Spectral Analysis of Block Structured Nonlinear Systems

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Abstract

A unified identification theory for nonlinear systems which can be represented by cascade, multiplicative and feedback connections of linear dynamic and static nonlinear subsystems is presented.

Introduction

Whilst a few years ago it was considered an acceptable practice to largely ignore the effects of nonlinearity on all but a few systems there is now an increasing awareness of the need to study, analyse and exploit nonlinear behaviour in systems design, CAE and modal analysis. A prerequisite to the analysis of nonlinear systems must be the development of methods of measuring or characterising the dynamical behaviour of such systems from input output measurement and this has been the objective of nonlinear systems identification [1,2,7] which has been an active area of research for some years now.

The aim of the present paper is to briefly review some identification results for block structured nonlinear systems. Although the results are not new[3], they have been known to the control engineering community for some years, they do not appear to have been noticed by researchers in other branches of engineering. A few authors such as Bendat and Piersol[4] have studied very special block structured systems which contain just square law operators. The results of the present study however show that the identification of a wide class of block structured systems can be unified in a very simple way by exploiting the theory of separable processes[5]. It would therefore appear timely to translate these results, which were developed in the time domain, into the frequency domain and to provide a unified theory for the spectral analysis of block structured nonlinear systems.

A detailed derivation of the results is given elsewhere and will not be repeated here. Our objective will be to present the main features of the results and to emphasise how they can be applied and interpreted in nonlinear spectral analysis.

2. Block Structured Systems:

Block structured systems[1] are systems which can be represented by interconnections of linear dynamic models and static nonlinear elements. Although these systems can, under fairly mild conditions, be represented by the

Volterra or Wiener series this approach tends to camouflage the very simple relationships which can be derived between the inputs and outputs of such systems. The basic philosophy underlying the study of block structured systems has been to try and identity each of the component subsystems in a manner which preserves the structure of the system under investigation[6]. This approach means that the identified model has a physical interpretation in relation to the real system and leads to identification algorithms which are much simpler than the functional series based methods[1,2].

Block structured systems have been studied by several authors, and an historical development of the field has been given elsewhere[1,3]. One of the first significant developments was by Korenberg[8,9] who analysed cascade systems and who derived two correlation results by analysing a Volterra representation of the system. Billings and Fakhouri[5,6] were able to generalise these results by introducing the theory of separable processes and showed how they could be used to determine the ordering of the linear dynamic and static nonlinear components or the system structure. This approach allowed the results for a whole family of cascade systems to be unified and provided a theoretical base on which to extend the ideas to feedback[10], multiplicative[11] and other block structured systems[12,13]. It is this latter approach which will therefore be reviewed in the present paper.

2.1 Cascade Systems

Because cascade block structured systems contain just two components, linear dynamic elements and static nonlinear characteristics, the easiest way to analyse them is to study each of these components in isolation initially and then to combine them.

The linear dynamic subsystems can of course be represented by the convolution integral

$$x(t) = \int_o^\infty h(\tau_1)u(t-\tau_1)d\tau_1 \qquad (1)$$

or the Wiener-Hopf eqn

$$\phi_{ux}(\tau) = \int_o^\infty h(\tau_1)\phi_{uu}(\tau-\tau_1)d\tau_1 \qquad (2)$$

where $\phi_{ux}(\tau)$ and $\phi_{uu}(\tau)$ represent the cross and auto-correlation functions respectively. It will be assumed that the nonlinear sybsystems can be represented by a single valued instantaneous nonlinear element $F[-]$ so that an input $x(t)$ would produce an output

$$y(t) = F[x(t)] \qquad (3)$$

which in theory could be represented as polynomial expansion

$$y(t) = \sum_{i=1}^{k} \gamma_i x^i(t) \tag{4}$$

Let $f(x_1,x_2;\tau)$ be the second order probability density function of the process $x(t)$ and define

$$g(x_2,\sigma) = \int_{-\infty}^{\infty} x_1 f(x_1,x_2;\sigma)\ dx_1 \tag{5}$$

If this function separates as

$$g(x_2,\sigma) = g_1(x_2)g_2(\sigma)\ \forall x,\sigma \tag{6}$$

then $x(t)$ is said to be a separable process [4,5] where

$$g_1(x_2) = \frac{x_2 f(x_2)}{g_2(o)} \quad ; \quad g_2(\sigma) = \frac{g_2(o)\phi_{xx}(\sigma)}{\phi_{xx}(o)} \tag{7}$$

Fortunately, the separable class of random processes is fairly wide and includes the Gaussian process, sine wave process and pseudorandom binary sequences[5].

Define the cross-correlation between the input and output of $F[\cdot]$ as

$$\phi_{xy}(\sigma) = \int_{-\infty}^{\infty} x_1 F[x_2]\ f(x_1,x_2;\sigma)\,dx_1 dx_2 \tag{8}$$

Substituting from eqn's (5) and (7) yields

$$\phi_{xy}(\sigma) = \int F[x_2]\ g_1(x_2)\ g_2(\sigma)\,dx_2$$

$$= \frac{\phi_{xx}(\sigma)}{\phi_{xx}(o)} \int F[x_2]\ x_2 f(x_2)\,dx_2$$

$$\phi_{xy}(\sigma) = C_F\ \phi_{xx}(\sigma) \tag{9}$$

where C_F is a constant.

Equation (9), which is known as the invariance property,[14], shows that $\phi_{xy}(\sigma)$ is directly proportional to $\phi_{xx}(\sigma)$ for any static nonlinear characteristic $F(.)$ providing $x(t)$ is separable.

The results of eqn's (2) and (9) represent the building blocks which can be used to construct a unified theory of spectral analysis for cascade systems[5].

2.1.1 The General Model

Consider the extension of these results to the cascade system illustrated in Fig.1 which is usually referred to as the general model[6].

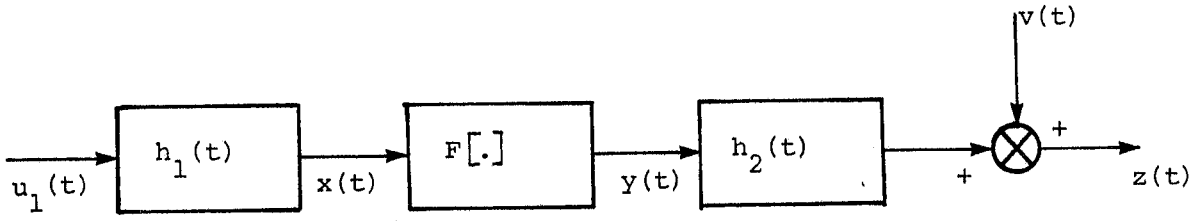

Fig.1 The General Model

The objective is to identify $h_1(t), h_2(t)$ and $F[.]$ from measurements of $u_1(t)$ and $z(t)$ only; the internal signals $x(t)$ and $y(t)$ are not available for measurement.

This can be achieved by computing the first and second degree correlation functions.

Considering the first degree correlation function initially the additional assumption that separability is preserved under linear transformation is required to ensure that if $u_1(t)$ is separable then $x(t)$ the input to the nonlinear element is also separable[5].

From Fig.1

$$y(t) = \int h_2(\theta)y(t-\theta)d\theta = F\{\int h_1(\tau_1)u_1(t-\tau_1)d\tau_1\}$$
$$= \int Q(t,\tau_1,u_1,h_1)u_1(t-\tau_1)d\tau_1$$

so that

$$z(t) = \int\int h_2(\theta)Q(t-\theta,\tau_1,u_1,h_1)u_1(t-\theta-\tau_1)d\theta d\tau_1 + v(t) \qquad (10)$$

where $Q(.)$ is a function of t,τ_1,u_1 and $h_1(t)$, which can be evaluated by deriving the Volterra expansion for $z(t)$. The first degree cross-correlation function between $u(t)$ and $z(t)$ is defined as

$$\phi_{uz}(\varepsilon) = \int\int h_2(\theta)\ \overline{Q(t-\theta,\tau_1,u_1,h_1)u_1(t-\theta-\tau_1)u(t-\varepsilon)}\ d\tau_1 d\theta$$
$$+ \overline{v(t)u(t-\varepsilon)} \qquad (11)$$

where the bar ——— indicates time average. Correlation with $u(t)$ which is a function of the input $u_1(t)$ is deliberate, the reason for this choice will become apparent shortly.

If $u(t)$ is separable with respect to $x(t)$ then from the invariance property eqn (9)

$$\phi_{uy}(\sigma) = C_{FG}\phi_{ux}(\sigma) \qquad (12)$$

holds across the nonlinearity where C_{FG} is a constant called Booton's equivalent given. Expanding (12) using the results from (10)

$$\overline{\int Q(t_1 \tau_1 u_1, h_1) u_1(t-\tau_1) u(t-\sigma) d\tau_1}$$

$$= C_{FG} \int h_1(\tau_1) \overline{u_1(t-\tau_1) u(t-\sigma)} d\tau_1 \tag{13}$$

and substituting into (11) yields

$$\phi_{uz}(\varepsilon) = C_{FG} \iint h_2(\theta) h_1(\tau_1) \phi_{uu_1}(\varepsilon-\theta-\tau_1) d\theta d\tau_1 + \phi_{uv}(\varepsilon) \tag{14}$$

This result represents an extension of the Wiener-Hopf eqn to a class of nonlinear systems[5,6]. Taking the Fourier Transform of eqn (14) and assuming that the noise is zero mean and independent of u(t) yields

$$S_{uz}(j\omega) = C_{FG} H_2(j\omega) H_1(j\omega) S_{uu_1}(\omega) \tag{15}$$

Providing the input $u_1(t)$ to the general model Fig.1 is separable under linear transformation therefore eqn (15) will hold for all static nonlinear characteristics $F[.]$. Indeed inspection of eqn (15) shows that the nonlinear characteristic only affects the constant C_{FG} in eqn (15) and consequently the experimenter may be misled into believing that the system under investigation is linear when in fact it may contain a severe nonlinearity

To derive the second degree correlation function for the system in Fig.1 the additional assumption that $u^2(t)$ is separable with respect to u(t) will be required so that by repeating the analysis for eqn (9) with this additional assumption yields

$$\phi_{u^2 y}(\sigma) = C_{FFG} \phi_{u^2 x}(\sigma) \tag{16}$$

across the nonlinear element $F[.]$ in Fig.1 where C_{FFG} is a constant. This is known as separability under double nonlinear transformation[5] and although this is not a general property of all separable processes it does hold when the input $u_1(t)$ is Gaussian. Defining the second degree correlation function

$$\phi_{u^2 z}(\varepsilon) = \iint h_2(\theta) \overline{Q(t-\theta, \tau_1, u_2, h_1) u_1(t-\theta-\tau_1) \overline{u^2(t-\varepsilon) d\theta d\tau_1}} \overline{+u^2(t-\varepsilon) v(t)} \tag{17}$$

and utilizing the invariance property eqn (16) for double nonlinear transforms

by following exactly the same analysis that was used in eqn (13) yields

$$\phi_{u^2z}(\varepsilon) = C_{FFG}\iiint h_2(\theta)h_1(\tau_1)h_1(\tau_2)\overline{u_1(t-\theta-\tau_1)u_1(t-\theta-\tau_2)}\,\overline{u^2(t-\varepsilon)}\,d\tau_1 d\tau_2 d\theta + \phi_{u^2v}(\varepsilon) \quad (18)$$

The noise term $\phi_{u^2v}(\varepsilon)$ tends to zero when v(t) is zero mean and independent of u(t). Eqn (18) provides a second relationship which, apart from the scale-factor C_{FFG} is independent of the nonlinear element F[.]. Finally, setting $u_1(t) = u(t)+b$ where u(t) is a zero mean Gaussian signal with a spectral density of P watts per cycle, b is a non zero constant representing the dc shift in $u_1(t)$ and correlating with $z'(t) = z(t) - \overline{z(t)}$ eqn's (14) and (18) reduce to

$$\phi_{uz'}(\varepsilon) = C_{FG}\int h_1(\tau_1)h_2(\varepsilon-\tau_1)d\tau_1 \quad (19)$$

$$\phi_{u^2z'}(\varepsilon) = C_{FFG}\int h_2(\tau_1)h_1^2(\varepsilon-\tau_1)d\tau_1 \quad (20)$$

Providing $h_1(t)$ is stable bounded inputs bounded outputs C_{FG} and C_{FFG} are constants given by

$$C_{FG} = P\gamma_1 + 2P\gamma_2 b\int h_1(\theta)d\theta + 3\gamma_3 P^2\int h_1^2(\theta)d\theta + 3P\gamma_3 b^2\iint h_1(\tau_1)\,h_1(\tau_2)d\tau_1 d\tau_2 + \ldots \quad (21)$$

$$C_{FFG} = 2\gamma_2 P^2 + 6\gamma_2 bP^2\int h_1(\theta)d\theta + \ldots \quad (22)$$

Taking the Fourier transform of eqn's (19) and (20) yields

$$S_{uz'}(j\omega) = C_{FG}\,H_1(j\omega)\,H_2(j\omega) \quad (23)$$

$$S_{u^2z'}(j\omega) = C_{FFG}\,H_2(j\omega)\,H_3(j\omega) \quad (24)$$

where $H_3(j\omega)$ is the Fourier Transform of $h_1^2(t)$ such that

$$H_3(j\omega) = 1/2\pi\int H_1(j\xi)\,H_1(j\omega-j\xi)d\xi$$

Notice that the estimates eqn's (23) and (24) are obtained by injecting an input u(t)+b, u(t) is zero mean Gaussian b≠o, into the system recording the output z(t) and then computing the cross-spectral densities between u(t) and $z'(t)$, and $u^2(t)$ and $z'(t)$ respectively. The estimates can therefore be readily obtained using a standard spectrum analyser with the only additional requirement that the input is squared to compute $S_{u^2z'}(j\omega)$. The input u(t)+b is used to ensure that the results are valid for both even and odd nonlinearities.

The dc offset b ensures that all terms in eqn's (21) and (22) contribute to C_{FG} and C_{FFG}[5,6].

Equations (23) and (24) provide two equations in terms of the two unknowns $H_1(j\omega)$ and $H_2(j\omega)$. Estimates of the linear system frequency response functions $\mu_1 H_1(j\omega)$ and $\mu_2 H_2(j\omega)$ can be obtained to within scale factors μ_1 and μ_2 by using a least squares decomposition routine [15,6,16]. Once the linear subsystems in Fig.1 have been identified the problem is reduced to fitting a polynomial[6], a series of straight line segments [17] or any other appropriate function to the nonlinear characteristic. Identification based on this analysis thus effectively decouples the estmation of the linear and nonlinear subsystems and this means that even systems with very violent nonlinearities can be readily identified[17]. This is a distinct advantage compared with the Volterra or Wiener functional series based methods which could be applied to the system of Fig.1. Application of the functional series methods however destroys the structure of the system, provides virtually no insight into the operation of the system and if the non-linearity $F[.]$ were violent would require the identification of at least the third, fourth and perhaps even an infinite number of kernels to provide an adequate description of the system[1,2,7].

The results of eqn's (23) and (24) are in fact estimates of the first and second order Volterra kernels for the general model. So that even though the Volterra series for this sytem may contain a large number of higher order terms they all collapse under the theory of separable processes to the form of eqn's (23) and (24) for this model structure. This provides a simple and elegant extension of the results for linear systems and demonstrates that systems within this class can be identified in a manner which provides a great deal of insight into the operation of the process.

The results can be extended to the case with pseudo random binary inputs [13] and an analysis showing the influence of record length, mean level b, power and bandwidth of the input, decomposition algorithm and noise on the accuracy of the estimates together with recommendations regarding the choice of these variables is available in the literature [16].

The results do of course apply to all subclasses of the general model and these are studied below.

2.1.2 The Wiener Model

The Wiener model[18] which is illustrated in Fig.2 consists of a linear system followed by a static nonlinear element.

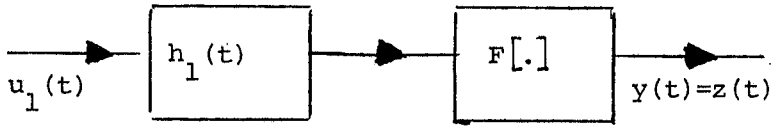

Fig.2 The Wiener Model

The Wiener model can clearly be obtained from the general model Fig.1 by setting $h_2(t)=\delta(t)$ so that $y(t) = z(t)$ and the results of eqn's (19), (20) and (23), (24) become

$$\phi_{uz'}(\varepsilon) = C_{FW}\, h_1(\varepsilon) \tag{25}$$

$$\phi_{u^2z'}(\varepsilon) = C_{FFW} h_1^2(\varepsilon)$$

such that

$$S_{uz'}(j\omega) = C_{FW} H_1(j\omega)$$

$$S_{u^2z'}(j\omega) = C_{FFW} H_3(j\omega) \tag{26}$$

$$H_3(j\omega) = 1/2\pi \int H_1(j\xi) H_1(j\xi)-j\xi)\,d\xi$$

where $C_{FW}=C_{FG}$, $C_{FFW}=C_{FFG}$ and these are defined by eqn's (21) and (22)

Identification of the Wiener model is therefore straightforward. Computation of the cross-spectral density $S_{uz'}(j\omega)$ thus provides an estimate of $H_1(j\omega)$ irrespective of $F[.]$. The nonlinear element can then be modelled as the second stage of the algorithm[18].

2.1.3 The Hammerstein Model

The Hammerstein model [19,20] which is illustrated in Fig.3 consists of a zero-memory nonlinear element followed by a linear dynamic subsystem.

Fig.3 The Hammerstein Model

Selling $h_1(t) = \delta(t)$ in eqn's (19), (20) and (23), (24) yields

$$\phi_{uz'}(\varepsilon) = C_{FH}h_2(\varepsilon)$$
$$\phi_{u^2z'}(\varepsilon) = 2C_{FFH}h_2(\varepsilon) \qquad (27)$$

such that

$$S_{uz'}(j\omega) = C_{FH}H_2(j\omega)$$
$$S_{u^2z'}(j\omega) = C_{FFH}H_2(j\omega) \qquad (28)$$

and $C_{FH} = P\gamma_1+2P\gamma_2b+3\gamma_3(P^2+b^2P)+.....$

$C_{FFH} = 2\gamma_2P^2+6\gamma_3bP^2+....$

Identification of the $H_2(j\omega)$ and $F[.]$ is therefore straightforward using $S_{uz'}(j\omega)$ only.

2.1.4 The Linear Model

If the general model is now reduced to a linear system by setting $F[.] = 1$ ($\gamma_1=1, \gamma_i=0\ \forall\ i>1$ in eqn (4)) in Fig.1 then the results of eqn's (19), (20), and (23) and (24) yield

$$\phi_{uz'}(\varepsilon) = \int h_1(\tau_1)h_2(\varepsilon-\tau_1)d\tau_1$$
$$\phi_{u^2z'}(\varepsilon) = 0\ \forall\ \varepsilon$$

and

$$S_{uz'}(j\omega) = H_1(j\omega)H_2(j\omega)$$
$$S_{u^2z}(j\omega) = 0 \qquad (30)$$

2.1.5 Structure Detection

All identification algorithms, specially those developed for nonlinear systems, should include a procedure which indicates the structure of the system under investigation. For the class of systems considered above this consists of determining the position of the nonlinearity with respect to the linear blocks. Billings and Fakhouri[5] showed that this information is implicit in the cross correlation and spectral estimates derived above, eqn's (19), (20) or (23) and (24).

The system is linear iff

$$\phi_{u^2z}(\varepsilon) = 0\ \forall\ \varepsilon \qquad (31)$$

or equivalently $S_{u^2z'}(j\omega) = 0 \;\forall\; \omega$

The second degree correlation or spectral density function thus provides a simple and convenient test for nonlinearity. Alternative tests for nonlinearity based on output measurements only are available in the literature[21].

If the system has the structure of a Hammerstein model, them from eqn's (27) and (28)

$$\frac{\phi_{uz'}(\varepsilon)}{\phi_{u^2z'}(\varepsilon)} = \text{Constant} \;\forall\; \varepsilon \tag{32}$$

or equivalently

$$\frac{S_{uz'}(j\omega)}{S_{u^2z'}(j\omega)} = \text{Constant} \;\forall\; \omega \tag{33}$$

If the system has the structure of a Wiener model, them from eqn's (25) and (26)

$$\frac{(\phi_{uz'}(\varepsilon))^2}{\phi_{u^2z'}(\varepsilon)} = \text{Constant} \;\forall\; \varepsilon \tag{34}$$

or equivalently

$$\frac{\int S_{uz'}(j\xi) S_{uz'}(j\omega - j\xi)\, d\xi}{S_{u^2z'}(j\omega)} = \text{Constant} \;\forall\; \omega \tag{35}$$

Finally, if none of the above conditions holds then the system may have the structure of the general model. This however is a necessary but not a sufficient condition which must be confirmed by analysing the residuals after $h_1(t)$, $h_2(t)$ and $F[.]$ have been identified[21]. Alternatively, a test devised by Chen et al[22] as an extension of the above ideas could be used but this is considerably more involved since it requires two-dimensional FFT's.

2.2 Feedback and Multiplicative Systems

The simplicity of the above results for cascade systems motivated study of feedback[10], multiplicative[11] and other block structured nonlinear systems[3,12]. As in the case of the general model the objective was to identify the individual elements of the system from input/output measurements only such that the structure of the process is preserved and truncation errors normally associated with a finite Volterra series description are avoided. Unfortunately, the results for these systems are not quite so straightforward.

For example the unity feedback system illustrated in Fig.4 can be identified by extending the cascade system results and computing the first and second degree cross correlation or spectral density functions[10]. However, multilevel inputs must be used and this can result in a tedious experimental procedure and complex identification algorithms.

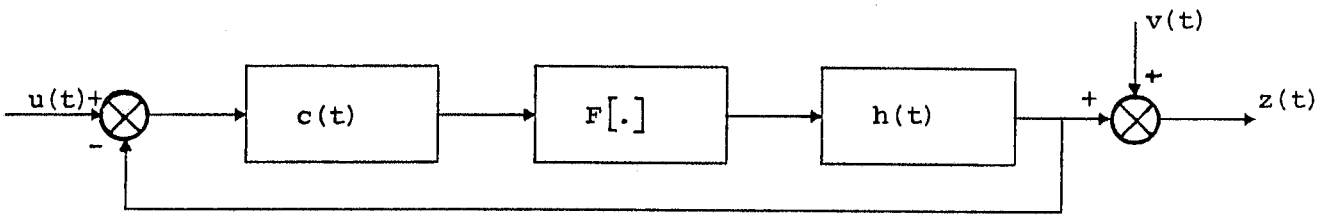

Fig.4 Nonlinear Feedback System

A similar algorithm can be derived for the $\mathbf{S}_m$ model[12] illustrated in Fig.5 which consists of a series of general models with reduced nonlinear elements connected in parallel with the outputs summated.

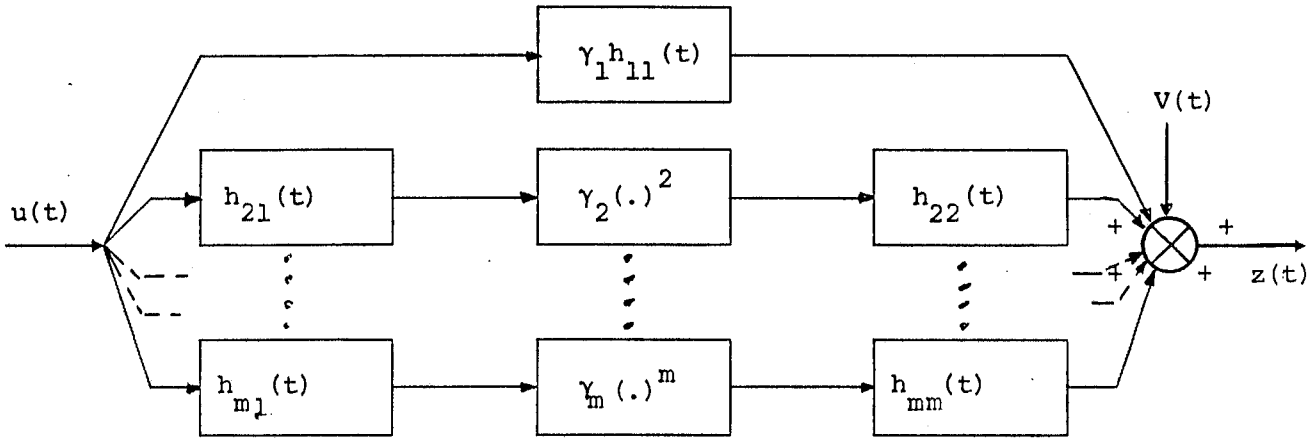

Fig.5 The S_m Model

To identify the individual component subsystems $h_{i,j}(t)$, from measurements of u(t) and z(t) only, for the multiplicative or factorable Volterra system[11] illustrated on Fig.6 however requires a slightly different approach.

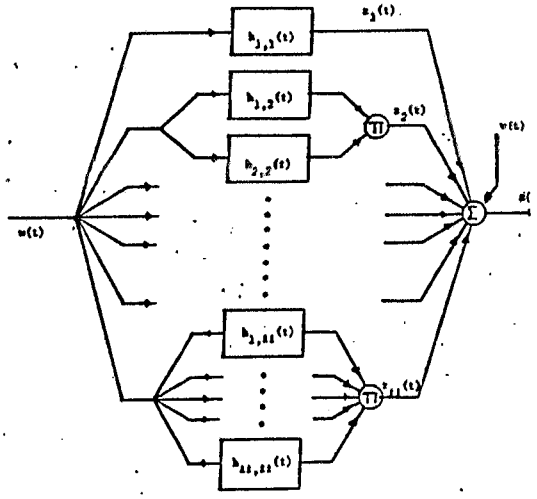

Fig.6 The factorable Volterra model.

Consider a factorable Volterra system which is composed of kernels up to order $\ell\ell$. If the input to this system is a compound input[11].

$$u(t) = \sum_{j=1}^{\ell\ell} x_j(t) \tag{36}$$

where $x_j(t)$, $j = 1,2,\ldots\ell\ell$ are zero mean independent processes with auto-correlation functions $\phi_{xx_j}(\tau) = \beta_j\delta(\tau)$, $j = 1,2,\ldots\ell\ell$ then estimates of the linear subsystems $h_{i,j}(t)$ can be easily obtained.

Computing the output correlation function $\psi^{\ell\ell}_{x_1\ldots x_{\ell\ell}z'}(\sigma_1,\sigma)$ based on a compound input eqn (36) yields

$$\begin{aligned}\psi^{\ell\ell}_{x_1\ldots x_{\ell\ell}z'}(\sigma_1,\sigma) &= \{z'(t)x_1(t-\sigma_1)\prod_{i=2}^{\ell\ell} x_i(t-\sigma)\}\Big/\{(\ell\ell-1)!(\prod_{n=1}^{\ell\ell}\beta_n)\} \\ &= \sum_{i=1}^{\ell\ell}\{h_{i,\ell\ell}(\sigma_i)\prod_{\substack{j=1\\ j\neq i}}^{\ell\ell} h_{j,\ell\ell}(\sigma)\} \\ &= \psi^{\ell\ell}_{x_1\ldots x_{\ell\ell}z'_{\ell\ell}}(\sigma_1,\sigma)\end{aligned} \tag{37}$$

Thus computing $\psi^{\ell\ell}_{x_1\ldots x_{\ell\ell}z'}(\sigma_1,\sigma)$ based on a compound input and the total system output $z'(t) = z(t) - \bar{z}(t)$ has effectively isolated the $\ell\ell$'th order kernel, or the subsystem with output $z_{\ell\ell}(t)$ in Fig.6 This result holds exactly even for compound prbs inputs[11]. The most surprising part of the result is the fact that $\psi^{\ell\ell}(\sigma_1,\sigma)$ is always a second order correlation function. The dimensionality of the correlation functions does not increase with the order of the kernel as is the case with the Lee and Schetzen[23] and all other algorithms. This is a direct consequence of the compound input.

Once $\psi^{\ell\ell}_{x_1\ldots x_{\ell\ell}z'}(\sigma_1,\sigma)$ has been measured the individual subsystems $h_{i,\ell\ell}(t)$ can be estimated by using a least squares routine[11] to decompose eqn.(37). The predicted output of the $\ell\ell$'th order subsystem can then be computed

$$\hat{z}_{\ell\ell}(t) = \int\ldots\int\hat{h}_{1,\ell\ell}(t_1)\ldots\hat{h}_{\ell\ell,\ell\ell}(t_{\ell\ell})\ (\prod_{j=1}^{\ell\ell}\sum_{i=1}^{\ell\ell} x_1(t-t_i))\ dt_1\ldots dt_{\ell\ell} \tag{38}$$

and a reduced system output $\hat{zz}'_{\ell\ell-1}(t) = z'(t) - \hat{z}'_{\ell\ell}(t)$ can be defined.

Repeating the above procedure the $(\ell\ell-1)$'th kernel can now be identified by computing the $(\ell\ell-1)$'th system output correlation function $\psi^{\ell\ell-1}_{x_1 \ldots x_{\ell\ell-1} zz'}(\sigma_1,\sigma)$ and estimating $\hat{h}_{i,\ell\ell-1}(t)$, $i=1,2,\ldots\ell\ell-1$ etc. The linear systems associated with all the remaining kernels or subsystems can be computed in an analogous manner.

Notice that the algorithm works backwards estimating the highest order kernel or subsystem first. This means that an estimate of the highest order kernel in the system must be available. If this is known however, from some simple tests on the system, then any order kernel can be obtained using second order correlation functions or equivalently a 2-D FFT algorithm only.

3. Simulation Result

Many simulation results are available in the literature[5,6,10,11,12] and it would serve no useful purpose to repeat these here. Because all the previous simulations have been concerned with the estimation of the equivalent system impulse response in the time domain it is however worthwhile considering a few representative simulations in the frequency domain. Two systems, a Hammerstein model and a Wiener model are therefore considered below.

The Hammerstein model illustrated in Fig.7a was simulated for a range of Gaussian inputs $u(t)+b$, where $\overline{u(t)}=0.0, b=0.1$ and $\sigma_B^2=0.01, \sigma_c^2=0.09$ and $\sigma_d^2=0.49$. In each case 1000 data pains were generated and $\hat{\phi}_{uz'}(\tau)$ and $\hat{\phi}_{uu}(\tau)$ were computed. An FFT algorithm together with a Hamming spectral window was then applied to the first 128 lags of the correlation functions. Fig.7b provides a comparison of the estimates $S_{uz'}(j\omega)/S_{uu}(j\omega)$ for each input and the theoretical frequency response function of the linear subsystem $H(j\omega)$. As expected from eqn (28) the gain estimates are, except for a constant multiplying factor, equal to $H(j\omega)$ because

$$\frac{S_{uz'}(j\omega)}{S_{uu}(j\omega)} = \frac{C_{FH}}{P} H(j\omega)$$

where $P = S_{uu}(j\omega)$ is the power spectral density of the Gaussian input. This also shows that all the phase estimates should be equal. Theoretically the estimates should differ by the constant term

$$CFH/P = \gamma_1 + 2\gamma_2 b + 3\gamma P_3 + 3B^2\gamma_3$$

where $\gamma_1=0.5$, $\gamma_2 = 0.0, \gamma_3 = 2$ so that for case B, $P = \sigma_B = 0.01$, $b = 0.1$

$$\therefore C_{FH}^{B}/P = 0.62$$

For case C, $P = \sigma_c = 0.09$, $b = 0.1$

$$\therefore C^{C}_{FH}/P = 1.1$$

and for case D, $P = \sigma_d = 0.49$, $b = 0.1$

$$\therefore C^{D}_{FH}/P = 3.5$$

These differences in the constant terms show that theoretically the gain estimate for case C should be 4.48dB above case B and case D should be 10.05dB above case C respectively and this is confirmed by the simulation results in Fig.7b.

The Wiener model illustrated in Fig.8a was also simulated for the same range of Gaussian inputs and spectral estimates were computed in exactly the same way as in the Hammerstein example, Fig.8b provides a comparison of the estimates $S_{uz'}(j\omega)/S_{uu}(j\omega)$ for each input and the theoretical frequency response function of the linear subsystem $H(j\omega)$. Clearly the gain estimates are, except for a constant scale factor, equal to $H(j\omega)$ and the phases are very similar as predicted by eqn(26). Theoretically the gain estimates should differ by the constant scale factor

$$C_{FW/P} = \gamma_1 + 2\gamma_2 b\int h(\theta)d\theta + 3\gamma_3 P\int h^2(\theta)d\theta + 3\gamma_3 b^2\iint h(\tau_1)h(\tau_2)d\tau_1 d\tau_2 + \ldots$$

when $\gamma_1 = 0.5, \gamma_2 = 0, \gamma_3 = 0.2$, $\int h(\tau_1)d\tau_1 = 7.5$, $\int h^2(\tau_1)d\tau_1 = 18.88$ such that for case B $C^{B}_{FW}/P = 0.95$, for case C $C^{C}_{FW}/P = 6.388$. These differences suggest that theoretically the gain estimate of case C should be 5.81dB above case B and case D should be 10.73dB above case C respectively and this is confirmed by the simulation results in Fig.8b.

4. Conclusions

A unified theory for the identification of nonlinear block structured systems has been presented. It has been shown that such systems can be identified in terms of the individual component subsystems often using very simple extensions of the well known correlation and spectral analysis algorithms.

The algorithms provide an alternative to the functional series methods based on the Volterra or Wiener expansions. Providing the system can be represented by one of the models in the block structured systems class the individual component subsystems can be identified in a manner which preserves the system structure and provides valuable insight into the operation of the system.

Even systems with very violent nonlinearities such as dead zone or saturation[17] can be identified. Because the algorithms decouple the identification of the linear and nonlinear component subsystems and essentially extract all the information from the first two Volterra kernels problems associated with truncation errors, multidimensional transforms and excessive computational requirements which are the curse of functional series methods are avoided[1,2].

Like the functional series methods which are based on Gaussian inputs the block structured algorithms are designed to work with certain classes of input signal, separable processes or compound inputs. Whilst early implementations of the algorithms suggested that typically 10,000 data points were required to obtain reliable estimates later analysis showed that by modifying the implementations slightly excellent estimates could be obtained with just a few thousand data pairs[16,25,26,26]. It should be emphasised that for brevity these modifications were not discussed in the present paper.

The results show quite clearly that whilst classical correlation and spectral analysis are excellent tools for analysing linear systems they must be applied with caution to systems with nonlinear characteristics. The results of previous sections have shown how averaging can be exploited to make certain nonlinear effects go to zero for certain classes of input. Classical correlation or spectral analysis for cascade systems for example yields estimates which are, apart from a scale factor, quite independent of the nonlinear characteristics. The nonlinearity, no matter how severe, becomes invisible with this type of analysis, it averages out to zero. The danger is that the extension of linear averaging techniques to nonlinear systems can lead to a totally false interpretation of the results. This will be all the more severe if the analyst has only one set of data to study because experimentation on the process is expensive. It would be very easy in such a situation to come to a totally misleading conclusion because certain terms have averaged out for that particular input.

The modern parametric methods[27] avoid many of these problems but much work remains to be done before nonlinearity can be measured in our systems and exploited in our designs.

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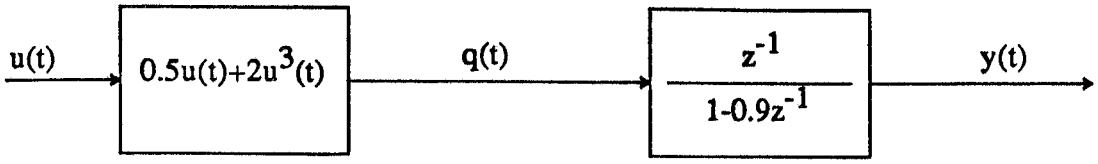
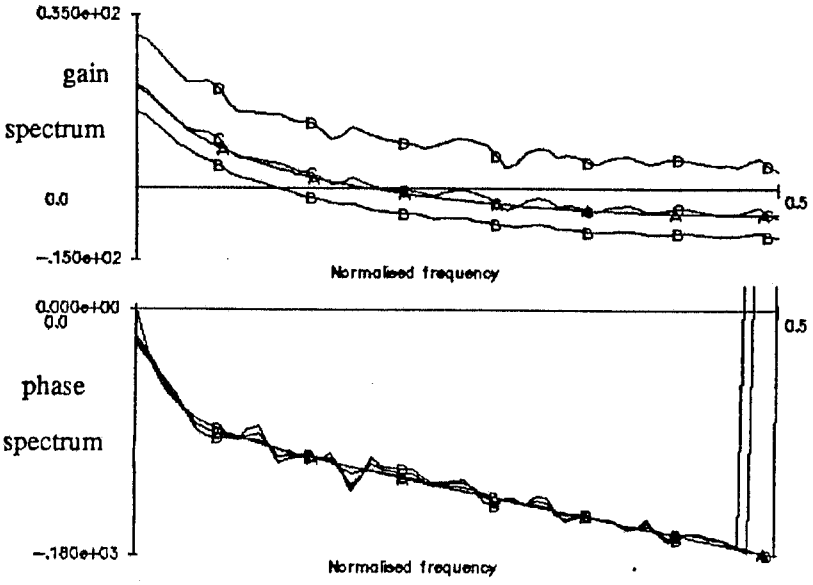


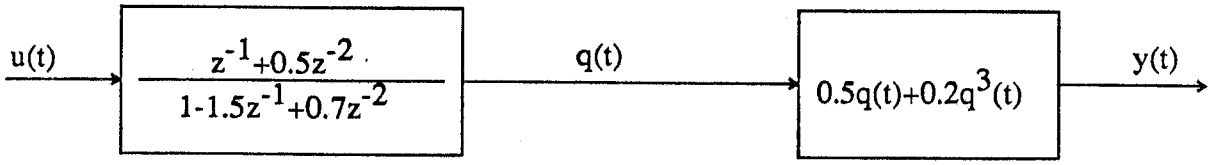
(a) Hammerstein model



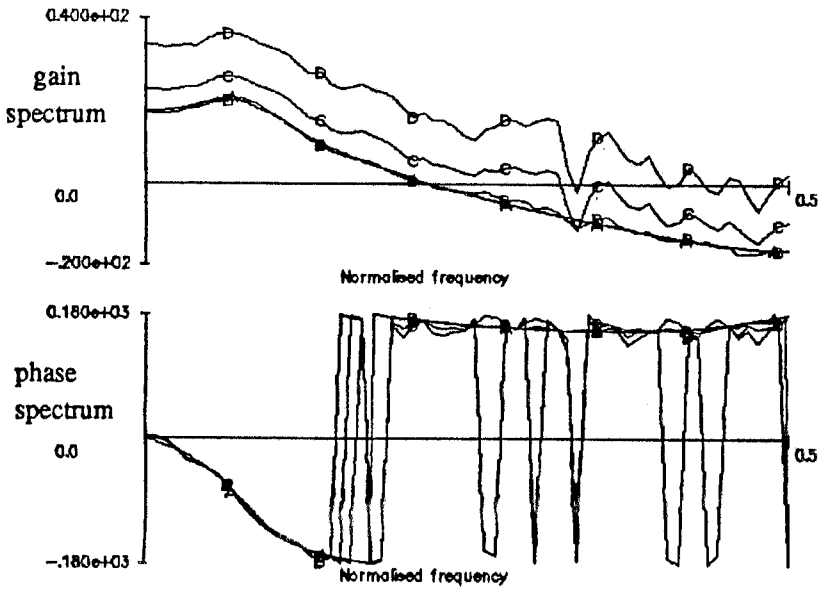
A:- H(jω)
B:- u(t), N(0.1, 0.01)
C:- u(t), N(0.1, 0.09)
D:- u(t), N(0.1, 0.49)

(b) Linear frequency response function estimates

Figure 7



(a) Wiener model



A:- H(jω)
B:- u(t), N(0.1, 0.01)
C:- u(t), N(0.1, 0.09)
D:- u(t), N(0.1, 0.49)

(b) Linear frequency response function estimates

Figure 8