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Majorization properties for certain new classes of analytic functions using the Salagean operator

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Abstract

In the present paper, we investigate the majorization properties for certain classes of multivalent analytic functions defined by the Salagean operator. Moreover, we point out some new and interesting consequences of our main result.

MSC: 30C45

Keywords: analytic functions; multivalent functions; α-uniformly starlike functions of order β; α-uniformly convex functions of order β; subordination; majorization property

1 Introduction and definitions

Let f and g be two analytic functions in the open unit disk

$$\Delta = \{z \in C : |z| < 1\}. \tag{1.1}$$

We say that f is majorized by g in Δ (see [1]) and write

$$f(z) \ll g(z) \quad (z \in \Delta) \tag{1.2}$$

if there exists a function φ, analytic in Δ, such that

$$\left|\varphi(z)\right| \le 1 \quad \text{and} \quad f(z) = \varphi(z)g(z) \quad (z \in \Delta). \tag{1.3}$$

It may be noted here that (1.2) is closely related to the concept of quasi-subordination between analytic functions.

For two functions f and g, analytic in Δ, we say that the function f is subordinate to g in Δ if there exists a Schwarz function ω, which is analytic in Δ with

$$\omega(0) = 0 \quad \text{and} \quad \left|\omega(z)\right| < 1 \quad (z \in \Delta),$$

such that

$$f(z) = g\left(\omega(z)\right) \quad (z \in \Delta).$$

We denote this subordination by $f(z) \prec g(z)$. Furthermore, if the function g is univalent in Δ, then

$$f(z) \prec g(z) \quad (z \in \Delta) \quad \Leftrightarrow \quad f(0) = g(0) \quad \text{and} \quad f(\Delta) \subset g(\Delta).$$

Let A_p denote the class of functions of the form

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k \quad \bigl(p \in N = \{1, 2, \dots\}\bigr), \tag{1.4}$$

that are analytic and p-valent in the open unit disk Δ. Also, let $A_1 = A$.

For a function $f \in A_p$, let $f^{(q)}$ denote a qth-order ordinary differential operator by

$$f^{(q)}(z) = \frac{p!}{(p-q)!} z^{p-q} + \sum_{k=p+1}^{\infty} \frac{k!}{(k-q)!} a_k z^{k-q}, \tag{1.5}$$

where $p > q$, $p \in N$, $q \in N_0 = N \cup \{0\}$ and $z \in \Delta$. Next, Frasin [2] introduced the differential operator $D^m f^{(q)}$ as follows:

$$D^m f^{(q)}(z) = \frac{p!(p-q)^m}{(p-q)!} z^{p-q} + \sum_{k=p+1}^{\infty} \frac{k!(k-q)^m}{(k-q)!} a_k z^{k-q}. \tag{1.6}$$

In view of (1.6), it is clear that $D^0 f^{(0)}(z) = f(z)$, $D^0 f^{(1)}(z) = zf'(z)$ and $D^m f^{(0)}(z) = D^m f(z)$ is a known operator introduced by Salagean [3].

Definition 1.1 A function $f(z) \in A_p$ is said to be in the class $L_{p,q}^{j,l}[A, B; \alpha, \gamma]$ of p-valent functions of complex order $\gamma \neq 0$ in Δ if and only if

$$\left[1 + \frac{1}{\gamma}\left(\frac{D^j f^{(q)}(z)}{D^l f^{(q)}(z)} - (p-q)^{j-l}\right) - \alpha \left|\frac{1}{\gamma}\left(\frac{D^j f^{(q)}(z)}{D^l f^{(q)}(z)} - (p-q)^{j-l}\right)\right|\right] \prec \frac{1+Az}{1+Bz}$$
$$\bigl(z \in \Delta; -1 \le B < A \le 1; j > l; p, j \in N; l, q \in N_0; 0 \le \alpha; \gamma \in C^* = C \setminus \{0\}\bigr). \tag{1.7}$$

Clearly, we have the following relationships:

(1) $L_{p,q}^{j,l}[A, B; 0, \gamma] = S_{p,q}^{j,l}[A, B; \gamma]$;
(2) $L_{1,0}^{m,n}[A, B; \alpha, 1] = U_{m,n}(\alpha, A, B)$;
(3) $L_{1,0}^{1,0}[1 - 2\beta, -1; \alpha, 1] = US(\alpha, \beta)$ $(0 \le \beta < 1)$ (α-uniformly starlike functions of order β);
(4) $L_{2,1}^{1,0}[1 - 2\beta, -1; \alpha, 1] = UK(\alpha, \beta)$ $(0 \le \beta < 1)$ (α-uniformly convex functions of order β);
(5) $L_{p,0}^{n+1,n}[1, -1; \alpha, \gamma] = S_n(p, \alpha, \gamma)$ $(n \in N_0)$;
(6) $L_{1,0}^{1,0}[1, -1; \alpha, \gamma] = S(\alpha, \gamma)$ $(0 \le \alpha < 1, \gamma \in C^*)$;
(7) $L_{1,0}^{2,1}[1, -1; \alpha, \gamma] = K(\alpha, \gamma)$ $(0 \le \alpha < 1, \gamma \in C^*)$;
(8) $L_{1,0}^{1,0}[1, -1; \alpha, 1 - \beta] = S^*(\alpha, \beta)$ $(0 \le \alpha < 1, 0 \le \beta < 1)$.

The classes $S_{p,q}^{j,l}[A, B; \gamma]$ and $U_{m,n}(\alpha, A, B)$ were introduced by Goswami and Aouf [4] and Li and Tang [5], respectively. The classes $US(\alpha, \beta)$ and $UK(\alpha, \beta)$ were studied recently in [6] (see also [7–12]). The class $S_n(p,0,\gamma) = S_n(p,\gamma)$ was introduced by Akbulut *et al.* [13]. Also, the classes $S(0,\gamma) = S(\gamma)$ and $K(0,\gamma) = K(\gamma)$ are said to be classes of starlike and convex of complex order $\gamma \neq 0$ in Δ which were considered by Nasr and Aouf [14] and Wiatrowski [15] (see also [16, 17]), and $S^*(0,\beta) = S^*(\beta)$ denotes the class of starlike functions of order β in Δ.

A majorization problem for the class $S(\gamma)$ has recently been investigated by Altintas *et al.* [18]. Also, majorization problems for the classes $S^*(\beta)$ and $S_{p,q}^{j,l}[A,B;\gamma]$ have been investigated by MacGregor [1] and Goswami and Aouf [4], respectively. Very recently, Goyal and Goswami [19] (see also [20]) generalized these results for the fractional derivative operator. In the present paper, we investigate a majorization problem for the class $L_{p,q}^{j,l}[A,B;\alpha,\gamma]$.

2 Majorization problem for the class $L_{p,q}^{j,l}[A,B;\alpha,\gamma]$

We begin by proving the following result.

Theorem 2.1 *Let the function $f \in A_p$ and suppose that $g \in L_{p,q}^{j,l}[A,B;\alpha,\gamma]$. If $D^j f^{(q)}(z)$ is majorized by $D^l g^{(q)}(z)$ in Δ, and*

$$(p-q)^{j-l} \geq \left[\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B|\right]\delta,$$

then

$$\left|D^{j+1} f^{(q)}(z)\right| \leq \left|D^{l+1} g^{(q)}(z)\right| \quad \bigl(|z| \leq r_0\bigr), \tag{2.1}$$

where $r_0 = r_0(p,q,\alpha,\gamma,j,l,A,B)$ is the smallest positive root of the equation

$$\begin{aligned}&\left[\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B|\right]r^3 - \left[(p-q)^{j-l} + 2|B|\right]r^2 \\ &\quad - \left[\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B| + 2\right]r + (p-q)^{j-l} = 0 \\ &\quad \bigl(-1 \leq B < A \leq 1; p,j \in N; q,l \in N_0; 0 \leq \alpha < 1; \gamma \in C^*, 0 \leq \delta \leq r_0\bigr).\end{aligned} \tag{2.2}$$

Proof Suppose that $g \in L_{p,q}^{j,l}[A,B;\alpha,\gamma]$. Then, making use of the fact that

$$\varpi - \alpha|\varpi - 1| \prec \frac{1+Az}{1+Bz} \quad \Leftrightarrow \quad \varpi\left(1 - \alpha e^{-i\phi}\right) + \alpha e^{-i\phi} \prec \frac{1+Az}{1+Bz} \quad (\phi \in R),$$

and letting

$$\varpi = 1 + \frac{1}{\gamma}\left(\frac{D^j g^{(q)}(z)}{D^l g^{(q)}(z)} - (p-q)^{j-l}\right)$$

in (1.7), we obtain

$$\left[1 + \frac{1}{\gamma}\left(\frac{D^j g^{(q)}(z)}{D^l g^{(q)}(z)} - (p-q)^{j-l}\right)\right]\left(1 - \alpha e^{-i\phi}\right) + \alpha e^{-i\phi} \prec \frac{1+Az}{1+Bz}$$

or, equivalently,

$$1+\frac{1}{\gamma}\left(\frac{D^j g^{(q)}(z)}{D^l g^{(q)}(z)}-(p-q)^{j-l}\right)\prec\frac{1+(\frac{A-\alpha Be^{-i\phi}}{1-\alpha e^{-i\phi}})z}{1+Bz}\tag{2.3}$$

which holds true for all $z\in\Delta$.

We find from (2.3) that

$$1+\frac{1}{\gamma}\left(\frac{D^j g^{(q)}(z)}{D^l g^{(q)}(z)}-(p-q)^{j-l}\right)=\frac{1+(\frac{A-\alpha Be^{-i\phi}}{1-\alpha e^{-i\phi}})\omega(z)}{1+B\omega(z)},\tag{2.4}$$

where $\omega(z)=c_1z+c_2z^2+\cdots$, $\omega\in P$, P denotes the well-known class of the bounded analytic functions in Δ and satisfies the conditions

$$\omega(0)=0\quad\text{and}\quad\left|\omega(z)\right|\le|z|\quad(z\in\Delta).$$

From (2.4), we get

$$\frac{D^j g^{(q)}(z)}{D^l g^{(q)}(z)}=\frac{(p-q)^{j-l}+[\frac{(A-B)\gamma}{1-\alpha e^{-i\phi}}+(p-q)^{j-l}B]\omega(z)}{1+B\omega(z)}.\tag{2.5}$$

By virtue of (2.5), we obtain

$$\begin{aligned}\left|D^l g^{(q)}(z)\right|&\le\frac{1+|B||z|}{(p-q)^{j-l}-|\frac{(A-B)\gamma}{1-\alpha e^{-i\phi}}+(p-q)^{j-l}B||z|}\left|D^j g^{(q)}(z)\right|\\&\le\frac{1+|B||z|}{(p-q)^{j-l}-[\frac{(A-B)|\gamma|}{1-\alpha}+(p-q)^{j-l}|B|]|z|}\left|D^j g^{(q)}(z)\right|.\end{aligned}\tag{2.6}$$

Next, since $D^j f^{(q)}(z)$ is majorized by $D^l g^{(q)}(z)$ in Δ, thus from (1.3), we have

$$D^j f^{(q)}(z)=\varphi(z)D^l g^{(q)}(z).$$

Differentiating the above equality with respect to z and multiplying by z, we get

$$D^{j+1}f^{(q)}(z)=z\varphi'(z)D^l g^{(q)}(z)+\varphi(z)D^{l+1}g^{(q)}(z).\tag{2.7}$$

Thus, by noting that $\varphi(z)\in P$ satisfies the inequality (see, *e.g.*, Nehari [21])

$$\left|\varphi'(z)\right|\le\frac{1-|\varphi(z)|^2}{1-|z|^2}\quad(z\in\Delta)\tag{2.8}$$

and making use of (2.6) and (2.8) in (2.7), we obtain

$$\begin{aligned}\left|D^{j+1}f^{(q)}(z)\right|\le{}&\left(\left|\varphi(z)\right|+\frac{1-|\varphi(z)|^2}{1-|z|^2}\cdot\frac{(1+|B||z|)|z|}{[(p-q)^{j-l}-(\frac{(A-B)|\gamma|}{1-\alpha}+(p-q)^{j-l}|B|)|z|]}\right)\\&\times\left|D^{l+1}g^{(q)}(z)\right|,\end{aligned}\tag{2.9}$$

which, upon setting

$$|z| = r \quad \text{and} \quad \left|\varphi(z)\right| = \rho \quad (0 \leq \rho \leq 1),$$

leads us to the inequality

$$\begin{aligned}&\left|D^{j+1}f^{(q)}(z)\right| \\ &\quad\leq \left(\frac{\psi(\rho)}{(1-r^2)[(p-q)^{j-l} - (\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B|)r]}\right)\left|D^{l+1}g^{(q)}(z)\right|,\end{aligned}$$

where

$$\begin{aligned}\psi(\rho) = {}&-r\bigl(1+|B|r\bigr)\rho^2 + \bigl(1-r^2\bigr)\left[(p-q)^{j-l} - \left(\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B|\right)r\right]\rho \\ &+ r\bigl(1+|B|r\bigr)\end{aligned} \tag{2.10}$$

takes its maximum value at $\rho = 1$ with $r_0 = r_0(p,q,\alpha,\gamma,j,l,A,B)$, where

$$r_0 = r_0(p,q,\alpha,\gamma,j,l,A,B)$$

is the smallest positive root of equation (2.2). Furthermore, if $0 \leq \delta \leq r_0(p,q,\alpha,\gamma,j,l,A,B)$, then the function $\psi(\rho)$ defined by

$$\begin{aligned}\psi(\rho) = {}&-\delta\bigl(1+|B|\delta\bigr)\rho^2 + \bigl(1-\delta^2\bigr)\left[(p-q)^{j-l} - \left(\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B|\right)\delta\right]\rho \\ &+ \delta\bigl(1+|B|\delta\bigr)\end{aligned} \tag{2.11}$$

is an increasing function on the interval $0 \leq \rho \leq 1$ so that

$$\begin{aligned}&\psi(\rho) \leq \psi(1) = \bigl(1-\delta^2\bigr)\left[(p-q)^{j-l} - \left(\frac{(A-B)|\gamma|}{1-\alpha} + (p-q)^{j-l}|B|\right)\delta\right] \\ &\bigl(0 \leq \rho \leq 1; 0 \leq \delta \leq r_0(p,q,\alpha,\gamma,j,l,A,B)\bigr).\end{aligned} \tag{2.12}$$

Hence, upon setting $\rho = 1$ in (2.11), we conclude that (2.1) of Theorem 2.1 holds true for $|z| \leq r_0(p,q,\alpha,\gamma,j,l,A,B)$, which completes the proof of Theorem 2.1. □

Setting $\alpha = 0$ in Theorem 2.1, we get the following result.

Corollary 2.1 *Let the function $f \in A_p$ and suppose that $g \in S_{p,q}^{j,l}[A,B;\gamma]$. If $D^j f^{(q)}(z)$ is majorized by $D^l g^{(q)}(z)$ in Δ, and*

$$(p-q)^{j-l} \geq \bigl[(A-B)|\gamma| + (p-q)^{j-l}|B|\bigr]\delta,$$

then

$$\left|D^{j+1}f^{(q)}(z)\right| \leq \left|D^{l+1}g^{(q)}(z)\right| \quad \bigl(|z| \leq r_0\bigr), \tag{2.13}$$

where $r_0 = r_0(p,q,\gamma,j,l,A,B)$ *is the smallest positive root of the equation*

$$\begin{aligned}&\bigl[(A-B)|\gamma| + (p-q)^{j-l}|B|\bigr]r^3 - \bigl[(p-q)^{j-l} + 2|B|\bigr]r^2 - \bigl[(A-B)|\gamma| + (p-q)^{j-l}|B| + 2\bigr]r \\ &\quad + (p-q)^{j-l} = 0 \\ &\quad \bigl(-1 \le B < A \le 1; p,j \in N; q,l \in N_0; \gamma \in C^*, 0 \le \delta \le r_0\bigr). \qquad (2.14)\end{aligned}$$

Remark 2.1 Corollary 2.1 improves the result of Goswami and Aouf [4, Theorem 1].

Putting $p = 1$, $q = 0$, $j = m$, $l = n$, $m > n$ and $\gamma = 1$ in Theorem 2.1, we obtain the following result.

Corollary 2.2 *Let the function* $f \in A$ *and suppose that* $g \in U_{m,n}(\alpha,A,B)$. *If* $D^m f(z)$ *is majorized by* $D^n g(z)$ *in* Δ, *then*

$$\bigl|D^{m+1}f(z)\bigr| \le \bigl|D^{n+1}g(z)\bigr| \quad \bigl(|z| \le r_0\bigr), \qquad (2.15)$$

where $r_0 = r_0(\alpha,A,B)$ *is the smallest positive root of the equation*

$$\begin{aligned}&\left[\frac{A-B}{1-\alpha} + |B|\right]r^3 - \bigl(1 + 2|B|\bigr)r^2 - \left[\frac{A-B}{1-\alpha} + |B| + 2\right]r + 1 = 0 \\ &\quad (-1 \le B < A \le 1; 0 \le \alpha < 1). \qquad (2.16)\end{aligned}$$

For $A = 1 - 2\beta$, $B = -1$, putting $m = 1$, $n = 0$ and $m = 2$, $n = 1$ in Corollary 2.2, respectively, we obtain the following Corollaries 2.3 and 2.4.

Corollary 2.3 *Let the function* $f \in A$ *and suppose that* $g \in US(\alpha,\beta)$. *If* $Df(z)$ *is majorized by* $g(z)$ *in* Δ, *then*

$$\bigl|f'(z) + zf''(z)\bigr| \le \bigl|g'(z)\bigr| \quad \bigl(|z| \le r_0\bigr),$$

where $r_0 = r_0(\alpha,\beta)$ *is the smallest positive root of the equation*

$$\left[\frac{2(1-\beta)}{1-\alpha} + 1\right]r^3 - 3r^2 - \left[\frac{2(1-\beta)}{1-\alpha} + 3\right]r + 1 = 0 \quad (0 \le \alpha < 1; 0 \le \beta < 1).$$

Corollary 2.4 *Let the function* $f \in A$ *and suppose that* $g \in UK(\alpha,\beta)$. *If* $D^2 f(z)$ *is majorized by* $Dg(z)$ *in* Δ, *then*

$$\bigl|D^3 f(z)\bigr| \le \bigl|D^2 g(z)\bigr| \quad \bigl(|z| \le r_0\bigr),$$

where $r_0 = r_0(\alpha,\beta)$ *is the smallest positive root of the equation*

$$\left[\frac{2(1-\beta)}{1-\alpha} + 1\right]r^3 - 3r^2 - \left[\frac{2(1-\beta)}{1-\alpha} + 3\right]r + 1 = 0 \quad (0 \le \alpha < 1; 0 \le \beta < 1).$$

Also, putting $A = 1$, $B = -1$, $q = 0$, $j = n + 1$ and $l = n$ in Theorem 2.1, we obtain the following result.

Corollary 2.5 *Let the function $f \in A_p$ and suppose that $g \in S_n(p,\alpha,\gamma)$. If $D^{n+1}f(z)$ is majorized by $D^n g(z)$ in Δ, then*

$$\left|D^{n+2}f(z)\right| \leq \left|D^{n+1}g(z)\right| \quad \bigl(|z| \leq r_0\bigr), \tag{2.17}$$

where $r_0 = r_0(p,\alpha,\gamma)$ is the smallest positive root of the equation

$$\left[\frac{2|\gamma|}{1-\alpha} + p\right]r^3 - (p+2)r^2 - \left[\frac{2|\gamma|}{1-\alpha} + p + 2\right]r + p = 0$$
$$\bigl(p \in N; \gamma \in C^*; 0 \leq \alpha < 1\bigr). \tag{2.18}$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors jointly worked on the results and they read and approved the final manuscript.

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