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Research Article

Finsler Geometry for Two-Parameter Weibull Distribution Function

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To construct the geometry in nonflat spaces in order to understand nature has great importance in terms of applied science. Finsler geometry allows accurate modeling and describing ability for asymmetric structures in this application area. In this paper, two-dimensional Finsler space metric function is obtained for Weibull distribution which is used in many applications in this area such as wind speed modeling. The metric definition for two-parameter Weibull probability density function which has shape (k) and scale (c) parameters in two-dimensional Finsler space is realized using a different approach by Finsler geometry. In addition, new probability and cumulative probability density functions based on Finsler geometry are proposed which can be used in many real world applications. For future studies, it is aimed at proposing more accurate models by using this novel approach than the models which have two-parameter Weibull probability density function, especially used for determination of wind energy potential of a region.

1. Introduction

Two-parameter Weibull function is one of the most used distribution functions for different purposes such as modeling, reliability analysis, life time data analysis, and many applied science areas such as mechanic, biosystem, nuclear, and energy system engineering [1–6]. In the literature studies, it is seen that two-parameter Weibull distribution is mainly used for the determination of wind energy potential in the different regions in the world [7–14]. The reasons that the usage of two-parameter Weibull distribution in this area are very good fit to the wind distribution, flexible structure of distribution, and having two-parameter. Also, after the determination of parameters for an observation height, parameters can be estimated for different height.

Before the installation of a wind energy conversion system in a region, the wind speed potential of that region needs to be determined and modeled. In line with this purpose, the most important problem of modeling by two-parameter Weibull distribution is accuracy estimation of parameters associated with designing optimal model. In accordance with this purpose, many different statistical and empirical methods are developed in the literature [15–19]. Also, different function structures such as Rayleigh, Lognormal, Gamma, and Burr are used for accurately modeling wind speed in the literature [20–22]. Determination of a new distribution function in order to develop correct and accurate model structure has importance for wind speed modeling in different regions and other real world application problems.

The fact that the wind speed and similar distributions have a nonsymmetrical and unstable character brings along many difficulties from the stand point of modeling. In this context, Finsler geometry is a very strong tool than well-known Riemann geometry for modeling physical phenomena that are genuinely asymmetric and/or nonisotropic [23–26].

Finsler metric function whose geodesics have two-parameter family of curve in Finsler space is obtained by Matsumoto [27–29]. In this paper, Finsler metrics which are associated with different n parameters, defined in nonnegative real numbers, are derived and they are obtained by Weibull distribution function which has two-parameter curve family. In addition, new probability and cumulative probability density functions based on Finsler geometry are proposed in this paper. Calculation of geodesics that have Finsler

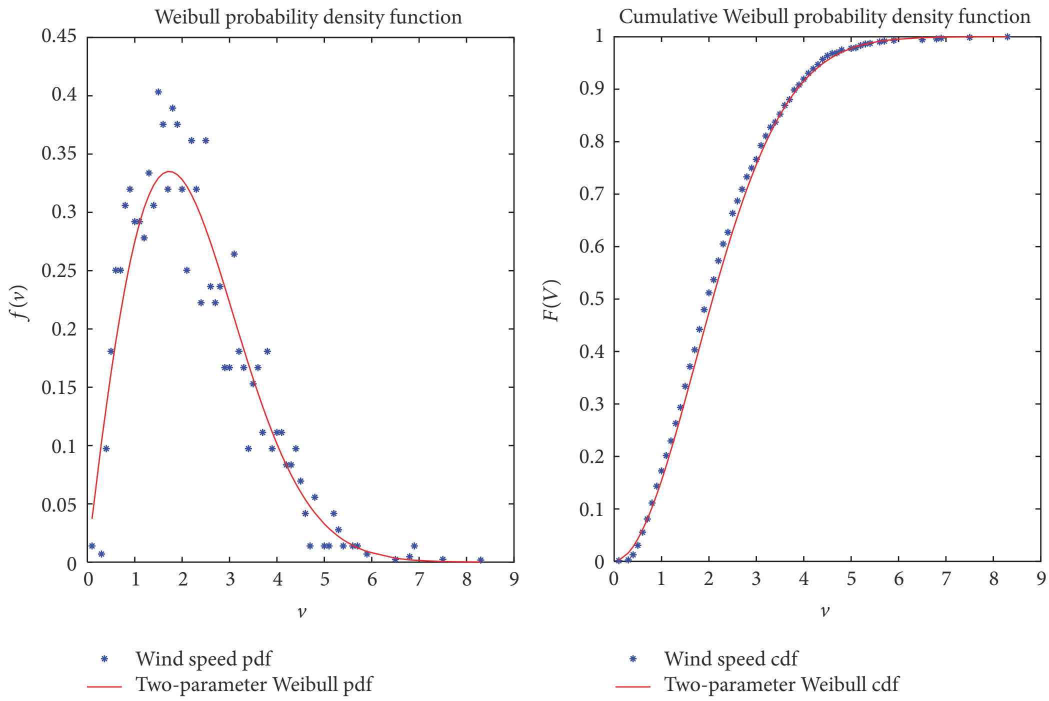

Figure 1: Probability and cumulative probability densities at 10 m height for sample wind speed data.

metrics and novel two-parameter probability and cumulative probability density functions is evaluated for chosen different nonnegative numbers, comparatively. Two-parameter Weibull distribution function structure is presented with an example in Section 2. Definition of Weibull distribution that has two-parameter family of curve and Finsler metrics that are obtained for two-parameter curve family are given in Section 3. In the last section, Finsler metrics that have two-parameter Weibull distribution function family of curve and their geodesics are evaluated for nonnegative different n numbers, comparatively. Finally, conclusions are given in Section 5.

2. Two-Parameter Weibull Distribution

Two-parameter Weibull distribution is used in many real world applications. In this section, the structure of the two-parameter Weibull distribution function will be discussed on the real world problem such as the wind speed distribution which has a nonlinear structure in the asymmetric platform.

Two-parameter Weibull distribution is one of the widely used statistical methods in the modeling of wind speed data. The Weibull distribution function is given by the following [30–36]:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-(v/c)^k}, \tag{1}$$

where $f(v)$ is the frequency or probability of occurrence of wind speed v, c is the Weibull scale parameter with unit equal to the wind speed unit (m/s), and k is the unitless Weibull shape parameter. The higher value of c indicates that the wind speed is higher, while the value of k shows the wind stability [37].

The cumulative Weibull distribution function $F(v)$ gives the probability of the wind speed exceeding the value v. It is expressed by the following [38, 39]:

$$F(v) = 1 - e^{-(v/c)^k}. \tag{2}$$

Probability and cumulative probability density function with sample wind speed data that is Bilecik region in Turkey are shown in Figure 1 for two-parameter Weibull distribution in which scale (k) and shape (c) parameters are calculated, 1.9416 and 2.5110, respectively, by maximum likelihood method.

3. Finsler Metrics for Two-Parameter Family of Curves

In a two-dimensional space, a continuous function $F : TM \to [0, \infty)$ is called a Finsler metric on a C^∞ manifold M if it satisfies the following conditions.

(i) $F(x, y; \dot{x}, \dot{y})$ is C^∞ on $TM \setminus \{0\}$.

(ii) $F(x, y; \lambda\dot{x}, \lambda\dot{y}) = \lambda F(x, y; \dot{x}, \dot{y})$, $\lambda > 0$.

(iii) $g_{ij}(x, y; \dot{x}, \dot{y})$, the fundamental metric tensor, is positively defined, where (x, y) denotes the coordinates of $p \in M$ and $(x, y; \dot{x}, \dot{y})$ denotes the local coordinates of $(\dot{x}, \dot{y}) \in T_pM$ [40, 41].

On the purpose of determination of Finsler metrics and their geodesics in two-dimensional Finsler space belonging to two-parameter Weibull distribution that has scale (k) and shape

(c) parameters calculations are made as follows based on Matsumoto article [28].

Two-parameter family of curves

$$y = f(x; a, b) \quad (3)$$

is given in (3). Let us take the family of $C(a, b)$ curve that is given with this equation in xy-plane. Our primary aim is to show how to find the $F^2 = (\mathbb{R}^2, L(x, y; \dot{x}, \dot{y})$ two-dimensional Finsler space. Darboux's method for solving of this problem has been discussed in our study.

First, from (3) we get

$$z\left(= y'\right) = f_x(x; a, b), \quad (4)$$

and solution of a and b as function of x, y, and z from (3) and (4) is as follows:

$$\begin{aligned} a &= \alpha(x, y, z), \\ b &= \beta(x, y, z). \end{aligned} \quad (5)$$

Then,

$$z' = f_{xx}(x; \alpha, \beta) = u(x, y, z) \quad (6)$$

is the second-order differential equation of y which characterizes $C(a, b)$ two-parameter curve family.

We will use (x, y) ve $(\dot{x}, \dot{y}) = (p, q)$ instead of (x^1, x^2) and (y^1, y^2), respectively, in F^2 Finsler space. Accordingly, differential of the arc length of the $C(a, b)$ curve is

$$\begin{aligned} ds &= L(x, y; \dot{x}, \dot{y})\,dt, \\ (\dot{x}, \dot{y}) &= \left(\frac{dx}{dt}, \frac{dy}{dt}\right). \end{aligned} \quad (7)$$

Assuming $\dot{x} > 0$, $ds = L(x, y; 1, \dot{y}/\dot{x})\dot{x}dt$. Using $y' = dy/dx$,

$$ds = A\left(x, y, y'\right) dx \quad (8)$$

is obtained. From here

$$A\left(x, y, y'\right) = L\left(x, y; 1, y'\right) \quad (9)$$

is acquired. Inversely, this can be written.

$$L(x, y; \dot{x}, \dot{y}) = A\left(x, y, \frac{\dot{y}}{\dot{x}}\right)\dot{x}. \quad (10)$$

Geodesic is curve that minimized $s = \int_{t_0}^{t} L(x, y; \dot{x}, \dot{y})dt$ length integral that it is calculated along a curve and obtained from Euler equations in (11).

$$\ddot{x}^i + 2G^i(x, y; \dot{x}, \dot{y}) = 0. \quad (11)$$

Defined in (11), spray coefficients are given as follows:

$$G^i(x, y; \dot{x}, \dot{y}) = \frac{1}{2}g^{il}\left\{\frac{\partial^2 F}{\partial x^j \partial y^l}y^j - \frac{\partial F}{\partial x^l}\right\}, \quad (12)$$

where $F(x, y; \dot{x}, \dot{y}) = L^2(x, y; \dot{x}, \dot{y})/2$ refers to Finsler metric.

Euler equations are rewritten in Rashevsky form as follows:

$$A_{zz}y'' + A_{yz}y' + A_{zz} - A_y = 0, \quad z = y'. \quad (13)$$

Here, defined in (9), $A(x, y, z)$ is associated with fundamental function.

According to Darboux's theorem definition of the basic metric [27],

$$A(x, y, z) = \int_0^z (z - t)\, H(t, y - tx)\, dt + zE_y + E_x \quad (14)$$

is given, where $H(\alpha, \beta)$ and $E(x, y)$ are arbitrarily chosen. Fundamental metric function is given another form by [28]

$$\begin{aligned} L(x, y, \dot{x}, \dot{y}) = {} & \dot{x}\int_0^z (z - t)\, H(t, y - tx)\, dt + \dot{x}E_x \\ & + \dot{y}E_y. \end{aligned} \quad (15)$$

The corresponding Finsler metric is derived by the two-parameter Weibull distribution function instead of two-parameter family of curves. First, the two-parameter cumulative Weibull distribution function in (2) is linearized to calculate α, β, u variables in a simpler form.

In (2), applying some mathematical calculations is as follows:

$$y = \log\left[-\log(1 - F(v))\right] = k\log\frac{v}{c}, \quad (16)$$

$$y = kx - k\log c \quad (17)$$

is obtained as linear equation, where $x = \log v$. Finally,

$$y = f(x; k, c) \quad (18)$$

is defined. When the given curve family is linear, some of the necessary quantities are obtained as follows [28]:

$$\begin{aligned} \alpha &= z, \\ \beta &= y - zx, \\ u &= 0. \end{aligned} \quad (19)$$

Different Finsler metrics and their geodesics resulting from selection $H(\alpha, \beta)$ and $E(x, y)$ arbitrary functions are discussed in the next section for family of curves that has two-parameter Weibull distribution.

4. Finsler Metrics and Geodesics for Two-Parameter Weibull Distribution

Different Finsler metrics for two-parameter Weibull distribution function will be obtained for n arbitrarily nonnegative real number by choice of $H(\alpha, \beta) = \beta^n$ in (15). Hence, with the selection of $H(z, y-zx) = \beta^n = (y-zx)^n$ and E = constant in (15), metric function that has Weibull distribution is obtained in the form of

$$L(x, y, \dot{x}, \dot{y}) = \frac{y^{(n+2)}\dot{x}}{x^2} \sum_{k=0}^{n} \binom{n+2}{k+2} \left(-\frac{x\dot{y}}{y\dot{x}}\right)^{(k+2)}. \quad (20)$$

It can be easily seen that the obtained function provides the Finsler metric conditions.

L_n and G_n^i, respectively, metric function defined in n value and spray coefficients, for integer selection of $n = 2$ and 5, are calculated as

$$L_2(x, y, p, q) = \frac{q^2x^2(q^2x^2 - 4pqxy + 6p^2y^2)}{x^2p^3},$$
$$L_5(x, y, p, q) = \frac{q^2x^2(-q^5x^5 + 7pq^4x^4y)}{x^2p^6} - 21p^2q^3x^3y^2 + 35p^3q^2x^2y^3 - 35p^4qxy^4 + 21p^5y^5. \quad (21)$$

Spray coefficients for Finsler metrics related to these equations are found by (12).

$$\begin{aligned}
{G_2}^1 &= \frac{p^2q(-qx + 4py)}{q^2x^2 - 4pqxy + 6p^2y^2}, \\
{G_2}^2 &= \frac{pq^2(-qx + 4py)}{q^2x^2 - 4pqxy + 6p^2y^2}, \\
{G_5}^1 &= \frac{pq^2(q^4x^4 - 7pq^3x^3y + 21p^2q^2x^2y^2 - 35p^3qxy^3 + 35p^4y^4)}{-q^5x^5 + 7pq^4x^4y - 21p^2q^3x^3y^2 + 35p^3q^2x^2y^3 - 35p^4qxy^4 + 21p^5y^5}, \\
{G_5}^2 &= \frac{p^2q(q^4x^4 - 7pq^3x^3y + 21p^2q^2x^2y^2 - 35p^3qxy^3 + 35p^4y^4)}{-q^5x^5 + 7pq^4x^4y - 21p^2q^3x^3y^2 + 35p^3q^2x^2y^3 - 35p^4qxy^4 + 21p^5y^5}.
\end{aligned} \quad (22)$$

As can be seen easily from the calculated values, spray coefficients are ${G_n}^1 = (p/q){G_n}^2$ for the n arbitrary nonnegative integer. In this case, y'' is always zero when substituted spray coefficients in (22) to (23) give the geodesics; we get

$$y'' = \frac{2 * (G^1q - G^2p)}{p^3}. \quad (23)$$

This gives us $y = C_1x + C_2$ linear function structure where C_1 and C_2 are the integration constants. In the calculation steps, if substituting this value to (17), two-parameter Weibull probability and cumulative probability functions that are the same in (1) and (2) are obtained for nonnegative all integer values of n.

If the same calculation steps are repeated for arbitrary positive rational numbers, $n = 1/2$ and 11/12, we get

$$L_{1/2}(x, y, p, q) = \frac{15q^2x^2\sqrt{y}}{8x^2p},$$
$$L_{11/12}(x, y, p, q) = \frac{805q^2x^2y^{11/12}}{288x^2p}. \quad (24)$$

Spray coefficients are found.

$${G_{1/2}}^1 = {G_{11/12}}^1 = 0,$$
$${G_{1/2}}^2 = \frac{q^2}{8y},$$
$${G_{11/12}}^2 = \frac{11q^2}{48y}. \quad (25)$$

Substituting spray coefficients in (25) to (23), we obtain second-order differential equation of y with respect to x.

$$y'' = K\frac{y'^2}{y}, \quad (26)$$

where K is a coefficient dependent on n. It is apparent that K are $-1/4$, $-11/24$ for $n = 1/2$, 11/12, respectively. In this case, it can be seen that relation between n and K is $K = -(1/2)n$. For all nonnegative rational numbers, when the differential equation in (26) is solved,

$$y = \left(C_2x + \frac{2}{n+2}C_1\right)^{2/(n+2)} \quad (27)$$

is found, where C_1 and C_2 are the integration constants. Substituting y to (17), new two-parameter cumulative function is

$$F_{\text{new}}(v; C_1, C_2) = 1 - e^{-(2/(n+2))v^{C_2}e^{(2/(n+2))C_1}}. \quad (28)$$

Setting $a = 2/(n+2)$, it is rewritten in the form

$$F_{\text{new}}(v; C_1, C_2) = 1 - e^{-av^{C_2}e^{aC_1}}. \quad (29)$$

Probability density function is calculated by $f_{\text{new}} = dF_{\text{new}}/dv$.

$$f_{\text{new}}(v; C_1, C_2) = aC_2e^{a(C_1 - V^{C_2}e^{aC_1})}v^{C_2-1}. \quad (30)$$

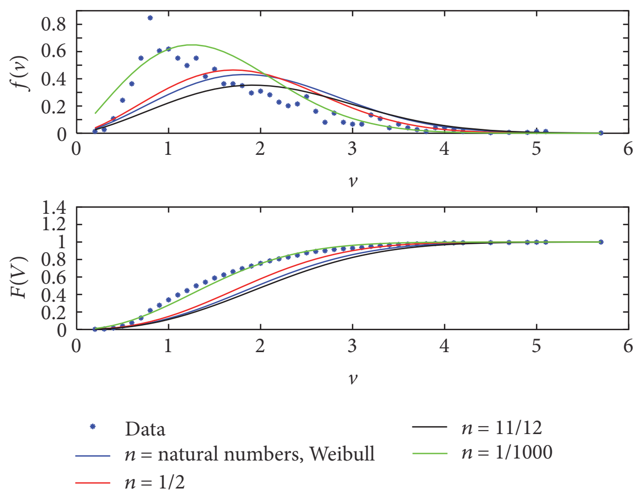

FIGURE 2: Comparative analysis for new function based on Finsler metric and Weibull function for different n values.

While the solution of the differential equation that is obtained by using the arbitrary values of nonnegative integer n gives the same geodesics as the two-parameter Weibull function, new function that defined nonnegative rational numbers of n is derived for two-dimensional family of curve.

On the real world application, it is foreseen that accurate modeling ability is given by estimation of $C1$ and $C2$ parameters in the new function. With an example on the real world application, it can be shown as Figure 2 that geodesics are based on Finsler geometry such as nonlinear wind speed modeling on real world problem, where the parameters in the new curve families based on Finsler metric using wind speed data for Bilecik/Turkey are determined by the boundary value problem and showed comparatively different n values.

We have already stated that it gives the same probability and cumulative probability density functions as the Weibull when n values are taken as nonnegative integers. For different nonnegative rational n values, while n goes to zero, the probability and cumulative probability density functions of models converge to observation values.

It can be said that optimal modeling can be applied to nonlinear structures for the different new family of curves that are obtained by choosing arbitrary n values and estimating of parameters in the many real world applications.

5. Conclusions

The two-parameter Weibull distribution presents the modeling opportunity for nonlinear structure in the real world problems. Modeling of wind speed which has nonsymmetrical and unstable character is one of these real world problems. With the help of Finsler geometry's modeling ability of physical phenomena that are genuinely asymmetric and/or nonisotropic more accurate modeling can be achieved. For this reason, Finsler metrics of Weibull distribution function with two-parameter family of curve are derived in this paper. The arbitrary function is chosen as $H(z, y - zx) = \beta^n$ in order to derive Finsler metrics that have family of two-parameter Weibull distribution functions. As a result of this selection, two-parameter new cumulative distribution function is derived as the geodesics obtained for the different nonnegative rational n values are examined. It is expected that the proposed Finsler metric based function can be applied in many real world problems such as wind speed modeling. It is foreseen that the performed analysis using this function will bring a new approach to the literature.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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