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# Common fixed point theorems for dominating and weak annihilator mappings in ordered metric spaces

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# Abstract

Very recently, Haghi *et al.* (Topol. Appl. 160:450-454, 2013) proved that some fixed point theorems in partial metric spaces can be obtained from metric spaces. In this paper, we prove some common fixed point theorems for four mappings *f*, *g*, *S* and *T* satisfying a nonlinear contraction in ordered metric spaces, where the mappings *f* and *g* are dominating and weak annihilators of the mappings *T* and *S*, respectively. We utilize the techniques of Haghi *et al.* to derive our main result, which is a generalization of the result of Shobkolaei *et al.* (Appl. Math. Comput. 219:443-452, 2012). Also, we introduce an example to support the usability of our results. **MSC:** Primary 54H25; secondary 47H10

**Keywords:** partial metric space; common fixed point; altering distance function; almost contraction; ordered metric space

## 1 Introduction

One of the most important problems in mathematical analysis is to establish existence and uniqueness theorems for some integral and differential equations. Fixed point theory, in ordered metric spaces, plays a major role in solving such kind of problems. The first result in this direction was obtained by Ran and Reurings [1]. This one was extended for nondecreasing mappings by Nieto and Lopez [2, 3]. Meanwhile, Agarwal *et al.* [5] and O'Regan and Petruşel [4] studied some results for generalized contractions in ordered metric spaces. Then, many authors obtained fixed point results in ordered metric spaces. For some works in ordered metric spaces, we refer the reader to [6-19].

Berinde [20–24] initiated the concept of almost contraction and studied existence fixed point results for almost contraction in complete metric spaces. Later, many authors studied different types of almost contractions and studied fixed point results; for example, see [25–31].

In 1994 Matthews [32] introduced the concept of partial metric spaces and proved the Banach contraction principle in these spaces. Then, many authors obtained interesting results in partial metric spaces [33, 36–38]. Very recently, Haghi *et al.* [33] proved that some fixed point theorems in partial metric spaces can be obtained from metric spaces.

The purpose of this paper is to prove some common fixed point theorems for four mappings f, g, S and T satisfying a nonlinear contraction in ordered metric spaces, where the

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mappings f and g are dominating and weak annihilators of the mappings T and S, respectively. We utilize the results of Haghi *et al.* [33] to derive Theorem 2.1.

## 2 Previous notations and results

In the sequel, we have to recall previous notations and results.

Let *f* and *g* be self-mappings of a set *X*. If w = fx = gx for some  $x \in X$ , then *x* is called a *coincidence point* of *f* and *g*, and *w* is called a *point of coincidence* of *f* and *g*. Two self-mappings *f* and *g* are said to be *weakly compatible* if they commute at their coincidence point, that is, if fx = gx, then fgx = gfx. For details, please see [34].

Now, consider  $(X, \leq)$  to be a partially ordered set. According to [35], a mapping *f* is called *weak annihilator* of *g* if  $fgx \leq x$  for all  $x \in X$  and *f* is called *dominating* if  $x \leq fx$  for all  $x \in X$ .

Recently, Shobkolaei *et al.* [38] initiated the concept of almost generalized (S, T)-contractive condition in a partial metric space and studied some common fixed point results in partial metric spaces.

**Definition 2.1** [38] Let f, g, S and T be self-maps on a partial metric space (X, p). Then f and g are said to satisfy *almost generalized* (S, T)-*contractive condition* if there exists  $\delta \in [0, 1)$  such that

$$p(fx,gy) \le \delta \max\left\{p(Sx,Ty), p(fx,Sx), p(gy,Ty), \frac{p(Sx,gy) + p(fx,Ty)}{2}\right\}$$
(2.1)

for all  $x, y \in X$ .

**Theorem 2.1** [38] Let  $(X, p, \leq)$  be a complete ordered partial metric space. Let f, g, T,  $S: X \to X$  be mappings with  $fX \subseteq TX$  and  $gX \subseteq SX$ , and the dominating mappings f and g are weak annihilators of T and S, respectively. Suppose that f and g satisfy almost generalized (S, T)-contractive condition (2.1) for any two comparable elements  $x, y \in X$ . If for a nondecreasing sequence  $\{x_n\}$  with  $x_n \leq y_n$  for all n but finitely many and  $y_n \to u$  implies that  $x_n \leq u$ , and furthermore

(1)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;

(2) one of fX, gX, SX and TX is a closed subspace of X,

then f, g, S and T have a common fixed point.

Moreover, the set of common fixed points of f, g, S and T is well ordered if and only if f, g, S and T have one and only one common fixed point.

#### 3 Main result

Let  $\Psi$  denote all functions  $\psi : [0, +\infty) \to [0, +\infty)$  such that

- (1)  $\psi$  is continuous and nondecreasing;
- (2)  $\psi(t) = 0$  if and only if t = 0.

Also, let  $\Phi$  denote all functions  $\phi : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \rightarrow [0, +\infty)$  such that

- (1)  $\phi$  is continuous;
- (2)  $\phi(t, s, u) = 0$  if and only if u = s = t = 0.
- If  $\psi \in \Psi$ , then  $\psi$  is called an *altering distance function* (see [39]).

Now, we introduce our definition.

**Definition 3.1** Let *f*, *g*, *S* and *T* be self-mappings on a metric space (*X*, *d*). Then *f* and *g* are said to *satisfy the almost nonlinear* (*S*, *T*, *L*,  $\psi$ ,  $\phi$ )*-contractive condition* if there exist  $\psi \in \Psi$ ,  $\phi \in \Phi$  and  $L \in [0, +\infty)$  such that

$$\psi(d(fx,gy)) \leq \psi\left(\max\left\{d(Sx,Ty),d(fx,Sx),d(gy,Ty),\frac{1}{2}(d(Sx,gy)+d(fx,Ty))\right\}\right)$$
$$-\phi(d(Sx,Ty),d(Sx,gy),d(fx,Ty))$$
$$+L\min\left\{d(Sx,Ty),d(Sx,gy),d(fx,Ty)\right\}$$
(3.1)

for all  $x, y \in X$ .

Now, let  $(X, d, \leq)$  be an ordered metric space. We say that *X* satisfies the property  $(\pi)$  if the following statement holds true.

( $\pi$ ) If { $x_n$ } is a nondecreasing sequence in X and { $y_n$ } is a sequence in X such that  $x_n \leq y_n$  for all n but finitely many and  $y_n \rightarrow u$ , then  $x_n \leq u$  for all n but finitely many.

In the rest of this paper,  $\mathbb{N}$  stands for the set of nonnegative integer numbers.

**Theorem 3.1** Let  $(X, d, \leq)$  be a complete ordered metric space. Let  $f, g, T, S : X \to X$  be four mappings such that f and g satisfy the nonlinear  $(S, T, L, \psi, \phi)$ -contractive condition (3.1) for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

- (1)  $fX \subseteq TX$ ;
- (2)  $gX \subseteq SX$ ;
- (3)  $La \phi(a, a, a) < 0$  for all a > 0;
- (4) f is dominating and weak annihilator of T;
- (5) g is dominating and weak annihilator of S;
- (6)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;
- (7) one of fX, gX, SX and TX is a closed subspace of X.

If X has the property  $(\pi)$ , then f, g, S and T have a common fixed point.

*Proof* Let  $x_0 \in X$ . Since  $fX \subseteq TX$ , we choose  $x_1 \in X$  such that  $fx_0 = Tx_1$ . Also, since  $gX \subseteq SX$ , we choose  $x_2 \in X$  such that  $gx_1 = Sx_2$ . Continuing this process, we can construct a sequence  $\{y_n\}$  in X such  $y_{2n} := fx_{2n} = Tx_{2n+1}$  and  $y_{2n+1} := gx_{2n+1} = Sx_{2n+2}$ . Since f is dominating and weak annihilator of T and g is dominating and weak annihilator of S,

$$x_{2n} \leq f x_{2n} = T x_{2n+1} \leq f T x_{2n+1} \leq x_{2n+1} \leq g x_{2n+1} = S x_{2n+2} \leq g S x_{2n+2} \leq x_{2n+2}.$$

Thus, for all  $n \ge 0$ , we have  $x_n \le x_{n+1}$ .

Suppose  $y_{2n} = y_{2n+1}$  for some  $n \in \mathbb{N}$ . We claim that  $y_{2n+1} = y_{2n+2}$ . Since  $x_{2n+2}$  and  $x_{2n+3}$  are comparable, we have

$$\begin{split} \psi \left( d(y_{2n+2}, y_{2n+1}) \right) \\ &= \psi \left( d(fx_{2n+2}, gx_{2n+1}) \right) \\ &\leq \psi \left( \max \left\{ d(Sx_{2n+2}, Tx_{2n+1}), d(fx_{2n+2}, Sx_{2n+2}), d(gx_{2n+1}, Tx_{2n+1}) \right\} \right) \end{split}$$

$$\begin{aligned} &\frac{1}{2} \Big( d(Sx_{2n+2}, gx_{2n+1}) + d(fx_{2n+2}, Tx_{2n+1}) \Big) \Big\} \Big) \\ &- \phi \Big( d(Sx_{2n+2}, Tx_{2n+1}), d(Sx_{2n+2}, gx_{2n+1}), d(fx_{2n+2}, Tx_{2n+1}) \Big) \\ &+ L \min \Big\{ d(Sx_{2n+2}, Tx_{2n+1}), d(Sx_{2n+2}, gx_{2n+1}), d(fx_{2n+2}, Tx_{2n+1}) \Big\} \\ &= \psi \Big( \max \Big\{ d(y_{2n+1}, y_{2n}), d(y_{2n+2}, y_{2n+1}), d(y_{2n+1}, y_{2n}), \\ &\frac{1}{2} \Big( d(y_{2n+1}, y_{2n+1}) + d(y_{2n+2}, y_{2n}) \Big) \Big\} \Big) - \phi \Big( d(y_{2n+1}, y_{2n}), d(y_{2n+2}, y_{2n}) \Big) \\ &+ L \min \Big\{ d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+1}), d(y_{2n+2}, y_{2n}) \Big\} \\ &= \psi \Big( \max \Big\{ 0, d(y_{2n+2}, y_{2n+1}), \frac{1}{2} \Big( d(y_{2n+2}, y_{2n+1}) \Big) \Big\} \Big) - \phi \Big( 0, 0, d(y_{2n+2}, y_{2n+1}) \Big) \\ &= \psi \Big( d(y_{2n+2}, y_{2n+1}), - \phi \Big( 0, 0, d(y_{2n+2}, y_{2n+1}) \Big) \Big\} \end{aligned}$$

Therefore,  $\phi(0, 0, d(y_{2n+2}, y_{2n+1})) = 0$  and hence  $y_{2n+2} = y_{2n+1}$ . Similarly, we may show that  $y_{2n+3} = y_{2n+2}$ . Thus  $\{y_n\}$  is a constant sequence in *X*, hence it is a Cauchy sequence in (*X*, *d*).

Suppose  $y_{2n} \neq y_{2n+1}$  for all  $n \in \mathbb{N}$ . Given  $n \in \mathbb{N}$ . If *n* is even, then n = 2t for some  $t \in \mathbb{N}$ . Since  $x_{2t}$  and  $x_{2t+1}$  are comparable, we have

$$\begin{split} \psi(d(y_n, y_{n+1})) &= \psi(d(y_{2t}, y_{2t+1})) \\ &= \psi(d(fx_{2t}, gx_{2t+1})) \\ &\leq \psi\left(\max\left\{d(Sx_{2t}, Tx_{2t+1}), d(fx_{2t}, Sx_{2t}), d(gx_{2t+1}, Tx_{2t+1}), \right. \\ &\left. \frac{1}{2}(d(Sx_{2t}, gx_{2t+1}) + d(fx_{2t}, Tx_{2t+1}))\right\}\right) \\ &- \phi(d(Sx_{2t}, Tx_{2t+1}), d(Sx_{2t}, gx_{2t+1}), d(fx_{2t}, Tx_{2t+1})) \\ &+ L \min\left\{d(Sx_{2t}, Tx_{2t+1}), d(Sx_{2t}, gx_{2t+1}), d(fx_{2t}, Tx_{2t+1})\right\} \\ &= \psi\left(\max\left\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1}), \frac{1}{2}(d(y_{2t-1}, y_{2t+1}))\right\}\right) \\ &- \phi(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0) \\ &\leq \psi\left(\max\left\{d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\right) \\ &= \psi\left(\max\left\{d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1})\right\}\right) - \phi\left(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1})\right)\right). \end{split}$$

If

 $\max\left\{d(y_{2t-1},y_{2t}),d(y_{2t},y_{2t+1})\right\}=d(y_{2t},y_{2t+1}),$ 

then

$$\phi(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0) = 0,$$

and hence  $d(y_{2t-1}, y_{2t}) = d(y_{2t-1}, y_{2t+1}) = 0$ . Thus  $y_{2t-1} = y_{2t}$ , a contradiction. Thus,

$$\max\left\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1})\right\} = d(y_{2t-1}, y_{2t}).$$
(3.2)

Therefore,

$$\psi(d(y_{2t}, y_{2t+1})) \le \psi(d(y_{2t-1}, y_{2t})) - \phi(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0).$$
(3.3)

If *n* is odd, then n = 2t + 1 for some  $t \in \mathbb{N}$ . Since  $x_{2t+2}$  and  $x_{2t+1}$  are comparable, we have

$$\begin{split} \psi(d(y_n, y_{n+1}) \\ &= \psi(d(y_{2t+2}, y_{2t+1})) \\ &= \psi(d(fx_{2t+2}, gx_{2t+1})) \\ &\leq \psi\left(\max\left\{d(Sx_{2t+2}, Tx_{2t+1}), d(fx_{2t+2}, Sx_{2t+2}), d(gx_{2t+1}, Tx_{2t+1}), \right. \\ &\left.\frac{1}{2}(d(Sx_{2t+2}, gx_{2t+1}) + d(fx_{2t+2}, Tx_{2t+1}))\right\}\right) \\ &- \phi(d(Sx_{2t+2}, Tx_{2t+1}), d(Sx_{2t+2}, gx_{2t+1}), d(fx_{2t+2}, Tx_{2t+1})) \\ &+ L\min\left\{d(Sx_{2t+2}, Tx_{2t+1}), d(Sx_{2t+2}, gx_{2t+1}), d(fx_{2t+2}, Tx_{2t+1})\right\} \\ &= \psi\left(\max\left\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1}), \frac{1}{2}(d(y_{2t+2}, y_{2t+1}) + d(d(y_{2t+1}, y_{2t})))\right\}\right) \\ &- \phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t+1}), \frac{1}{2}(d(y_{2t+2}, y_{2t+1}) + d(d(y_{2t+1}, y_{2t}))))\right\}) \\ &= \psi\left(\max\left\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1}), \frac{1}{2}(d(y_{2t+1}, y_{2t+1}) + d(d(y_{2t+1}, y_{2t})))\right)\right\}\right) \\ &- \phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t+1}), \frac{1}{2}(d(y_{2t+1}, y_{2t+1}) + d(d(y_{2t+1}, y_{2t}))))\right\}) \\ &= \psi\left(\max\left\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\right\}\right) - \phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t+1})\right) \\ &= \psi\left(\max\left\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\right\}\right) - \phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t+1})\right) \right\}. \end{split}$$

If

$$\max\left\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\right\} = d(y_{2t+2}, y_{2t+1}),$$

then

 $\phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})) = 0,$ 

and hence  $d(y_{2t+1}, y_{2t}) = d(y_{2t+2}, y_{2t}) = 0$ . Thus  $y_{2t+1} = y_{2t}$ , a contradiction. So,

$$\max\left\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\right\} = d(y_{2t+1}, y_{2t}). \tag{3.4}$$

Therefore

$$\psi(d(y_{2t+2}, y_{2t+1})) \le \psi(d(y_{2t+1}, y_{2t})) - \phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})).$$
(3.5)

From (3.2) and (3.4), we have

$$d(y_n, y_{n+1}) \le d(y_{n-1}, y_n). \tag{3.6}$$

Therefore  $\{d(y_{n+1}, y_n) : n \in \mathbb{N}\}$  is a nonincreasing sequence. Thus there exists  $r \ge 0$  such that

$$\lim_{n\to+\infty}d(y_n,y_{n+1})=r.$$

On taking lim sup in (3.3) and (3.5), we have

$$\psi(r) \leq \psi(r) - \liminf_{t \to +\infty} \phi(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0)$$

and

$$\psi(r) \leq \psi(r) - \liminf_{t \to +\infty} \phi(d(y_{2t+1}, y_{2t}), 0, d(y_{2t}, y_{2t+2})).$$

Thus

$$\liminf_{t \to +\infty} d(y_{2t-1}, y_{2t}) = \liminf_{t \to +\infty} d(y_{2t-1}, y_{2t+1}) = \liminf_{t \to +\infty} d(y_{2t}, y_{2t+2}) = \liminf_{t \to +\infty} d(y_{2t}, y_{2t+1}) = 0.$$

Therefore, r = 0 and hence

$$\lim_{n \to +\infty} d(y_n, y_{n+1}) = 0.$$
(3.7)

Now, we show that  $\{y_n\}$  is a Cauchy sequence in the metric space (X, d). It is sufficient to show that  $\{y_{2n}\}$  is a Cauchy sequence in (X, d). Suppose to the contrary; that is,  $\{y_{2n}\}$  is not a Cauchy sequence in (X, d). Then there exists  $\epsilon > 0$  for which we can find two subsequences  $\{y_{2m(i)}\}$  and  $\{y_{2n(i)}\}$  of  $\{y_{2n}\}$  such that n(i) is the smallest index for which

$$n(i) > m(i) > i, \quad d(y_{2m(i)}, y_{2n(i)}) \ge \epsilon$$
(3.8)

and

$$d(y_{2m(i)}, y_{2n(i)-2}) < \epsilon.$$

$$(3.9)$$

From (3.8), (3.9) and the triangular inequality, we get that

$$\begin{aligned} \epsilon &\leq d(y_{2m(i)}, y_{2n(i)}) \\ &\leq d(y_{2m(i)}, y_{2n(i)-2}) + d(y_{2n(i)-2}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2n(i)}) \\ &< \epsilon + d(y_{2n(i)-2}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2n(i)}). \end{aligned}$$

On letting  $i \to +\infty$  in the above inequalities and using (3.7), we have

$$\lim_{i \to +\infty} d(y_{2m(i)}, y_{2n(i)}) = \epsilon.$$
(3.10)

Again, from (3.8) and the triangular inequality, we get that

$$\begin{aligned} \epsilon &\leq d(y_{2m(i)}, y_{2n(i)}) \\ &\leq d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2m(i)}) \\ &\leq d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2m(i)+1}) + d(y_{2m(i)+1}, y_{2m(i)}) \\ &\leq d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2m(i)}) + 2d(y_{2m(i)+1}, y_{2m(i)}) \\ &\leq 2d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)}, y_{2m(i)}) + 2d(y_{2m(i)+1}, y_{2m(i)}). \end{aligned}$$

Letting  $i \to +\infty$  in the above inequalities and using (3.7) and (3.10), we get that

$$\lim_{i \to +\infty} d(y_{2m(i)}, y_{2n(i)}) = \lim_{i \to +\infty} d(y_{2m(i)+1}, y_{2n(i)-1})$$
$$= \lim_{i \to +\infty} d(y_{2m(i)+1}, y_{2n(i)})$$
$$= \lim_{i \to +\infty} d(y_{2m(i)}, y_{2n(i)-1})$$
$$= \epsilon.$$

Since  $x_{2n(i)}$  and  $x_{2m(i)+1}$  are comparable, we have

$$\begin{split} \psi \left( d(y_{2n(i)}, y_{2m(i)+1}) \right) \\ &= \psi \left( d(fx_{2n(i)}, gx_{2m(i)+1}) \right) \\ &\leq \psi \left( \max \left\{ d(Sx_{2n(i)}, Tx_{2m(i)+1}), d(fx_{2n(i)}, Sx_{2n(i)}), d(gx_{2m(i)+1}, Tx_{2m(i)+1}), \right. \\ &\left. \frac{1}{2} \left( d(Sx_{2n(i)}, gx_{2m(i)+1}) + d(fx_{2n(i)}, Tx_{2m(i)+1}) \right) \right\} \right) \\ &- \phi \left( d(Sx_{2n(i)}, Tx_{2m(i)+1}), d(Sx_{2n(i)}, gx_{2m(i)+1}), d(fx_{2n(i)}, Tx_{2m(i)+1}) \right) \\ &+ L \min \left\{ d(Sx_{2n(i)}, Tx_{2m(i)+1}), d(Sx_{2n(i)}, gx_{2m(i)+1}), d(fx_{2n(i)}, Tx_{2m(i)+1}) \right\} \\ &= \psi \left( \max \left\{ d(y_{2n(i)-1}, y_{2m(i)}), d(y_{2n(i)}, y_{2n(i)-1}), d(y_{2m(i)+1}, y_{2m(i)}), \right. \\ &\left. \frac{1}{2} \left( d(y_{2n(i)}, y_{2m(i)}) + d(y_{2n(i)-1}, y_{2m(i)+1}) \right) \right\} \right) \\ &- \phi \left( d(y_{2n(i)-1}, y_{2m(i)}), d(y_{2n(i)-1}, y_{2m(i)+1}), d(y_{2n(i)}, y_{2m(i)}) \right) \\ &+ L \min \left\{ d(y_{2n(i)-1}, y_{2m(i)}), d(y_{2n(i)-1}, y_{2m(i)+1}), d(y_{2n(i)}, y_{2m(i)}) \right\} \right]. \end{split}$$

Letting  $i \to +\infty$  and using the continuity of  $\psi$ , we get that

$$\psi(\epsilon) \leq \psi(\epsilon) - \phi(\epsilon, \epsilon, \epsilon) + L\epsilon.$$

By condition (3), we get  $\psi(\epsilon) = 0$  and hence  $\epsilon = 0$ , a contradiction. Thus  $\{y_n\}$  is a Cauchy sequence in (X, d).

Since (X, d) is complete, there is  $y \in X$  such that  $y_n \to y$  in the metric space (X, d). Thus

$$\lim_{n \to +\infty} d(y_n, y) = 0. \tag{3.11}$$

Now we show that *y* is the fixed point of *g* and *T*. Assume that *TX* is closed, since  $\{y_{2n} = Tx_{2n+1}\}$  is a sequence in *TX* converging to *y*, we have  $y \in TX$ . So, there exists  $u \in X$  such that y = Tu. Therefore,

$$\lim_{n \to +\infty} f x_{2n} = \lim_{n \to +\infty} g x_{2n+1} = \lim_{n \to +\infty} T x_{2n+1} = \lim_{n \to +\infty} S x_{2n+2} = y = T u.$$

Now, we show that gu = y. Since  $x_{2n} \leq fx_{2n}$  and  $y_{2n} = fx_{2n} \rightarrow y$ , we have  $x_{2n} \leq y$ . Since the mapping f is dominating and weak annihilator of T, we obtain  $x_{2n} \leq y = Tu \leq fTu \leq u$ . Thus

$$\begin{split} \psi(d(y_{2n},gu)) \\ &= \psi(d(fx_{2n},gu)) \\ &\leq \psi\left(\max\left\{d(Sx_{2n},Tu),d(fx_{2n},Sx_{2n}),d(gu,Tu),\frac{1}{2}(d(Sx_{2n},gu)+d(fx_{2n},Tu))\right\}\right) \\ &\quad -\phi(d(Sx_{2n},Tu),d(Sx_{2n},gu),d(fx_{2n},Tu)) \\ &\quad +L\min\{d(Sx_{2n},Tu),d(Sx_{2n},gu),d(fx_{2n},Tu)\} \\ &= \psi\left(\max\left\{d(y_{2n-1},y),d(y_{2n},y_{2n-1}),d(gu,y),\frac{1}{2}(d(y_{2n-1},gu)+d(y_{2n},y))\right\}\right) \\ &\quad -\phi(d(y_{2n-1},y),d(y_{2n-1},gu),d(y_{2n},y)) + L\min\{d(y_{2n-1},y),d(y_{2n-1},gu),d(y_{2n},y)\}. \end{split}$$

Letting  $n \to +\infty$  in the above inequalities and using (3.7), we get that

$$\psi(d(y,gu)) \leq \psi(d(gu,y)) - \phi(0,d(y,gu),0).$$

Therefore  $\phi(0, d(y, gu), 0) = 0$  and hence d(gu, y) = 0. Thus gu = y. Since g and T are weakly compatible and gu = gTu = Tgu = Ty, we have

$$gy = gTu = Tgu = Ty.$$

Again, since  $x_{2n}$  and y are comparable, we have

$$\begin{split} \psi \left( d(y_{2n}, gy) \right) \\ &= \psi \left( d(fx_{2n}, gy) \right) \\ &\leq \psi \left( \max \left\{ d(Sx_{2n}, Ty), d(fx_{2n}, Sx_{2n}), d(gy, Ty), \frac{1}{2} \left( d(Sx_{2n}, gy) + d(fx_{2n}, Ty) \right) \right\} \right) \\ &- \phi \left( d(Sx_{2n}, Ty), d(Sx_{2n}, gy), d(fx_{2n}, Ty) \right) \\ &+ L \min \left\{ d(Sx_{2n}, Ty), d(Sx_{2n}, gy), d(fx_{2n}, Ty) \right\} \end{split}$$

$$=\psi\left(\max\left\{d(y_{2n-1},gy),d(y_{2n},y_{2n-1}),d(gy,gy),\frac{1}{2}(d(y_{2n-1},gy)+d(y_{2n},gy))\right\}\right)\\-\phi\left(d(y_{2n-1},gy),d(y_{2n-1},gy),d(y_{2n},gy)\right)+\min\left\{d(y_{2n-1},gy),d(y_{2n-1},gy),d(y_{2n},gy)\right\}.$$

On letting  $n \to +\infty$  in the above inequalities and using (3.7), we have

$$\psi(d(y,gy)) \leq \psi(d(y,gy)) - \phi(d(y,gy),d(y,gy),d(y,gy)) + Ld(y,gy).$$

Using condition (3), we get  $\psi(d(y, gy)) = 0$ . Thus d(y, gy) = 0 and hence gy = y.

Finally, we have to show that *y* is also a fixed point of *f* and *T*. Since  $gX \subseteq SX$ , there exists  $v \in X$  such that y = gy = Sv. Since the mapping *g* is dominating and weak annihilator of *S*, we have  $y \leq gy = Sv \leq gSv \leq v$ . Thus *y* and *v* are comparable, and hence

$$\begin{split} \psi(d(fv, Sv)) \\ &= \psi(d(fv, gy)) \\ &\leq \psi\left(\max\left\{d(Sv, Ty), d(fv, Sv), d(gy, Ty), \frac{1}{2}(d(Sv, gy) + d(fv, Ty))\right\}\right) \\ &- \phi(d(Sv, Ty), d(Sv, gy), d(fv, Ty)) + L\min\{d(Sv, Ty), d(Sv, gy), d(fv, Ty)\} \\ &= \psi\left(\max\left\{d(Sv, Sv), d(fv, Sv), d(Sv, Sv), \frac{1}{2}(d(Sv, Sv) + d(fv, Sv))\right\}\right) \\ &- \phi(d(Sv, Sv), d(Sv, Sv), d(fv, Sv)) + L\min\{d(Sv, Sv), d(Sv, Sv), d(fv, Sv)\} \\ &= \psi(d(fv, Sv)) - \phi(0, 0, d(fv, Sv)) + Ld(fv, Sv). \end{split}$$

Using condition (3), we get  $\psi(d(fv, Sv)) = 0$ . Thus d(fv, Sv) = 0 and hence fv = Sv = gy = y. Since *f* and *S* are weakly compatible, we have fy = fSv = Sfv = Sy. Since *y* and *y* are comparable, we have

$$\begin{split} \psi(d(fy,y)) &= \psi(d(fy,gy)) \\ &\leq \psi\left(\max\left\{d(Sy,Ty), d(fy,Sy), d(gy,Ty), \frac{1}{2}(d(Sy,gy) + d(fy,Ty))\right\}\right) \\ &- \phi(d(Sy,Ty), d(Sy,gy), d(fy,Ty)) + L\min\{d(Sy,Ty), d(Sy,gy), d(fy,Ty)\} \\ &= \psi\left(\max\left\{d(fy,y), d(fy,fy), d(y,y), \frac{1}{2}(d(fy,y) + d(fy,y))\right\}\right) \\ &- \phi(d(fy,y), d(fy,y), d(fy,y)) + L\min\{d(fy,y), d(fy,y), d(fy,y)\} \\ &= \psi(d(fy,y)) - \phi(d(fy,y), d(fy,y), d(fy,y)) + Ld(fy,y). \end{split}$$

Using condition (3), we get  $\psi(d(fy, y)) = 0$ . Thus d(fy, y) = 0 and hence fy = y. So, y is a common fixed point of f, g, T and S. In case SX, fX or gX is closed, the proof of the existence of a common fixed point is similar to the arguments above.

**Corollary 3.1** Let  $(X, d, \preceq)$  be a complete ordered metric space, and let  $f, g, T, S : X \rightarrow X$  be four mappings. Assume that there exist  $\psi \in \Psi$  and  $\phi \in \Phi$  such that

$$\psi(d(fx,gy)) \le \psi\left(\max\left\{d(Sx,Ty), d(fx,Sx), d(gy,Ty), \frac{1}{2}(d(Sx,gy) + d(fx,Ty))\right\}\right) - \phi\left(\max\left\{d(Sx,Ty), d(Sx,gy), d(fx,Ty)\right\}\right)$$

holds for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

- (1)  $fX \subseteq TX$ ;
- (2)  $gX \subseteq SX$ ;
- (3) f is dominating and weak annihilator of T;
- (4) g is dominating and weak annihilator of S;
- (5)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;
- (6) one of fX, gX, SX and TX is a closed subspace of X.

If X satisfies the property  $(\pi)$ , then f, g, S and T have a common fixed point.

**Corollary 3.2** Let  $(X, d, \preceq)$  be a complete ordered metric space, and let  $f, g, T, S : X \rightarrow X$  be four mappings. Assume that there exist  $\phi_1, \psi \in \Psi$  such that

$$\psi(d(fx,gy)) \le \psi\left(\max\left\{d(Sx,Ty),d(fx,Sx),d(gy,Ty),\frac{1}{2}(d(Sx,gy)+d(fx,Ty))\right\}\right) - \phi_1\left(\max\left\{d(Sx,Ty),d(Sx,gy),d(fx,Ty)\right\}\right)$$

holds for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

(1)  $fX \subseteq TX$ ;

(2)  $gX \subseteq SX$ ;

- (3) f is dominating and weak annihilator of T;
- (4) g is dominating and weak annihilator of S;
- (5)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;
- (6) one of fX, gX, SX and TX is a closed subspace of X.

If X satisfies the property  $(\pi)$ , then f, g, S and T have a common fixed point.

*Proof* Follows from Corollary 3.1 by defining  $\phi : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \rightarrow [0, +\infty)$ via  $\phi(s, t, u) = \phi_1(\max\{s, t, u\})$  and noting that  $\phi \in \Phi$ .

**Corollary 3.3** Let  $(X, d, \leq)$  be a complete ordered metric space, and let  $f, g, T, S : X \to X$  be four mappings. Assume that there exist  $\phi_1, \psi \in \Psi$  such that

$$\psi\left(d(fx,gy)\right) \leq \psi\left(\max\left\{d(Sx,Ty),d(fx,Sx),d(gy,Ty),\frac{1}{2}\left(d(Sx,gy)+d(fx,Ty)\right)\right\}\right)$$
$$-\phi_1\left(\max\left\{d(Sx,Ty),d(fx,Sx),d(gy,Ty),\frac{1}{2}\left(d(Sx,gy)+d(fx,Ty)\right)\right\}\right)$$

holds for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

- (1)  $fX \subseteq TX$ ;
- (2)  $gX \subseteq SX$ ;
- (3) f is dominating and weak annihilator of T;
- (4) g is dominating and weak annihilator of S;
- (5)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;
- (6) one of fX, gX, SX and TX is a closed subspace of X.

If X satisfies the property  $(\pi)$ , then f, g, S and T have a common fixed point.

*Proof* By noting that

$$\phi_1\left(\max\left\{d(Sx, Ty), d(Sx, gy), d(fx, Ty)\right\}\right)$$
  
$$\leq \phi_1\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}(d(Sx, gy) + d(fx, Ty))\right\}\right),$$

the proof follows from Corollary 3.2.

Jachymski [40] proved that some conditions for generalized contractions in (ordered) metric spaces are equivalent. By the aid of Lemma 1 [40], we have the following result.

**Theorem 3.2** Let f, g, S and T be self-mappings on a partial ordered metric space (X, d), and set

$$M(x,y) = \max\left\{d(Sx,Ty), d(fx,Sx), d(gy,Ty), \frac{1}{2}(d(Sx,gy) + d(fx,Ty))\right\}.$$

Then the following are equivalent:

(i) There exist  $\alpha \in [0,1)$  and  $\psi \in \Psi$  such that

$$\psi(d(fx,gy)) \le \alpha \psi(M(x,y))$$

for all comparable elements  $x, y \in X$ .

(ii) There exist  $\psi, \phi \in \Psi$  such that

$$\psi(d(fx,gy)) \le \psi(M(x,y)) - \phi(M(x,y))$$

for all comparable elements  $x, y \in X$ .

(iii) There exists a continuous nondecreasing function  $\phi : [0, +\infty) \rightarrow [0, +\infty)$  such that  $\phi(t) < t$  for all t > 0 and for any  $x, y \in X$ ,

$$d(fx,gy) \le \phi(M(x,y)).$$

*Proof* Set  $D = \{(M(x, y), d(fx, gy)) : x, y \text{ are two comparable elements in } X\}$ . Then the proof follows from Lemma 1 of [40].

By the aid of Theorem 3.2 and Corollary 3.3, we have the following results.

**Theorem 3.3** Let  $(X, d, \preceq)$  be a complete ordered metric space, and let  $f, g, T, S : X \rightarrow X$  be four mappings. Assume that there exist  $\psi \in \Psi$  and  $k \in [0, 1)$  such that

$$\psi\left(d(fx,gy)\right) \le k\psi\left(\max\left\{d(Sx,Ty),d(fx,Sx),d(gy,Ty),\frac{1}{2}\left(d(Sx,gy)+d(fx,Ty)\right)\right\}\right)$$

holds, for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

- (1)  $fX \subseteq TX$ ;
- (2)  $gX \subseteq SX$ ;
- (3) f is dominating and weak annihilator of T;
- (4) g is dominating and weak annihilator of S;
- (5)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;
- (6) one of fX, gX, SX and TX is a closed subspace of X.

If X satisfies the property  $(\pi)$ , then f, g, S and T have a common fixed point.

**Theorem 3.4** Let  $(X, d, \leq)$  be a complete ordered metric space, and let  $f, g, T, S : X \rightarrow X$  be four mappings. Assume that there exists a continuous and nondecreasing function  $\phi : [0, +\infty) \rightarrow [0, +\infty)$  with  $\phi(t) < t$  for all t > 0 such that

$$d(fx,gy) \le \phi\left(\max\left\{d(Sx,Ty),d(fx,Sx),d(gy,Ty),\frac{1}{2}(d(Sx,gy)+d(fx,Ty))\right\}\right)$$

holds for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

- (1)  $fX \subseteq TX$ ;
- (2)  $gX \subseteq SX$ ;
- (3) f is dominating and weak annihilator of T;
- (4) g is dominating and weak annihilator of S;
- (5)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;
- (6) one of fX, gX, SX and TX is a closed subspace of X.

If X satisfies the property  $(\pi)$ , then f, g, S and T have a common fixed point.

As a direct result of our theorems, we have the following result.

**Corollary 3.4** Let  $(X, d, \leq)$  be a complete ordered metric space, and let  $f, g, T, S : X \rightarrow X$  be four mappings. Assume that there exists  $k \in [0, 1)$  such that

$$d(fx,gy) \le k \max\left\{d(Sx,Ty), d(fx,Sx), d(gy,Ty), \frac{1}{2}(d(Sx,gy) + d(fx,Ty))\right\}$$

holds for any two comparable elements  $x, y \in X$ . Suppose that f, g, S and T satisfy the following conditions:

(1)  $fX \subseteq TX$ ;

- (2)  $gX \subseteq SX$ ;
- (3) f is dominating and weak annihilator of T;
- (4) g is dominating and weak annihilator of S;
- (5)  $\{f, S\}$  and  $\{g, T\}$  are weakly compatible;

(6) one of fX, gX, SX and TX is a closed subspace of X. If X satisfies the property  $(\pi)$ , then f, g, S and T have a common fixed point.

**Remark 3.1** By using the method of Haghi *et al.* [33], its an easy matter to show that Theorem 2.1 is a consequence result of Corollary 3.4.

To support our results, we introduce the following example.

**Example 3.1** On  $X = \{0, 1, 2, 3, ...\}$ , define

$$d: X \times X \to X, \qquad d(x, y) = \begin{cases} 0 & \text{if } x = y; \\ \max\{x, y\} & \text{if } x \neq y. \end{cases}$$

We introduce a relation on *X* by  $x \leq y$  if and only if  $y \leq x$ . Also, define  $f, g, S, T : X \rightarrow X$  by the formulas

$$fx = gx = \begin{cases} 0 & \text{if } x \in \{0, 1, 2\}; \\ 3 & \text{if } x \in \{3, 4\}; \\ x - 2 & \text{if } x \ge 5 \end{cases}$$

and

$$Tx = Sx = \begin{cases} 0 & \text{if } x = 0; \\ x + 2 & \text{if } x \ge 1. \end{cases}$$

Consider  $\psi : [0, +\infty) \rightarrow [0, +\infty)$ ,  $\psi(t) = t^2$  and  $\phi : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \rightarrow [0, +\infty)$ ,  $\phi(t, s, u) = \max\{t, s, u\}$ . Then, for every two comparable elements  $x, y \in X$ , we have

$$\psi\left(d(fx,fy)\right) \leq \psi\left(\max\left\{d(Tx,Ty),d(fx,Tx),d(fy,Ty),\frac{1}{2}\left(d(Tx,fy)+d(fx,Ty)\right)\right\}\right) -\phi\left(d(Tx,Ty),d(Tx,fy),d(fx,Ty)\right).$$
(3.12)

Proof Let

$$M(x, y) = \max\left\{ d(Tx, Ty), d(fx, Tx), d(fy, Ty), \frac{1}{2} (d(Tx, fy) + d(fx, Ty)) \right\}$$

and

$$N(x, y) = \phi(d(Tx, Ty), d(Tx, fy), d(fx, Ty)) = \max\{d(Tx, Ty), d(Tx, fy), d(fx, Ty)\}$$

Given  $x, y \in X$ , without loss of generality, we assume that  $x \le y$ .

Now, we divide the proof into the following cases:

- CASE I: x = y. Here, we have  $\psi(d(fx, fy)) = 0$  and get (3.12).
- CASE II: x < y and  $x, y \in \{0, 1, 2\}$ . Here,  $\psi(d(fx, fy)) = 0$ , hence (3.12) holds.
- CASE III:  $x \in \{0, 1, 2\}$  and y = 3. Here, fx = 0, fy = 3,  $Tx \in \{0, 3, 4\}$  and Ty = 5. Thus, d(fx, fy) = 3, M(x, y) = 5 and N(x, y) = 5. Since  $9 \le 25 5$ , we obtain (3.12).

- CASE IV:  $x \in \{0, 1, 2\}$  and y = 4. Here, fx = 0, fy = 3,  $Tx \in \{0, 3, 4\}$  and Ty = 6. Thus, d(fx, fy) = 3, M(x, y) = 6 and N(x, y) = 6. Since  $9 \le 36 6$ , we deduce (3.12).
- CASE V:  $x \in \{0, 1, 2\}$  and  $y \ge 5$ . Here, fx = 0, fy = y 2,  $Tx \in \{0, 3, 4\}$  and Ty = y + 2. Thus, d(fx, fy) = y - 2, M(x, y) = y + 2 and N(x, y) = y + 2. Since  $(y - 2)^2 \le (y + 2)^2 - (y + 2)$ , we have (3.12).
- CASE VI: x = 3 and y = 4. Here, fx = 0, fy = 3, Tx = 5 and Ty = 6. Thus, d(fx, fy) = 3, M(x, y) = 6 and N(x, y) = 6. Since  $9 \le 36 6$ , we get (3.12).
- CASE VII: x = 3 and  $y \ge 5$ . Here, fx = 3, fy = y 2, Tx = 5 and Ty = y + 2. Thus, d(fx, fy) = y - 2, M(x, y) = y + 2 and N(x, y) = y + 2. Since  $(y - 2)^2 \le (y + 2)^2 - (y + 2)$ , we have (3.12) and

$$\psi(p(fx,gy))$$

$$\leq e^{-1}\psi\left(\max\left\{p(Sx,Ty),p(fx,Sx),p(gy,Ty),\frac{1}{2}(p(Sx,gy)+p(fx,Ty))\right\}\right).$$
(3.13)

- CASE VIII: x = 4 and  $y \ge 5$ . Here, fx = 3, fy = y 2, Tx = 6 and Ty = y + 2. Thus, d(fx, fy) = y - 2, M(x, y) = y + 2 and N(x, y) = y + 2. Since  $(y - 2)^2 \le (y + 2)^2 - (y + 2)$ , we have (3.12) and (3.13).
- CASE IX:  $y > x \ge 5$ . Here, fx = x 2, fy = y 2, Tx = x + 2 and Ty = y + 2. Thus, d(fx, fy) = y - 2, M(x, y) = y + 2 and N(x, y) = y + 2. Since  $(y - 2)^2 \le (y + 2)^2 - (y + 2)$ , we have (3.12) and (3.13).

**Remark 3.2** Note that Example 3.1 satisfies all the hypotheses of Corollary 3.1. But Example 3.1 does not satisfy the hypotheses of Theorem 2.1.

# 4 Conclusions

In this paper, we proved some common fixed point theorems for four mappings f, g, S and T satisfying a nonlinear contraction in ordered metric spaces, where the mappings f and g are dominating and weak annihilators of the mappings T and S, respectively. We utilized the techniques of Haghi *et al.* [33] to derive our main result, which is a generalization of the result of Shobkolaei *et al.* [38]. Also, we introduced an example to support the usability of our results.

#### **Competing interests**

The authors declare that they have no competing interests.

#### Authors' contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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