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Semi-active Damping using a Hybrid Control Approach

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Abstract

In this paper, a hybrid control framework is used to design semi-active controllers for vibration reduction. It is shown that the semi-active skyhook damper, typically used for vibration reduction, can be recast in the framework of an event-driven intermittent controller. By doing this we can then exploit the well developed techniques associated with hybrid control theory to design the semi-active control system. Illustrative simulation examples are based on a two degree-of-freedom system, often used to model the dynamics of a quarter car body model. The simulation results demonstrate how hybrid control design techniques can improve the overall performance of the semi-active control system.

Keywords: Semi-active control, hybrid control, skyhook

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1 Introduction

Semi-active control is an increasingly important control method which is used in a wide range of structural control applications. For example, semi-active control methods have been used extensively for automotive semi-active suspension systems (Besinger et al., 1992; Kitching et al., 2000; Hong et al., 2002; Jalili, 2002; Sammier et al., 2003; Verros et al., 2005; Shen et al., 2006; Giorgetti et al., 2006; Biglarbegian *et al.*, 2008; Ihsan *et al.*, 2008; Turnip *et al.*, 2010; Collette and Preumont, 2010; Ahmadian and Blanchard, 2011; Crews *et al.*, 2011; Chen *et al.*, 2011). A more general introduction to semi-active control techniques can be found in Casciati et al. (2006) or Preumont (2002) and other relevant review articles are given by Housner et al. (1997), Spencer and Nagarajaiah (2003) and Wagg and Neild (2011). Semi-active control is also used in many civil/structural engineering applications — see for example, Chen and Chen (2004); Xu et al. (2006); Casciati and Ubertini (2008); Bani-Hani and Sheban (2006); Nagarajaiah and Narasimhan (2007); Bahar et al. (2010), and references therein.

There are two main reasons why semi-active control is of increasing importance; (i) semi-active approaches have some significant advantages over the passive or active control alternatives, and (ii) the advent of new damper technologies, particularly magneto-rheological (MR) dampers, have made practical implementations of semi-active controllers more straightforward (Jansen and Dyke, 2000; Yang et al., 2004). Magneto-rheological (MR) dampers have been successfully used for applications such as seat and vehicle suspensions (Choi and Han, 2003), helicopter dampers (Ngatu et al., 2010) and reducing vibration in bridges (Sahasrabudhe and Nagarajaiah, 2005). MR dampers belong to a wider class of fluid filled dampers which have been widely studied for semi-active control applications (starting with (Karnopp et al., 1974)). One of the advantages of using this type of device is that it will have some inbuilt passive capability which acts as a fail-safe in the system, should the semi-active control become disabled for some reason.

Of course, passive dampers have been used in a wide range of engineering applications for many years to reduce the effect of shock and vibration on mechanical systems (Soong and Dargush, 1997). Such devices are designed to dissipate unwanted vibration energy without requiring any external source of power to operate. They also possess the mathematical property of passivity, as defined, for example, by Anderson and Vongpanitlerd (2006). As discussed by Willems (1972), passive systems are a subset of dissipative systems. In the context of this paper the word passive will be used.

However, there are limitations to the effectiveness of passive damping techniques, particularly when the system being damped has multiple vibration frequencies or varying parameters. One solution is to use a control actuator instead of the damper, to create an actively controlled system. In general, this requires external power and can introduce issues with stability or robustness, which is unacceptable for many engineering applications. This paper shows how such active controllers can be rendered passive in the sense that they never inject power into the controlled system. This restricted active design can then be used to design semi-active controllers.

It was mentioned above that semi-active approaches have some significant advantages over the passive or active control alternatives. The most important advantage of semi-active controllers over active control approaches is that they are designed and implemented using, or as if using, passive devices. This means that semi-active control cannot add energy to the system, and the resulting control is normally unconditionally stable. The most important advantage of semi-active controllers over passive techniques is that the amount of control exerted over the system is far greater, and in some situations can approach that of a fully active control system. In this context semi-active control can be thought of as an excellent compromise between passive and active control, combining some of the major benefits of both whilst limiting the disadvantages.

The semi-active control effect using a semi-active element, such as a damper, is achieved by varying physical parameters within the semi-active element. For example, by varying the viscosity of a fluid inside a damper system. This approach was first introduced by Karnopp et al. (1974) to give a device that dissipates power whilst retaining some advantages of active control, without the problem of ensuring stability and robustness. The semi-active damper described by Karnopp et al. (1974) is a mechanical damper with a variable damping ratio implemented by a hydraulic valve. Many of the more recent studies have made use of MR dampers for a similar purpose (Preumont, 2002; Potter et al., 2010; Dong et al., 2010; Bahar et al., 2010).

However, semi-active damping can also be implemented using electromechanical transducers such as piezo-electric devices (Harari et al., 2009) or DC motors whilst still retaining the passivity property that power never flows into the controlled system although it is extracted some of the time. The technique also includes the possibility of using non-collocated sensor-actuator pairs — see for example Preumont (2002, Chapter 12).

A common semi-active control strategy is based on active skyhook control (Karnopp, 1995; Preumont et al., 2002; Hong et al., 2002), of which two main versions exist; continuous semi-active skyhook control and on-off skyhook control. The strategy is based on the idea that a mass can be effectively isolated from the support input by attaching a grounded passive damper, resulting in a damping force which resists the absolute velocity of the mass. As a grounding point is normally not available the semi-active damper is mounted between the mass and the support. For the continuous version, the damper is controlled to generate a damping force equal to that a grounded damper would impose on the mass. This is not however always possible as semi-active dampers are passive which results in the control signal being set to zero when a non-passive force is desired. The on-off version switches the damper between a high and a low damping value — see for example Potter et al. (2010) and references therein. The key control issue is determining when the resulting damper force is deleterious for vibration reduction and minimise its effect at these times via the switching mechanism in the controller. These two versions of semi-active skyhook control, continuous and on-off, are generalised in this paper. The former is seen as active controller which is switched off to prevent energy injection into the system; the latter is seen as a passive controller which is switched off when not acting like an active controller. Both can be implemented using a modulated damper or a force transducer.

In this paper we will reformulate both forms of semi-active control as hybrid control systems (van der Schaft and Schumacher, 2000; Haddad et al., 2006; Goebel et al., 2009).

By doing this we can then exploit the well developed techniques associated with hybrid control theory to design the semi-active control systems. One such hybrid controller is the event-driven intermittent controller which has been developed not only in the engineering context (Gawthrop and Wang, 2009b) but also in the physiological context (Asai et al., 2009; Gawthrop et al., 2011). In this paper the intermittent controller design framework will be used to enhance the design possibilities of a semi-active control system. In particular we use the example of Preumont (2002) to demonstrate how event-driven intermittent control can be used to redesign continuous and on-off semi-active skyhook control. The advantage is that the intermittent controller contains additional parameters and features which can be used to modify the switching behaviour while maintaining its passive nature and obtain an improved semi-active behaviour.

Section 2 gives background methods and results relating to passive damping, active damping, semi-active damping and intermittent control. Section 3 studies the differences between semi-active damping and intermittent control and the enhancements needed to each to give equivalent algorithms; an illustrative example is used throughout. Section 4 concludes the paper.

2 Background

This section provides the background material for the paper, which is illustrated using an example of Preumont (2002, Fig. 6.18). Section 2.1 looks at standard passive and active damping based control and Section 2.2 looks at the semi-active case and reinterprets it in a state-space control context. Section 2.3 summarises the features of event-driven intermittent control required in the rest of the paper.

2.1 Passive and active damping

To aid discussion of control of the various damper devices, namely passive, active and semi-active, the quarter car model used by Preumont (2002, Fig. 6.18) is considered.

Figure 1 shows the quarter car model, where mass M represents the tyre and mass m represents the quarter body. This system has four states which may be conveniently chosen as: the absolute velocities of the two masses, v_1 and v_2, the displacement of mass M relative to ground, z_1, and the displacement of mass m relative to mass M, z_2. Between the two masses are two damper devices: a passive damper with coefficient c and a controllable damper which exerts a force f on the masses. The controllable damper accepts a control signal u which directly generates the damper force, $u = f$. It is the generation of this control signal u that is of interest.

Using these four states, the system of Figure 1 can be written in state-space form as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{B}_d d(t) \tag{1}$$

$$y(t) = \mathbf{C}\mathbf{x}(t) \tag{2}$$

where the output y is the body velocity, $y = v_2$, the disturbance is the ground velocity, $d = v_0$, and the state vector, $\mathbf{x}$, is defined as

$$\mathbf{x} = \begin{bmatrix} v_2 \\ z_2 \\ v_1 \\ z_1 \end{bmatrix} \tag{3}$$

The matrices A, B, B_d and C are then given by:

$$\mathbf{A} = \begin{bmatrix} -\frac{c}{m} & -\frac{k}{m} & \frac{c}{m} & 0 \\ 1 & 0 & -1 & 0 \\ \frac{c}{M} & \frac{k}{M} & -\frac{c}{M} & -\frac{K}{M} \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \frac{1}{m} \\ 0 \\ -\frac{1}{M} \\ 0 \end{bmatrix}, \ \mathbf{B}_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \tag{4}$$

In the following discussion two further vectors are used

$$\mathbf{k}_r = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{k}_a = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

such that the velocity of mass m relative to mass M and the absolute velocity of mass m may be written as $\mathbf{k}_r\mathbf{x}$ and $\mathbf{k}_a\mathbf{x}$ respectively.

If the controllable damper is purely passive then the control signal may be written as

$$u = -c_p(v_2 - v_1) = -c_p\mathbf{k}_r\mathbf{x}, \tag{6}$$

where c_p is the damper coefficient; as u is explicitly generated by a damper, this is a passive controller.

Alternatively the controllable damper could mimic a skyhook damper via active control. A skyhook damper produces a damper force proportional to the velocity of the mass m, i.e. as if a damper with coefficient c_s were attached between the mass and a fixed mount. In this case it is represented by the state-space feedback equation:

$$u = -c_s v_2 = -c_s\mathbf{k}_a\mathbf{x}. \tag{7}$$

In this case the controller is *not* passive: although the actuator force is applied to both masses, the control signal is generated from just the velocity of mass m. This will be discussed further when considering the semi-active skyhook controller in Section 2.2.

Following Preumont (2002, Figure 6.18), we set

$$\begin{aligned} M &= 36\text{kg} \quad K = 160\text{kN/m} \\ m &= 240\text{kg} \quad k = 16\text{kN/m} \quad c = 980\text{Ns/m} \end{aligned} \tag{8}$$

The force generated by the controllable device, f, will depend on the control strategy chosen, but the damping constant used is 2000 Ns/m.

The velocity transmissibility $G_v(j\omega)$ is defined as:

$$G_v(j\omega) = \frac{v_2(j\omega)}{v_0(j\omega)} \tag{9}$$

Figure 2(a) shows $|G_v(j\omega)|$ corresponding to four cases:

1. No explicit control, $u = f = 0$, with minimal damping.
2. No explicit control, $u = f = 0$, with $c = 980$Ns/m.
3. Passive control, given by Equation (6), with $c_p = 2000$ and
4. Active skyhook control, given by Equation (7), with $c_s = 2000$.

The control transmissibility $G_f(j\omega)$ is defined as:

$$G_f(j\omega) = \frac{f(j\omega)}{v_0(j\omega)} \tag{10}$$

$G_f(j\omega)$ is zero for the first two cases, Figure 2(b) shows $|G_f(j\omega)|$ corresponding to the last two cases.

As discussed by Preumont (2002), Figures 2(a) and 2(b) illustrate the following properties. The physical damper c "open-loop" damps out the resonance at about 11Hz shown in the "minimal damper" plot; but, by itself, has little effect on the resonance at about 1.2Hz. Increasing the effective passive damping in case 3 reduces the resonant peak at about 1.2Hz but leads to a large control signal $u = f$; the skyhook active damper, case 4, is effective at removing the resonant peak at about 1.2Hz and, in conjunction with the physical damper c gives a well-damped response at all frequencies.

The rest of the paper will look at replacing the active controllable device with a semi-active device. To motivate this discussion it is helpful to consider a semi-active damper, i.e. a damper with a controllable damping coefficient, mounted between the two masses. To implement a control strategy, the device must generate an equivalent damping coefficient c_e such that the control force is imposed on the masses. For the passive controller, case 3, this is straightforward; $c_e = c_p$. For the active skyhook controller, case 4, the equivalent damping coefficient is given by

$$c_e = -\frac{u}{v_r} = \frac{\mathbf{k}_a\mathbf{x}}{\mathbf{k}_r\mathbf{x}} c_s. \tag{11}$$

In this case it can be seen that c_e can be negative as well as positive, corresponding to times when energy is injected into the system via the control device. In contrast for a semi-active device the equivalent damping is constrained such that $c_e \geq 0$. The transition from active control to semi-active control will now be considered with the aim of retaining the performance advantages of the active damper whilst being constrained to a purely passive device.

2.2 Semi-active damping

There are two approaches to *designing* a semi-active damper: (i) an active controller is made passive by switching off the control when in a non-passive region and (ii) a passive controller is forced to approximate an active control device by switching off the control when the sign of control signal generated by the passive and active controllers are different. Furthermore, there are two ways of *implementing* the semi-active damper: (i) explicitly generating the control signal and passing it to an actuator and (ii) implicitly applying the control signal using a modulated damper.

The first semi-active design approach is now considered. Any passive, or semi-active damper must satisfy the passive constraint (Karnopp et al., 1974, Equation (3)) or (Preumont, 2002, Sec.12.4):

$$uv_r \leq 0 \tag{12}$$

where u is the damper force (or control signal) and $v_r = \mathbf{k}_r\mathbf{x}$ the relative velocity of the ends of the damper.

In the case of the passive damper considered in the quarter body car example the control signal is given by $u = -c_p\mathbf{k}_r\mathbf{x}$, Equation (6), and therefore Equation (12) is satisfied. In general, however, a state-feedback control law (for example, that of Equation (7), or one designed using optimal control theory) does not correspond to a passive system. In state space form, the control signal may be expressed as

$$u = -\mathbf{k}\mathbf{x}. \tag{13}$$

As a result, the passive constraint (12) can be written as:

$$-uv_r = \mathbf{x}^T\mathbf{k}^T\mathbf{k}_r\mathbf{x} \geq 0 \tag{14}$$

This can be rewritten as:

$$\mathbf{x}^T\mathbf{Q}_t\mathbf{x} \geq 0, \quad \mathbf{Q}_t = \frac{1}{\mathbf{k}\mathbf{k}_r^T}\mathbf{k}^T\mathbf{k}_r, \tag{15}$$

where the scalar factor $\mathbf{k}\mathbf{k}_r^T$ has been included to normalise Q_t. Thus the passive constraint (12) defines regions in state space via the quadratic constraint (15); this point is discussed further in Sections 3.1 and 3.2.

Finally, the control signal, Equation (13), can be modified to meet the passive constraint, Equation (15), by writing

$$u = \begin{cases} -\mathbf{k}\mathbf{x} & \text{if } \mathbf{x}^T\mathbf{Q}_t\mathbf{x} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

Alternatively this control signal can be rewritten in terms of the equivalent damping c_e to give

$$c_e = -\frac{u}{v_r} = \begin{cases} \frac{\mathbf{k}\mathbf{x}}{\mathbf{k}_r\mathbf{x}} & \text{if } \mathbf{x}^T\mathbf{Q}_t\mathbf{x} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

Note that for the inequality $\mathbf{x}^T\mathbf{Q}_t\mathbf{x} \geq 0$ to be satisfied u and v_r must be of opposite sign and hence $c \geq 0$ is satisfied for all time. For the purposes of implementation, the maximum value of equivalent damping c_e can be limited to c_{max}. This constraint can also be included when the implementation is of the form of equation (16) by additionally limiting the control signal u in terms of the current state $\mathbf{x}$ so that:

$$|u| \leq c_{max}|\mathbf{k}_r\mathbf{x}| \tag{18}$$

The second semi-active design approach is now considered. If the sign of the control signal from the passive controller, $u = -c_p \mathbf{k}_r \mathbf{x} = -c_p v_r$ (Equation (6)), is the same as the sign of the control signal from the active controller, $u = -\mathbf{kx}$ (Equation (13)), then $uv_r \geq 0$ which is identical to inequality Equation (14). It follows that exactly the same quadratic switching surface as used in the first design approach, given by Equation (15), is required for the second design approach. However, we note that the control signal is different as it is based on a passive controller. When (15) is satisfied $u = -c_p \mathbf{k}_r \mathbf{x}$. When the inequality is not satisfied the control signal generated by the passive system opposes that generated by an active one and thus the control signal is set to zero. That is:

$$u = \begin{cases} -c_p \mathbf{k}_r \mathbf{x} & \text{if } \mathbf{x}^T \mathbf{Q}_t \mathbf{x} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Or in terms of the equivalent damping of the device

$$c_e = \begin{cases} c_p & \text{if } \mathbf{x}^T \mathbf{Q}_t \mathbf{x} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Considering the skyhook controller, where $\mathbf{k} = c_s \mathbf{k}_a$ (comparing Equations (13) and (7)), then Equation (17) corresponds to the continuous semi-active skyhook controller of Preumont (2002, Section 12.6.1) and Equation (20) to the on-off semi-active skyhook controller of Preumont (2002, Section 12.6.2). Liu et al. (2005) also discuss these two types of semi-active controller.

2.3 Hybrid control and Intermittent control

The intermittent controller was introduced (in the engineering field) by Ronco et al. (1999) and subsequently developed by Gawthrop and Wang (2007). The intermittent controller has both event driven (Gawthrop and Wang, 2009b) and constrained (Gawthrop and Wang, 2009a) versions.

The intermittent controller combines both continuous-time and discrete-time features and is thus a hybrid controller and results in a hybrid dynamical system. Early books on hybrid systems include Flugge-Lotz (1953, 1968) and more recent books include those of van der Schaft and Schumacher (2000) and Haddad et al. (2006) and a recent tutorial is given by Goebel et al. (2009). As discussed by, for example, van der Schaft and Schumacher (2000), hybrid systems can use the methodology of discrete-event systems and finite state machines; but this aspect is not relevant to intermittent control. As discussed by Haddad et al. (2006) and Goebel et al. (2009) a subset of hybrid systems is described by *impulsive* systems; the impulsive framework provides a natural description of intermittent control.

Intermittent control has an interpretation which contains a generalised hold (Gawthrop and Wang, 2007). One particular form of hold is based on the closed-loop system dynamics of an underlying continuous control design: this will be called the *System-Matched Hold* (SMH) in this note. As discussed by Gawthrop and Wang (2009b), this form of hold is also related to the *control signal generator* of Astrom (2008).

Intermittent control makes use of two time frames:

1. **continuous-time** t, within which the controlled system, Equations (1) and (2), evolves.

2. **discrete-time** points t_i at which feedback occurs. Thus, for example, the ordered discrete-time time instants are denoted t_i and the ith intermittent interval is defined as

$$\Delta_i = t_{i+1} - t_i > 0 \tag{21}$$

   It is reasonable to design intermittent control so that Δ_i is bounded above and below so that:

$$0 < \Delta_{min} \leq \Delta_i \leq \Delta_{max} \tag{22}$$

The intermittent controller considered here is based on the continuous-time controller of the form of Equation (13) but a sample and hold element with state $\mathbf{x}_h$ is interposed between state and controller to give:

$$u(t) = -\mathbf{k}\mathbf{x}_h(t) \tag{23}$$

$$\text{where } \dot{\mathbf{x}}_h = A_h\mathbf{x}_h(t) \qquad t \neq t_i \tag{24}$$

$$\text{and } \mathbf{x}_h(t) = \mathbf{x}(t) \qquad t = t_i \tag{25}$$

The hold matrix A_h is chosen to match the closed-loop system defined by the controlled system (1) and feedback controller (13) and is thus given by:

$$A_h = A - B\mathbf{k} \tag{26}$$

In the event driven case (Gawthrop and Wang, 2009b), the event detector continually monitors the system state $\mathbf{x}$ and generates an event whenever a quadratic function of the system state exceeds a threshold q_t^2:

$$E_x = \mathbf{x}^T(t)\mathbf{Q}_t\mathbf{x}(t) - q_t^2 \geq 0 \tag{27}$$

where $\mathbf{Q}_t$ is a positive semi-definite matrix. A sample is taken at time $t_i = t$ when the following logical statement is true:

$$(E_x(t) \geq 0) \text{ AND } (t - t_{i-1}) \geq \Delta_{min} \tag{28}$$

3 Combining Semi-active Damping and Hybrid Control

Semi-active control (Section 2.2) and intermittent control (Section 2.3) have many similarities but also some differences. This section focuses on the differences and how they can be used to cross-fertilise the two areas. The main differences are:

1. Intermittent control has a discrete-time aspect due to the minimum interval Δ_{min} between the feedback times t_i whereas semi-active control is continuous-time.

2. Intermittent control is *open-loop* when the inequality, Equation (27), is not satisfied whereas semi-active control is *zero* (see Equation (16)).

3. The inequality in Equation (27) contains the positive constant q_t whereas the inequality in Equation (15) does not.

The following sections provide a rapprochement between the two types of control and, more importantly, show how the semi-active controller can be extended to give useful properties.

3.1 Intermittent control implementation of semi-active damping

This section looks at how intermittent control can be modified to yield the same algorithm as semi-active control; the following sections look at how the features of intermittent control which are discarded in this section can be used to enhance semi-active control.

The first difference can be simply overcome by setting Δ_{max} to be a small value with respect to closed-loop system time constants. Such a discrete-time approximation of continuous time control is required for digital implementation anyway.

The second difference requires an alternative version of the intermittent control model; in particular, when the relevant inequality is not satisfied, "open-loop" should be replaced by "zero". With this in mind, a modified version of intermittent control is suggested where an additional jump equation is added to Equations (23)–(25) to reset the hold state to zero at times t'_k where the switching function $E_x(t)$, Equation (27), changes from positive to negative. In particular the modified intermittent control is given by:

$$u(t) = -\mathbf{k}\mathbf{x}_h(t) \tag{29}$$

$$\text{where } \dot{\mathbf{x}}_h = A_h \mathbf{x}_h(t) \qquad (t \neq t_i) AND (t \neq t'_k) \tag{30}$$

$$\mathbf{x}_h(t) = \mathbf{x}(t) \qquad t = t_i \tag{31}$$

$$\text{and } \mathbf{x}_h(t) = \mathbf{0} \qquad t = t'_k \tag{32}$$

The hold is reset to zero taken at time $t = t'_k$ when the quadratic switching function $E_x(t)$ first becomes zero after a positive period. In other words the following logical statement is true:

$$(E_x(t - \Delta_{min}) \geq 0) \text{ AND } (E_x(t) < 0) \tag{33}$$

The new jump equation (32) together with the hold equation (30) ensures that $\mathbf{x}_h = \mathbf{0}$ (and thus $u(t) = 0$) for times when $E_x(t)$ is negative. The replacement set of intermittent control equations, Equations (29)–(32), will be used for the examples in the rest of this paper.

The third difference can be eliminated by using $q_t = 0$ in Equation (27). However, $q_t > 0$ is exploited in Section 3.2.

3.1.1 Example

Again considering the skyhook control strategy for the controllable damper, where $\mathbf{k} = c_s \mathbf{k}_a$, the switching matrix $\mathbf{Q}_t$, defined in Equation (15), is:

$$\mathbf{Q}_t = \frac{1}{\mathbf{k}\mathbf{k}_r^T} \mathbf{k}^T \mathbf{k}_r = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{34}$$

The inequality, Equation (27), then becomes the quadratic form:

$$x_1^2 - x_1 x_3 = v_2^2 - v_1 v_2 \geq q_t^2 \tag{35}$$

As Equation (35) only includes two of the four state components, it can be represented in two dimensions. In particular, the shaded regions of Figure 3(a) correspond to the values of x_1 and x_3 that satisfy (35) for $q_t = 0$.

The controlled system, Equations (1) and (2), was simulated using the intermittent controller. The ground velocity v_0 was sinusoidal. The intermittent control equivalent of each of three controllers were simulated: the semi-active continuous control of (17) limited to a maximum damping of $c_{max} = 4c_s$ using (18) and shown in Figure 4; semi-active continuous control of (17) limited to a maximum damping of $c_{max} = c_s$ and shown in Figure 5 and the semi-active continuous control of (20) and shown in Figure 6. The switching process means that neither the control signal u nor the output y is sinusoidal. The frequency responses of Figures 4– 6 were computed from the fundamental frequency of the corresponding Fourier series. In each of Figures 4(a)– 6(a), the firm black line shows the transmissibility from the ground velocity v_0 to the mass velocity (the "x" symbols are discussed in Section 3.2.1). The transmissibility corresponding to the passive damper system is shown as the dashed (upper) line for comparison. In addition, to assess the loss of performance when compared to an active system, the performance of the active skyhook controller, case 4 in Figure 2, is shown as the dotted (lower) line. Figures 4(b)– 6(b) show the corresponding transmissibilities from ground to control signal u.

Figures 4– 6 indicates that the performance of the three semi-active controllers lies between the performance of the purely passive and the purely active controllers. The two continuous controllers behave like the passive controller at low frequencies and the active controller at high frequencies; the passivity constraint (12) limits the performance at low frequencies. Furthermore, comparison of Figures 4– 5 shows that reducing the maximum allowed damping coefficient further reduces mid frequency performance whilst reducing the control amplitude at those frequencies. Unlike the two continuous controllers, the switched controller behaves like the passive controller at high frequencies; this is because it cannot give the smaller equivalent damping required by the active controller at these frequencies.

To examine this behaviour further, the black line in Figures 7– 9 show the equivalent damping coefficient over one period at four different frequencies for the three controllers – the grey line is discussed in Section 3.2.1. These signals show the switching corresponding to the system state crossing the boundaries of the shaded regions of Figure 3(a). In the case of Figure 7, the equivalent damping c_e is bounded above by $4c_s$; in the case of Figure 8, the equivalent damping c_e is bounded above by c_s and in the case of Figure 9, the equivalent damping $c_e = c_s$.

Both semi-active controllers are non-linear and thus, as discussed above, the frequency plots only show the fundamental frequency response for each individual sinusoidal input. Similarly, the non-linearity means that the shape of a time domain response plot is amplitude dependent. Nevertheless, the step response of the controlled system is of interest. The system was simulated with a pulse disturbance velocity v_0 applied to the model of Figure 1 where:

$$v_0 = \begin{cases} 100 & t < 0.01 \\ 0 & t \geq 0.01 \end{cases} \tag{36}$$

This is equivalent to an approximate unit step in position applied gradually over a time of 0.01sec.

Figure 10 shows the response of the switched-damper controller and the continuously-modulated damper (with maximum damping $4c_s$). Figures 10(a)&(b) show the velocity v_2 of the upper mass of Figure 1 superimposed on the corresponding responses of the (linear) passive and skyhook dampers. Figures 10(c)&(d) show the corresponding forces f and Figures 10(e)&(f) show the corresponding equivalent damping c_e. As already indicated in the frequency domain, both semi-active controllers have a response which is a compromise between the passive and the active skyhook controller. The extra freedom of the continuously-modulated damper, compared with that of the switched-damper, allows a closer match to the skyhook controller in this case.

3.2 Relaxed-switching semi-active damping

The shaded regions of Figures 3(b)–3(d) show the values of x_1 and x_3 that satisfy (35) for three values of $q_t > 0$. Compared to Figure 3(a), the regions are smaller and do not include a region around the origin. In this sense, the switching criterion is relaxed and the corresponding intermittent controller will give zero control when the state elements x_1 and x_3 are sufficiently small.

Switching controllers can give rise to "chatter" where the state repeatedly crosses the switching boundary at a high frequency. This can be avoided by using two switching surfaces: the one with smaller $q_t = q_{t1}$ used to switch from on to off and the one with larger $q_t = q_{t2}$ used to switch from off to on. This is a form of hysteresis.

3.2.1 Example

To examine the consequences of this relaxation of switching surfaces, the example of Section 3.1 was repeated but with non zero threshold: $q_{t1} = 0.25$ and $q_{t2} = 0.5$. The "x" symbols of Figures 4–6 show the results of the same three controllers as those of Section 3.1.1 except for the non-zero threshold. In the case of continuous control (Figures 4–5), the equivalent damping is actually zero at low and high frequencies giving zero control at these frequencies; but the performance is only slightly reduced. The use of a threshold means that control is only applied when needed.

To examine this point further, the grey line in Figures 7–9 shows the equivalent damping coefficient over one period at four different frequencies for the three controllers when the threshold is used. In Figure 7, the equivalent damping c_e, and thus the control signal u is zero at $f = 0.5, 5\&10$Hz but is very similar to the control with zero threshold at $f = 1$Hz except that it is zero for longer[1]. In Figure 8, the control signal u is zero at $f = 5\&10$Hz and in Figure 9, the control signal u is zero at $f = 5$Hz.

The use of relaxed switching thus provides an approach to reducing control activity in regions of low control benefit. Moreover, the use of hysteresis ($q_{t1} \neq q_{t2}$) can eliminate chattering behaviour.

4 Conclusion

The well-known semi-active damping approach to vibration control is shown to be closely related to a form of hybrid control known as intermittent control. In particular semi-active damping, in both the continuous and switched forms, is shown to be a form of linear intermittent control with a quadratic switching surface. This insight leads to a new form of vibration control called relaxed-switching semi-active damping in this paper. Simulation results indicate that this reduces control activity without significant reduction in performance.

This cross fertilisation of semi-active control and hybrid control suggests other research directions not explored in this paper. For example, intermittent control has features which have potential application to semi-active damping. These features include the use of an observer to replace direct state measurement and to allow multisensor fusion; prediction to overcome time-delay (Gawthrop and Wang, 2007) and hard constraints on control action and state-trajectories (Gawthrop and Wang, 2006, 2009a, 2010). More generally, the wider field of hybrid control contains many design approaches, analysis methods and stability results which may well be applicable to semi-active damping.

This paper focusses on the simplest passive controller: the damper and a simple active controller: the skyhook controller. However the basic principles of Section 2.2, and their reinterpretation as a form of hybrid control, apply equally to any passive controller and any active controller. Thus, for example, the passive controller could be a mass-spring-damper system and the active controller could arise from optimal control theory.

The fundamental methodology of this paper is based on the idea of extracting energy from a dynamic system whilst avoiding injecting energy into the system. Therefore, in addition to their role in vibration suppression, the hybrid controllers advocated in this paper have potential application to energy harvesting.

[1]Simulations with no hysteresis ($q_{t1} = q_{t2} = 0.5$) exhibited chattering which is not present here

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List of Figures

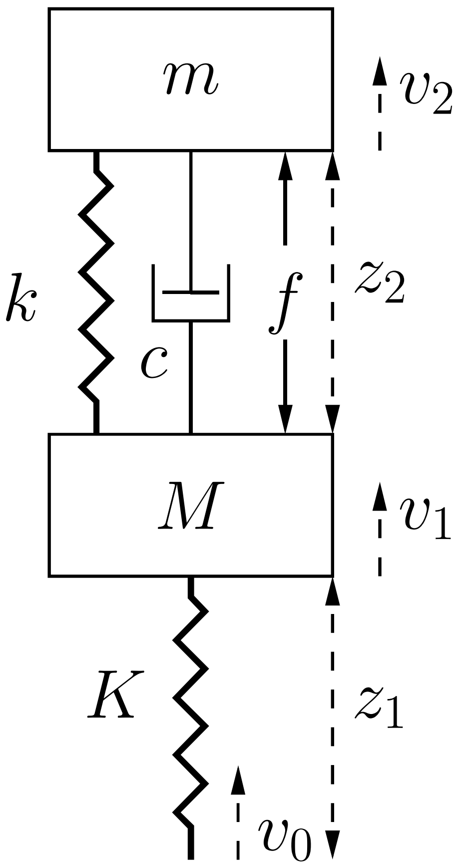
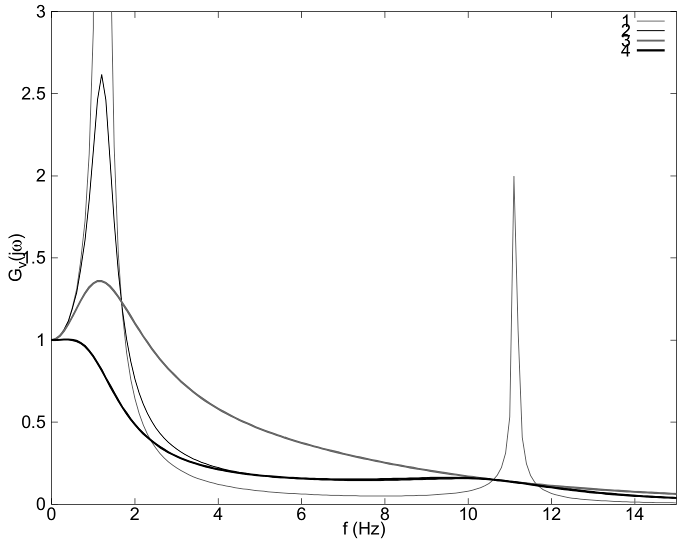
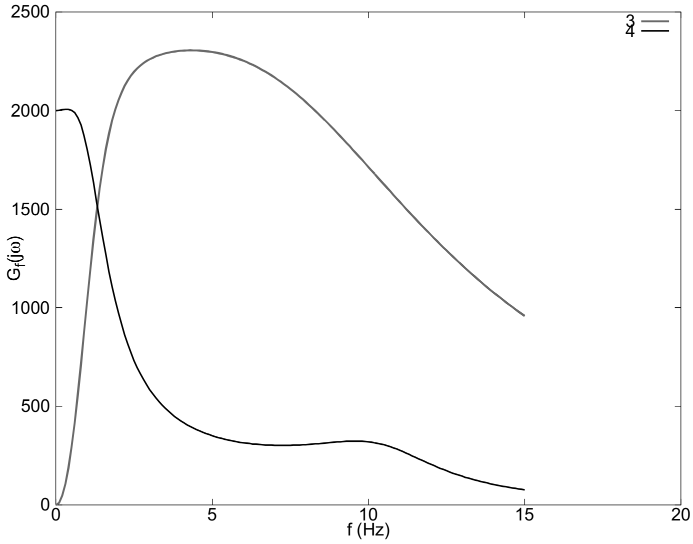


Figure 1: Quarter car model. $m = 240$kg, $M = 36$kg, $k = 16$kN/m, $K = 160$kN/m, $c = 980$Nsec/m. z_1 and z_2 are the spring extensions in m and v_1 and v_2 are the mass velocities in m/s. f is an external applied force and $u = -f$ where u is the active control signal.

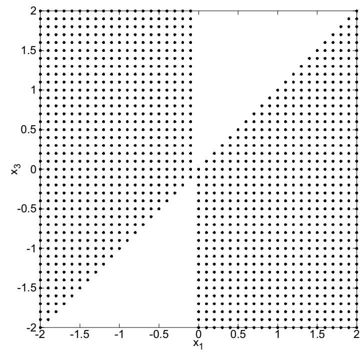


(a) Velocity transmissibility

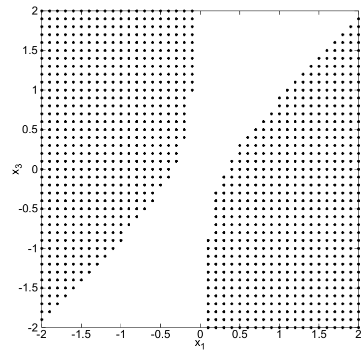


(b) Force transmissibility

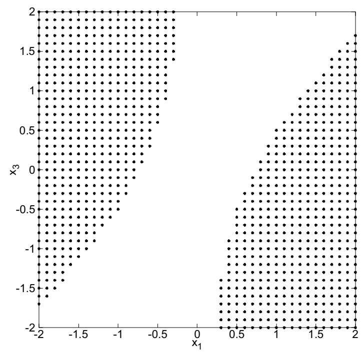
Figure 2: Active and passive control. (a) Shows the velocity transmissibility from the disturbance velocity v_0 to v_2 for four cases: 1. no explicit control, $u = f = 0$, and minimal damping; 2. no explicit control, $u = f = 0$, with $c = 980$Ns/m; 3. passive control, given by Equation (6), with $c_p = 2000$; 4. active skyhook control, given by Equation (7), with $c_s = 2000$. (b) Shows the corresponding control signal u (which is zero for the first two cases).



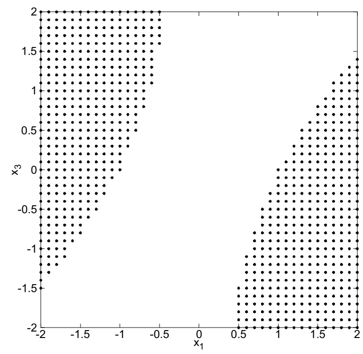
(a) $q_t^2 = 0.0$



(b) $q_t^2 = 0.1$

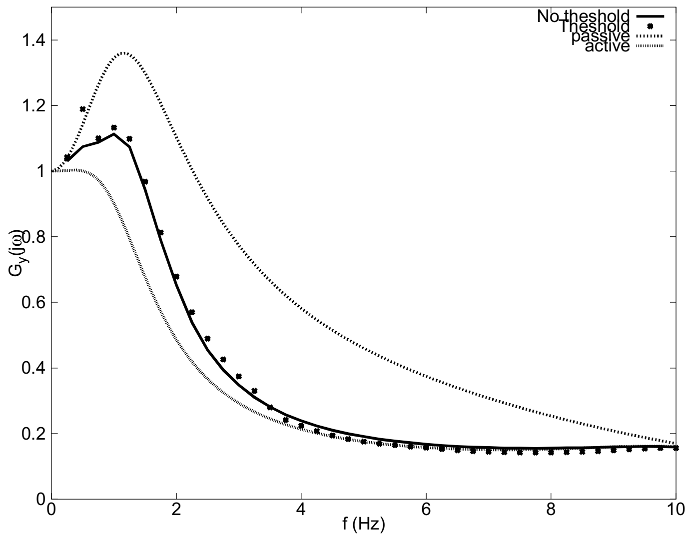


(c) $q_t^2 = 0.5$

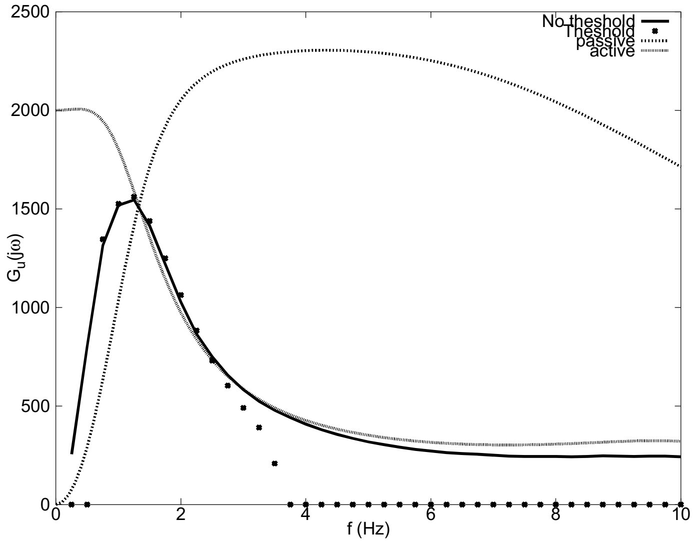


(d) $q_t^2 = 1.0$

Figure 3: Switching surfaces. $k = k_s$

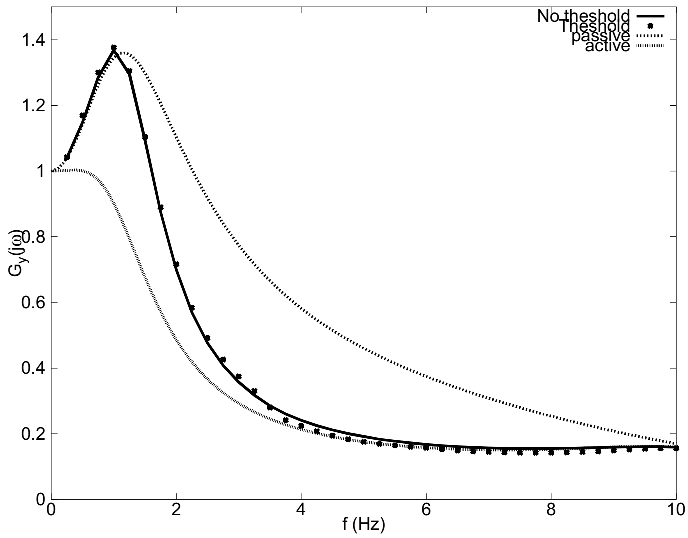


(a) Velocity

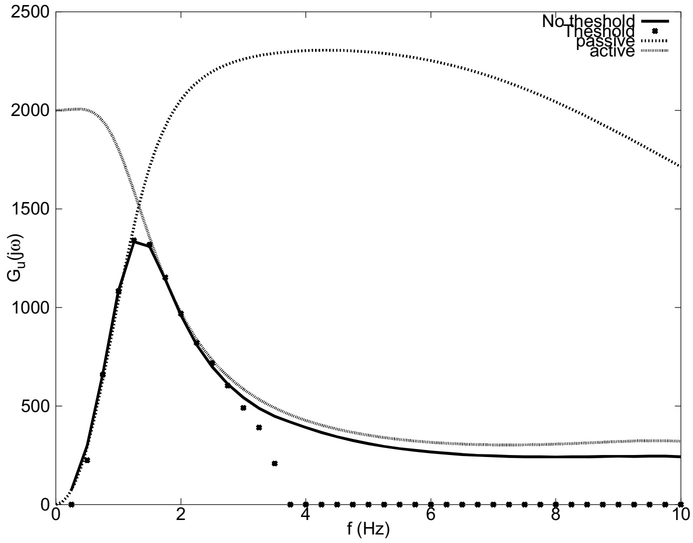


(b) Force

Figure 4: Continuous semi-active control. Limit $4c_s$

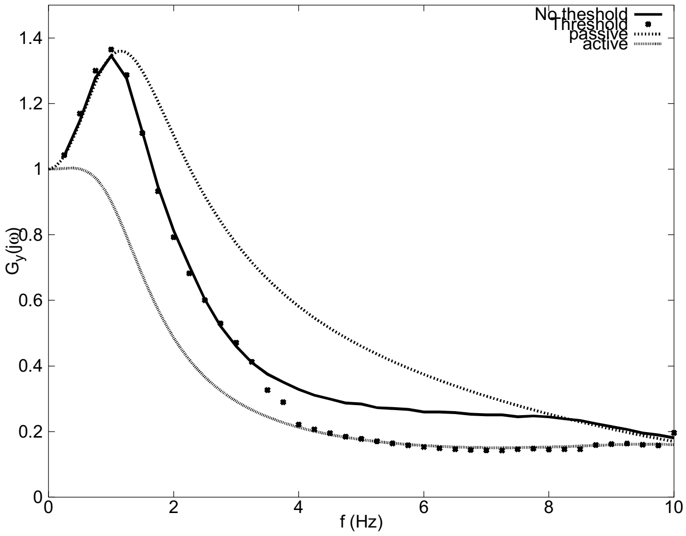


(a) Velocity

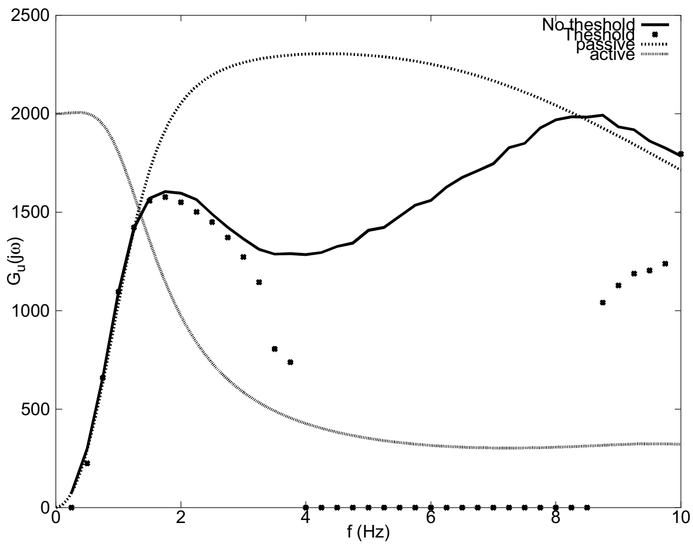


(b) Force

Figure 5: Continuous semi-active control. Limit c_s

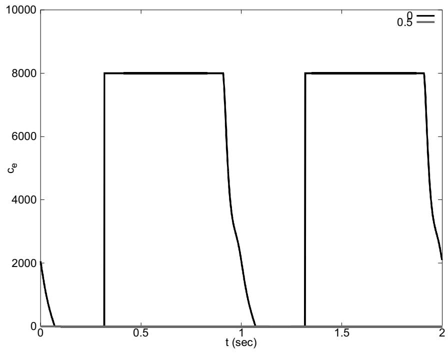


(a) Velocity

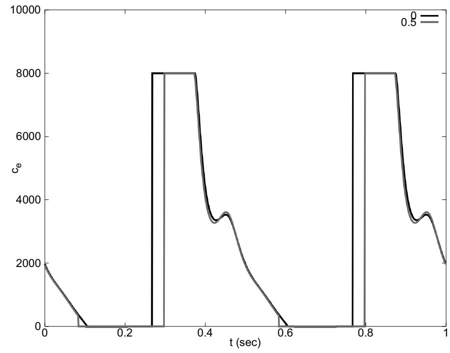


(b) Force

Figure 6: Switched semi-active control.



(a) $f = 0.5$Hz



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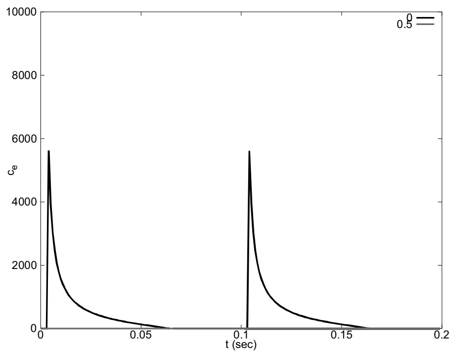

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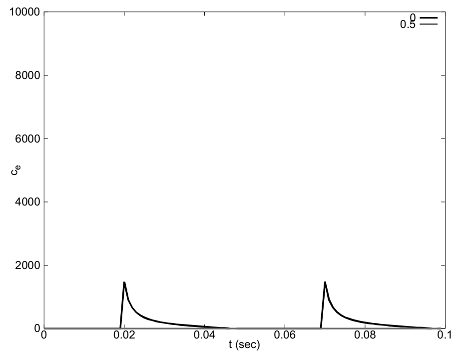
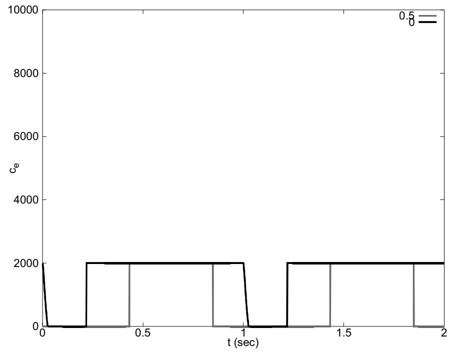


(d) $f = 10$Hz

Figure 7: Continuous (limit $4c_s$) semi-active control: equivalent damping



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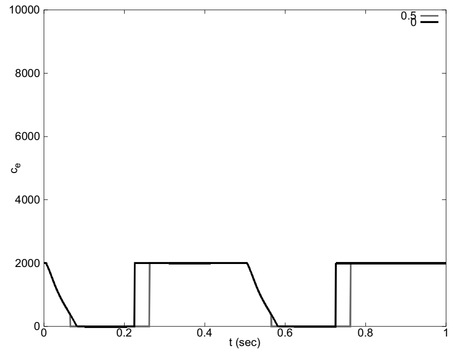

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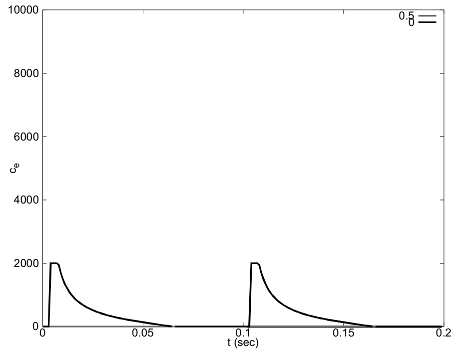

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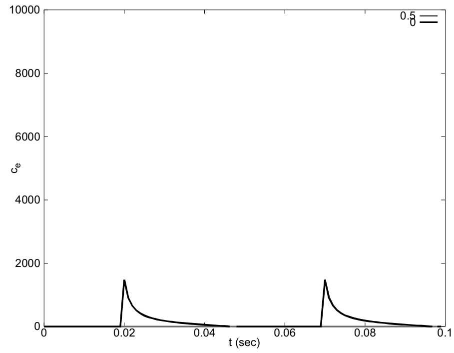
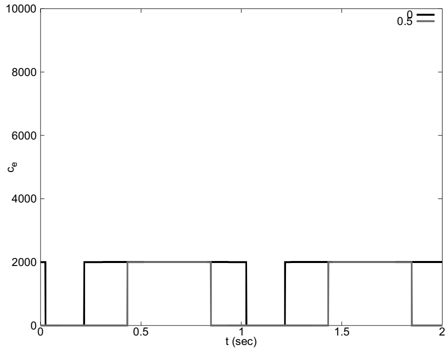
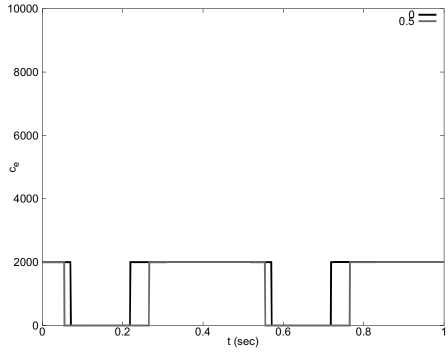


(d) $f = 10$Hz

Figure 8: Continuous (limit c_s) semi-active control: equivalent damping



(a) $f = 0.5$Hz



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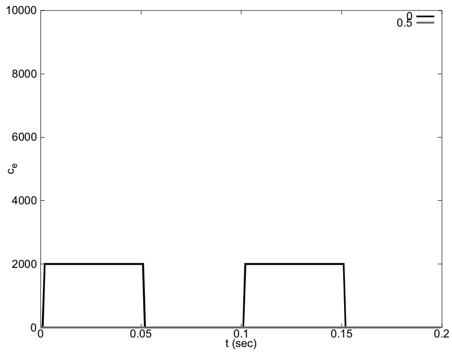
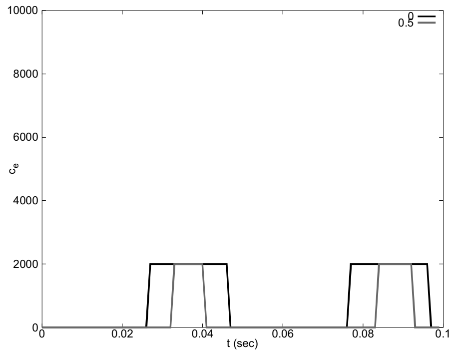
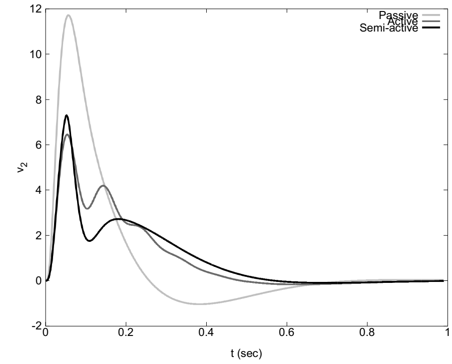


(c) $f = 5$Hz

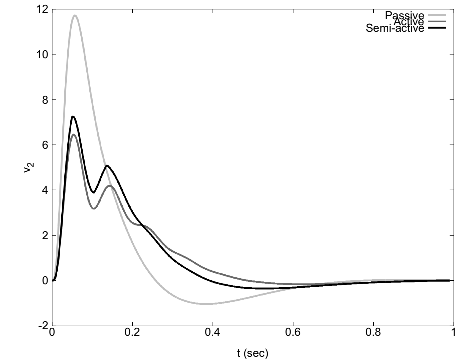


(d) $f = 10$Hz

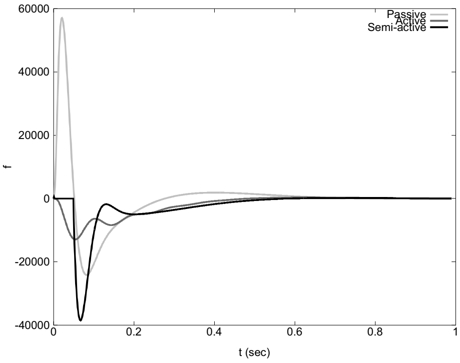
Figure 9: Switched semi-active control: equivalent damping



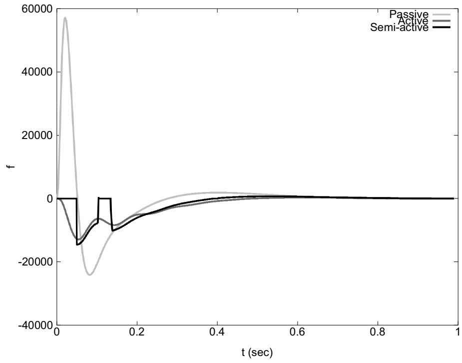
(a) Switched: v_2



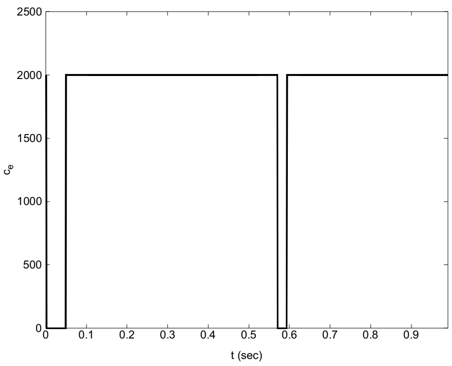
(b) Continuous: v_2



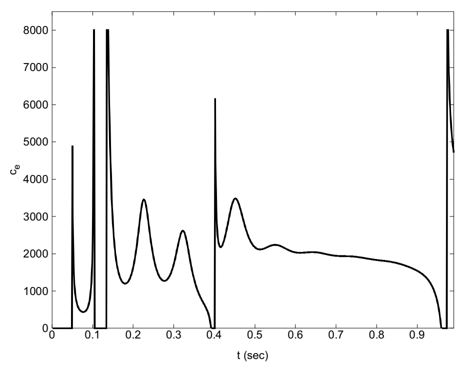
(c) Switched: f



(d) Continuous: f



(e) Switched: c_e



(f) Continuous: c_e

Figure 10: Response to step disturbance