Li et al. *Advances in Difference Equations* 2012, **2012**:197
http://www.advancesindifferenceequations.com/content/2012/1/197

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Improved criterion for the elimination of overflow oscillations in digital filters with external disturbance

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Abstract

In this paper, an improved criterion for the H_∞ elimination of overflow oscillations in digital filters with external disturbance is presented. Compared with some existing results in Ahn (AEU, Int. J. Electron. Commun. 65:750-752, 2011) and Kokil *et al.* (AEU, Int. J. Electron. Commun. doi:10.1016/j.aeue.2012.01.004, 2012), a distinct feature of the proposed criterion is that it can include the existing results as special cases or be less restrictive than them. Finally, an example is given to show this improvement over the existing conditions.

Keywords: exponential stability; digital filters; overflow arithmetic; LMI

1 Introduction

When designing digital filters by using a fixed-point arithmetic, one encounters overflow nonlinearities [1, 2]. Such nonlinearities may lead to the instability and possibly the zero-input limit cycles of the designed filters. But the global asymptotic stability of the null solution guarantees the absence of limit cycles in the designed filter [1, 3]. Therefore, the problem of the global asymptotic stability of fixed-point digital filters using saturation arithmetic has received considerable attention from many researchers [3–18]. However, most existing criteria are not available under the unfavorable environments with external disturbance. Recently, [15, 16] have considered this case, but the method given in [15, 16] seems still restrictive. So, there is room for further investigation on this topic.

Motivated by the preceding discussion, in this paper, an improved criterion for the H_∞ elimination of overflow oscillations in digital filters with external disturbance is proposed. It is shown that the presented criterion is less restrictive or more general than those in [15, 16].

Notation Throughout this paper, I is used to denote an identity matrix with appropriate dimension. For a real symmetric matrix P, the notation $P > 0$ (≥ 0) means that the matrix P is a positive definite (positive semi-definite), and $A > (\geq) B$ means $A - B > (\geq) 0$. The superscript τ denotes the transpose of a vector or matrix.

2 Problem formulation and preliminaries

Consider the following digital filter:

$$\begin{cases} x(r+1) = f(y(r)) + \omega(r) \\ \qquad = [f_1(y_1(r))f_2(y_2(r))\cdots f_n(y_n(r))]^\tau + [\omega_1(r)\omega_2(r)\cdots\omega_n(r)]^\tau, \\ y(r) = [y_1(r)y_2(r)\cdots y_n(r)]^\tau = Ax(r), \end{cases} \tag{1}$$

where $x(r) \in \mathbb{R}^n$ is the state vector, $\omega(r) \in \mathbb{R}^n$ is an external disturbance, $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the coefficient matrix, $y(r) \in \mathbb{R}^n$ is the system output vector, and $f_i(\cdot)$, $i = 1, \ldots, n$, are given by

$$\begin{cases} -1 \le f_i(y_i(r)) \le 1, & |y_i(r)| > 1, \\ f_i(y_i(r)) = y_i(r), & -1 \le y_i(r) \le 1. \end{cases} \tag{2}$$

The nonlinearities given by (2) include two's complement overflow arithmetic and saturation arithmetic as special cases.

The purpose of this paper is to develop an improved exponential stability with H_∞ performance γ under zero-initial conditions for all nonzero $\omega(r)$ for the system (1) satisfying conditions (2) and to compare it with the existing results in [15, 16]. To obtain our main results, we need the following lemma.

Lemma 1 *For the given system* (1)-(2), *if there are a diagonal matrix* $L = \mathrm{diag}\{l_1, l_2, \ldots, l_n\} \in R^{n\times n}$, *a matrix* $N = [n_{ij}] \in R^{n\times n}$ *which satisfy* $l_j \ge \sum_{i=1}^n |n_{ji}|$, $i, j = 1, \ldots, n$, *then the following inequality holds*:

$$\left[y^\tau(r)L + f^\tau\big(y(r)\big)N^\tau\right]\left[y(r) - f\big(y(r)\big)\right] \ge 0. \tag{3}$$

Proof The inequality (3) can be rewritten as

$$\sum_{j=1}^n \left[y_j(r)l_j + \sum_{i=1}^n f_i\big(y_i(r)\big)n_{ji}\right]\left[y_j(r) - f_j\big(y_j(r)\big)\right] \ge 0. \tag{4}$$

When $-1 \le y_i(r) \le 1$, it means $y_j(r) - f_j(y_j(r)) = 0$. It is clear that (4) holds.

When $|y_i(r)| > 1$, it can obtained that

$$\left[y_j(r)l_j + \sum_{i=1}^n f_i\big(y_i(r)\big)n_{ji}\right]\left[y_j(r) - f_j\big(y_j(r)\big)\right] = y_j^2(r)\left[l_j + \sum_{i=1}^n \frac{f_i(y_i(r))}{y_j(r)}n_{ji}\right]\left[1 - \frac{f_j(y_j(r))}{y_j(r)}\right]. \tag{5}$$

From (2), it is easy to see that $|f_i(y_i(r))| \le 1$ holds. Hence, $|\frac{f_i(y_i(r))}{y_j(r)}| \le 1$ and $|\frac{f_j(y_j(r))}{y_j(r)}| \le 1$. Because $l_j \ge \sum_{i=1}^n |n_{ji}|$, then the following inequality holds:

$$\left[y_j(r)l_j + \sum_{i=1}^n f_i\big(y_i(r)\big)n_{ji}\right]\left[y_j(r) - f_j\big(y_j(r)\big)\right] \ge 0. \tag{6}$$

According to (6), hence, (4) holds. This completes the proof of Lemma 1. □

3 Improved stability criterion

First of all, we present the main result, *i.e.*, a new criterion for the H_∞ elimination of overflow oscillations in digital filters with external disturbance.

Theorem 1 *For a given level* γ *and the system* (1) *satisfying condition* (2), *if there exist positive definite matrices* $P = P^\tau \in \mathbb{R}^{n\times n}$, $S = S^\tau \in \mathbb{R}^{n\times n}$, *a positive diagonal matrix* $L = \operatorname{diag}\{l_1, l_2, \ldots, l_n\} \in R^{n\times n}$, *and a matrix* $N = [n_{ij}] \in \mathbb{R}^{n\times n}$ *such that*

$$W = \begin{bmatrix} S - P + 2A^\tau LA & A^\tau N - A^\tau L & 0 \\ * & P - N - N^\tau & P \\ * & * & P - \gamma^2 I \end{bmatrix} < 0 \tag{7a}$$

and

$$l_j \ge \sum_{i=1}^{n} |n_{ji}|, \quad i, j = 1, \ldots, n \tag{7b}$$

hold, then the system (1) *is exponentially stable with* H_∞ *performance* γ.

Proof Consider the quadratic Lyapunov function

$$V\bigl(x(r)\bigr) = x^\tau(r)Px(r), \tag{8}$$

where $P = P^\tau = [p_{ij}]_{n\times n} > 0$. Along the trajectories of the system (1), one has

$$\begin{aligned} \triangle V\bigl(x(r)\bigr) &= x^\tau(r+1)Px(r+1) - x^\tau(r)Px(r) \\ &= \bigl[f\bigl(y(r)\bigr) + \omega(r)\bigr]^\tau P\bigl[f\bigl(y(r)\bigr) + \omega(r)\bigr] - x^\tau(r)Px(r) \\ &\quad + 2\bigl[y^\tau(r)L + f^\tau\bigl(y(r)\bigr)N^\tau\bigr]\bigl[y(r) - f\bigl(y(r)\bigr)\bigr] \\ &\quad - 2\bigl[y^\tau(r)L + f^\tau\bigl(y(r)\bigr)N^\tau\bigr]\bigl[y(r) - f\bigl(y(r)\bigr)\bigr] \\ &= \eta^\tau(r)W\eta(r) - x^\tau(r)Sx(r) + \gamma^2\omega^\tau(r)\omega(r) + \Phi(r), \end{aligned} \tag{9}$$

where $\eta(r) = [x^\tau(r)f^\tau(y(r))\omega^\tau(r)]^\tau$, $S > 0$ and $\Phi(r) = -2[y^\tau(r)L + f^\tau(y(r))N^\tau][y(r) - f(y(r))]$. Moreover, from Lemma 1, one can infer that $\Phi(r)$ is nonpositive. If LMI (7a), (7b) is satisfied, we have

$$\triangle V\bigl(x(r)\bigr) \le -x^\tau(r)Sx(r) + \gamma^2\omega^\tau(r)\omega(r). \tag{10}$$

By summation of both sides of (10) from 0 to ∞ and computing $V(x(\infty)) \ge 0$ and $V(x(0)) = 0$, we have $\sum_{r=0}^{\infty} x^\tau(r)Sx(r) < \gamma^2 \sum_{r=0}^{\infty} \omega^\tau(r)\omega(r)$.

Next, we show the system (1) with $\omega(r) = 0$ is exponentially stable. Note that

$$\lambda_{\min}(P)\bigl\|x(r)\bigr\|^2 \le V(x(r)) \le \lambda_{\max}(P)\bigl\|x(r)\bigr\|^2,$$

we have

$$\triangle V\bigl(x(r)\bigr) < -x^\tau(r)Sx(r) \le -\lambda_{\min}(S)\bigl\|x(r)\bigr\|^2. \tag{11}$$

According to (10) and Theorem 1 of [15], this guarantees the exponential stability. This completes the proof. □

4 Comparison with the existing results

In order to show the improvement of the proposed criterion over the existing results in [15, 16], we will first recall them.

Lemma 2 (Ahn 2011 [15]) *For a given level $\gamma > 0$, if we assume that there exist symmetric positive definite matrices P, S, a positive diagonal matrix M, and a positive scalar δ such that*

$$\begin{bmatrix} \delta A^\tau A + S - P & A^\tau M & 0 \\ * & P - \delta I - 2M & P \\ * & * & P - \gamma^2 I \end{bmatrix} < 0, \tag{12}$$

then the system (1) *with* (2) *is exponentially stable with H_∞ performance γ.*

Lemma 3 (Kokil 2012 [16]) *For a given level $\gamma > 0$, if we assume that there exist symmetric positive definite matrices P, S, and a positive scalar δ such that*

$$\begin{bmatrix} \delta A^\tau A + S - P & A^\tau C & 0 \\ * & P - \delta I - C - C^\tau & P \\ * & * & P - \gamma^2 I \end{bmatrix} < 0, \tag{13}$$

where C is a row diagonally dominant matrix with positive diagonal elements, then the system (1) *with* (2) *is exponentially stable with H_∞ performance γ.*

Remark 1 As noted in [16], when C in Lemma 3 is identified as the positive definite diagonal matrix M in Lemma 2, Lemma 2 can be recovered as a special case of Lemma 3.

Comparing with Lemma 3, Theorem 1 may further relax the condition. We will show the improvement by using Remark 2.

Remark 2 Letting $L - N = Z$ in Theorem 1, it is clear that Z is a row diagonally dominant matrix with positive diagonal elements. Then LMI (7a) can be rewritten as

$$\begin{bmatrix} S - P + 2A^\tau LA & -A^\tau Z & 0 \\ * & P - 2L + Z + Z^\tau & P \\ * & * & P - \gamma^2 I \end{bmatrix} < 0. \tag{14}$$

When $Z = -C$ and $2l_1 = 2l_2 = \cdots = 2l_n = \delta$ with C is defined in (13), it is worth pointing out that Lemma 3 can be recovered as a special case of Theorem 1.

5 Numerical examples

In this section, we use an example to illustrate the usefulness of our result.

Example 1 Consider the second-order system (1) with

$$A = \begin{bmatrix} 0.99 & 1 \\ 0 & -0.1 \end{bmatrix}, \qquad w(r) = \begin{bmatrix} \cos(r) \\ \sin(r) \end{bmatrix}.$$

The minimum lower bound of γ for this system as obtained via Theorem 1 is 12.9062. On the other hand, Lemma 2 and Lemma 3 give the minimum lower bound of γ as 72.9310 and 35.1760. It is clear that the minimum lower bound of γ in our paper is smaller than Lemma 1 and Lemma 2. In other words, Theorem 1 provides improved results over the previous results in [15, 16]. And a feasible solution is in the following:

$$P = \begin{bmatrix} 0.8671 & 0.6795 \\ 0.6795 & 35.8751 \end{bmatrix}, \qquad S = \begin{bmatrix} 0.0025 & -0.1094 \\ -0.1094 & 15.5430 \end{bmatrix},$$
$$N = \begin{bmatrix} 0.4386 & 0.0000 \\ 0.8342 & 32.4671 \end{bmatrix}, \qquad L = \operatorname{diag}\{0.4398, 57.0005\}.$$

6 Conclusion

In this paper, an LMI-based criterion for the H_∞ elimination of overflow oscillations in fixed-point state-space digital filters with external disturbance has been established. The obtained criterion has been shown to be less restrictive than some existing results or to cover them as special cases.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
TL carried out the main part of this manuscript. The others participated in the discussion and gave the examples. All authors read and approved the final manuscript.

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Acknowledgements
This work was supported in part by the Major State Basic Research Development Program of China (973 Program) under Grant No. 2012CB720003, in part by the National Science Foundation of China under Grants No. 61125306, 60904025, 60904026.

Received: 24 August 2012 Accepted: 5 November 2012 Published: 14 November 2012

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doi:10.1186/1687-1847-2012-197
Cite this article as: Li et al.: **Improved criterion for the elimination of overflow oscillations in digital filters with external disturbance.** *Advances in Difference Equations* 2012 **2012**:197.