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Outage performance of relay-assisted primary and secondary transmissions in cognitive relay networks

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Abstract

In this paper, a two-phase protocol for relay-assisted spectrum sharing in cognitive relay network is proposed. The primary system comprises a transmitter-receiver pair (PT-PR) and a decode-and-forward relay (Relay), while the secondary system comprises a transmitter-receiver pair (ST-SR). In the proposed protocol, the Relay assists the transmissions of the primary users as well as the secondary users where there is no direct link between the ST and the SR. Outage probabilities of the primary system and the secondary system are derived and verified through simulations. Compared with the protocol without cooperation, better outage performance of the primary system is achieved. Meanwhile, though the ST causes interference to the PR, the interference is compensated by the cooperation of the Relay. Moreover, the proposed protocol realizes the communications of the secondary users on the condition that the secondary transmission link is not ideal.

Keywords: Spectrum sharing; Cognitive relay network; Decode-and-forward relaying; Outage probability

1 Introduction

Traditional spectrum management policy, which is based on licensing of spectrum, allows the licensed users to access the frequency bands allocated to them. However, this policy cannot satisfy the increasing demand of users in wireless networks. The contradiction between the increasing demand and the limited spectrum resources compels us to improve the spectral efficiency.

For this purpose, some new technologies have been proposed. Cooperative diversity, as a new form of spatial diversity, was proposed to fight against channel fading and enhance throughput. The advantages of some cooperative diversity protocols have been studied in [1-3], and an adaptive cooperation diversity scheme has been studied in [4] which considered the mutual interference between the primary and secondary users. Cooperative relay, which uses intermediate entities to carry information

from a source node to its destination node, is emerging as a promising technology to improve the reliability, throughput, and coverage areas of wireless networks. The most common relaying protocols are amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying. The performance of DF relaying in spectrum sharing has been widely investigated in [5]. With the purpose of improving spectral efficiency, the concept of cognitive radio [6] was proposed by J. Mitola. In cognitive radio networks, secondary users transmit signals by detecting spectrum holes, or share the same frequency bands with primary users as long as the induced interference from secondary users to primary users is below a threshold [7]. The authors of [8] have studied a more general multi-user scenario which was extended from [7], and the authors of [9] have considered cooperative relaying for spectrum sensing in a two-user cognitive radio environment. Recently, cognitive relay network, which combines cognitive radio and cooperative relay, has attracted significant interests, and it is able to improve secondary users' transmission performance. The difference between cognitive relay networks and conventional relay networks has been studied in [10], and it showed that outage performance of cognitive relay networks is better than that of conventional relay networks.

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The authors of [11] have obtained a tight lower bound of the outage probability for underlay cognitive relay networks. A three-phase cognitive two-way relaying scenario has been analyzed in [12], and three different two-hop relaying protocols according to the amount of a prior knowledge have been investigated in [13]. Cognitive radio networks with opportunistic DF and uncoded DF-relaying schemes were considered and analyzed in [14]. When the achievements on the transmission performance of primary users increase, more and more attention has been paid to the transmission performance of secondary users. In [15], the authors have studied the capacity of the secondary users in cognitive relay networks, and proved that the reactive DF scheme is able to achieve the similar capacity of the proactive DF scheme. The performances of the secondary users in cognitive relay networks with the consideration of the primary users' interference have been analyzed in [16-18]. Especially, the authors of [17] have considered the outage performances of the primary and secondary systems on the condition that there exists a direct link between the secondary users. Power allocation has been applied to improve outage performance of the secondary system in [19].

Some other relevant researches have been done. Outage performances with relay selection in Rayleigh fading channels were studied in [20], and the full diversity order has been obtained for Rayleigh fading channels in [21,22]. Moreover, the authors of [23] have analyzed the impact of cooperation communications on the outage probability of cognitive radio networks over the Nakagami-m fading channel and generalized the contributions in [21,22]. While more and more achievements on outage performance with perfect channel state information have been obtained, a lot of attention has been put into the one with imperfect channel state information. The transmissions of the secondary users will cause more interference at the primary users with imperfect channel state information. The authors of [24-28] have studied the transmission performances of the primary and secondary systems under imperfect channel state information.

Most of the existing research works focused on the separate performance of the primary system or the secondary system, and little research work has considered the performances of both primary and secondary systems. In view of this situation, we propose a protocol which employs a relay to help the transmissions of the primary and secondary systems contemporarily. With the cooperation of the relay, we try to decrease outage probability of the primary system, and compensate for the interference produced by the secondary users. Meanwhile, the relay assists the transmissions of the secondary users on the condition that there is no direct link between the secondary transmitter and the secondary receiver.

The rest of this paper is organized as follows: Section 2 describes the system model. In Section 3, we express the transmission signals in every transmission phase, and calculate outage probabilities of the primary and secondary systems. Section 4 presents the simulation results and discussions, and Section 5 concludes the paper.

2 System model

In cognitive radio systems, there is unlicensed spectrum for the transmissions of secondary users, whose priorities are lower than primary users. Secondary users are allowed to access the licensed spectrum in two cases: one is that secondary users utilize spectrum holes to transmit signals, and the other is that secondary users share licensed spectrum with primary users on the condition of causing tolerable harmful interference to primary users. In this case, it is impossible that there exists a portion of licensed spectrum for secondary users all the time. With the purposes of improving outage performance of the primary system and realizing communications of the secondary users without a direct link, we propose a spectrum sharing protocol where a relay assists the communications of the primary and secondary users, as depicted in Figure 1.

The primary system comprises a transmitter (PT), a receiver (PR), and a decode-and-forward relay (Relay), while the secondary system comprises a transmitter (ST) and a receiver (SR). The Relay assists the transmissions of the primary users and secondary users simultaneously. In the proposed system model, outage performance of the primary system benefits from the assistance of the Relay, and communications of the secondary users are achieved under the assistance of the Relay when there is no direct link between the ST and the SR.
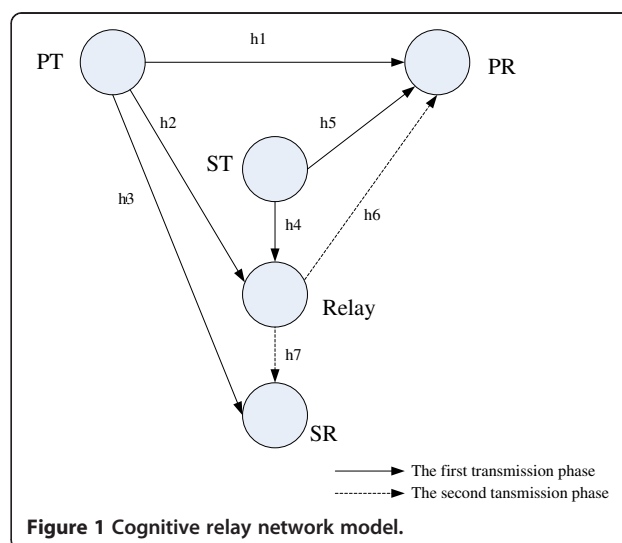

Figure 1 Cognitive relay network model.

The whole transmission process is divided into two phases. In the first transmission phase, the PT and the ST transmit the primary and secondary signals, respectively. The PR, the Relay, and the SR receive the primary signal, and the Relay and the PR receive the secondary signal. Then, the primary signal and the secondary signal are decoded and superimposed at the Relay. In the second transmission phase, the PR and the SR receive a weighted linear composite signal which is transmitted by the Relay. At the PR, the secondary signal which is regarded as an interfering signal will be removed, and the primary signal will be restored if the PR is able to decode the secondary signal successfully. Then, the primary signal is retrieved by maximal-ratio combining (MRC) of the received signals from the two transmission phases at the PR. Similarly, the primary signal is regarded as an interfering signal and removed at the SR if the SR is successful to decode the primary signal, and the secondary signal is retrieved from the signal which is sent from the Relay.

All the channels are assumed to experience Rayleigh fading. The channel coefficients of the links $PT \rightarrow PR$, $PT \rightarrow Relay$, $PT \rightarrow SR$, $ST \rightarrow Relay$, $ST \rightarrow PR$, $Relay \rightarrow PR$, and $Relay \rightarrow SR$ are recorded by h_1, h_2, h_3, h_4, h_5, h_6, and h_7, respectively. Moreover, we assume $h_i \sim N(0, d_i^{-v})$ ($i = 1, 2, 3, 4, 5, 6, 7$), which means that h_i is a circularly symmetric complex Gaussian random variable with variance d_i^{-v}. Here, d_i represents the normalized distance between two nodes, and v represents the path loss exponent. That is to say d_1, d_2, d_3, d_4, d_5, d_6, and d_7 denote the normalized distances between PT and PR, PT and Relay, PT and SR, ST and Relay, ST and PR, Relay and PR, and Relay and SR, respectively. This distance normalization is done with respect to the distance between PT and PR, i.e., $d_1 = 1$.

3 Signal description and outage performance analysis

3.1 Outage performance of the proposed protocol (scheme A)

In this subsection, outage performances of the primary system and the secondary system will be analyzed.

3.1.1 Outage probability of the primary system

In the first transmission phase, the PT transmits the primary signal x_p, and the PR, the Relay, and the SR receive the signal, respectively. The received signals of the PR, the Relay, and the SR are denoted by y_{11}, y_{21}, y_{31}, respectively, and we have

$$y_{a1} = \sqrt{P_p} h_a x_p + n_{a1}, \tag{1}$$

where $a = 1, 2, 3$. Here, P_p represents transmission power of the PT, h_a is a channel coefficient, and $n_{a1} \sim N$

$(0, \sigma^2)$ is an additive white Gaussian noise with zero mean and variance σ^2. According to [7], the achievable rate between the PT and the Relay is calculated by

$$R_2 = \frac{1}{2} \log_2 \left(1 + \frac{P_p |h_2|^2}{\sigma^2} \right) = \frac{1}{2} \log_2 \left(1 + \frac{P_p \gamma_2}{\sigma^2} \right). \tag{2}$$

Here, the value $\frac{1}{2}$ is due to the fact that the whole transmission process is divided into two transmission phases, and for ease of presentation, we record $\gamma_j = |h_j|^2$ ($j = 1, 2, 3, 4, 5, 6, 7$).

Likewise, the ST transmits the secondary signal x_s, and the signal is received by the Relay, as well as the PR. The received signals of the Relay and the PR are recorded as y_{41} and y_{51}, respectively, which can be expressed as

$$y_{c1} = \sqrt{P_s} h_c x_s + n_{c1}, \tag{3}$$

where $c = 4, 5$. Here, P_s represents transmission power of the ST, h_c is a channel coefficient, and $n_{c1} \sim N(0, \sigma^2)$ is an additive white Gaussian noise with zero mean and variance σ^2. The achievable rate between the ST and the Relay is recorded as R_4, and the one between the ST and the PR is recorded as R_5. They are calculated by

$$R_d = \frac{1}{2} \log_2 \left(1 + \frac{P_s \gamma_d}{\sigma^2} \right), \tag{4}$$

where $d = 4, 5$.

Then, the Relay decodes the signals sent from the PT and the ST, respectively. If the Relay is able to decode both x_p and x_s, a composite signal x_R is generated by linearly combining the regenerated signal x_p with power αP_r and the secondary signal x_s with power $(1 - \alpha)P_r$, where α ($0 \le \alpha \le 1$) is the power allocation factor at the Relay, and P_r is transmission power of the Relay. So, the composite signal is described by

$$x_R = \sqrt{\alpha P_r} x_p + \sqrt{(1-\alpha)P_r} x_s. \tag{5}$$

In the second transmission phase, the PR receives the signal which is sent from the Relay if the Relay decodes both x_p and x_s successfully. Otherwise, the Relay will keep silent. The received signal at the PR is recorded as y_{12}, which is given by

$$y_{12} = \sqrt{\alpha P_r} h_6 x_p + \sqrt{(1-\alpha)P_r} h_6 x_s + n_{12}, \tag{6}$$

where h_6 is the channel coefficient of the link between the Relay and PR, and $n_{12} \sim N(0, \sigma^2)$ is an additive white Gaussian noise.

Then, the interference signal $\sqrt{(1-\alpha)P_r} h_6 x_s$ will be removed from y_{12} on the condition that the PR is successful to decode x_s. So, we have $y_{12}^* = \sqrt{\alpha P_r} h_6 x_p + n_{12}$.

Thus, MRC will be applied to combine y_{11} and y_{12}^*. According to [7], the achievable rate is given by

$$R_{11}^{\mathrm{MRC}} = \frac{1}{2} \log_2 \left(1 + \frac{P_p \gamma_1}{\sigma^2} + \frac{\alpha P_r \gamma_6}{\sigma^2} \right). \quad (7)$$

On the other condition that the PR fails to decode x_s, the secondary signal will be regarded as noise at the PR. Then, MRC will also be applied to combine y_{11} and y_{12}. The achievable rate turns to be

$$R_{12}^{\mathrm{MRC}} = \frac{1}{2} \log_2 \left(1 + \frac{P_p \gamma_1}{\sigma^2} + \frac{\alpha P_r \gamma_6}{(1-\alpha) P_r \gamma_6 + \sigma^2} \right). \quad (8)$$

For the primary system, there are two cases where the primary signal is transmitted successfully. The former is that x_p is transmitted through the direct link from the PT to the PR on the condition that the Relay fails to decode either of x_p or x_s. In this case, the achievable rate between the PT and the PR is given by $R_1 = \log_2 \left(1 + \frac{P_p \gamma_1}{\sigma^2} \right)$. The latter is that x_p is transmitted from the relay-assisted link if the Relay decodes both x_p and x_s successfully. As mentioned, the former case includes three sub-cases, they are listed as follows: First, x_p is transmitted through the direct link on the condition that the Relay succeeds to decode x_p and fails to decode x_s. Second, x_p is transmitted through the direct link on the condition that the Relay fails to decode x_p and succeeds to decode x_s. Third, x_p is transmitted through the direct link on the condition that the Relay fails to decode both x_p and x_s.

The latter case includes two sub-cases, and they are listed as follows: One case is that the Relay succeeds to decode both x_p and x_s, but the PR fails to decode x_s. Then, the primary signal x_p is transmitted through the relay-assisted link. The other case is that the Relay succeeds to decode both x_p and x_s, and the PR decodes x_s successfully. Then, the primary signal x_p is transmitted through the relay-assisted link.

The transmissions will be interrupted when the achievable rate is low than the target rate. So, outage probability of the primary system with target rate R_{pt} and R_{st} for the primary and secondary systems respectively, is given by

$$P_{\mathrm{out}}^p = 1 - \begin{bmatrix} P_r\{R_2 > R_{\mathrm{pt}}\} P_r\{R_4 < R_{\mathrm{st}}\} P_r\left\{\frac{1}{2} R_1 > R_{\mathrm{pt}}\right\} \\ + P_r\{R_2 < R_{\mathrm{pt}}\} P_r\{R_4 > R_{\mathrm{st}}\} P_r\left\{\frac{1}{2} R_1 > R_{\mathrm{pt}}\right\} \\ + P_r\{R_2 < R_{\mathrm{pt}}\} P_r\{R_4 < R_{\mathrm{st}}\} P_r\left\{\frac{1}{2} R_1 > R_{\mathrm{pt}}\right\} \\ + P_r\{R_2 > R_{\mathrm{pt}}\} P_r\{R_4 > R_{\mathrm{st}}\} P_r\{R_5 > R_{\mathrm{st}}\} P_r\{R_{11}^{\mathrm{MRC}} > R_{\mathrm{pt}}\} \\ + P_r\{R_2 > R_{\mathrm{pt}}\} P_r\{R_4 > R_{\mathrm{st}}\} P_r\{R_5 < R_{\mathrm{st}}\} P_r\{R_{12}^{\mathrm{MRC}} > R_{\mathrm{pt}}\} \end{bmatrix}. \quad (9)$$

According to [7], since $\gamma_1 \sim \varepsilon(1)$ and $\gamma_e \sim \varepsilon(d_e^v)$ ($e = 2$, 4, 5, 6), which means γ_e is an exponentially distributed random variable with mean $\frac{1}{d_e^v}$, we have

$$P_r\left\{\frac{1}{2} R_1 > R_{\mathrm{pt}}\right\} = P_r\left\{\gamma_1 > \frac{\rho_1 \sigma^2}{P_p}\right\} = \exp\left(-\frac{\rho_1 \sigma^2}{P_p}\right), \quad (10)$$

$$P_r\{R_2 > R_{\mathrm{pt}}\} = P_r\left\{\gamma_2 > \frac{\rho_1 \sigma^2}{P_p}\right\} = \exp\left(-d_2^v \frac{\rho_1 \sigma^2}{P_p}\right), \quad (11)$$

$$P_r\{R_4 > R_{\mathrm{st}}\} = P_r\left\{\gamma_4 > \frac{\rho_3 \sigma^2}{P_s}\right\} = \exp\left(-d_4^v \frac{\rho_3 \sigma^2}{P_s}\right), \quad (12)$$

$$P_r\{R_5 > R_{\mathrm{st}}\} = P_r\left\{\gamma_5 > \frac{\rho_3 \sigma^2}{P_s}\right\} = \exp\left(-d_5^v \frac{\rho_3 \sigma^2}{P_s}\right), \quad (13)$$

$$\begin{aligned} P_r\{R_{11}^{\mathrm{MRC}} > R_{\mathrm{pt}}\} &= P_r\left\{\frac{1}{2} \log_2\left(1 + \frac{P_p \gamma_1}{\sigma^2} + \frac{\alpha P_r \gamma_6}{\sigma^2}\right) > R_{\mathrm{pt}}\right\} \\ &= \{P_p \gamma_1 + \alpha P_r \gamma_6 > \rho_1 \sigma^2\} \\ &= 1 - \{P_p \gamma_1 + \alpha P_r \gamma_6 \leq \rho_1 \sigma^2\} \\ &= \exp\left(-\frac{\rho_1 \sigma^2}{P_p}\right) + \left(-1 + d_6^v \frac{P_p}{\alpha P_r}\right)^{-1} \\ &\quad \times \left[\exp\left(-\frac{\rho_1 \sigma^2}{P_p}\right) - \exp\left(-d_6^v \frac{\rho_1 \sigma^2}{\alpha P_r}\right)\right], \end{aligned} \quad (14)$$

where $\rho_1 = 2^{2R_{\mathrm{pt}}} - 1$. Assuming $P_r \gg \sigma^2$, we have

$$\begin{aligned} P_r\{R_{12}^{\mathrm{MRC}} > R_{\mathrm{pt}}\} &\approx P_r\left\{\frac{1}{2} \log_2\left[1 + \frac{P_p \gamma_1}{\sigma^2} + \frac{\alpha}{1-\alpha}\right] > R_{\mathrm{pt}}\right\} \\ &= P_r\left\{\gamma_1 > \frac{\sigma^2}{P_p}\left(\rho_1 - \frac{\alpha}{1-\alpha}\right)\right\} \\ &= \begin{cases} \exp\left[-\frac{\sigma^2}{P_p}\left(\rho_1 - \frac{\alpha}{1-\alpha}\right)\right] & 0 \leq \alpha < \hat{\alpha} \\ 1 & \hat{\alpha} \leq \alpha \leq 1 \end{cases}, \end{aligned} \quad (15)$$

where $\hat{\alpha} = \frac{\rho_1}{1+\rho_1}$. Substituting (10) to (15) into (9), we have

$$P_{\mathrm{out}}^p = \begin{cases} P_{\mathrm{out}}^{p,1} & 0 \leq \alpha < \hat{\alpha} \\ P_{\mathrm{out}}^{p,2} & \hat{\alpha} \leq \alpha \leq 1 \end{cases}. \quad (16)$$

Here,

$$P_{\text{out}}^{p,1} = 1 - \left\{ \begin{array}{l} \exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\left(1-\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\right)\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right) \\[2ex] +\left(1-\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\right)\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right) \\[2ex] +\left(1-\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\right)\left(1-\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\right)\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right) \\[2ex] +\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\exp\left(-d_5^v \frac{\rho_3\sigma^2}{P_s}\right) \\[2ex] \left(\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right)+\left(-1+d_6^v \frac{P_p}{\alpha P_r}\right)^{-1}\exp\left(-d_6^v \frac{\rho_1\sigma^2}{\alpha P_r}\right)\left(\exp\left(\left(-1+d_6^v \frac{P_p}{\alpha P_r}\right)\frac{\rho_1\sigma^2}{P_p}\right)-1\right)\right) \\[2ex] +\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\left[1-\exp\left(-d_5^v \frac{\rho_3\sigma^2}{P_s}\right)\right]\exp\left(-\frac{\sigma^2}{P_p}\left(\rho_1-\frac{\alpha}{1-\alpha}\right)\right) \end{array} \right\},$$

and

$$P_{\text{out}}^{p,2} = 1 - \left\{ \begin{array}{l} \exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\left(1-\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\right)\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right) \\[2ex] +\left(1-\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\right)\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right) \\[2ex] +\left(1-\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\right)\left(1-\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\right)\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right) \\[2ex] +\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\exp\left(-d_5^v \frac{\rho_3\sigma^2}{P_s}\right) \\[2ex] \left(\exp\left(-\frac{\rho_1\sigma^2}{P_p}\right)+\left(-1+d_6^v \frac{P_p}{\alpha P_r}\right)^{-1}\exp\left(-d_6^v \frac{\rho_1\sigma^2}{\alpha P_r}\right)\left(\exp\left(\left(-1+d_6^v \frac{P_p}{\alpha P_r}\right)\frac{\rho_1\sigma^2}{P_p}\right)-1\right)\right) \\[2ex] +\exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right)\exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right)\left[1-\exp\left(-d_5^v \frac{\rho_3\sigma^2}{P_s}\right)\right] \end{array} \right\}.$$

3.1.2 Outage probability of the secondary system

In the first transmission phase, the SR receives the signal which is sent from the PT, and the received signal is given by $y_{31} = \sqrt{P_p}h_3 x_p + n_{31}$. The achievable rate between the PT and the SR is $R_3 = \frac{1}{2}\log_2\left(1+\frac{P_p\gamma_3}{\sigma^2}\right)$. In the second transmission phase, the SR receives the signal x_R which is sent from the Relay on the condition that the Relay succeeds to decode x_p and x_s. We record the received signal as y_{22}, which is given by

$$\begin{aligned} y_{22} &= h_7 x_R + n_{22} \\ &= \sqrt{\alpha P_r}h_7 x_p + \sqrt{(1-\alpha)P_r}h_7 x_s + n_{22}. \end{aligned} \quad (17)$$

Meanwhile, if the SR decodes the primary signal x_p successfully, the component $\sqrt{\alpha P_r}h_7 x_p$ will be removed

from y_{22} as an interfering signal. So we have $y_{22}^* = \sqrt{(1-\alpha)P_r}h_7 x_s + n_{22}$. The achievable rate between the Relay and the SR is calculated by

$$R_7 = \frac{1}{2}\log_2\left(1+\frac{(1-\alpha)P_r\gamma_7}{\sigma^2}\right). \quad (18)$$

For the secondary system, the transmissions will be interrupted except the only case where the Relay decodes both x_p and x_s successfully, and the SR decodes x_p successfully. So outage probability of the secondary system with the target rates R_{pt} and R_{st} is given by

$$P_{\text{out}}^s = 1 - P_r\{R_2 > R_{\text{pt}}\}P_r\{R_3 > R_{\text{pt}}\}P_r\{R_4 > R_{\text{st}}\}P_r\{R_7 > R_{\text{st}}\}. \quad (19)$$

Since $\gamma_f \sim \varepsilon\left(d_f^v\right)$ ($f = 3, 7$), which means an exponentially distributed random variable with mean $\frac{1}{d_f^v}$, we have

$$P_r\{R_3 > R_{\text{pt}}\} = P_r\left\{\gamma_3 > \frac{\rho_1\sigma^2}{P_p}\right\} = \exp\left(-d_3^v \frac{\rho_1\sigma^2}{P_p}\right), \quad (20)$$

$$P_r\{R_7 > R_{\text{st}}\} = P_r\left\{\gamma_7 > \frac{\rho_3\sigma^2}{(1-\alpha)P_r}\right\} = \exp\left(-d_7^v \frac{\rho_3\sigma^2}{(1-\alpha)P_r}\right). \quad (21)$$

So, substituting (11), (12), (20), and (21) into (19), we have

$$P_{\text{out}}^s = 1 - \exp\left(-d_2^v \frac{\rho_1\sigma^2}{P_p}\right) \exp\left(-d_3^v \frac{\rho_1\sigma^2}{P_p}\right)$$
$$\times \exp\left(-d_4^v \frac{\rho_3\sigma^2}{P_s}\right) \exp\left(-d_7^v \frac{\rho_3\sigma^2}{(1-\alpha)P_r}\right). \quad (22)$$

3.2. Outage performance of the protocol without cooperation (scheme B)

The system without cooperation comprises of a transmitter-receiver pair (PT-PR), and the primary signal is transmitted through the direct link from the PT to the PR. Outage probability of the primary system is given by

$$P_{\text{out}}^o = P_r\{R_1 < R_{\text{pt}}\} = 1 - \exp\left(-\frac{\sigma^2}{P_p}\rho_2\right). \quad (23)$$

where $\rho_2 = 2^{R_{\text{pt}}} - 1$. This protocol will be compared with our proposed protocol in Section 4.

4 Simulation results and discussions

The MATLAB tool is used to simulate the outage probabilities. The theoretical and numerical results for outage probabilities of the primary system and the secondary system versus the power allocation factor α in the proposed protocol (scheme A) are shown in Figure 2, and the results of the primary system's outage probability in the protocol without cooperation (scheme B) are also plotted for comparison. The topological structure of the cognitive relay network is constructed like this: the PT, PR, ST, SR, and Relay are collinear. In the two-dimensional plane, the PT and the PR are located at the points (0, 0) and (1, 0), respectively. The Relay moves on the positive X-axis. The ST is located at the midpoint of the PT and the Relay, while the SR is located at the midpoint of the Relay and the PR. In this topology, we let $R_{\text{pt}} = R_{\text{st}} = 1$, $v = 4$, $P_p = P_s = P_r = 10$ W, $\sigma^2 = 1$, $d_1 = 1$, $d_3 = \frac{1}{2}(1 + d_2)$, $d_4 = \frac{1}{2}d_2$, $d_5 = 1 - \frac{1}{2}d_2$, $d_6 = |1 - d_2|$, $d_7 = \frac{1}{2}|1 - d_2|$, and the values of d_2 are selected as 0.5, 0.8, and
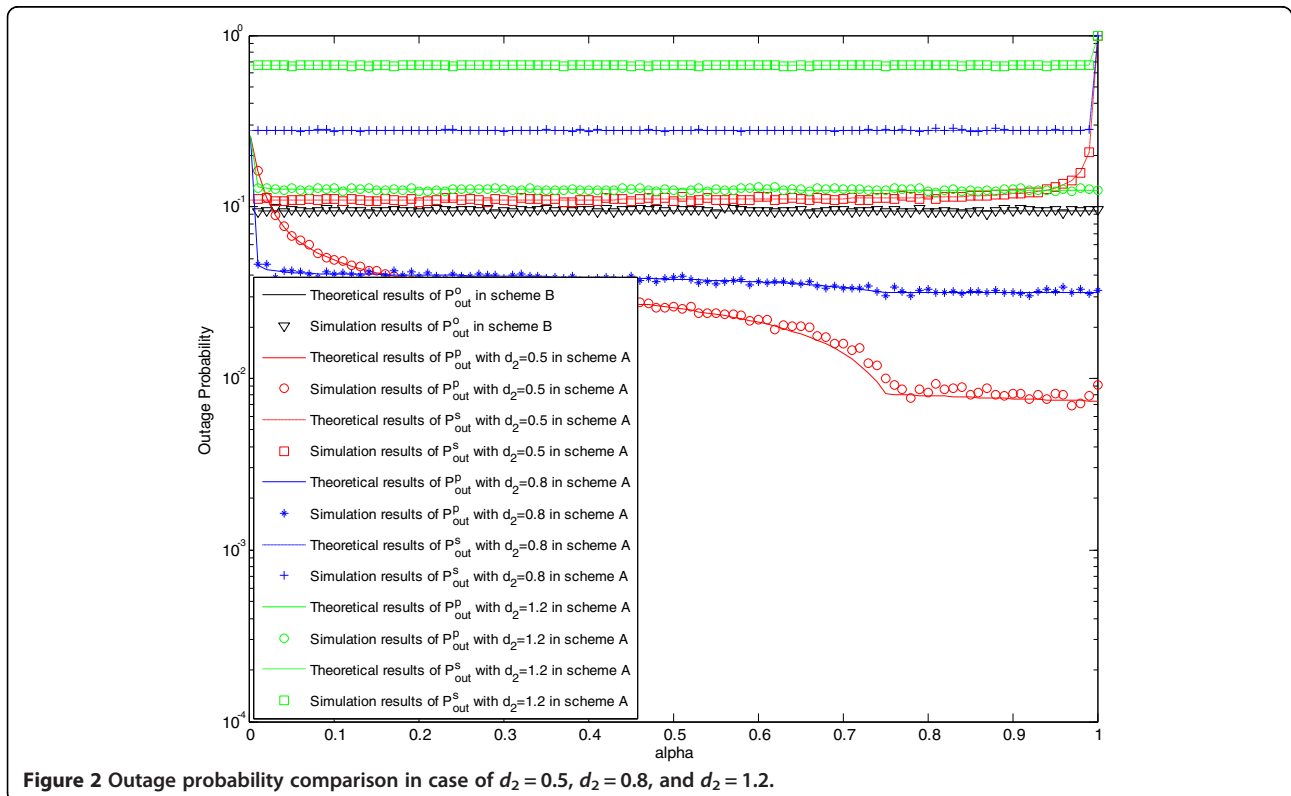

Figure 2 Outage probability comparison in case of $d_2 = 0.5$, $d_2 = 0.8$, and $d_2 = 1.2$.

1.2. It is easy to find that the theoretical results match well with the numerical results from Figure 2. For $d_2 = 0.5$ and 0.8, outage probabilities of the primary system in scheme A are lower than those in scheme B when $\alpha \geq \alpha^* \approx 0.05$, i.e., $P_{out}^p < P_{out}^o$. It means that outage performance of the primary system benefits from the cooperation of the Relay. For $d_2 = 1.2$, outage probability in scheme A is a little higher than that in scheme B, i.e., $P_{out}^p > P_{out}^o$. Through analysis and discussions, we know that the Relay is able to assist the transmissions of the primary and secondary users, and interference will be introduced to the primary users when the Relay assists the transmissions of the secondary users. For $d_2 = 1.2$, the distance between the PT and Relay is far away from that between the PT and PR, and the benefit of cooperation from the Relay is not obvious compared with the introduced interference.

From Figure 2, we also find that the secondary users are able to communicate smoothly with the cooperation of the Relay when $d_2 = 0.5$, 0.8, 1.2. Moreover, we can get that there exist two extreme cases, i.e., $\alpha = 0$ and $\alpha = 1$. The former ($\alpha = 0$) corresponds to the protocol where the primary signal is only transmitted from the PT to the PR, and the whole transmission power of the Relay is used to transmit the secondary signal, which is regarded as an interference signal at the PR. It is easy to get that outage probability of the secondary system is

satisfactory, but outage probability of the primary system is unsatisfactory. With the increasing of α, outage performance of the primary system becomes better, and there is no obvious effect on outage performance of the secondary system. That is to say, outage performance of the primary system in the proposed protocol is better than that in the protocol where the Relay assists the transmissions of the secondary users only. The latter ($\alpha = 1$) means the protocol where the Relay helps the transmissions of the primary users only. In this protocol, though outage probability of the primary system is low than that in the proposed protocol, outage probability of the secondary system is equal to 1, which means that the ST cannot communicate with the SR. So, we are able to better balance outage performances of the primary system and the secondary system in the proposed protocol.

It is easy to find that outage probabilities of the primary system and the secondary system are affected obviously by the variation of d_2. So we show the effects of d_2 on outage performances of the primary and secondary systems in Figure 3. Here, we adopt the same topological structure as in Figure 2, and let $R_{pt} = R_{st} = 1$, $v = 4$, $P_p = P_s = P_r = 10$ W, $\sigma^2 = 1$, $\alpha = 0.5$. According to the results from Figure 2, d_2 is set to be less than d_1, which means that the distance between the PT and the Relay is less than that between the PT and the PR. From Figure 3, it is obvious that the outage probability of the primary
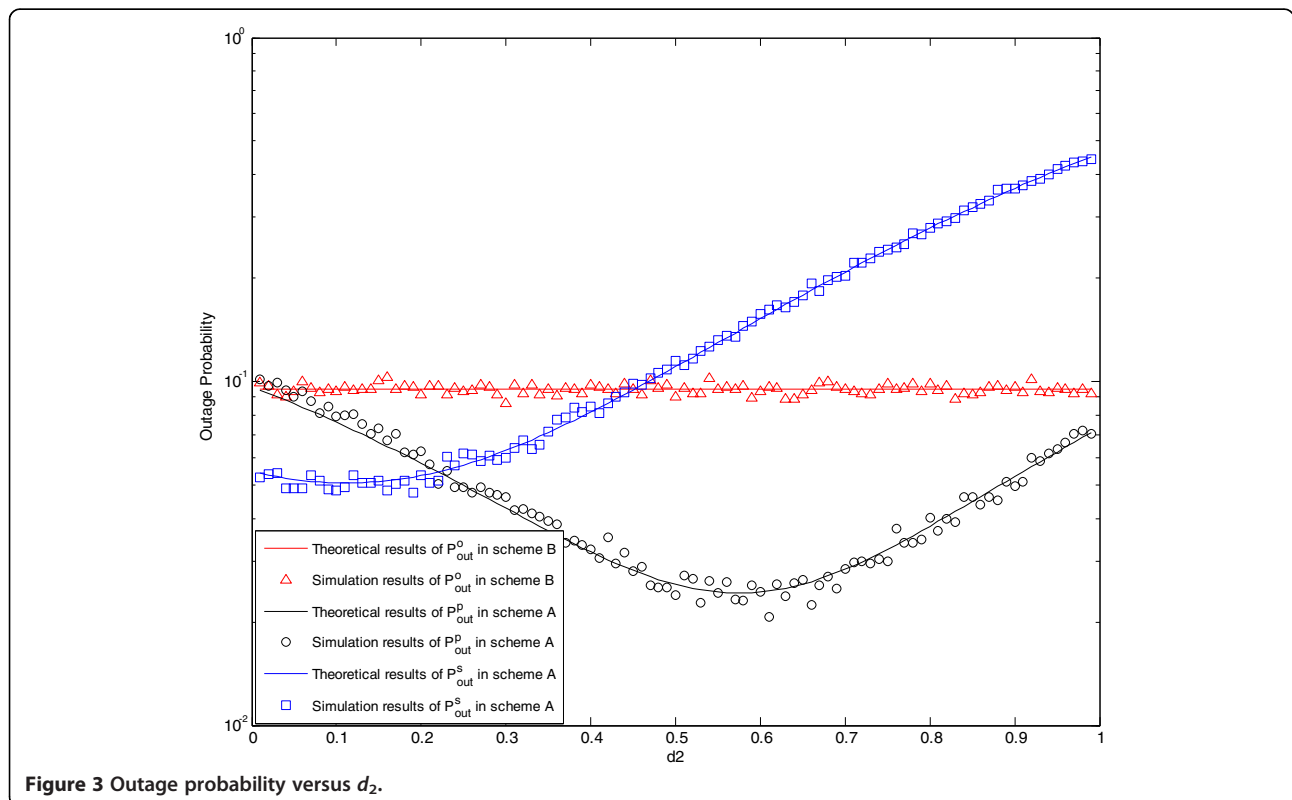

Figure 3 Outage probability versus d_2.

system in scheme A is less than or equal to that in scheme B. When $d_2 \leq d_2^* \approx 0.5$, the outage probability of the primary system decreases with the increasing of d_2, but it increases with the increasing of d_2 when $d_2 > d_2^* \approx 0.5$. From the results, we know that outage performance of the primary system will be better if the distance between the PT and Relay is less than that between the PT and PR, and it coincides with the analysis in Figure 2. Moreover, outage probability of the secondary system increases with the increasing of d_2.

Then, we consider the effects of transmission power on the outage performances. We show outage probabilities of the primary and secondary systems versus P_r, P_p, and P_s in Figures 4, 5, and 6, respectively. Here, we adopt the same topological structure as in Figure 2, and let $R_{pt} = R_{st} = 1$, $v = 4$, $\sigma^2 = 1$, $\alpha = 0.5$. Meanwhile, when one of P_r, P_p, and P_s is regarded as a variable, the other two parameters are set to 10 W. From Figure 4, it is easy to get that outage probabilities of the primary and secondary systems decrease with the increasing of P_r. The reason for these results is that the Relay has the ability to assist the transmissions of the primary users as well as the secondary users, but there exists the mutual interference between the primary and secondary users. With the increasing of P_r, both of the cooperative capability and mutual interference become more and more, so outage probabilities of the primary and secondary systems tend to be stable.

In Figure 5, we show the effect of P_p on the outage performances. Through analysis, we know that P_{out}^p, P_{out}^s, and P_{out}^o decrease with the increasing of P_p from (16), (22), and (23), and they meet the simulation results in Figure 5. Because P_p is the transmission power of the PT, the outage probability of the primary system will be better with the increasing of P_p. For the secondary users, the decoding of the primary signal benefits from the increase of P_p, so the outage probability of the secondary system also decreases with the increasing of P_p.

The effects of P_s on outage performances are shown in Figure 6. It can be seen that both P_{out}^p and P_{out}^s decrease with the increasing of P_s. For the secondary system, P_{out}^s decreases with the increasing of P_s from (22). Even though, there is not an obvious diminution of P_{out}^s from Figure 6. The reason is that there is not a direct link between the ST and the SR, which means that transmissions of the secondary users must be assisted by the Relay. So the effects of P_s cannot be reflected obviously on P_{out}^s. For the primary system, the interference caused by the secondary users will become larger, but the increase of P_s also contributes to the decoding of secondary signal at the PR and Relay. Then, it helps the cooperative transmissions of the primary signal at the Relay. That is to say, the increase of P_s will cause larger interference as well as better cooperation for the primary system, and the benefit of cooperation is far more than
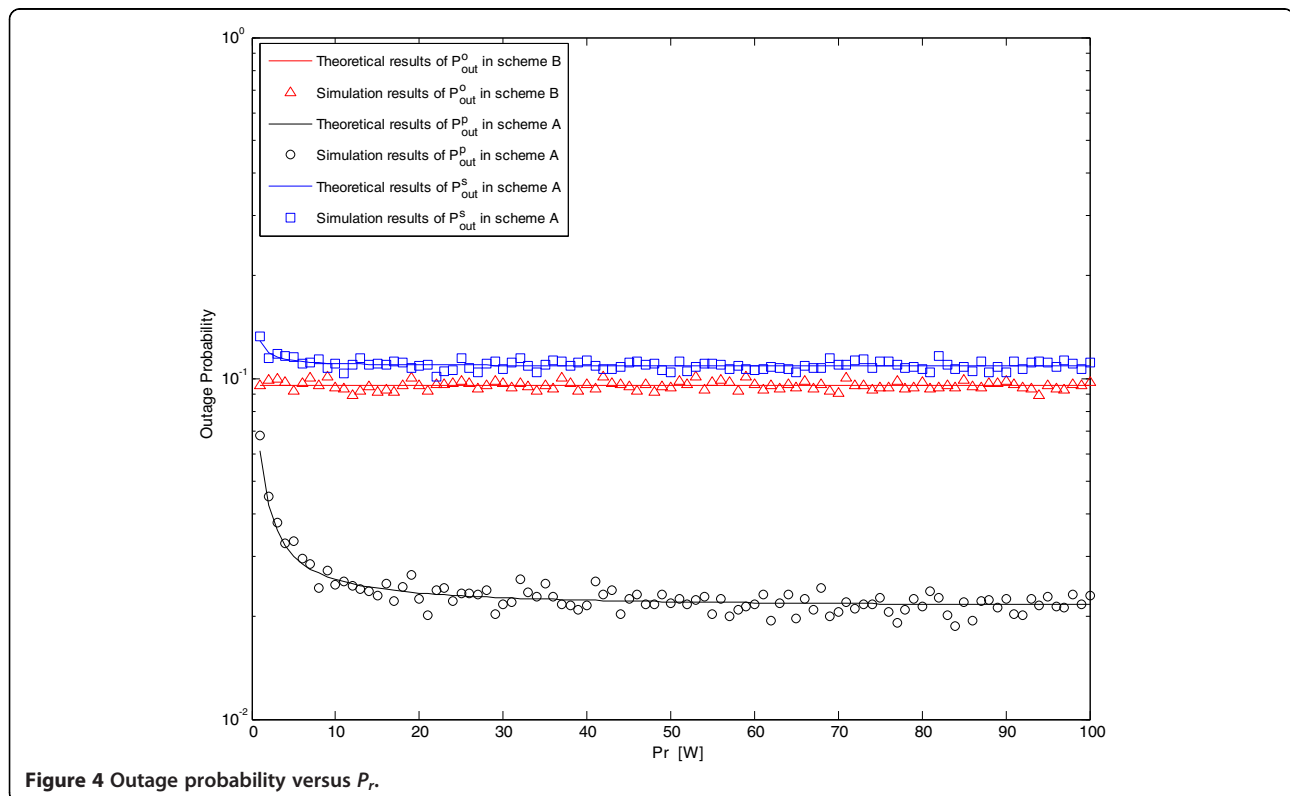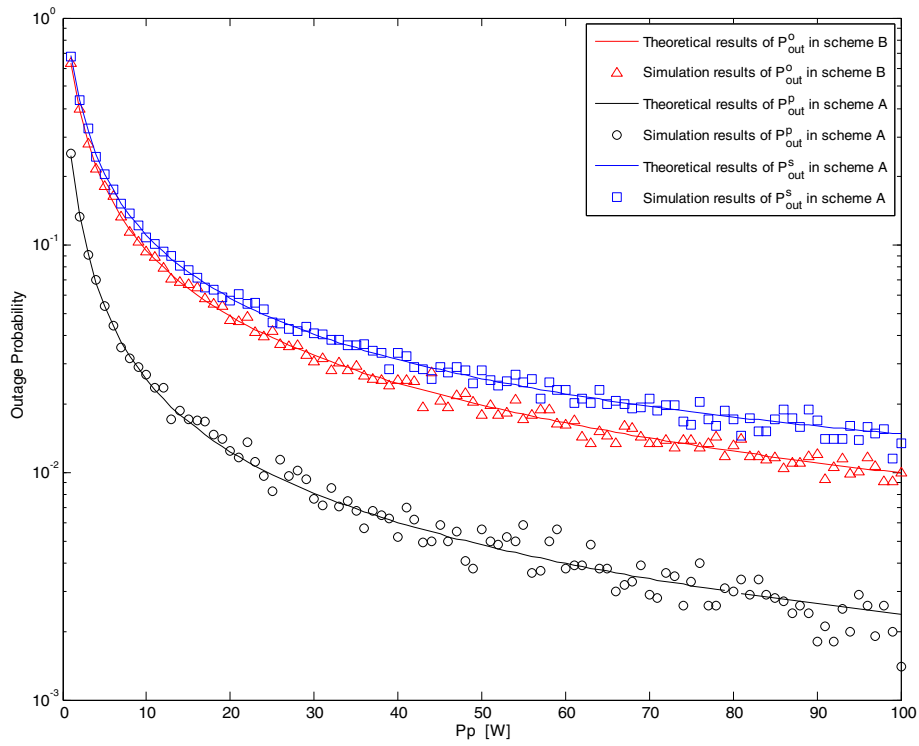

Figure 4 Outage probability versus P_r.

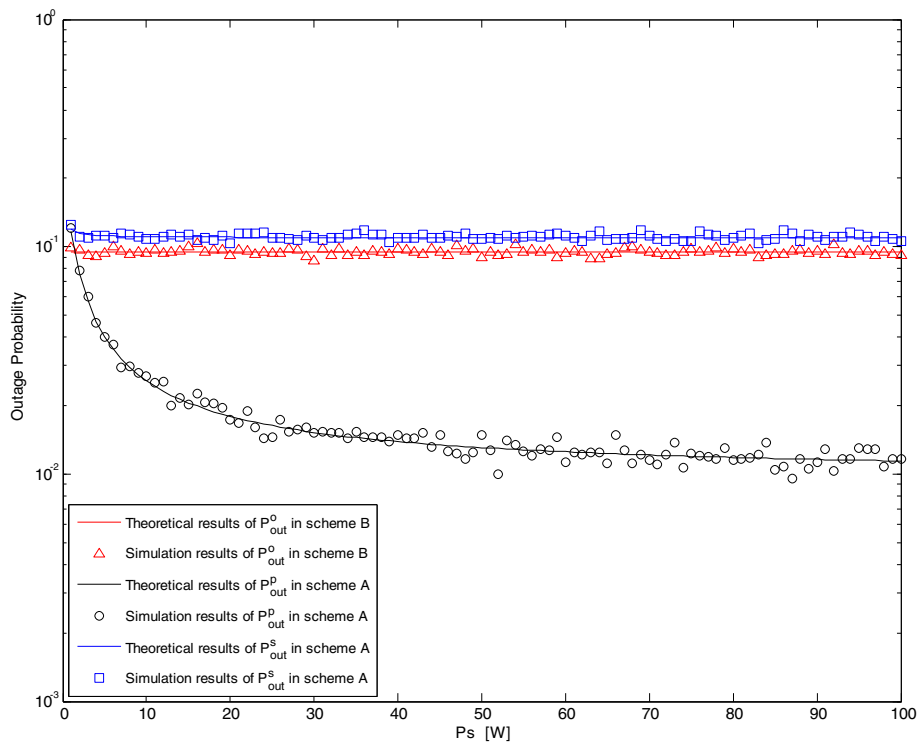**Figure 5 Outage probability versus P_p.**

Figure 6 Outage probability versus P_s.

the interference when P_s increases. So there is an obvious improvement on outage performance of the primary system with the increasing of P_s.

5 Conclusions

In this paper, a relay-assisted spectrum sharing protocol in cognitive relay networks has been presented. In the proposed protocol, better outage performance of the primary users is achieved, and communications of the secondary users without a direct link are realized under the assistance of a decode-and-forward relay.

Competing interests
The authors declare that they have no competing interests.

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