R)$ as $y_t = (y_t^1, y_t^2)$ where $y_t^i(\theta) = y_i(t+\theta)$ for $\theta \in [-\tau,0]$ and $i = 1,2$.

Due to the biological interpretation of system (1.3), only positive solutions are meaningful and admissible. Thus, we shall only consider the admissible initial conditions:

$$y_{i_{t_0}}(s) = \varphi_i(s), \quad s \in [-\tau,0], \tag{1.4}$$

where $\varphi_i(s) \in C([-\tau,0],(0,\infty))$. We write $y(t) = y_t(t_0,\varphi)$ for a solution of the initial value problems (1.3) and (1.4).

The remaining parts of this paper is organized as follows. In Section 2, we introduce some notation, definitions, and lemmas. In Section 3, we first establish the equivalence between the solution (or positive periodic solution) of a Nicholson-type delay system with impulses and that of the corresponding Nicholson-type delay system without impulses. Then, we give some criteria ensuring the existence and uniqueness of positive periodic solutions of Nicholson-type delay systems with and without impulses. In Section 4, an example and its simulation are provided to illustrate our results obtained in the previous sections. Finally, some conclusions are drawn in Section 5.

2 Preliminaries

For convenience, in the following discussion, we always use the notation

$$g^- = \min_{0\le t\le\omega} g(t), \qquad g^+ = \max_{0\le t\le\omega} g(t),$$

where g is a continuous ω-periodic function defined on R.

Definition 2.1 A function $y(t) = (y_1(t), y_2(t))^T$ defined on $[t_0 - \tau, \infty)$ is said to be a solution of Equation (1.3) with initial condition (1.4) if

(i) $y(t)$ is absolutely continuous on the intervals $(t_0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \ldots$;

(ii) for all t_k, $k = 1, 2, \ldots$, $y(t_k^+)$ and $y(t_k^-)$ exist, and $y(t_k^-) = y(t_k)$;

(iii) $y(t)$ satisfies the differential equation of (1.3) in $[t_0, \infty)\backslash\{t_k\}$ and the impulsive conditions for all $t = t_k$, $k = 1, 2, \ldots$;

(iv) $y_{i_{t_0}}(s) = \varphi_i(s)$, $s \in [-\tau, 0]$.

Under hypotheses (H_1)-(H_3), we consider the following Nicholson-type delay systems without impulsive effects:

$$\begin{cases} x_1'(t) = -\alpha_1(t)x_1(t) + \beta_1(t)x_2(t) + \sum_{j=1}^m p_{1j}(t)x_1(t-\tau_{1j}(t))e^{-q_{1j}(t)x_1(t-\tau_{1j}(t))}, \\ x_2'(t) = -\alpha_2(t)x_2(t) + \beta_2(t)x_1(t) + \sum_{j=1}^m p_{2j}(t)x_2(t-\tau_{2j}(t))e^{-q_{2j}(t)x_2(t-\tau_{2j}(t))}, \\ \quad t \ge t_0 > 0, \end{cases} \tag{2.1}$$

with initial conditions

$$x_{i_{t_0}}(s) = \varphi_i(s) \quad \text{for } s \in [-\tau, 0], \varphi \in C\bigl([-\tau, 0], (0, \infty)\bigr), \tag{2.2}$$

where

$$p_{ij}(t) = \prod_{t-\tau_{ij}(t)\le t_k<t} (1+b_k)^{-1} c_{ij}(t) \quad \text{and} \quad q_{ij}(t) = \prod_{0<t_k<t-\tau_{ij}(t)} (1+b_k)\gamma_{ij}(t),$$

$i = 1, 2$, $j = 1, 2, \ldots, m$.

By a solution $x(t)$ of Equation (2.1) with initial condition (2.2) we mean an absolutely continuous function $x(t) = (x_1(t), x_2(t))^T$ defined on $[t_0, \infty)$ satisfying Equation (2.1) for $t \ge t_0$ and initial condition (2.2) on $[-\tau, 0]$.

Similarly to the method of [18], we have the following:

Lemma 2.1 *Assume that* (H_1)-(H_3) *hold. Then*

(i) *if* $x(t) = (x_1(t), x_2(t))^T$ *is a solution* (*or positive* ω*-periodic solution*) *of Equation* (2.1) *with initial condition* (2.2), *then* $y(t) = (\prod_{0<t_k<t}(1+b_k)x_1(t), \prod_{0<t_k<t}(1+b_k)x_2(t))^T$ *is a solution* (*or positive* ω*-periodic solution*) *of Equation* (1.3) *with initial condition* (1.4) *on* $[-\tau,\infty)$;

(ii) *if* $y(t) = (y_1(t), y_2(t))^T$ *is a solution* (*or positive* ω*-periodic solution*) *of Equation* (1.3) *with initial condition* (1.4), *then* $x(t) = (\prod_{0<t_k<t}(1+b_k)^{-1}y_1(t), \prod_{0<t_k<t}(1+b_k)^{-1}y_2(t))^T$ *is a solution* (*or positive* ω*-periodic solution*) *of Equation* (2.1) *with initial condition* (2.2) *on* $[-\tau,\infty)$.

Proof (i) If $x(t) = (x_1(t), x_2(t))^T$ is a solution (or positive ω-periodic solution) of Equation (2.1) on $[t_0,\infty)$, then it is easy to see that $y(t)$ is absolutely continuous on all intervals $(t_0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \ldots$, and for any $t \neq t_k$,

$$\begin{aligned}
&y_1'(t) + \alpha_1(t)y_1(t) - \beta_1(t)y_2(t) - \sum_{j=1}^{m} c_{1j}(t)y_1\bigl(t-\tau_{1j}(t)\bigr)e^{-\gamma_{1j}(t)y_1(t-\tau_{1j}(t))} \\
&\quad= \prod_{0<t_k<t}(1+b_k)x_1'(t) + \alpha_1(t)\prod_{0<t_k<t}(1+b_k)x_1(t) - \beta_1(t)\prod_{0<t_k<t}(1+b_k)x_2(t) \\
&\qquad - \sum_{j=1}^{m} c_{1j}(t)\prod_{0<t_k<t-\tau_{1j}(t)}(1+b_k)x_1\bigl(t-\tau_{1j}(t)\bigr)e^{-\gamma_{1j}(t)\prod_{0<t_k<t-\tau_{1j}(t)}(1+b_k)x_1(t-\tau_{1j}(t))} \\
&\quad= \prod_{0<t_k<t}(1+b_k)\Biggl[x_1'(t) + \alpha_1(t)x_1(t) - \beta_1(t)x_2(t) \\
&\qquad - \sum_{j=1}^{m} c_{1j}(t)\prod_{t-\tau_{1j}(t)\le t_k<t}(1+b_k)^{-1}x_1\bigl(t-\tau_{1j}(t)\bigr)e^{-\gamma_{1j}(t)\prod_{0<t_k<t-\tau_{1j}(t)}(1+b_k)x_1(t-\tau_{1j}(t))}\Biggr] \\
&\quad= \prod_{0<t_k<t}(1+b_k)\Biggl[x_1'(t) + \alpha_1(t)x_1(t) - \beta_1(t)x_2(t) \\
&\qquad - \sum_{j=1}^{m} p_{1j}(t)x_1\bigl(t-\tau_{1j}(t)\bigr)e^{-q_{1j}(t)x_1(t-\tau_{1j}(t))}\Biggr] \\
&\quad= 0.
\end{aligned} \tag{2.3}$$

Similarly, we have

$$y_2'(t) + \alpha_2(t)y_2(t) - \beta_2(t)y_1(t) - \sum_{j=1}^{m} c_{2j}(t)y_2\bigl(t-\tau_{2j}(t)\bigr)e^{-\gamma_{2j}(t)y_2(t-\tau_{2j}(t))} = 0. \tag{2.4}$$

On the other hand, for every $t = t_k$, $k = 1, 2, \ldots$, and t_k situated in $[0,\infty)$,

$$y_i\bigl(t_k^+\bigr) = \lim_{t\to t_k^+}\prod_{0<t_j<t}(1+b_j)x_i(t) = \prod_{0<t_j\le t_k}(1+b_j)x_i(t_k), \quad i = 1, 2,$$

and

$$y_i(t_k) = \prod_{0<t_j<t_k}(1+b_j)x_i(t_k), \quad i = 1, 2.$$

Thus, for every $t = t_k$, $k = 1, 2, \ldots$,

$$y_i(t_k^+) = (1 + b_k)y_i(t_k), \quad i = 1, 2. \tag{2.5}$$

Therefore, we arrive at the conclusion that $y(t)$ is the solution (or positive ω-periodic solution) of Equation (1.3) with initial condition (1.4). In fact, if $x(t)$ is the solution (or positive ω-periodic solution) of Equation (2.1) with initial condition (2.2), then $y_i(t) = \prod_{0<t_k<t}(1 + b_k)x_i(t) = x_i(t) = \varphi_i(t)$ on $[-\tau, 0]$, $i = 1, 2$.

(ii) Since $y(t) = (y_1(t), y_2(t))^T$ is a solution (or positive ω-periodic solution) of Equation (1.3) with initial condition (1.4), it follows that $y(t)$ is absolutely continuous on all intervals $(t_0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \ldots$. Therefore, $x_i(t) = \prod_{0<t_k<t}(1 + b_k)^{-1}y_i(t)$ is absolutely continuous on all intervals $(t_0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \ldots$. Moreover, it follows that, for any $t = t_k$, $k = 1, 2, \ldots$,

$$\begin{aligned} x_i(t_k^+) &= \lim_{t\to t_k^+} \prod_{0<t_j<t} (1 + b_j)^{-1}y_i(t) \\ &= \prod_{0<t_j\le t_k} (1 + b_j)^{-1}y_i(t_k^+) = \prod_{0<t_j<t_k} (1 + b_j)^{-1}y_i(t_k) = x_i(t_k) \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} x_i(t_k^-) &= \lim_{t\to t_k^-} \prod_{0<t_j<t} (1 + b_j)^{-1}y_i(t) \\ &= \prod_{0<t_j<t_k} (1 + b_j)^{-1}y_i(t_k^-) = \prod_{0<t_j<t_k} (1 + b_j)^{-1}y_i(t_k) = x_i(t_k), \quad i = 1, 2, \end{aligned} \tag{2.7}$$

which implies that $x(t)$ is continuous and easy to prove absolutely continuous on $[0, \infty)$. Now, similarly to the proof in case (i), we can easily check that $x(t) = \prod_{0<t_k<t}(1 + b_k)^{-1}y(t)$ is a solution (or positive ω-periodic solution) of Equation (2.1) with initial condition (2.2) on $[-\tau, \infty]$.

From the above analysis we know that the conclusion of Lemma 2.1 is true. This completes the proof. □

Lemma 2.2 *Suppose that*

(H$_4$) $\frac{\beta_1^+\beta_2^+}{\alpha_1^-\alpha_2^-} < 1$.

Then every solution $x(t)$ *of Equation* (2.1) *with* (2.2) *and every solution* $y(t)$ *of Equation* (1.3) *with* (1.4) *are positive and bounded on* $[t_0, \infty)$.

Proof Clearly, by Lemma 2.1, we only need to prove that every solution $x(t)$ of Equation (2.1) with (2.2) is positive and bounded on $[t_0, \infty)$. In order to show that, we only need to see Lemma 2.3 in [11].

Furthermore, from the proof of Lemma 2.3 in [11] we also obtain the following conclusions: Under the condition (H$_4$), for every solution $x(t) = (x_1(t), x_2(t))^T$ of Equation (2.1) with (2.2), when $t > t_0$,

$$\max_{t_0\le s\le t} x_1(s) \le \left(1-\frac{\beta_1^+\beta_2^+}{\alpha_1^-\alpha_2^-}\right)^{-1} \times \left[\varphi_1(0)+\sum_{j=1}^m \frac{p_{1j}^+}{\alpha_1^- q_{1j}^- e}+\frac{\beta_1^+}{\alpha_1^-}\left(\varphi_2(0)+\sum_{j=1}^m \frac{p_{2j}^+}{\alpha_2^- q_{2j}^- e}\right)\right] \triangleq b_1 \tag{2.8}$$

and

$$\max_{t_0\le s\le t} x_2(s) \le \left(1-\frac{\beta_1^+\beta_2^+}{\alpha_1^-\alpha_2^-}\right)^{-1} \times \left[\varphi_2(0)+\sum_{j=1}^m \frac{p_{2j}^+}{\alpha_2^- q_{2j}^- e}+\frac{\beta_2^+}{\alpha_2^-}\left(\varphi_1(0)+\sum_{j=1}^m \frac{p_{1j}^+}{\alpha_1^- q_{1j}^- e}\right)\right] \triangleq b_2. \tag{2.9}$$

□

Lemma 2.3 (Cone fixed point theorem [19]) *Suppose that Ω_1, Ω_2 are open bounded subsets in Banach space X, and $\theta \in \Omega_1$, $\overline{\Omega_1} \subset \Omega_2$. Let P be a cone in X, and $T : P\cap(\overline{\Omega_2}\setminus\Omega_1) \to P$ be a completely continuous operator. If*

(i) $\|Tx\| \le \|x\|$ *for* $x \in P\cap\partial\Omega_1$ *and* $\|Tx\| \ge \|x\|$ *for* $x \in P\cap\partial\Omega_2$, *or*

(ii) $\|Tx\| \le \|x\|$ *for* $x \in P\cap\partial\Omega_2$ *and* $\|Tx\| \ge \|x\|$ *for* $x \in P\cap\partial\Omega_1$,

then the operator T has at least one fixed point in $P\cap(\overline{\Omega_2}\setminus\Omega_1)$.

3 Existence and uniqueness of positive periodic solution

For ease of exposition, throughout this paper, we adopt the following notation:

$$|x_i|_\infty = \max_{0\le t\le\omega}\left|x_i(t)\right|, \qquad x(t) = \left(x_1(t), x_2(t)\right)^T, \quad i = 1, 2.$$

We denote by X the set of all continuously ω-periodic functions $x(t)$ defined on R, *i.e.*, $X = \{x(t)|x(t) = (x_1(t), x_2(t))^T \in C(R, R^2), x(t+\omega) = x(t)\}$, and denote

$$\|x\| = \max\left\{|x_1|_\infty, |x_2|_\infty\right\}.$$

Then, X endowed with the norm $\|x\|$ is a Banach space. Let P be the cone of X defined by $P = \{x(t) \in X|x(t) \ge 0, t \in [t_0, t_0+\omega]\}$.

Define the operator T by

$$(Tx)(t) = \begin{pmatrix} \int_t^{t+\omega} G_1(t,s)[\beta_1(s)x_2(s) + \sum_{j=1}^m p_{1j}(s)x_1(s-\tau_{1j}(s))e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}]\,ds \\ \int_t^{t+\omega} G_2(t,s)[\beta_2(s)x_1(s) + \sum_{j=1}^m p_{2j}(s)x_2(s-\tau_{2j}(s))e^{-q_{2j}(s)x_2(s-\tau_{2j}(s))}]\,ds \end{pmatrix}, \tag{3.1}$$

where

$$G_1(t,s) = \frac{e^{\int_t^s \alpha_1(u)\,du}}{e^{\int_0^\omega \alpha_1(u)\,du}-1}, \qquad G_2(t,s) = \frac{e^{\int_t^s \alpha_2(u)\,du}}{e^{\int_0^\omega \alpha_2(u)\,du}-1}, \quad s \in [t, t+\omega].$$

It is easy to check that Equation (2.1) has positive ω-periodic solution if and only if the operator T has a fixed point in $P^0 = \{x(t) \in X|x(t) > 0, t \in [t_0, t_0+\omega]\}$. In addition, we have $0 < N_i \triangleq \frac{1}{e^{\int_0^\omega \alpha_i(u)\,du}-1} = G_i(t,t) \le G_i(t,s) \le G_i(t,t+\omega) = \frac{e^{\int_0^\omega \alpha_i(u)\,du}}{e^{\int_0^\omega \alpha_i(u)\,du}-1} \triangleq M_i$, $i = 1, 2$.

Lemma 3.1 *Assume that* (H_1)-(H_4) *hold. Then $T : P \to P$ is completely continuous.*

Proof First, we prove $T : P \to P$. From (H_3) we know that $\alpha_i(t)$, $i = 1, 2$, are continuous ω-periodic functions. Further, we find

$$G_i(t+\omega, s+\omega) = G_i(t,s), \quad s \in [t, t+\omega]. \tag{3.2}$$

In view of (H_3), (3.1), (3.2), and the definition of P, for any $x \in P$ and $t \in R$, we have

$$\begin{aligned}
(Tx)_1(t+\omega) &= \int_{t+\omega}^{t+2\omega} G_1(t+\omega, s)\left[\beta_1(s)x_2(s) + \sum_{j=1}^{m} p_{1j}(s)x_1\big(s-\tau_{1j}(s)\big)e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}\right] ds \\
&= \int_{t}^{t+\omega} G_1(t+\omega, u+\omega)\left[\beta_1(u+\omega)x_2(u+\omega)\right. \\
&\quad \left. + \sum_{j=1}^{m} p_{1j}(u+\omega)x_1\big(u+\omega-\tau_{1j}(u+\omega)\big)e^{-q_{1j}(u+\omega)x_1(u+\omega-\tau_{1j}(u+\omega))}\right] du \\
&= \int_{t}^{t+\omega} G_1(t,u)\left[\beta_1(u)x_2(u) + \sum_{j=1}^{m} p_{1j}(u)x_1\big(u-\tau_{1j}(u)\big)e^{-q_{1j}(u)x_1(u-\tau_{1j}(u))}\right] du \\
&= (Tx)_1(t).
\end{aligned}$$

Similarly, we have

$$(Tx)_2(t+\omega) = (Tx)_2(t).$$

In addition, it is clear that $Tx \in C(R, R^2)$ and $(Tx)(t) \geq 0$ for any $x \in P$, $t \in R$. Hence, $Tx \in P$ for any $x \in P$. Thus, $T : P \to P$.

Second, we show that $T : P \to P$ is completely continuous. Obviously, $T : P \to P$ is continuous. Since $\sup_{u \geq 0} ue^{-u} = \frac{1}{e}$, by (2.8) and (2.9), for any $x \in P$ and $t \in [t_0, t_0+\omega]$, we have

$$\begin{aligned}
(Tx)_1(t) &= \int_{t}^{t+\omega} G_1(t,s)\left[\beta_1(s)x_2(s) + \sum_{j=1}^{m} p_{1j}(s)x_1\big(s-\tau_{1j}(s)\big)e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}\right] ds \\
&\leq M_1 \int_{0}^{\omega} \left[\beta_1(s)x_2(s) + \sum_{j=1}^{m} p_{1j}(s)x_1\big(s-\tau_{1j}(s)\big)e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}\right] ds \\
&\leq M_1\omega\left[\beta_1^+ b_2 + \sum_{j=1}^{m} \frac{p_{1j}^+}{q_{1j}^- e}\right] \triangleq B_1
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
(Tx)_2(t) &= \int_{t}^{t+\omega} G_2(t,s)\left[\beta_2(s)x_1(s) + \sum_{j=1}^{m} p_{2j}(s)x_2\big(s-\tau_{2j}(s)\big)e^{-q_{2j}(s)x_2(s-\tau_{2j}(s))}\right] ds \\
&\leq M_2 \int_{0}^{\omega} \left[\beta_2(s)x_1(s) + \sum_{j=1}^{m} p_{2j}(s)x_2\big(s-\tau_{2j}(s)\big)e^{-q_{2j}(s)x_2(s-\tau_{2j}(s))}\right] ds \\
&\leq M_2\omega\left[\beta_2^+ b_1 + \sum_{j=1}^{m} \frac{p_{2j}^+}{q_{2j}^- e}\right] \triangleq B_2.
\end{aligned} \tag{3.4}$$

Moreover,

$$\begin{aligned} \left|(Tx)_1'(t)\right| &= \Bigg| G_1(t,t+\omega)\Bigg[\beta_1(t+\omega)x_2(t+\omega)+\sum_{j=1}^{m}p_{1j}(t+\omega)x_1\big(t+\omega-\tau_{1j}(t+\omega)\big) \\ &\quad\times e^{-q_{1j}(t+\omega)x_1(t+\omega-\tau_{1j}(t+\omega))}\Bigg] \\ &\quad - G_1(t,t)\Bigg[\beta_1(t)x_2(t)+\sum_{j=1}^{m}p_{1j}(t)x_1\big(t-\tau_{1j}(t)\big)e^{-q_{1j}(t)x_1(t-\tau_{1j}(t))}\Bigg] \\ &\quad - \alpha_1(t)\int_t^{t+\omega}G_1(t,s)\Bigg[\beta_1(s)x_2(s)+\sum_{j=1}^{m}p_{1j}(s)x_1\big(s-\tau_{1j}(s)\big)e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}\Bigg]\,ds\Bigg| \\ &= \Bigg|-\alpha_1(t)(Ax)_1(t)+\Bigg[\beta_1(t)x_2(t)+\sum_{j=1}^{m}p_{1j}(t)x_1\big(t-\tau_{1j}(t)\big)e^{-q_{1j}(t)x_1(t-\tau_{1j}(t))}\Bigg]\Bigg| \\ &\le \alpha_1^+B_1+\beta_1^+b_2+\sum_{j=1}^{m}\frac{p_{1j}^+}{q_{1j}^-e}. \end{aligned} \tag{3.5}$$

Similarly, we have

$$\left|(Tx)_2'(t)\right| \le \alpha_2^+B_2+\beta_2^+b_1+\sum_{j=1}^{m}\frac{p_{2j}^+}{q_{2j}^-e}. \tag{3.6}$$

In view of (3.3)-(3.6), $\{Tx : x \in P\}$ is a family of uniformly bounded and equicontinuous functions on $[t_0, t_0+\omega]$. By the Ascoli-Arzela theorem, $T : P \to P$ is compact. Therefore, $T : P \to P$ is completely continuous. The proof of Lemma 3.1 is complete. □

Theorem 3.1 *Assume that* (H$_1$)-(H$_4$) *hold. Then Equation* (1.3) *with* (1.4) *has at least one positive ω-periodic solution.*

Proof By (3.3) and (3.4), for any $x \in P$ and $t > t_0$, we have

$$(Tx)_1(t) \le B_1 \quad \text{and} \quad (Tx)_2(t) \le B_2.$$

Therefore,

$$\|Tx\| \le \max\{B_1, B_2\} \triangleq B > 0. \tag{3.7}$$

For any $x \in P$ and $t > t_0$, we have

$$\begin{aligned} (Tx)_1(t) &= \int_t^{t+\omega}G_1(t,s)\Bigg[\beta_1(s)x_2(s)+\sum_{j=1}^{m}p_{1j}(s)x_1\big(s-\tau_{1j}(s)\big)e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}\Bigg]\,ds \\ &\ge N_1\int_0^{\omega}\Bigg[\beta_1(s)x_2(s)+\sum_{j=1}^{m}p_{1j}(s)x_1\big(s-\tau_{1j}(s)\big)e^{-q_{1j}(s)x_1(s-\tau_{1j}(s))}\Bigg]\,ds. \end{aligned} \tag{3.8}$$

Let $\tau^- = \min_{j=1,2,\ldots,m}\{\tau_{1j}^-, \tau_{2j}^-\}$. There are two possible cases to consider.

Case 1. $\tau^- \geq \omega$. In view of (3.8), we have

$$\begin{aligned}(Tx)_1(t) &\geq N_1 \int_0^{\omega} \left[\sum_{j=1}^{m} p_{1j}(s) x_1\big(s - \tau_{1j}(s)\big) e^{-q_{1j}(s) x_1(s-\tau_{1j}(s))} \right] ds \\ &\geq N_1 \omega \sum_{j=1}^{m} p_{1j}^- \varphi_1^- e^{-q_{1j}^+ \varphi_1^+} \triangleq A_{11} > 0,\end{aligned}$$

where $\varphi_1^- = \min_{-\tau \leq s \leq 0} \varphi_1(t)$, $\varphi_1^+ = \max_{-\tau \leq s \leq 0} \varphi_1(t)$.

Case 2. $\tau^- < \omega$. In view of (3.8), we have

$$\begin{aligned}(Tx)_1(t) &\geq N_1 \int_0^{\tau^-} \left[\sum_{j=1}^{m} p_{1j}(s) x_1\big(s - \tau_{1j}(s)\big) e^{-q_{1j}(s) x_1(s-\tau_{1j}(s))} \right] ds \\ &\geq N_1 \tau^- \sum_{j=1}^{m} p_{1j}^- \varphi_1^- e^{-q_{1j}^+ \varphi_1^+} \triangleq A_{12} > 0.\end{aligned}$$

Therefore,

$$(Tx)_1(t) \geq \min\{A_{11}, A_{12}\} \triangleq A_1 > 0.$$

Similarly, we have

$$(Tx)_2(t) \geq \min\{A_{21}, A_{22}\} \triangleq A_2 > 0,$$

where $A_{21} = N_2 \omega \sum_{j=1}^{m} p_{2j}^- \varphi_2^- e^{-q_{2j}^+ \varphi_2^+}$, $A_{22} = N_2 \tau^- \sum_{j=1}^{m} p_{2j}^- \varphi_2^- e^{-q_{2j}^+ \varphi_2^+}$, $\varphi_2^- = \min_{-\tau \leq s \leq 0} \varphi_2(t)$, $\varphi_2^+ = \max_{-\tau \leq s \leq 0} \varphi_2(t)$.

Then, for any $x \in P$ and $t > t_0$,

$$\|Tx\| \geq \min\{A_1, A_2\} \triangleq A > 0. \tag{3.9}$$

Let

$$\Omega_1 = \big\{ x \in X : \|x\| < A \big\}$$

and

$$\Omega_2 = \big\{ x \in X : \|x\| < B \big\}.$$

Clearly, Ω_1 and Ω_2 are open bounded subsets in X, and $\theta \in X$, $\overline{\Omega_1} \subset \Omega_2$. By Lemma 3.1, $T : P \cap (\overline{\Omega_2} \setminus \Omega_1) \to P$ is completely continuous.

If $x \in P \cap \partial\Omega_2$, which implies that $\|x\| = B$, then from (3.7) we have $\|Tx\| \leq B$, and hence $\|Tx\| \leq \|x\|$ for $x \in P \cap \partial\Omega_2$.

If $x \in P \cap \partial\Omega_1$, which implies that $\|x\| = A$, then from (3.9) we have $\|Tx\| \geq A$, and hence $\|Tx\| \geq \|x\|$ for $x \in P \cap \partial\Omega_1$.

By Lemma 2.3 the operator T has at least one fixed point in $P \cap (\overline{\Omega_2} \setminus \Omega_1)$, *i.e.*, Equation (2.1) with (2.2) has at least one ω-periodic solution. Since $\theta \overline{\in} P \cap (\overline{\Omega_2} \setminus \Omega_1)$, Equation (2.1) with (2.2) has at least one positive ω-periodic solution. Therefore, Equation (1.3) with (1.4) has at least one positive ω-periodic solution by Lemma 2.1. This completes the proof of Theorem 3.1. □

Theorem 3.2 *Let* (H_1)-(H_4) *hold. Suppose further that the following condition holds*:

(H_5) $\alpha_i^- - \beta_i^+ - \sum_{j=1}^m p_{ij}^+ > 0$, $i = 1,2$.

Then Equation (1.3) *with* (1.4) *has a unique positive* ω*-periodic solution*.

Proof By Theorem 3.1 we know that Equation (2.1) with (2.2) has at least one positive ω-periodic solution. Thus, in order to prove Theorem 3.2, we only need to prove the uniqueness of a positive ω-periodic solution for Equation (2.1) with (2.2).

The following proof is similar to that of Theorem 3.2 in [11].

Assume that $x(t)$ and $\widetilde{x}(t)$ are two positive ω-periodic solutions of Equation (2.1). Set $z_i(t) = x_i(t) - \widetilde{x}_i(t)$, where $t \in [t_0 - \tau, \infty)$, $i = 1,2$. Then

$$\begin{cases} z_1'(t) = -\alpha_1(t)z_1(t) + \beta_1(t)z_2(t) + \sum_{j=1}^m p_{1j}(t)[x_1(t-\tau_{1j}(t))e^{-q_{1j}(t)x_1(t-\tau_{1j}(t))} \\ \qquad - \widetilde{x}_1(t-\tau_{1j}(t))e^{-q_{1j}(t)\widetilde{x}_1(t-\tau_{1j}(t))}], \\ z_2'(t) = -\alpha_2(t)z_2(t) + \beta_2(t)z_1(t) + \sum_{j=1}^m p_{2j}(t)[x_2(t-\tau_{2j}(t))e^{-q_{2j}(t)x_2(t-\tau_{2j}(t))} \\ \qquad - \widetilde{x}_2(t-\tau_{2j}(t))e^{-q_{2j}(t)\widetilde{x}_2(t-\tau_{2j}(t))}], \quad t \geq t_0 > 0. \end{cases} \tag{3.10}$$

Set

$$\Gamma_i(u) = -(\alpha_i^- - u) + \beta_i^+ + \sum_{j=1}^m p_{ij}^+ e^{u\tau_i^+}, \quad u \in [0,1], \tau_i^+ = \max_{1\leq j\leq m} \tau_{ij}^+, i = 1,2.$$

Clearly, $\Gamma_i(u)$, $i = 1,2$, are continuous functions on $[0,1]$. From (H_5) we have

$$\Gamma_i(0) = -\alpha_i^- + \beta_i^+ + \sum_{j=1}^m p_{ij}^+ < 0, \quad i = 1,2.$$

Hence, we can choose two constants $\eta > 0$ and $0 < \lambda \leq 1$ such that

$$\Gamma_i(\lambda) = (\lambda - \alpha_i^-) + \beta_i^+ + \sum_{j=1}^m p_{ij}^+ e^{\lambda\tau_i^+} < -\eta < 0, \quad i = 1,2. \tag{3.11}$$

Consider the Lyapunov functions

$$V_1(t) = |z_1(t)|e^{\lambda t}, \qquad V_2(t) = |z_2(t)|e^{\lambda t}.$$

Calculating the upper right derivative of $V_i(t)$ ($i = 1,2$) along the solution $z(t)$ of (3.10), we obtain

$$D^+(V_1(t)) \leq \Bigg[(\lambda - \alpha_1(t))|z_1(t)| + \beta_1(t)|z_2(t)| + \sum_{j=1}^m p_{1j}(t)\Big|x_1(t-\tau_{1j}(t))e^{-q_{1j}(t)x_1(t-\tau_{1j}(t))} \\ - \widetilde{x}_1(t-\tau_{1j}(t))e^{-q_{1j}(t)\widetilde{x}_1(t-\tau_{1j}(t))}\Big| \Bigg] e^{\lambda t} \quad \text{for all } t \geq t_0, \tag{3.12}$$

and

$$D^+\big(V_2(t)\big) \le \Bigg[\big(\lambda - \alpha_2(t)\big)\big|z_2(t)\big| + \beta_2(t)\big|z_1(t)\big| + \sum_{j=1}^{m} p_{2j}(t)\big|x_2\big(t-\tau_{2j}(t)\big)e^{-q_{2j}(t)x_2(t-\tau_{2j}(t))} - \widetilde{x}_2\big(t-\tau_{2j}(t)\big)e^{-q_{2j}(t)\widetilde{x}_2(t-\tau_{2j}(t))}\big|\Bigg]e^{\lambda t} \quad \text{for all } t \ge t_0. \tag{3.13}$$

We claim that there is $M > 0$ such that

$$V_i(t) = \big|z_i(t)\big|e^{\lambda t} \le M \quad \text{for all } t > t_0, i = 1, 2. \tag{3.14}$$

Otherwise, one of the following cases must occur.

Case 1. There exists $T_1 > t_0$ such that

$$V_1(T_1) = M \quad \text{and} \quad V_i(t) < M \quad \text{for all } t \in [t_0 - \tau, T_1], i = 1, 2. \tag{3.15}$$

Case 2. There exists $T_2 > t_0$ such that

$$V_2(T_2) = M \quad \text{and} \quad V_i(t) < M \quad \text{for all } t \in [t_0 - \tau, T_2], i = 1, 2. \tag{3.16}$$

We will need the inequality

$$\big|xe^{-x} - ye^{-y}\big| \le |x - y| \quad \text{for } x, y \in [0, +\infty). \tag{3.17}$$

Indeed, by the mean value theorem we have

$$\big|xe^{-x} - ye^{-y}\big| = \left|\frac{1-\xi}{e^{\xi}}\right| \cdot |x - y|, \quad \text{where } \xi \text{ is between } x \text{ and } y.$$

For $\xi > 1$, we have $|\frac{1-\xi}{e^{\xi}}| = \frac{\xi-1}{e^{\xi}} \le \frac{1}{e^2} < 1$, and for $0 \le \xi \le 1$, we have $|\frac{1-\xi}{e^{\xi}}| = \frac{1-\xi}{e^{\xi}} \le 1$. Therefore, inequality (3.17) holds.

In case 1, in view of (3.12) and inequality (3.17), (3.15) implies that

$$\begin{aligned}
0 \le D^+\big(V_1(T_1) - M\big) &\le \Bigg[\big(\lambda - \alpha_1(T_1)\big)\big|z_1(T_1)\big| + \beta_1(T_1)\big|z_2(T_1)\big| \\
&\quad + \sum_{j=1}^{m} p_{1j}(T_1)\big|x_1\big(T_1 - \tau_{1j}(T_1)\big)e^{-q_{1j}(T_1)x_1(T_1-\tau_{1j}(T_1))} \\
&\quad - \widetilde{x}_1\big(T_1 - \tau_{1j}(T_1)\big)e^{-q_{1j}(T_1)\widetilde{x}_1(T_1-\tau_{1j}(T_1))}\big|\Bigg]e^{\lambda T_1} \\
&= \Bigg[\big(\lambda - \alpha_1(T_1)\big)\big|z_1(T_1)\big| + \beta_1(T_1)\big|z_2(T_1)\big| + \sum_{j=1}^{m} \frac{p_{1j}(T_1)}{q_{1j}(T_1)} \\
&\quad \times \big|q_{1j}(T_1)x_1\big(T_1 - \tau_{1j}(T_1)\big)e^{-q_{1j}(T_1)x_1(T_1-\tau_{1j}(T_1))} \\
&\quad - q_{1j}(T_1)\widetilde{x}_1\big(T_1 - \tau_{1j}(T_1)\big)e^{-q_{1j}(T_1)\widetilde{x}_1(T_1-\tau_{1j}(T_1))}\big|\Bigg]e^{\lambda T_1}
\end{aligned}$$

$$
\begin{aligned}
&\le \left(\lambda - \alpha_1(T_1)\right)\left|z_1(T_1)\right|e^{\lambda T_1} + \beta_1(T_1)\left|z_2(T_1)\right|e^{\lambda T_1} \\
&\quad + \sum_{j=1}^{m} p_{1j}(T_1)\left|z_1\left(T_1 - \tau_{1j}(T_1)\right)\right|e^{\lambda(T_1-\tau_{1j}(T_1))}e^{\lambda\tau_{1j}(T_1)} \\
&\le \left[\left(\lambda - \alpha_1^-\right) + \beta_1^+ + \sum_{j=1}^{m} p_{1j}^+ e^{\lambda\tau_1^+}\right]M.
\end{aligned}
$$

Thus,

$$
\left(\lambda - \alpha_1^-\right) + \beta_1^+ + \sum_{j=1}^{m} p_{1j}^+ e^{\lambda\tau_1^+} \ge 0,
$$

which contradicts (3.11). Hence, (3.14) holds.

In case 2, in view of (3.13) and (3.17), (3.16) yields that

$$
\begin{aligned}
0 &\le D^+\left(V_2(T_2) - M\right) \le \Bigg[\left(\lambda - \alpha_2(T_2)\right)\left|z_2(T_2)\right| + \beta_2(T_2)\left|z_1(T_2)\right| \\
&\quad + \sum_{j=1}^{m} p_{2j}(T_2)\Big|x_2\left(T_2 - \tau_{2j}(T_2)\right)e^{-q_{2j}(T_2)x_2(T_2-\tau_{2j}(T_2))} \\
&\quad - \widetilde{x}_2\left(T_2 - \tau_{2j}(T_2)\right)e^{-q_{2j}(T_2)\widetilde{x}_2(T_2-\tau_{2j}(T_2))}\Big|\Bigg]e^{\lambda T_2} \\
&= \Bigg[\left(\lambda - \alpha_2(T_2)\right)\left|z_2(T_2)\right| + \beta_2(T_2)\left|z_1(T_2)\right| + \sum_{j=1}^{m} \frac{p_{2j}(T_2)}{q_{2j}(T_2)} \\
&\quad \times \Big|q_{2j}(T_2)x_2\left(T_2 - \tau_{2j}(T_2)\right)e^{-q_{2j}(T_2)x_2(T_2-\tau_{2j}(T_2))} \\
&\quad - q_{2j}(T_2)\widetilde{x}_2\left(T_2 - \tau_{2j}(T_2)\right)e^{-q_{2j}(T_2)\widetilde{x}_2(T_2-\tau_{2j}(T_2))}\Big|\Bigg]e^{\lambda T_2} \\
&\le \left(\lambda - \alpha_2(T_2)\right)\left|z_2(T_2)\right|e^{\lambda T_2} + \beta_2(T_2)\left|z_1(T_2)\right|e^{\lambda T_2} \\
&\quad + \sum_{j=1}^{m} p_{2j}(T_2)\left|z_2\left(T_2 - \tau_{2j}(T_2)\right)\right|e^{\lambda(T_2-\tau_{2j}(T_2))}e^{\lambda\tau_{2j}(T_2)} \\
&\le \left[\left(\lambda - \alpha_2^-\right) + \beta_2^+ + \sum_{j=1}^{m} p_{2j}^+ e^{\lambda\tau_2^+}\right]M.
\end{aligned}
$$

Thus,

$$
\left(\lambda - \alpha_2^-\right) + \beta_2^+ + \sum_{j=1}^{m} p_{2j}^+ e^{\lambda\tau_2^+} \ge 0,
$$

which contradicts (3.11). Hence, (3.14) holds. It follows that

$$
\left|z_i(t)\right| < Me^{-\lambda t} \quad \text{for all } t > t_0, i = 1, 2. \tag{3.18}
$$

In view of (3.18) and the periodicity of $z(t)$, we have

$$z_i(t) = x_i(t) - \widetilde{x}_i(t) = 0 \quad \text{for all } t \in [t_0 - \tau, \infty), i = 1, 2.$$

This completes the proof. □

Remark 3.1 In Theorems 3.1 and 3.2, the conditions that ensure the existence and uniqueness of a positive ω-periodic solution for Nicholson-type delay systems with and without impulses are simple and easily to test, which is less conservative than the conditions required in some previous works [11, 12]. Moreover, the main results in this paper are totally different from that of [17].

4 An example

Example 4.1 Consider the following impulsive Nicholson-type system with delays

$$\begin{cases} y_1'(t) = -(9 + \sin^2 \pi t)y_1(t) + (5 + \cos^2 \pi t)y_2(t) + (\frac{3}{16} + \frac{1}{2}|\sin \pi t|)y_1(t - e^{|\cos \pi t|}) \\ \qquad \times e^{-(\frac{7}{3} + |\sin \pi t|)y_1(t - e^{|\cos \pi t|})} \\ \qquad + (\frac{5}{8} - \frac{1}{2}|\sin \pi t|)y_1(t - e^{|\sin \pi t|})e^{-(\frac{5}{2} + |\cos \pi t|)y_1(t - e^{|\sin \pi t|})}, \\ y_2'(t) = -(9 + \cos^2 \pi t)y_1(t) + (5 + \sin^2 \pi t)y_2(t) + (\frac{3}{16} + \frac{1}{2}|\cos \pi t|)y_1(t - e^{|\sin \pi t|}) \\ \qquad \times e^{-(\frac{7}{3} + |\cos \pi t|)y_1(t - e^{|\sin \pi t|})} \\ \qquad + (\frac{5}{8} - \frac{1}{2}|\cos \pi t|)y_1(t - e^{|\cos \pi t|})e^{-(\frac{5}{2} + |\sin \pi t|)y_1(t - e^{|\cos \pi t|})}, \quad t \geq 0, \\ y_i(t_k^+) = (1 + b_k)y_i(t_k), \quad i = 1, 2, k = 1, 2, \ldots, \end{cases} \tag{4.1}$$

with initial condition

$$y_i(s) = \ln(3 + t)) = \varphi_i(t), \quad t \in [-e, 0], i = 1, 2, \tag{4.2}$$

where $b_k = 2^{\sin \frac{\pi}{2} k} - 1$, and $t_k = k$, $k = 1, 2, \ldots$.

Let $f(t) = \prod_{0<t_k<t}(1 + b_k) = \prod_{0<t_k<t} 2^{\sin \frac{\pi}{2} k}$. Then

$$\begin{aligned} f(t + 4) &= \prod_{0<t_k<t+4} 2^{\sin \frac{\pi}{2} k} = \prod_{0<t_k \leq 4} 2^{\sin \frac{\pi}{2} k} \cdot \prod_{4<t_k<t+4} 2^{\sin \frac{\pi}{2} k} \\ &= 2^{\sum_{k=1}^{4} \sin \frac{\pi}{2} k} \cdot \prod_{0<t_k<t} 2^{\sin \frac{\pi}{2}(k-4)} = 2^0 \cdot \prod_{0<t_k<t} 2^{\sin \frac{\pi}{2} k} = f(t), \end{aligned}$$

which implies that $f(t)$ is a periodic function with period 4.

Since $\alpha_1(t) = 9 + \sin^2 \pi t$, $\alpha_2(t) = 9 + \cos^2 \pi t$, $\beta_1(t) = 5 + \cos^2 \pi t$, $\beta_2(t) = 5 + \sin^2 \pi t$, we have $\alpha_1^- = \alpha_2^- = 9$, $\beta_1^+ = \beta_2^+ = 6$, and thus $\frac{\beta_1^+ \beta_2^+}{\alpha_1^- \alpha_2^-} = \frac{4}{9} < 1$.

It is obvious that

$$p_{11}(t) = \prod_{t - e^{|\cos \pi t|} \leq t_k < t} 2^{\sin \frac{\pi}{2} k}\left(\frac{3}{16} + \frac{1}{2}|\sin \pi t|\right),$$

$$p_{12}(t) = \prod_{t - e^{|\sin \pi t|} \leq t_k < t} 2^{\sin \frac{\pi}{2} k}\left(\frac{5}{8} - \frac{1}{2}|\sin \pi t|\right),$$

$$p_{21}(t) = \prod_{t - e^{|\sin \pi t|} \leq t_k < t} 2^{\sin \frac{\pi}{2} k}\left(\frac{3}{16} + \frac{1}{2}|\cos \pi t|\right),$$

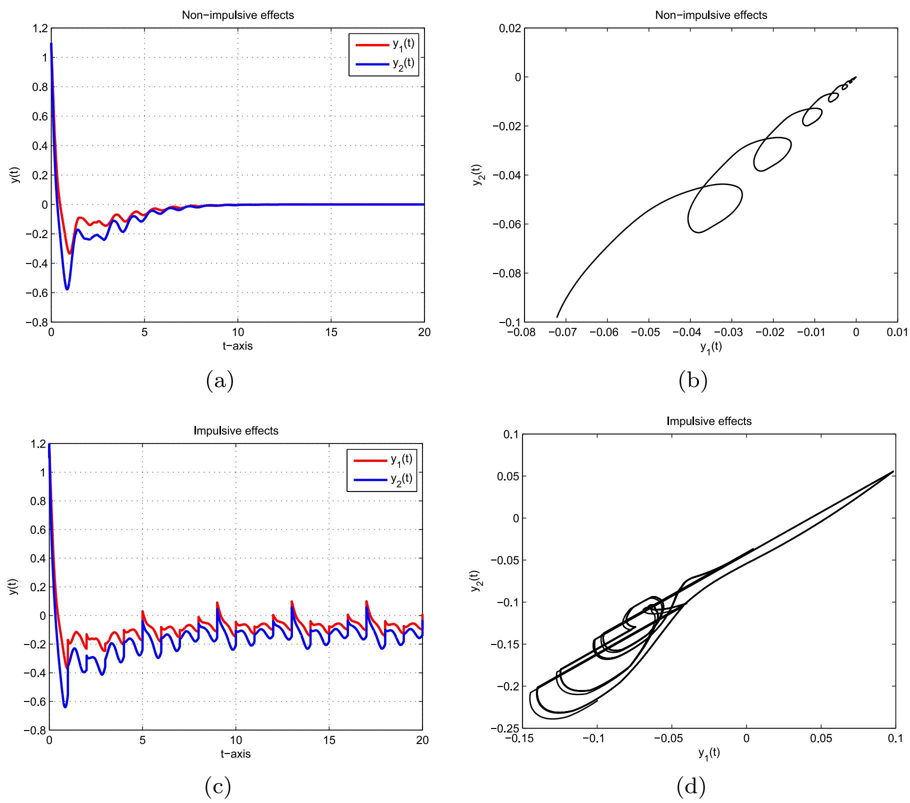

Figure 1 The dynamic behavior of the system (4.1) with the initial condition (4.2). (a) Time-series of the y_1, y_2 of system (4.1) without impulsive effects for $t \in [0, 20]$. **(b)** Phase portrait of solutions of system (4.1) without impulsive effects for $t \in [3, 20]$. **(c)** Times-series of the y_1, y_2 of impulsive system (4.1) for $t \in [0, 20]$. **(d)** Phase portrait of solutions of impulsive system (4.1) for $t \in [5, 20]$.

$$p_{22}(t) = \prod_{t-e^{|\cos\pi t|} \le t_k < t} 2^{\sin\frac{\pi}{2}k}\left(\frac{5}{8} - \frac{1}{2}|\cos\pi t|\right),$$

$$q_{11}(t) = \prod_{0<t_k<t-e^{|\cos\pi t|}} 2^{\sin\frac{\pi}{2}k}\left(\frac{7}{3} + |\sin\pi t|\right),$$

$$q_{12}(t) = \prod_{0<t_k<t-e^{|\sin\pi t|}} 2^{\sin\frac{\pi}{2}k}\left(\frac{5}{2} + |\cos\pi t|\right),$$

$$q_{21}(t) = \prod_{0<t_k<t-e^{|\sin\pi t|}} 2^{\sin\frac{\pi}{2}k}\left(\frac{7}{3} + |\cos\pi t|\right),$$

$$q_{22}(t) = \prod_{0<t_k<t-e^{|\cos\pi t|}} 2^{\sin\frac{\pi}{2}k}\left(\frac{5}{2} + |\sin\pi t|\right).$$

Therefore,

$$\alpha_i^- - \beta_i^+ - \sum_{j=1}^{2} p_{ij}^+ = \frac{3}{4} > 0, \quad i = 1, 2.$$

It follows from Theorem 3.2 that Equation (4.1) with initial condition (4.2) has a unique 4-periodic solution. This fact is verified by the numerical simulation in Figure 1.

Remark 4.1 System (4.1) is a simple form of impulsive Nicholson-type system with delays. Since $q_{11}^{-} = q_{21}^{-} = \frac{7}{6} > 1$, $q_{12}^{-} = q_{22}^{-} = \frac{5}{4} > 1$, it is clear that the condition of Theorem 3.1 in [11] and Theorem 2.1 in [12] are not satisfied. Therefore, all the results obtained in [11, 12] and the references therein cannot be applicable to system (4.1). This implies that the results of this paper are essentially new.

5 Conclusion

In this paper, a class of Nicholson-type delay systems with impulsive effects are investigated. First, an equivalence relation between the solution (or positive periodic solution) of a Nicholson-type delay system with impulses and that of the corresponding Nicholson-type delay system without impulses is established. Then, by applying the cone fixed point theorem, some criteria are established for the existence and uniqueness of a positive periodic solution of the given system. The fixed point theorem in cones is very popular in investigation of positive periodic solutions to impulsive functional differential equations [20, 21]. Our results imply that under the appropriate linear periodic impulsive perturbations, the Nicholson-type delay systems with impulses preserve the original periodic property of the Nicholson-type delay systems without impulses. Finally, an example and its simulation are provided to illustrate the main results. It is worth noting that there are only very few results [17] on Nicholson-type delay systems with impulses, and our results extend and improve greatly some earlier works reported in the literature. Furthermore, our results are important in applications of periodic oscillatory Nicholson-type delay systems with impulsive control.

Competing interests

The authors have declared that no competing interests exist.

Authors' contributions

RZ came up with the main idea of the theorems and gave an example. Moreover, RZ and YH completed the proofs of the results, and TW designed a MATLAB program to simulate the results of example. RZ and YH wrote the manuscript. All authors read and approved the final manuscript.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Grant No. 11171374) and the Scientific Research Fund of Shandong Provincial of P.R. China (Grant No. ZR2011AZ001).

Received: 4 June 2015 Accepted: 19 November 2015 Published online: 04 December 2015

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