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Research Article

Nonfragile Finite-Time Extended Dissipative Control for a Class of Uncertain Switched Neutral Systems

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This paper is concerned with finite-time extended dissipative analysis and nonfragile control for a class of uncertain switched neutral systems with time delay, and the controller is assumed to have either additive or multiplicative form. By employing the average dwell-time and linear matrix inequality technique, sufficient conditions for finite-time boundedness of the switched neutral system are provided. Then finite-time extended dissipative performance for the switched neutral system is addressed, where we can solve H_∞, $L_2 - L_\infty$, Passivity, and (Q, S, R)-dissipativity performance in a unified framework based on the concept of extended dissipative. Furthermore, nonfragile state feedback controllers are proposed to guarantee that the closed-loop system is finite-time bounded with extended dissipative performance. Finally, numerical examples are given to demonstrate the effectiveness of the proposed method.

1. Introduction

Switched system is an important class of hybrid systems, which consists of a family of subsystems and a logical rule that orchestrates switching between them. For its practical importance, switched systems have received considerable attention in the last decades [1–5]. Meanwhile, time delay exists widely in many practical systems and may cause undesirable system performance or even instability [6–9]. Switched system with time delay is a main issue in recent years. As a special time delay system, switched neutral systems have received much attention [10–14]. For example, the problem of stability analysis and H_∞ control for several switched neutral systems were considered in [10] and [13], respectively.

Up to now, most researches for switched neutral systems focus on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in practice, the transient performance of a system is also of great significance. In many practical applications such as missile systems and robot control systems, the main concern is the system behavior over a finite-time interval. Therefore, finite-time analysis of switched systems is worth researching. Recently, some related research results were published in the literatures [15–20]. More specifically, finite-time H_∞ control of switched systems was addressed in [16], and finite-time stabilization and boundedness of switched linear system were investigated in [19].

On the other hand, the controller coefficients are generally exact values when designing a desired controller. However, in practice, uncertainty cannot be avoided in controller design, and it may be caused by many reasons, such as numerical round-off errors and actuator degradation. The existence of uncertainty motivates the study of nonfragile control. Over decades, much attention has been devoted to the issue of controller fragility and related remedies [21–24]. To name a few, the problem of passivity-based nonfragile control for Markovian jump systems with aperiodic sampling is studied in [22], and nonfragile H_∞ control for linear systems with multiplicative controller gain variations is investigated in [24], respectively. More recently, an effective tool named extended dissipative was firstly proposed by Zhang et al. in [25] to deal with the problem of robust control. By adjusting weighting matrices, the extended dissipative covers some well-known performance indices such as H_∞ performance, $L_2 - L_\infty$ performance, Passivity performance, and (Q, S, R)-dissipativity performance. This concept has been successfully applied to the stability analysis for several neural networks [26–30]. Could this concept be applied to switched systems? To the best of our knowledge, the topic of nonfragile finite-time extended dissipative control for a class of uncertain switched neutral systems has not been investigated yet, which motivates our study.

This paper is organized as follows. In Section 2, preliminaries and problem statement are formulated and some necessary lemmas are given. In Section 3, by employing the average dwell-time and linear matrix inequality approach, some sufficient conditions of finite-time boundness and finite-time extended dissipative performance for switched neutral systems are established. Furthermore, existence and the design method of the nonfragile state feedback controllers are proposed. All of the results are in terms of a set of linear matrix inequalities which can be easily resolved using the LMIs toolbox. In Section 4, numerical examples are given to show the effectiveness of the proposed approach. The main contributions of this paper include the following. (1) We firstly apply the concept of extended dissipative to the nonfragile finite-time analysis and control to the uncertain switched neutral systems. (2) More general switched systems are considered in our paper, including the time-varying delay and distributed delay, neutral parameters, and additive and multiplicative form controller.

Notation. The notations used in this paper are standard. R^n denotes the n-dimensional Euclidean space and M^T represents the transpose of the matrix M. The notation $X > 0$ (≥ 0) is used to denote a symmetric positive definite (positive-semidefinite) matrix. $\|X\|$ represents the Euclidean norm of the matrix X; $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalue of matrix P, respectively. I is the identity matrix with appropriate dimension. diag$\{\cdot\}$ stands for a block-diagonal matrix. The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry.

2. Preliminaries and Problem Statement

Consider the following switched neutral system with time-varying delay:

$$\begin{aligned} &\dot{x}(t) - \widehat{C}_{\sigma(t)}\dot{x}(t-\tau(t)) \\ &\quad = \widehat{A}_{\sigma(t)}x(t) + \widehat{B}_{\sigma(t)}x(t-h(t)) + D_{\sigma(t)}w(t) \\ &\qquad + E_{\sigma(t)}u(t) + \widehat{G}_{\sigma(t)}\int_{t-r(t)}^{t} x(s)\,ds, \qquad (1) \\ &z(t) = F_{\sigma(t)}x(t), \\ &x(\theta) = \varphi(\theta), \quad \forall\theta \in [-\tau, 0], \end{aligned}$$

where $x(t) \in R^n$ is the state vector, $u(t)$ is the control input, $w(t)$ is the exogenous disturbance which belongs to $L_2[0,\infty)$, and $z(t) \in R^n$ is the output. The switching signal $\sigma(t) : [0,\infty) \mapsto M = \{1, 2, \ldots, l\}$ is a piecewise continuous function, where l is the number of subsystems and $\sigma(t) = i$ means that the ith subsystem is activated. $\varphi(\theta)$ is the initial condition, and $h(t)$, $r(t)$, and $\tau(t)$ denote the time-varying delay and satisfy

$$\begin{aligned} 0 &\leq h(t) \leq h_m, \\ \dot{h}(t) &\leq \widehat{h} < 1, \\ 0 &\leq \tau(t) \leq \tau_m, \qquad (2) \\ \dot{\tau}(t) &\leq \widehat{\tau} < 1, \\ 0 &\leq r(t) \leq r_m. \end{aligned}$$

Furthermore, $-\tau = \max\{h_m, \tau_m\}$, and for each $\sigma(t) = i$, $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{C}_i$, and $\widehat{G}_i$ are uncertain real-valued matrices with appropriate dimensions. We assume that the uncertainties are norm-bounded and of the form

$$\begin{aligned} \left[\widehat{A}_i, \widehat{B}_i, \widehat{C}_i, \widehat{G}_i\right] &= \left[A_i, B_i, C_i, G_i\right] \\ &\quad + L_i \Xi_i(t)\left[M_{1i}, M_{2i}, M_{3i}, M_{4i}\right], \end{aligned} \qquad (3)$$

where A_i, B_i, C_i, G_i, M_{1i}, M_{2i}, M_{3i}, and M_{4i} are known real-valued constant matrices with appropriate dimensions and $\Xi_i(t)$ is unknown and possibly time-varying matrix with Lebesgue measurable elements satisfying $\Xi_i^T(t)\Xi_i(t) \leq I$.

In this paper, we consider the following nonfragile state feedback controller: $u(t) = K_{\sigma(t)}(t)x(t)$, where $K_{\sigma(t)}(t) = K_i + \Delta K_i(t)$, K_i is the controller gain and $\Delta K_i(t)$ is a perturbed matrix with the following forms.

Case 1. $\Delta K_i(t)$ has an additive uncertainty which is assumed to be

$$\Delta K_i(t) = J_{1i} U_i(t) J_{2i}, \qquad (4)$$

where J_{1i} and J_{2i} are known real constant matrices with appropriate dimensions and the time-varying matrix $U_i(t)$ satisfies $U_i(t)U_i^T(t) \leq I$.

Case 2. $\Delta K_i(t)$ has a multiplicative uncertainty

$$\Delta K_i(t) = \begin{bmatrix} k_{11i}(\sigma_{1i} + \sigma_{11i}) & \cdots & k_{1ni}(\sigma_{1i} + \sigma_{1ni}) \\ k_{21i}(\sigma_{2i} + \sigma_{21i}) & \cdots & k_{2ni}(\sigma_{2i} + \sigma_{2ni}) \\ \vdots & \ddots & \vdots \\ k_{m1i}(\sigma_{mi} + \sigma_{m1i}) & \cdots & k_{mni}(\sigma_{mi} + \sigma_{mni}) \end{bmatrix}, \qquad (5)$$

where k_{pqi} is the element of K_i and σ_{pi} and σ_{pqi} ($p = 1, \ldots, m$; $q = 1, \ldots, n$) are real uncertain parameters which satisfy

$$\begin{aligned} \left|\sigma_{pi}\right| &\leq \overline{\sigma}_{1pi} \leq 1, \\ \left|\sigma_{pqi}\right| &\leq \overline{\sigma}_{pqi} \leq \overline{\sigma}_{2pi} \leq 1. \end{aligned} \qquad (6)$$

Assumption 1. For a given time constant T_f, the external disturbance satisfies

$$\int_0^{T_f} w^T(t)\,w(t)\,dt \leq d, \quad d \geq 0. \qquad (7)$$

Assumption 2. For a given time constant T_f, the state vector $x(t)$ is time-varying and satisfies the constraint

$$\int_0^{T_f} x^T(t)\,x(t)\,dt \le k, \tag{8}$$

where k is a fixed sufficient large constant number.

Assumption 3 (see [25]). Matrices ψ_1, ψ_2, ψ_3, and ψ_4 satisfy the following conditions:

(1) $\psi_1 = \psi_1^T \le 0$, $\psi_3 = \psi_3^T > 0$, $\psi_4 = \psi_4^T \ge 0$;
(2) $(\|\psi_1\| + \|\psi_2\|)\psi_4 = 0$.

Assumption 4. For $\forall \alpha \ge 0$, $\mu \ge 1$, $\forall t \in [0, T_f]$, we have

$$e^{\alpha t}\mu^{N_\sigma(0,t)} \le b, \tag{9}$$

where $N_\sigma(0,t)$ denotes the switching number of $\sigma(t)$ over $(0,t)$ and b denotes a positive number.

Definition 5 (see [25]). For given matrices ψ_1, ψ_2, ψ_3, and ψ_4 satisfying Assumption 3, system (1) is said to be extended dissipative if the following inequality holds for any $T_f \ge 0$ and all $\mu(t) \in L_2[0,\infty)$:

$$\int_0^{T_f} J(t)\,dt - \sup_{0\le t\le T_f} z^T(t)\,\psi_4 z(t) \ge 0, \tag{10}$$

where

$$\begin{aligned} J(t) &= z^T(t)\,\psi_1 z(t) + 2z^T(t)\,\psi_2 w(t) \\ &\quad + w^T(t)\,\psi_3 w(t). \end{aligned} \tag{11}$$

Remark 6. The concept of extended dissipative introduced in Definition 5 contains a few of well-known performance indices as special cases by setting the weighting matrices:

(1) $L_2 - L_\infty$ performance: $\psi_1 = 0$, $\psi_2 = 0$, $\psi_3 = \gamma^2 I$, and $\psi_4 = I$;
(2) H_∞ performance: $\psi_1 = -I$, $\psi_2 = 0$, $\psi_3 = \gamma^2 I$, and $\psi_4 = 0$;
(3) Passivity performance: $\psi_1 = 0$, $\psi_2 = I$, $\psi_3 = \gamma I$, and $\psi_4 = 0$;
(4) (Q,S,R)-dissipativity performance: $\psi_1 = Q$, $\psi_2 = S$, $\psi_3 = R - \beta I$, and $\psi_4 = 0$.

Definition 7 (see [17]). Given three positive constants c_1, c_2, and T_f with $c_1 < c_2$, a positive definite matrix R and a switching signal $\sigma(t)$, assume that $\mu(t) \equiv 0$, $\forall t \in [0,T_f]$, and switched neutral system (1) is said to be finite-time bounded with respect to $(c_1, c_2, R, T_f, \sigma)$, if, $\forall t \in [0,T_f]$,

$$\begin{aligned} \sup_{-\tau\le\theta\le 0} \left\{x^T(\theta)\,Rx(\theta), \dot{x}^T(\theta)\,R\dot{x}(\theta)\right\} \le c_1 \Longrightarrow \\ x^T(t)\,Rx(t) \le c_2. \end{aligned} \tag{12}$$

Furthermore, if the condition above holds with $w(t) \equiv 0$, $\forall t \in [0, T_f]$, the system is said to be finite-time stable.

Definition 8 (see [17]). For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1,T_2)$ denote the switching number of $\sigma(t)$ over (T_1,T_2). If

$$N_\sigma(T_1,T_2) \le N_0 + \frac{T_2 - T_1}{\tau_a} \tag{13}$$

holds for $\tau_a > 0$ and an integer $N_0 \ge 0$, then τ_a is called an average dwell-time. Without loss of generality, in this paper we choose $N_0 = 0$.

Lemma 9 (see [29]). *Let a and b be real matrices of appropriate dimensions and satisfy $2a^T b \le a^T a + b^T b$.*

Lemma 10 (see [31]). *For any positive definite symmetric matrix $N \in R^{n\times n}$, scalar $\tau > 0$, and a vector function $x(\bullet) : [-\tau, 0] \to R^n$, the following integral inequality is satisfied:*

$$\begin{aligned} &-\tau\int_{t-\tau}^{t} x^T(s)\,Nx(s)\,ds \\ &\quad \le -\int_{t-\tau}^{t} x^T(s)\,ds N\int_{t-\tau}^{t} x(s)\,ds. \end{aligned} \tag{14}$$

Lemma 11 (see [32]). *Let J and L be real matrices of appropriate dimensions. Then, for any scalar $\epsilon > 0$, one has*

$$JU(t)\,L + (JU(t)\,L)^T \le \epsilon JJ^T + \epsilon^{-1}L^T L, \tag{15}$$

when $U(t)$ satisfies $U^T(t)U(t) \le I$.

Lemma 12 (see [33]). *Let G be real positive definite symmetric matrices and let B and L be appropriate dimensional real matrices. Then, one has*

$$BL + (BL)^T \le BG^{-1}B^T + L^T GL. \tag{16}$$

3. Main Results

3.1. Finite-Time Boundedness Analysis. Consider the following unforced switched neutral system without uncertainties:

$$\begin{aligned} &\dot{x}(t) - C_{\sigma(t)}\dot{x}(t-\tau(t)) \\ &\quad = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h(t)) + D_{\sigma(t)}w(t) \\ &\quad + G_{\sigma(t)}\int_{t-r(t)}^{t} x(s)\,ds, \\ &z(t) = F_{\sigma(t)}x(t), \\ &x(t_0+\theta) = \varphi(\theta), \quad \forall\theta \in [-\tau, 0]. \end{aligned} \tag{17}$$

In this section, the problem of finite-time boundedness analysis of the switched neutral system is proposed, by using the average dwell-time approach, sufficient conditions are derived by solving some linear matrix inequalities, and the results are shown as follows.

Theorem 13. *For given positive scalars α, $\widehat{h}$, $\widehat{\tau}$, h_m, and r_m, if there exist positive definite symmetric matrices $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$, $\widetilde{T}_i$, and $\widetilde{M}_i$ and matrices N_i, X_{11i}, X_{12i}, X_{22i}, H_{1i}, and H_{2i} with appropriate dimensions, then*

$$X_i = \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geq 0, \tag{18}$$

$$\Delta_i = \begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \widetilde{T}_i \end{bmatrix} \geq 0, \tag{19}$$

$$\Theta_i = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \widetilde{P}_i G_i + A_i^T \widetilde{Z}_i G_i + h_m A_i^T \widetilde{T}_i G_i \\ * & \phi_{22} & \phi_{23} & B_i^T \widetilde{Z}_i D_i + h_m B_i^T \widetilde{T}_i D_i & B_i^T \widetilde{Z}_i G_i + h_m B_i^T \widetilde{T}_i G_i \\ * & * & \phi_{33} & C_i^T \widetilde{Z}_i D_i + h_m C_i^T \widetilde{T}_i D_i & C_i^T \widetilde{Z}_i G_i + h_m C_i^T \widetilde{T}_i G_i \\ * & * & * & \phi_{44} & D_i^T \widetilde{Z}_i G_i + h_m D_i^T \widetilde{T}_i G_i \\ * & * & * & * & G_i^T \widetilde{Z}_i G_i + h_m G_i^T \widetilde{T}_i G_i - \dfrac{\widetilde{M}_i}{r_m} \end{bmatrix} < 0, \tag{20}$$

hold, where

$$\begin{aligned}
\phi_{11} &= -\alpha \widetilde{P}_i + \widetilde{P}_i A_i + A_i^T \widetilde{P}_i + \widetilde{Q}_i + A_i^T \widetilde{Z}_i A_i \\
&\quad + h_m A_i^T \widetilde{T}_i A_i + r_m \widetilde{M}_i + H_{1i} + H_{1i}^T + h_m X_{11i}, \\
\phi_{12} &= \widetilde{P}_i B_i + A_i^T \widetilde{Z}_i B_i + h_m A_i^T \widetilde{T}_i B_i - H_{1i}^T + H_{2i} \\
&\quad + h_m X_{12i}, \\
\phi_{13} &= \widetilde{P}_i C_i + A_i^T \widetilde{Z}_i C_i + h_m A_i^T \widetilde{T}_i C_i, \\
\phi_{14} &= \widetilde{P}_i D_i + A_i^T \widetilde{Z}_i D_i + h_m A_i^T \widetilde{T}_i D_i, \\
\phi_{22} &= -\left(1 - \widehat{h}\right) \widetilde{Q}_i + B_i^T \widetilde{Z}_i B_i + h_m B_i^T \widetilde{T}_i B_i - H_{2i} - H_{2i}^T \\
&\quad + h_m X_{22i}, \\
\phi_{23} &= B_i^T \widetilde{Z}_i C_i + h_m B_i^T \widetilde{T}_i C_i, \\
\phi_{33} &= -(1 - \widehat{\tau}) \widetilde{Z}_i + C_i^T \widetilde{Z}_i C_i + h_m C_i^T \widetilde{T}_i C_i, \\
\phi_{44} &= -N_i + D_i^T \widetilde{Z}_i D_i + h_m D_i^T \widetilde{T}_i D_i,
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\left(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6\right) \\
&\quad \cdot c_1 + \lambda_7 d < c_2 \lambda_1 e^{-\alpha T_f};
\end{aligned} \tag{22}$$

meanwhile, the average dwell-time satisfies

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln\left[\left(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6\right) c_1 + \lambda_7 d\right] - \alpha T_f}. \tag{23}$$

We define

$$\begin{aligned}
\widetilde{P}_i &= R^{1/2} P_i R^{1/2}, \\
\widetilde{Q}_i &= R^{1/2} Q_i R^{1/2}, \\
\widetilde{Z}_i &= R^{1/2} Z_i R^{1/2}, \\
\widetilde{T}_i &= R^{1/2} T_i R^{1/2}, \\
\widetilde{M}_i &= R^{1/2} M_i R^{1/2},
\end{aligned} \tag{24}$$

$$\begin{aligned}
\lambda_{min}(P_i) &= \lambda_1, \\
\lambda_{max}(P_i) &= \lambda_2, \\
\lambda_{max}(Q_i) &= \lambda_3, \\
\lambda_{max}(Z_i) &= \lambda_4, \\
\lambda_{max}(T_i) &= \lambda_5, \\
\lambda_{max}(M_i) &= \lambda_6, \\
\lambda_{max}(N_i) &= \lambda_7,
\end{aligned} \tag{25}$$

where $\mu > 1$ *satisfies*

$$\begin{aligned}
\widetilde{P}_i &< \mu \widetilde{P}_j, \\
\widetilde{Q}_i &< \mu \widetilde{Q}_j, \\
\widetilde{Z}_i &< \mu \widetilde{Z}_j, \\
\widetilde{T}_i &< \mu \widetilde{T}_j,
\end{aligned}$$

$$\widetilde{M}_i < \mu \widetilde{M}_j, \quad \forall i, j \in M. \tag{26}$$

Then, switched system (17) is finite-time bounded with respect to $(c_1, c_2, d, R, T_f, \sigma)$.

Proof. Choose the piecewise Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(t) &= V_{\sigma(t)}(t) = V_i(t) \\ &= V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t) + V_{5i}(t), \end{aligned} \tag{27}$$

where

$$\begin{aligned} V_{1i}(t) &= x^T(t)\widetilde{P}_i x(t), \\ V_{2i}(t) &= \int_{t-h(t)}^{t} e^{\alpha(t-s)} x^T(t)\widetilde{Q}_i x(t)\, ds, \\ V_{3i}(t) &= \int_{t-\tau(t)}^{t} e^{\alpha(t-s)} \dot{x}^T(t)\widetilde{Z}_i \dot{x}(t)\, ds, \\ V_{4i}(t) &= \int_{-h_m}^{0}\int_{t+\epsilon}^{t} e^{\alpha(t-s)} \dot{x}^T(t)\widetilde{T}_i \dot{x}(t)\, ds\, d\epsilon, \\ V_{5i}(t) &= \int_{-r_m}^{0}\int_{t+\epsilon}^{t} e^{\alpha(t-s)} x^T(t)\widetilde{M}_i x(t)\, ds\, d\epsilon, \end{aligned} \tag{28}$$

in which α is a given scalar and $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$, $\widetilde{T}_i$, and $\widetilde{M}_i$ are positive definite matrices to be determined. Taking the derivative of $V(t)$ with respect to t along the trajectory of system (17) yields

$$\begin{aligned} \dot{V}_{1i}(t) &= 2x^T(t)\widetilde{P}_i \dot{x}(t), \\ \dot{V}_{2i}(t) &= \alpha V_{2i}(t) + x^T(t)\widetilde{Q}_i x(t) \\ &\quad - e^{\alpha h(t)}\left(1-\dot{h}(t)\right) x^T(t-h(t))\widetilde{Q}_i x(t-h(t)) \\ &\le \alpha V_{2i}(t) + x^T(t)\widetilde{Q}_i x(t) \\ &\quad - \left(1-\widehat{h}\right) x^T(t-h(t))\widetilde{Q}_i x(t-h(t)), \\ \dot{V}_{3i}(t) &= \alpha V_{3i}(t) + \dot{x}^T(t)\widetilde{Z}_i \dot{x}(t) \\ &\quad - e^{\alpha\tau(t)}(1-\dot{\tau}(t))\dot{x}^T(t-\tau(t))\widetilde{Z}_i \dot{x}(t-\tau(t)) \\ &\le \alpha V_{3i}(t) + \dot{x}^T(t)\widetilde{Z}_i \dot{x}(t) \\ &\quad - (1-\widehat{\tau})\dot{x}^T(t-\tau(t))\widetilde{Z}_i \dot{x}(t-\tau(t)), \\ \dot{V}_{4i}(t) &= \alpha V_{4i}(t) + h_m \dot{x}^T(t)\widetilde{T}_i \dot{x}(t) \\ &\quad - \int_{t-h(t)}^{t} e^{\alpha(t-s)}\dot{x}^T(s)\widetilde{T}_i \dot{x}(s)\, ds \\ &\le \alpha V_{4i}(t) + h_m \dot{x}^T(t)\widetilde{T}_i \dot{x}(t) \\ &\quad - \int_{t-h(t)}^{t} \dot{x}^T(s)\widetilde{T}_i \dot{x}(s)\, ds, \\ \dot{V}_{5i}(t) &= \alpha V_{5i}(t) + r_m x^T(t)\widetilde{M}_i x(t) \\ &\quad - \int_{t-r(t)}^{t} e^{\alpha(t-s)} x^T(s)\widetilde{M}_i x(s)\, ds \\ &\le \alpha V_{5i}(t) + r_m x^T(t)\widetilde{M}_i x(t) \\ &\quad - \int_{t-r(t)}^{t} x^T(s)\widetilde{M}_i x(s)\, ds. \end{aligned} \tag{29}$$

Using the Leibniz-Newton formula, we have

$$\begin{aligned} &2\left[x^T(t)H_{1i} + x^T(t-h(t))H_{2i}\right] \\ &\quad \cdot \left[x(t) - \int_{t-h(t)}^{t}\dot{x}(s)\, ds - x(t-h(t))\right] = 0. \end{aligned} \tag{30}$$

Let

$$\chi(t) = \left[x^T(t) \quad x^T(t-h(t))\right]^T, \tag{31}$$

and it obviously holds that

$$h_m \chi^T(t) X_i \chi(t) - \int_{t-h(t)}^{t} \chi^T(t) X_i \chi(t)\, ds \ge 0. \tag{32}$$

By Lemma 10, it is easy to obtain

$$\begin{aligned} &-\int_{t-r(t)}^{t} x^T(s)\widetilde{M}_i x(s)\, ds \\ &\quad \le -\frac{1}{r_m}\int_{t-r(t)}^{t} x^T(s)\, ds \widetilde{M}_i \int_{t-r(t)}^{t} x(s)\, ds. \end{aligned} \tag{33}$$

Thus we have

$$\begin{aligned} &\dot{V}(t) - \alpha V(t) - w^T(t) N_i w(t) \\ &\quad \le X^T(t)\Theta_i X(t) - \int_{t-h(t)}^{t} \vartheta^T(t,s)\Delta_i \vartheta(t,s)\, ds, \end{aligned} \tag{34}$$

where

$$\begin{aligned} &X(t) \\ &= \left[x^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(t-\tau(t)) \quad w^T(t) \quad \int_{t-r(t)}^{t} x^T(s)\, ds\right]^T, \\ &\vartheta(t,s) = \left[x^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(s)\right]^T. \end{aligned} \tag{35}$$

Considering (19) and (20), we can obtain that

$$\dot{V}(t) - \alpha V(t) - w^T(t) N_i w(t) < 0. \tag{36}$$

Integrating (36), it can be obtained from (26) and (36) that, $\forall t \in [t_k, t_{k+1})$,

$$\begin{aligned}
V(t) &< e^{\alpha(t-t_k)}V(t_k) + \int_{t_k}^{t} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&< e^{\alpha(t-t_k)} \mu V(t_k^-) + \int_{t_k}^{t} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&< e^{\alpha(t-t_k)} \mu \Big[e^{\alpha(t_k - t_{k-1})} V(t_{k-1}) \\
&+ \int_{t_{k-1}}^{t_k} e^{\alpha(t_k-s)} w^T(s) N_i w(s)\, ds \Big] \\
&+ \int_{t_k}^{t} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&= e^{\alpha(t-t_{k-1})} \mu V(t_{k-1}) \\
&+ \mu \int_{t_{k-1}}^{t_k} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&+ \int_{t_k}^{t} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds < \cdots \\
&< e^{\alpha(t-0)} \mu^{N_\sigma(0,t)} V(0) \\
&+ \mu^{N_\sigma(0,t)} \int_{0}^{t_1} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&+ \mu^{N_\sigma(t_1,t)} \int_{t_1}^{t_2} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds + \cdots \\
&+ \mu \int_{t_{k-1}}^{t_k} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&+ \int_{t_k}^{t} e^{\alpha(t-s)} w^T(s) N_i w(s)\, ds \\
&= e^{\alpha(t-0)} \mu^{N_\sigma(0,t)} V(0) \\
&+ \int_{0}^{t} e^{\alpha(t-s)} \mu^{N_\sigma(s,t)} w^T(s) N_i w(s)\, ds \\
&< e^{\alpha t} \mu^{N_\sigma(0,t)} V(0) + \mu^{N_\sigma(0,t)} e^{\alpha t} \int_{0}^{t} w^T(s) N_i w(s)\, ds \\
&< e^{\alpha T_f} \mu^{N_\sigma(0,T_f)} \left[V(0) + \int_{0}^{T_f} w^T(s) N_i w(s)\, ds \right] \\
&< e^{\alpha T_f} \mu^{N_\sigma(0,T_f)} \left[V(0) + \lambda_{\max}(N_i) d \right].
\end{aligned} \tag{37}$$

From Definition 8, we can deduce that $N_\sigma(0, T_f) < T_f/\tau_a$, and then we can obtain

$$V(t) < e^{(\alpha + \ln \mu/\tau_a)T_f} \left[V(0) + \lambda_7 d \right]. \tag{38}$$

On the other hand

$$\begin{aligned}
V(t) &> x^T(t) \widetilde{P}_i x(t) = x^T(t) R^{1/2} P_i R^{1/2} x(t) \\
&\geq \lambda_{\min}(P_i) x^T(t) R x(t) = \lambda_1 x^T(t) R x(t), \\
V(0) &\leq \lambda_{\max}(P_i) x^T(0) R x(0) + h_m e^{\alpha h_m} \lambda_{\max}(Q_i) \\
&\cdot \sup_{-\tau \leq \theta \leq 0} \left\{ x^T(\theta) R x(\theta), \dot{x}^T(\theta) R \dot{x}(\theta) \right\} \\
&+ \tau_m e^{\alpha \tau_m} \lambda_{\max}(Z_i) \\
&\cdot \sup_{-\tau \leq \theta \leq 0} \left\{ x^T(\theta) R x(\theta), \dot{x}^T(\theta) R \dot{x}(\theta) \right\} \\
&+ h_m e^{\alpha h_m} \lambda_{\max}(T_i) \\
&\cdot \sup_{-\tau \leq \theta \leq 0} \left\{ x^T(\theta) R x(\theta), \dot{x}^T(\theta) R \dot{x}(\theta) \right\} \\
&+ r_m e^{\alpha r_m} \lambda_{\max}(M_i) \\
&\cdot \sup_{-\tau \leq \theta \leq 0} \left\{ x^T(\theta) R x(\theta), \dot{x}^T(\theta) R \dot{x}(\theta) \right\} \leq \big[\lambda_{\max}(P_i) \\
&+ h_m e^{\alpha h_m} \lambda_{\max}(Q_i) + \tau_m e^{\alpha \tau_m} \lambda_{\max}(Z_i) \\
&+ h_m e^{\alpha h_m} \lambda_{\max}(T_i) + r_m e^{\alpha r_m} \lambda_{\max}(M_i) \big] \\
&\cdot \sup_{-\tau \leq \theta \leq 0} \left\{ x^T(\theta) R x(\theta), \dot{x}^T(\theta) R \dot{x}(\theta) \right\} \leq \big[\lambda_2 \\
&+ h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6 \big] \\
&\cdot c_1.
\end{aligned} \tag{39}$$

From (38)-(39), we can obtain

$$x^T(t) R x(t) \leq \frac{V(t)}{\lambda_1} < \frac{\left[\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6 \right] c_1 + \lambda_7 d}{\lambda_1} e^{\alpha T_f} \mu^{T_f/\tau_a}. \tag{40}$$

When $\mu = 1$, it is obvious that $x^T(t)Rx(t) < c_2$ by (22).

When $\mu > 1$, by virtue of (22), we have that

$$\begin{aligned}
&\ln(\lambda_1 c_2) - \ln \big[\big(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 \\
&+ h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6 \big) c_1 + \lambda_7 d \big] - \alpha T_f > 0.
\end{aligned} \tag{41}$$

From (23), we can obtain

$$\frac{T_f}{\tau_a} < \frac{\ln(\lambda_1 c_2) - \ln\left[\left(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + \tau_m e^{\alpha\tau_m}\lambda_4 + h_m e^{\alpha h_m}\lambda_5 + r_m e^{\alpha r_m}\lambda_6\right)c_1 + \lambda_7 d\right] - \alpha T_f}{\ln(\mu)}$$
$$= \frac{\ln\left[\lambda_1 c_2 e^{-\alpha T_f} / \left(\left(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + \tau_m e^{\alpha\tau_m}\lambda_4 + h_m e^{\alpha h_m}\lambda_5 + r_m e^{\alpha r_m}\lambda_6\right)c_1 + \lambda_7 d\right)\right]}{\ln(\mu)}. \quad (42)$$

Substituting (42) into (40) yields

$$x^T(t) R x(t) < \left[\frac{\left[\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + \tau_m e^{\alpha\tau_m}\lambda_4 + h_m e^{\alpha h_m}\lambda_5 + r_m e^{\alpha r_m}\lambda_6\right]c_1 + \lambda_7 d}{\lambda_1}\right] e^{\alpha T_f}$$
$$\left[\frac{\lambda_1 c_2 e^{-\alpha T_f}}{(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + \tau_m e^{\alpha\tau_m}\lambda_4 + h_m e^{\alpha h_m}\lambda_5 + r_m e^{\alpha r_m}\lambda_6)c_1 + \lambda_7 d}\right] = c_2. \quad (43)$$

The proof is completed. □

Based on Theorem 13, when we set $w(t) = 0$, the following corollary is proposed to solve the finite-time stable problem.

Corollary 14. *Consider system (17) with $w(t) = 0$. For given positive scalars α, $\hat{h}$, $\hat{\tau}$, h_m, and r_m, if there exist positive definite symmetric matrices $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$, $\widetilde{T}_i$, and $\widetilde{M}_i$ and matrices X_{11i}, X_{12i}, X_{22i}, H_{1i}, and H_{2i} with appropriate dimensions, then*

$$\begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \widetilde{T}_i \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \widetilde{P}_i G_i + A_i^T \widetilde{Z}_i G_i + h_m A_i^T \widetilde{T}_i G_i \\ * & \Delta_{22} & \Delta_{23} & B_i^T \widetilde{Z}_i G_i + h_m B_i^T \widetilde{T}_i G_i \\ * & * & \Delta_{33} & C_i^T \widetilde{Z}_i G_i + h_m C_i^T \widetilde{T}_i G_i \\ * & * & * & G_i^T \widetilde{Z}_i G_i + h_m G_i^T \widetilde{T}_i G_i - \dfrac{\widetilde{M}_i}{r_m} \end{bmatrix} < 0 \quad (44)$$

hold, where

$$\begin{aligned} \Delta_{11} &= -\alpha\widetilde{P}_i + \widetilde{P}_i A_i + A_i^T \widetilde{P}_i + \widetilde{Q}_i + A_i^T \widetilde{Z}_i A_i \\ &\quad + h_m A_i^T \widetilde{T}_i A_i + r_m \widetilde{M}_i + H_{1i} + H_{1i}^T + h_m X_{11i}, \\ \Delta_{12} &= \widetilde{P}_i B_i + A_i^T \widetilde{Z}_i B_i + h_m A_i^T \widetilde{T}_i B_i - H_{1i}^T + H_{2i} \\ &\quad + h_m X_{12i}, \\ \Delta_{13} &= \widetilde{P}_i C_i + A_i^T \widetilde{Z}_i C_i + h_m A_i^T \widetilde{T}_i C_i, \\ \Delta_{22} &= -\left(1 - \hat{h}\right)\widetilde{Q}_i + B_i^T \widetilde{Z}_i B_i + h_m B_i^T \widetilde{T}_i B_i - H_{2i} \\ &\quad - H_{2i}^T + h_m X_{22i}, \\ \Delta_{23} &= B_i^T \widetilde{Z}_i C_i + h_m B_i^T \widetilde{T}_i C_i, \\ \Delta_{33} &= -(1 - \hat{\tau})\widetilde{Z}_i + C_i^T \widetilde{Z}_i C_i + h_m C_i^T \widetilde{T}_i C_i, \\ &\left(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + \tau_m e^{\alpha\tau_m}\lambda_4 + h_m e^{\alpha h_m}\lambda_5 + r_m e^{\alpha r_m}\lambda_6\right) \\ &\quad \cdot c_1 < c_2 \lambda_1 e^{-\alpha T_f}; \end{aligned} \quad (45)$$

meanwhile, the average dwell-time satisfies

$$\tau_a > \tau_a^* = \frac{T_f \ln\mu}{\ln(\lambda_1 c_2) - \ln\left[(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + \tau_m e^{\alpha\tau_m}\lambda_4 + h_m e^{\alpha h_m}\lambda_5 + r_m e^{\alpha r_m}\lambda_6)c_1\right] - \alpha T_f}, \quad (46)$$

where $\mu > 1$ satisfying (26) and $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$, $\widetilde{T}_i$, $\widetilde{M}_i$, λ_1, λ_2, λ_3, λ_4, λ_5, and λ_6 are defined just the same as (25). Then the switched system (17) is finite-time stable with respect to $(c_1, c_2, d, R, T_f, \sigma)$.

Proof. The proof is similar to that of Theorem 13, and it is omitted here. □

3.2. Finite-Time Extended Dissipative Analysis. In this section, the finite-time extended dissipative analysis is considered in the following theorem.

Theorem 15. *For given positive scalars α, $\hat{h}$, $\hat{\tau}$, h_m, r_m, and b, if there exist positive definite symmetric matrices $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$,*

$\widetilde{T}_i$, and $\widetilde{M}_i$ and matrices X_{11i}, X_{12i}, X_{22i}, H_{1i}, and H_{2i} with appropriate dimensions, then

$$\frac{1}{b}\widetilde{P}_i - F_i^T \psi_4 F_i > 0, \tag{47}$$

$$X_i = \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geq 0, \tag{48}$$

$$\Delta_i = \begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \widetilde{T}_i \end{bmatrix} \geq 0, \tag{49}$$

$$\Phi_i = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \widetilde{P}_i G_i + A_i^T \widetilde{Z}_i G_i + h_m A_i^T \widetilde{T}_i G_i \\ * & \Phi_{22} & \Phi_{23} & B_i^T \widetilde{Z}_i D_i + h_m B_i^T \widetilde{T}_i D_i & B_i^T \widetilde{Z}_i G_i + h_m B_i^T \widetilde{T}_i G_i \\ * & * & \Phi_{33} & C_i^T \widetilde{Z}_i D_i + h_m C_i^T \widetilde{T}_i D_i & C_i^T \widetilde{Z}_i G_i + h_m C_i^T \widetilde{T}_i G_i \\ * & * & * & \Phi_{44} & D_i^T \widetilde{Z}_i G_i + h_m D_i^T \widetilde{T}_i G_i \\ * & * & * & * & G_i^T \widetilde{Z}_i G_i + h_m G_i^T \widetilde{T}_i G_i - \dfrac{\widetilde{M}_i}{r_m} \end{bmatrix} < 0 \tag{50}$$

hold, where

$$\begin{aligned}
\Phi_{11} &= -\alpha\widetilde{P}_i + \widetilde{P}_i A_i + A_i^T \widetilde{P}_i + \widetilde{Q}_i + A_i^T \widetilde{Z}_i A_i \\
&\quad + h_m A_i^T \widetilde{T}_i A_i - F_i^T \psi_1 F_i + r_m \widetilde{M}_i + H_{1i} + H_{1i}^T \\
&\quad + h_m X_{11i}, \\
\Phi_{12} &= \widetilde{P}_i B_i + A_i^T \widetilde{Z}_i B_i + h_m A_i^T \widetilde{T}_i B_i - H_{1i}^T + H_{2i} \\
&\quad + h_m X_{12i}, \\
\Phi_{13} &= \widetilde{P}_i C_i + A_i^T \widetilde{Z}_i C_i + h_m A_i^T \widetilde{T}_i C_i, \\
\Phi_{14} &= \widetilde{P}_i D_i + A_i^T \widetilde{Z}_i D_i + h_m A_i^T \widetilde{T}_i D_i - F_i^T \psi_2, \\
\Phi_{22} &= -\left(1 - \widehat{h}\right) \widetilde{Q}_i + B_i^T \widetilde{Z}_i B_i + h_m B_i^T \widetilde{T}_i B_i - H_{2i} - H_{2i}^T \\
&\quad + h_m X_{22i}, \\
\Phi_{23} &= B_i^T \widetilde{Z}_i C_i + h_m B_i^T \widetilde{T}_i C_i, \\
\Phi_{33} &= -(1 - \widehat{\tau}) \widetilde{Z}_i + C_i^T \widetilde{Z}_i C_i + h_m C_i^T \widetilde{T}_i C_i, \\
\Phi_{44} &= -\psi_3 + D_i^T \widetilde{Z}_i D_i + h_m D_i^T \widetilde{T}_i D_i;
\end{aligned} \tag{51}$$

meanwhile, the average dwell-time satisfies

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln\left[\lambda_8 k + (\lambda_9 + \lambda_{10}) d\right] - \alpha T_f}, \tag{52}$$

where

$$\begin{aligned}
\lambda_{min}(P_i) &= \lambda_1, \\
\lambda_{max}\left(F_i^T F_i\right) &= \lambda_8, \\
\lambda_{max}\left(\psi_2^T \psi_2\right) &= \lambda_9, \\
\lambda_{max}(\psi_3) &= \lambda_{10}, \\
\widetilde{P}_i &= R^{1/2} P_i R^{1/2}, \\
\widetilde{Q}_i &= R^{1/2} Q_i R^{1/2}, \\
\widetilde{Z}_i &= R^{1/2} Z_i R^{1/2}, \\
\widetilde{T}_i &= R^{1/2} T_i R^{1/2}, \\
\widetilde{M}_i &= R^{1/2} M_i R^{1/2}.
\end{aligned} \tag{53}$$

Then, system (17) is finite-time bounded with extended dissipative performance with respect to $(0, c_2, d, R, T_f, \sigma)$.

Proof. Choose the same Lyapunov-Krasovskii function as in Theorem 13, similar to the proof of Theorem 13, and we obtain

$$\begin{aligned}
\dot{V}(t) - \alpha V(t) - J(t) &\leq X^T(t) \Phi_i X(t) \\
&\quad - \int_{t-h(t)}^{t} \vartheta^T(t,s) \Delta_i \vartheta(t,s)\, ds,
\end{aligned} \tag{54}$$

where

$$\begin{aligned}
&X(t) \\
&= \left[x^T(t) \;\; x^T(t-h(t)) \;\; \dot{x}^T(t-\tau(t)) \;\; w^T(t) \;\; \int_{t-r(t)}^{t} x^T(s)\, ds\right]^T,
\end{aligned} \tag{55}$$

$$\vartheta(t,s) = \left[x^T(t) \;\; x^T(t-h(t)) \;\; \dot{x}^T(s)\right]^T;$$

by virtue of (49)-(50) we can obtain that

$$\dot{V}(t) - \alpha V(t) - J(t) < 0 \tag{56}$$

follows the proof line of (37); it is easy to obtain the following inequality:

$$V(t) < e^{\alpha t}\mu^{N_\sigma(0,t)}V(0) + \int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}J(s)\,ds; \quad (57)$$

under zero initial condition $V(0) = 0$, it can be calculated that

$$V(t) < e^{\alpha t}\mu^{N_\sigma(0,t)}\int_0^t J(s)\,ds, \quad (58)$$

and it is equivalent to

$$\frac{V(t)}{e^{\alpha t}\mu^{N_\sigma(0,t)}} < \int_0^t J(s)\,ds; \quad (59)$$

by Assumption 4, we have

$$\frac{V(t)}{b} < \int_0^t J(s)\,ds, \quad (60)$$

so we obtain

$$\int_0^t J(s)\,ds > \frac{V(t)}{b} > \frac{1}{b}x^T(t)\widetilde{P}_i x(t) > 0, \quad (61)$$

considering inequality

$$\int_0^{T_f} J(t)\,dt - \sup_{0\le t\le T_f} z^T(t)\psi_4 z(t) \ge 0; \quad (62)$$

when $\psi_4 = 0$, one obtains

$$\int_0^{T_f} J(t)\,dt \ge 0; \quad (63)$$

when $\psi_4 > 0$, by Assumption 3 we have $\psi_1 = 0$, $\psi_2 = 0$, and $\psi_3 > 0$, and then we obtain

$$\int_0^t J(s)\,ds = \int_0^t w^T(s)\psi_3 w(s)\,ds; \quad (64)$$

thus, for $\forall t \in [0, T_f]$, we have

$$\int_0^{T_f} J(s)\,ds > \int_0^t J(s)\,ds \ge \frac{1}{b}x^T(t)\widetilde{P}_i x(t) > 0; \quad (65)$$

it follows from (47) that

$$\begin{aligned}\int_0^{T_f} J(s)\,ds &\ge \frac{1}{b}x^T(t)\widetilde{P}_i x(t) \ge x^T(t)F_i^T\psi_4 F_i x(t)\\ &= z^T(t)\psi_4 z(t),\end{aligned} \quad (66)$$

so we get

$$\int_0^{T_f} J(t)\,dt - \sup_{0\le t\le T_f} z^T(t)\psi_4 z(t) \ge 0. \quad (67)$$

Thus the proof of extended dissipative is completed.

Next, we proof finite-time boundedness. Following the proof above, we can deduce that

$$\begin{aligned}V(t) &< e^{\alpha t}\mu^{N_\sigma(0,t)}\int_0^t J(s)\,ds,\\ V(t) &< e^{(\alpha+\ln\mu/\tau_a)T_f}\int_0^{T_f} J(s)\,ds.\end{aligned} \quad (68)$$

When $\psi_1 \le 0$, we can obtain

$$\begin{aligned}&\int_0^{T_f} J(s)\,ds \le \int_0^{T_f}\left[2z^T(s)\psi_2 w(s) + w^T(s)\cdot\psi_3 w(s)\right]ds,\\ &V(t) < e^{(\alpha+\ln\mu/\tau_a)T_f}\left[\int_0^{T_f}\left[2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)\right]ds\right],\end{aligned} \quad (69)$$

so we get

$$\begin{aligned}x^T(t)Rx(t) &\le \frac{V(t)}{\lambda_1}\\ &< \frac{e^{(\alpha+\ln\mu/\tau_a)T_f}}{\lambda_1}\left[\int_0^{T_f}\left[2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)\right]ds\right],\end{aligned} \quad (70)$$

by

$$\begin{aligned}&\int_0^{T_f}\left[2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)\right]ds\\ &\quad = \int_0^{T_f}\left[2x^T(s)F_i^T\psi_2 w(s) + w^T(s)\psi_3 w(s)\right]ds,\end{aligned} \quad (71)$$

and by Lemma 9, we have

$$\begin{aligned}2x^T(s)F_i^T\psi_2 w(s) &\le x^T(s)F_i^T F_i x(s)\\ &\quad + w^T(s)\psi_2^T\psi_2 w(s).\end{aligned} \quad (72)$$

According to (106), we can obtain

$$\begin{aligned}x^T(t)Rx(t) &\le \frac{V(t)}{\lambda_1}\\ &< \frac{e^{(\alpha+\ln\mu/\tau_a)T_f}}{\lambda_1}\left[\int_0^{T_f}\left[2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)\right]ds\right]\\ &< \frac{e^{(\alpha+\ln\mu/\tau_a)T_f}}{\lambda_1}\left[\int_0^{T_f}\left[x^T(s)F_i^T F_i x(s)\right.\right.\end{aligned}$$

$$+ w^T(s)\psi_2^T\psi_2 w(s) + w^T(s)\psi_3 w(s)\Big] ds\Big]$$
$$< \frac{e^{(\alpha+\ln\mu/\tau_a)T_f}}{\lambda_1}\left[\lambda_8 k + (\lambda_9 + \lambda_{10}) d\right]. \tag{73}$$

From (52), we can conclude that $x^T(t)Rx(t) \leq c_2$. Thus the proof is completed. □

3.3. Nonfragile Finite-Time Extended Dissipative Control. Consider system (1), under the controller $\mu(t) = K_{\sigma(t)}x(t)$, the corresponding closed-loop system is given by

$$\begin{aligned}
&\dot{x}(t) - \widehat{C}_{\sigma(t)}\dot{x}(t-\tau(t)) \\
&\quad = \left(\widehat{A}_{\sigma(t)} + E_{\sigma(t)}K_{\sigma(t)}(t)\right)x(t) + \widehat{B}_{\sigma(t)}x(t-h(t)) \\
&\quad + D_{\sigma(t)}w(t) + \widehat{G}_{\sigma(t)}\int_{t-r(t)}^{t} x(s)\,ds, \\
&z(t) = F_{\sigma(t)}x(t), \\
&x(t_0+\theta) = \varphi(\theta), \quad \forall\theta \in [-\tau, 0].
\end{aligned} \tag{74}$$

Firstly, for the additive gain variation model satisfying the form of Case 1, that is, $\Delta K_i(t) = J_{1i}U_i(t)J_{2i}$, we have the following theorem.

Theorem 16. *For given positive scalars α, $\widehat{h}$, $\widehat{\tau}$, h_m, r_m, δ, ϵ, and b, if there exist positive definite symmetric matrices $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$, $\widetilde{T}_i$, and $\widetilde{M}_i$ and matrices L_i, M_{1i}, M_{2i}, M_{3i}, M_{4i}, J_{1i}, and J_{2i} with appropriate dimensions, then*

$$\frac{1}{b}\widetilde{P}_i - F_i^T\psi_4 F_i > 0, \tag{75}$$

$$\begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geq 0, \tag{76}$$

$$\begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \widetilde{T}_i \end{bmatrix} \geq 0, \tag{77}$$

$$\begin{bmatrix}
\Theta_{11} & \Theta_{12} & C_iR_i & D_i - R_iF_i^T\psi_2 & G_i & \Theta_{16} & \Theta_{17} & R_iM_{1i}^T & R_iJ_{2i}^T & R_iF_i^T \\
* & \Theta_{22} & 0 & 0 & 0 & R_iB_i^T & h_mR_iB_i^T & R_iM_{2i}^T & 0 & 0 \\
* & * & \Theta_{33} & 0 & 0 & R_iC_i^T & h_mR_iC_i^T & R_iM_{3i}^T & 0 & 0 \\
* & * & * & -\psi_3 & 0 & D_i^T & h_mD_i^T & 0 & 0 & 0 \\
* & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & G_i^T & h_mG_i^T & M_{4i}^T & 0 & 0 \\
* & * & * & * & * & \Theta_{66} & \Theta_{67} & 0 & 0 & 0 \\
* & * & * & * & * & * & \Theta_{77} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\delta I & 0 & 0 \\
* & * & * & * & * & * & * & * & -\epsilon I & 0 \\
* & * & * & * & * & * & * & * & * & \psi_1^{-1}
\end{bmatrix} < 0 \tag{78}$$

hold, where

$$\begin{aligned}
\Theta_{11} &= -\alpha R_i + A_iR_i + R_iA_i^T + E_iY_i + Y_i^TE_i^T + \widehat{Q}_i \\
&\quad + r_m\widehat{M}_i + \widehat{H}_{1i} + \widehat{H}_{1i}^T + h_m\widehat{X}_{11i} + \delta L_iL_i^T \\
&\quad + \epsilon E_iJ_{1i}J_{1i}^TE_i^T, \\
\Theta_{12} &= B_iR_i - \widehat{H}_{1i}^T + \widehat{H}_{2i} + h_m\widehat{X}_{12i}, \\
\Theta_{16} &= R_iA_i^T + Y_i^TE_i^T + \delta L_iL_i^T + \epsilon E_iJ_{1i}J_{1i}^TE_i^T, \\
\Theta_{17} &= h_mR_iA_i^T + h_mY_i^TE_i^T + h_m\delta L_iL_i^T \\
&\quad + \epsilon h_mE_iJ_{1i}J_{1i}^TE_i^T, \\
\Theta_{22} &= -\left(1-\widehat{h}\right)\widehat{Q}_i - \widehat{H}_{2i} - \widehat{H}_{2i}^T + h_mX_{22i}, \\
\Theta_{33} &= -(1-\widehat{\tau})\widehat{Z}_i, \\
\Theta_{66} &= -W_i + \delta L_iL_i^T + \epsilon E_iJ_{1i}J_{1i}^TE_i^T, \\
\Theta_{67} &= h_m\delta L_iL_i^T + \epsilon h_mE_iJ_{1i}J_{1i}^TE_i^T,
\end{aligned}$$

$$\Theta_{77} = -h_m V_i + h_m \delta L_i L_i^T + \epsilon h_m^2 E_i J_{1i} J_{1i}^T E_i^T; \tag{79}$$

the matrices are defined as follows:

$$\begin{aligned}
K_i \widetilde{P}_i^{-1} &= Y_i, \\
\widetilde{P}_i^{-1} &= R_i, \\
\widetilde{Z}_i^{-1} &= W_i, \\
\widetilde{T}_i^{-1} &= V_i, \\
\widetilde{P}_i^{-1} \widetilde{Q}_i \widetilde{P}_i^{-1} &= \widehat{Q}_i, \\
\widetilde{P}_i^{-1} \widetilde{Z}_i \widetilde{P}_i^{-1} &= \widehat{Z}_i, \\
\widetilde{P}_i^{-1} \widetilde{M}_i \widetilde{P}_i^{-1} &= \widehat{M}_i, \\
\widetilde{P}_i^{-1} H_{1i} \widetilde{P}_i^{-1} &= \widehat{H}_{1i}, \\
\widetilde{P}_i^{-1} H_{2i} \widetilde{P}_i^{-1} &= \widehat{H}_{2i}, \\
\widetilde{P}_i^{-1} X_{11i} \widetilde{P}_i^{-1} &= \widehat{X}_{11i}, \\
\widetilde{P}_i^{-1} X_{12i} \widetilde{P}_i^{-1} &= \widehat{X}_{12i}, \\
\widetilde{P}_i^{-1} X_{22i} \widetilde{P}_i^{-1} &= \widehat{X}_{22i};
\end{aligned} \tag{80}$$

meanwhile, the average dwell-time satisfies

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln[\lambda_8 k + (\lambda_9 + \lambda_{10}) d] - \alpha T_f}, \tag{81}$$

where

$$\begin{aligned}
\lambda_{min}(P_i) &= \lambda_1, \\
\lambda_{max}(F_i^T F_i) &= \lambda_8, \\
\lambda_{max}(\psi_2^T \psi_2) &= \lambda_9, \\
\lambda_{max}(\psi_3) &= \lambda_{10}, \\
\widetilde{P}_i &= R^{1/2} P_i R^{1/2}, \\
\widetilde{Q}_i &= R^{1/2} Q_i R^{1/2}, \\
\widetilde{Z}_i &= R^{1/2} Z_i R^{1/2}, \\
\widetilde{T}_i &= R^{1/2} T_i R^{1/2}, \\
\widetilde{M}_i &= R^{1/2} M_i R^{1/2}.
\end{aligned} \tag{82}$$

Then the switched linear neutral system is finite-time bounded with extended dissipative performance. Furthermore, the non-fragile controller can be chosen by

$$u(t) = K_{\sigma(t)}(t) x(t). \tag{83}$$

Proof. Replacing A_i, B_i, C_i, and G_i in (50) with $\widehat{A}_i + E_i K_i + E_i \Delta K_i(t)$, $\widehat{B}_i$, $\widehat{C}_i$, and $\widehat{G}_i$ and by Schur complement, we obtain

$$\Lambda_i = \begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \widetilde{P}_i \widehat{C}_i & \widetilde{P}_i D_i - F_i^T \psi_2 & \widetilde{P}_i \widehat{G}_i & \Lambda_{16} & h_m \left(\widehat{A}_i + E_i K_i + E_i \Delta K_i(t)\right)^T \widetilde{T}_i \\
* & \Lambda_{22} & 0 & 0 & 0 & \widehat{B}_i^T \widetilde{Z}_i & h_m \widehat{B}_i^T \widetilde{T}_i \\
* & * & \Lambda_{33} & 0 & 0 & \widehat{C}_i^T \widetilde{Z}_i & h_m \widehat{C}_i^T \widetilde{T}_i \\
* & * & * & -\psi_3 & 0 & D_i^T \widetilde{Z}_i & h_m D_i^T \widetilde{T}_i \\
* & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & \widehat{G}_i^T \widetilde{Z}_i & h_m \widehat{G}_i^T \widetilde{T}_i \\
* & * & * & * & * & -\widetilde{Z}_i & 0 \\
* & * & * & * & * & * & -h_m \widetilde{T}_i
\end{bmatrix}, \tag{84}$$

where

$$\begin{aligned}
\Lambda_{11} &= -\alpha \widetilde{P}_i + \widetilde{P}_i \left(\widehat{A}_i + E_i K_i + E_i \Delta K_i(t)\right) \\
&\quad + \left(\widehat{A}_i + E_i K_i + E_i \Delta K_i(t)\right)^T \widetilde{P}_i + \widetilde{Q}_i \\
&\quad + r_m \widetilde{M}_i - F_i^T \psi_1 F_i + H_{1i} + H_{1i}^T + h_m X_{11i}, \\
\Lambda_{12} &= \widetilde{P}_i \widehat{B}_i - H_{1i}^T + H_{2i} + h_m X_{12i}, \\
\Lambda_{16} &= \left(\widehat{A}_i + E_i K_i + E_i \Delta K_i(t)\right)^T \widetilde{Z}_i, \\
\Lambda_{22} &= -\left(1 - \widehat{h}\right) \widetilde{Q}_i - H_{2i} - H_{2i}^T + h_m X_{22i}, \\
\Lambda_{33} &= -(1 - \widehat{\tau}) \widetilde{Z}_i;
\end{aligned} \tag{85}$$

Λ_i can be rewritten as

$$\Omega_i + \Gamma_{1i} U_i(t) \Gamma_{2i} + \Gamma_{2i}^T U_i^T(t) \Gamma_{1i}^T, \tag{86}$$

where

$$\Omega_i = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \widetilde{P}_i\widehat{C}_i & \widetilde{P}_iD_i - F_i^T\psi_2 & \widetilde{P}_i\widehat{G}_i & \Omega_{16} & h_m\left(\widehat{A}_i + E_iK_i\right)^T\widetilde{T}_i \\ * & \Omega_{22} & 0 & 0 & 0 & \widehat{B}_i^T\widetilde{Z}_i & h_m\widehat{B}_i^T\widetilde{T}_i \\ * & * & \Omega_{33} & 0 & 0 & \widehat{C}_i^T\widetilde{Z}_i & h_m\widehat{C}_i^T\widetilde{T}_i \\ * & * & * & -\psi_3 & 0 & D_i^T\widetilde{Z}_i & h_mD_i^T\widetilde{T}_i \\ * & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & \widehat{G}_i^T\widetilde{Z}_i & h_m\widehat{G}_i^T\widetilde{T}_i \\ * & * & * & * & * & -\widetilde{Z}_i & 0 \\ * & * & * & * & * & * & -h_m\widetilde{T}_i \end{bmatrix}, \quad (87)$$

with

$$\begin{aligned} \Omega_{11} &= -\alpha\widetilde{P}_i + \widetilde{P}_i\left(\widehat{A}_i + E_iK_i\right) + \left(\widehat{A}_i + E_iK_i\right)^T\widetilde{P}_i + \widetilde{Q}_i \\ &\quad + r_m\widetilde{M}_i - F_i^T\psi_1F_i + H_{1i} + H_{1i}^T + h_mX_{11i}, \\ \Omega_{12} &= \widetilde{P}_i\widehat{B}_i - H_{1i}^T + H_{2i} + h_mX_{12i}, \\ \Omega_{16} &= \left(\widehat{A}_i + E_iK_i\right)^T\widetilde{Z}_i, \\ \Omega_{22} &= -\left(1-\widehat{h}\right)\widetilde{Q}_i - H_{2i} - H_{2i}^T + h_mX_{22i}, \\ \Omega_{33} &= -\left(1-\widehat{\tau}\right)\widetilde{Z}_i; \end{aligned} \quad (88)$$

by using Lemma 11, there exists a scalar $\epsilon > 0$, such that

$$\begin{aligned} &\Omega_i + \Gamma_{1i}U_i(t)\Gamma_{2i} + \Gamma_{2i}^TU_i^T(t)\Gamma_{1i}^T \\ &\quad < \Omega_i + \epsilon\Gamma_{1i}\Gamma_{1i}^T + \epsilon^{-1}\Gamma_{2i}^T\Gamma_{2i}, \end{aligned} \quad (89)$$

where

$$\begin{aligned} \Gamma_{1i} &= \left[J_{1i}^TE_i^T\widetilde{P}_i \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; J_{1i}^TE_i^T\widetilde{Z}_i \;\; h_mJ_{1i}^TE_i^T\widetilde{T}_i\right]^T, \\ \Gamma_{2i} &= \left[J_{2i} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]. \end{aligned} \quad (90)$$

Here we consider the norm-bounded uncertainties, and we set $\Omega_i = \Omega_{1i} + \Omega_{2i}$, where

$$\Omega_{1i} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \widetilde{P}_iC_i & \widetilde{P}_iD_i - F_i^T\psi_2 & \widetilde{P}_iG_i & \Xi_{16} & h_m\left(A_i + E_iK_i\right)^T\widetilde{T}_i \\ * & \Xi_{22} & 0 & 0 & 0 & B_i^T\widetilde{Z}_i & h_mB_i^T\widetilde{T}_i \\ * & * & \Xi_{33} & 0 & 0 & C_i^T\widetilde{Z}_i & h_mC_i^T\widetilde{T}_i \\ * & * & * & -\psi_3 & 0 & D_i^T\widetilde{Z}_i & h_mD_i^T\widetilde{T}_i \\ * & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & G_i^T\widetilde{Z}_i & h_mG_i^T\widetilde{T}_i \\ * & * & * & * & * & -\widetilde{Z}_i & 0 \\ * & * & * & * & * & * & -h_m\widetilde{T}_i \end{bmatrix}, \quad (91)$$

with

$$\begin{aligned} \Xi_{11} &= -\alpha\widetilde{P}_i + \widetilde{P}_i\left(A_i + E_iK_i\right) + \left(A_i + E_iK_i\right)^T\widetilde{P}_i + \widetilde{Q}_i \\ &\quad + r_m\widetilde{M}_i - F_i^T\psi_1F_i + H_{1i} + H_{1i}^T + h_mX_{11i}, \\ \Xi_{12} &= \widetilde{P}_iB_i - H_{1i}^T + H_{2i} + h_mX_{12i}, \\ \Xi_{16} &= \left(A_i + E_iK_i\right)^T\widetilde{Z}_i, \\ \Xi_{22} &= -\left(1-\widehat{h}\right)\widetilde{Q}_i - H_{2i} - H_{2i}^T + h_mX_{22i}, \\ \Xi_{33} &= -\left(1-\widehat{\tau}\right)\widetilde{Z}_i, \end{aligned}$$

$$\Omega_{2i} = \begin{bmatrix} \widetilde{P}_iL_i \\ 0 \\ 0 \\ 0 \\ 0 \\ \widetilde{Z}_iL_i \\ h_m\widetilde{T}_iL_i \end{bmatrix} \Xi_i(t)\left[M_{1i} \;\; M_{2i} \;\; M_{3i} \;\; 0 \;\; M_{4i} \;\; 0 \;\; 0\right]$$

$$+\begin{bmatrix} M_{1i}^T \\ M_{2i}^T \\ M_{3i}^T \\ 0 \\ M_{4i}^T \\ 0 \\ 0 \end{bmatrix} \Xi_i^T(t)\left[L_i^T\widetilde{P}_i \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ L_i^T\widetilde{Z}_i \ \ h_m L_i^T\widetilde{T}_i\right]. \tag{92}$$

By Lemma 11, there exists a scalar $\delta > 0$, such that

$$\Omega_{2i} \le \delta \begin{bmatrix} \widetilde{P}_i L_i \\ 0 \\ 0 \\ 0 \\ 0 \\ \widetilde{Z}_i L_i \\ h_m\widetilde{T}_i L_i \end{bmatrix} \left[L_i^T\widetilde{P}_i \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ L_i^T\widetilde{Z}_i \ \ h_m L_i^T\widetilde{T}_i\right]$$

$$+\delta^{-1}\begin{bmatrix} M_{1i}^T \\ M_{2i}^T \\ M_{3i}^T \\ 0 \\ M_{4i}^T \\ 0 \\ 0 \end{bmatrix}\left[M_{1i} \ \ M_{2i} \ \ M_{3i} \ \ 0 \ \ M_{4i} \ \ 0 \ \ 0\right]. \tag{93}$$

Then pre- and postmultiplying (78) by diag$\{\widetilde{P}_i, \widetilde{P}_i, \widetilde{P}_i, I, I, \widetilde{Z}_i, \widetilde{T}_i, I, I, I\}$, we have

$$\Pi_{1i} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \widetilde{P}_i C_i & \widetilde{P}_i D_i - F_i^T\psi_2 & \widetilde{P}_i G_i & \Pi_{16} & \Pi_{17} & M_{1i}^T & J_{2i}^T & F_i^T \\ * & \Pi_{22} & 0 & 0 & 0 & B_i^T\widetilde{Z}_i & h_m B_i^T\widetilde{T}_i & M_{2i}^T & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & C_i^T\widetilde{Z}_i & h_m C_i^T\widetilde{T}_i & M_{3i}^T & 0 & 0 \\ * & * & * & -\psi_3 & 0 & D_i^T\widetilde{Z}_i & h_m D_i^T\widetilde{T}_i & 0 & 0 & 0 \\ * & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & G_i^T\widetilde{Z}_i & h_m G_i^T\widetilde{T}_i & M_{4i}^T & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\delta I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & * & * & * & \psi_1^{-1} \end{bmatrix} < 0, \tag{94}$$

where

$$\begin{aligned}
\Pi_{11} &= -\alpha\widetilde{P}_i + \widetilde{P}_i\left(A_i + E_iK_i\right) + \left(A_i + E_iK_i\right)^T\widetilde{P}_i + \widetilde{Q}_i \\
&\quad + r_m\widetilde{M}_i + H_{1i} + H_{1i}^T + h_m X_{11i} + \delta\widetilde{P}_i L_i L_i^T\widetilde{P}_i \\
&\quad + \epsilon\widetilde{P}_i E_i J_{1i} J_{1i}^T E_i^T\widetilde{P}_i, \\
\Pi_{12} &= \widetilde{P}_i B_i - H_{1i}^T + H_{2i} + h_m X_{12i}, \\
\Pi_{16} &= \left(A_i + E_iK_i\right)^T\widetilde{Z}_i + \delta\widetilde{P}_i L_i L_i^T\widetilde{Z}_i \\
&\quad + \epsilon\widetilde{P}_i E_i J_{1i} J_{1i}^T E_i^T\widetilde{Z}_i, \\
\Pi_{17} &= h_m\left(A_i + E_iK_i\right)^T\widetilde{T}_i + h_m\delta\widetilde{P}_i L_i L_i^T\widetilde{T}_i \\
&\quad + \epsilon h_m\widetilde{P}_i E_i J_{1i} J_{1i}^T E_i^T\widetilde{T}_i, \\
\Pi_{22} &= -\left(1 - \widehat{h}\right)\widetilde{Q}_i - H_{2i} - H_{2i}^T + h_m X_{22i}, \\
\Pi_{33} &= -\left(1 - \widehat{\tau}\right)\widetilde{Z}_i, \\
\Pi_{66} &= -\widetilde{Z}_i + \delta\widetilde{Z}_i^T L_i L_i^T\widetilde{Z}_i + \epsilon\widetilde{Z}_i^T E_i J_{1i} J_{1i}^T E_i^T\widetilde{Z}_i, \\
\Pi_{67} &= h_m\delta\widetilde{Z}_i^T L_i L_i^T\widetilde{T}_i + \epsilon h_m\widetilde{Z}_i^T E_i J_{1i} J_{1i}^T E_i^T\widetilde{T}_i, \\
\Pi_{77} &= -h_m\widetilde{T}_i + h_m\delta\widetilde{T}_i^T L_i L_i^T\widetilde{T}_i + \epsilon h_m^2\widetilde{T}_i^T E_i J_{1i} J_{1i}^T E_i^T\widetilde{T}_i.
\end{aligned} \tag{95}$$

Based on above discussion, from $\Pi_{1i} < 0$, by Schur complement, we can conclude that $\Lambda_i < 0$. Similar to the proof of Theorem 15, we can obtain

$$\begin{aligned}
\dot{V}(t) - \alpha V(t) - J(t) &\le X^T(t)\Lambda_i X(t) \\
&\quad - \int_{t-h(t)}^{t}\vartheta^T(t,s)\Delta_i\vartheta(t,s)\,ds,
\end{aligned} \tag{96}$$

where

$$X(t) = \left[x^T(t) \;\; x^T(t-h(t)) \;\; \dot{x}^T(t-\tau(t)) \;\; w^T(t) \;\; \int_{t-r(t)}^{t} x^T(s)\,ds\right]^T, \quad (97)$$

$$\vartheta(t,s) = \left[x^T(t) \;\; x^T(t-h(t)) \;\; \dot{x}^T(s)\right]^T;$$

Δ_i is given in (77). The following proof is similar to that of Theorem 15; it is omitted here. □

Furthermore, for the multiplicative gain variation model $\Delta K_i(t)$ with the form in (5) of Case 2, we have the following theorem.

Theorem 17. *For given positive scalars α, $\widehat{h}$, $\widehat{\tau}$, h_m, r_m, δ, ϵ, b, and $0 \le c \le 1$, if there exist positive definite symmetric matrices $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{Z}_i$, $\widetilde{T}_i$, and $\widetilde{M}_i$ and matrices L_i, M_{1i}, M_{2i}, M_{3i}, and M_{4i}, then*

$$\frac{1}{b}\widetilde{P}_i - F_i^T\psi_4F_i > 0, \quad (98)$$

$$\begin{bmatrix} -cI & W_{qi} \\ W_{qi} & -Q_{qi} \end{bmatrix} < 0, \quad (99)$$

$$\begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \ge 0, \quad (100)$$

$$\begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \widetilde{T}_i \end{bmatrix} \ge 0, \quad (101)$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & C_iR_i & \Gamma_{14} & G_i & \Gamma_{16} & \Gamma_{17} & Y_i^T & R_iF_i^T & R_iM_{1i}^T & \Psi_{1i}K_i \\ * & \Gamma_{22} & 0 & 0 & 0 & R_iB_i^T & h_mR_iB_i^T & 0 & 0 & R_iM_{2i}^T & 0 \\ * & * & \Gamma_{33} & 0 & 0 & R_iC_i^T & h_mR_iC_i^T & 0 & 0 & R_iM_{3i}^T & 0 \\ * & * & * & -\psi_3 & 0 & D_i^T & h_mD_i^T & 0 & 0 & 0 & 0 \\ * & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & G_i^T & h_mG_i^T & 0 & 0 & M_{4i}^T & 0 \\ * & * & * & * & * & \Gamma_{66} & \Gamma_{67} & 0 & 0 & 0 & \Psi_{2i}K_i \\ * & * & * & * & * & * & \Gamma_{77} & 0 & 0 & 0 & \Psi_{3i}K_i \\ * & * & * & * & * & * & * & -\epsilon I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \psi_1^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\delta I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\sum\limits_{q=1}^{n}\overline{\sigma}_{2qi}I \end{bmatrix} < 0, \quad (102)$$

where

$$\Gamma_{11} = -\alpha R_i + A_iR_i + R_iA_i^T + E_iY_i + Y_i^TE_i^T + \widehat{Q}_i + r_m\widehat{M}_i + \widehat{H}_{1i} + \widehat{H}_{1i}^T + h_m\widehat{X}_{11i} + \delta L_iL_i^T + \epsilon E_i\emptyset_i^2E_i^T + m\sum_{q=1}^{n}\overline{\sigma}_{2qi}\widehat{Q}_{qi},$$

$$\Gamma_{12} = B_iR_i - \widehat{H}_{1i}^T + \widehat{H}_{2i} + h_m\widehat{X}_{12i},$$

$$\Gamma_{14} = D_i - R_iF_i^T\psi_2,$$

$$\Gamma_{16} = R_iA_i^T + Y_i^TE_i^T + \delta L_iL_i^T + \epsilon E_i\emptyset_i^2E_i^T,$$

$$\Gamma_{17} = h_mR_iA_i^T + h_mY_i^TE_i^T + h_m\delta L_iL_i^T + \epsilon h_mE_i\emptyset_i^2E_i^T,$$

$$\Gamma_{22} = -\left(1-\widehat{h}\right)\widehat{Q}_i - \widehat{H}_{2i} - \widehat{H}_{2i}^T + h_m\widehat{X}_{22i},$$

$$\Gamma_{33} = -\left(1-\widehat{\tau}\right)\widehat{Z}_i,$$

$$\Gamma_{66} = -W_i + \delta L_iL_i^T + \epsilon E_i\emptyset_i^2E_i^T,$$

$$\Gamma_{67} = \epsilon h_mE_i\emptyset_i^2E_i^T + h_m\delta L_iL_i^T,$$

$$\Gamma_{77} = -h_mV_i + h_m\delta L_iL_i^T + \epsilon h_m^2E_i\emptyset_i^2E_i^T,$$

$$\emptyset_i = \operatorname{diag}\left\{\overline{\sigma}_{11i}, \overline{\sigma}_{12i}, \ldots, \overline{\sigma}_{1mi}\right\},$$

$$\Psi_{1i}=\left[E_iV_{1i}\ \ E_iV_{2i}\ \ \cdots\ \ E_iV_{mi}\right],$$
$$\Psi_{2i}=\left[E_iV_{1i}\ \ E_iV_{2i}\ \ \cdots\ \ E_iV_{mi}\right],$$
$$\Psi_{3i}=\left[h_mE_iV_{1i}\ \ h_mE_iV_{2i}\ \ \cdots\ \ h_mE_iV_{mi}\right],$$
$$V_{pi}=\operatorname{diag}\left\{\underbrace{0,0,\ldots,0}_{p-1},1,0,\ldots,0\right\}; \tag{103}$$

the matrices are defined as follows:

$$\begin{aligned}
K_i\widetilde{P}_i^{-1}&=Y_i,\\
\widetilde{P}_i^{-1}&=R_i,\\
\widetilde{Z}_i^{-1}&=W_i,\\
\widetilde{T}_i^{-1}&=V_i,\\
\widetilde{P}_i^{-1}Q_{qi}\widetilde{P}_i^{-1}&=\widehat{Q}_{qi},\\
\widetilde{P}_i^{-1}\widetilde{Q}_i\widetilde{P}_i^{-1}&=\widehat{Q}_i,\\
\widetilde{P}_i^{-1}\widetilde{Z}_i\widetilde{P}_i^{-1}&=\widehat{Z}_i,\\
\widetilde{P}_i^{-1}\widetilde{M}_i\widetilde{P}_i^{-1}&=\widehat{M}_i,\\
\widetilde{P}_i^{-1}H_{1i}\widetilde{P}_i^{-1}&=\widehat{H}_{1i},\\
\widetilde{P}_i^{-1}H_{2i}\widetilde{P}_i^{-1}&=\widehat{H}_{2i},\\
\widetilde{P}_i^{-1}X_{11i}\widetilde{P}_i^{-1}&=\widehat{X}_{11i},\\
\widetilde{P}_i^{-1}X_{12i}\widetilde{P}_i^{-1}&=\widehat{X}_{12i},\\
\widetilde{P}_i^{-1}X_{22i}\widetilde{P}_i^{-1}&=\widehat{X}_{22i};
\end{aligned} \tag{104}$$

the average dwell-time satisfies

$$\tau_a>\tau_a^*=\frac{T_f\ln\mu}{\ln\left(\lambda_1c_2\right)-\ln\left[\lambda_8k+\left(\lambda_9+\lambda_{10}\right)d\right]-\alpha T_f}, \tag{105}$$

$$\begin{aligned}
\lambda_{min}\left(P_i\right)&=\lambda_1,\\
\lambda_{max}\left(F_i^TF_i\right)&=\lambda_8,\\
\lambda_{max}\left(\psi_2^T\psi_2\right)&=\lambda_9,\\
\lambda_{max}\left(\psi_3\right)&=\lambda_{10},
\end{aligned} \tag{106}$$

$$\begin{aligned}
\widetilde{P}_i&=R^{1/2}P_iR^{1/2},\\
\widetilde{Q}_i&=R^{1/2}Q_iR^{1/2},\\
\widetilde{Z}_i&=R^{1/2}Z_iR^{1/2},\\
\widetilde{T}_i&=R^{1/2}T_iR^{1/2},\\
\widetilde{M}_i&=R^{1/2}M_iR^{1/2}.
\end{aligned} \tag{107}$$

The controller gains can be given by $K_i=Y_i\widetilde{P}_i$. *Then the switched linear neutral system is finite-time bounded with extended dissipative performance under the nonfragile controller* $u(t)=K_{\sigma(t)}(t)x(t)$.

Proof. Replacing A_i, B_i, C_i, and G_i in (50) with $\widehat{A}_i+E_iK_i+E_i\Delta K_i(t)$, $\widehat{B}_i$, $\widehat{C}_i$, and $\widehat{G}_i$ and by Schur complement, we obtain

$$\Gamma_i=\begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \widetilde{P}_i\widehat{C}_i & \widetilde{P}_iD_i-F_i^T\psi_2 & \widetilde{P}_i\widehat{G}_i & \Lambda_{16} & h_m\left(\widehat{A}_i+E_iK_i+E_i\Delta K_i(t)\right)^T\widetilde{T}_i\\
* & \Lambda_{22} & 0 & 0 & 0 & \widehat{B}_i^T\widetilde{Z}_i & h_m\widehat{B}_i^T\widetilde{T}_i\\
* & * & \Lambda_{33} & 0 & 0 & \widehat{C}_i^T\widetilde{Z}_i & h_m\widehat{C}_i^T\widetilde{T}_i\\
* & * & * & -\psi_3 & 0 & D_i^T\widetilde{Z}_i & h_mD_i^T\widetilde{T}_i\\
* & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & \widehat{G}_i^T\widetilde{Z}_i & h_m\widehat{G}_i^T\widetilde{T}_i\\
* & * & * & * & * & -\widetilde{Z}_i & 0\\
* & * & * & * & * & * & -h_m\widetilde{T}_i
\end{bmatrix}, \tag{108}$$

where

$$\begin{aligned}
\Lambda_{11}&=-\alpha\widetilde{P}_i+\widetilde{P}_i\left(\widehat{A}_i+E_iK_i+E_i\Delta K_i(t)\right)\\
&\quad+\left(\widehat{A}_i+E_iK_i+E_i\Delta K_i(t)\right)^T\widetilde{P}_i+\widetilde{Q}_i\\
&\quad+r_m\widetilde{M}_i-F_i^T\psi_1F_i+H_{1i}+H_{1i}^T+h_mX_{11i},\\
\Lambda_{12}&=\widetilde{P}_i\widehat{B}_i-H_{1i}^T+H_{2i}+h_mX_{12i},\\
\Lambda_{16}&=\left(\widehat{A}_i+E_iK_i+E_i\Delta K_i(t)\right)^T\widetilde{Z}_i,\\
\Lambda_{22}&=-\left(1-\widehat{h}\right)\widetilde{Q}_i-H_{2i}-H_{2i}^T+h_mX_{22i},\\
\Lambda_{33}&=-\left(1-\widehat{\tau}\right)\widetilde{Z}_i;
\end{aligned} \tag{109}$$

(108) can be rewritten as

$$\Omega_i + \begin{bmatrix} \widetilde{P}_i E_i \Delta K_i(t) + (E_i \Delta K_i(t))^T \widetilde{P}_i & 0 & 0 & 0 & 0 & (E_i \Delta K_i(t))^T \widetilde{Z}_i & h_m (E_i \Delta K_i(t))^T \widetilde{T}_i \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}, \tag{110}$$

where

$$\Omega_i = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \widetilde{P}_i \widehat{C}_i & \widetilde{P}_i D_i - F_i^T \psi_2 & \widetilde{P}_i \widehat{G}_i & \Omega_{16} & h_m \left(\widehat{A}_i + E_i K_i\right)^T \widetilde{T}_i \\ * & \Omega_{22} & 0 & 0 & 0 & \widehat{B}_i^T \widetilde{Z}_i & h_m \widehat{B}_i^T \widetilde{T}_i \\ * & * & \Omega_{33} & 0 & 0 & \widehat{C}_i^T \widetilde{Z}_i & h_m \widehat{C}_i^T \widetilde{T}_i \\ * & * & * & -\psi_3 & 0 & D_i^T \widetilde{Z}_i & h_m D_i^T \widetilde{T}_i \\ * & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & \widehat{G}_i^T \widetilde{Z}_i & h_m \widehat{G}_i^T \widetilde{T}_i \\ * & * & * & * & * & -\widetilde{Z}_i & 0 \\ * & * & * & * & * & * & -h_m \widetilde{T}_i \end{bmatrix}, \tag{111}$$

with

$$\begin{aligned} \Omega_{11} &= -\alpha \widetilde{P}_i + \widetilde{P}_i \left(\widehat{A}_i + E_i K_i\right) + \left(\widehat{A}_i + E_i K_i\right)^T \widetilde{P}_i + \widetilde{Q}_i \\ &\quad + r_m \widetilde{M}_i - F_i^T \psi_1 F_i + H_{1i} + H_{1i}^T + h_m X_{11i}, \\ \Omega_{12} &= \widetilde{P}_i \widehat{B}_i - H_{1i}^T + H_{2i} + h_m X_{12i}, \\ \Omega_{16} &= \left(\widehat{A}_i + E_i K_i\right)^T \widetilde{Z}_i, \\ \Omega_{22} &= -\left(1 - \widehat{h}\right) \widetilde{Q}_i - H_{2i} - H_{2i}^T + h_m X_{22i}, \\ \Omega_{33} &= -(1 - \widehat{\tau}) \widetilde{Z}_i; \end{aligned} \tag{112}$$

considering (5), (110) can be rewritten as

$$\begin{aligned} &\Omega_i + \widetilde{B}_i \widetilde{H}_i \widetilde{L}_i + \left(\widetilde{B}_i \widetilde{H}_i \widetilde{L}_i\right)^T \\ &\quad + \sum_{p=1}^{m} \sum_{q=1}^{n} \sigma_{pqi} \left[\widetilde{K}_{pqi} W_{qi} \widetilde{I}_i + \left(\widetilde{K}_{pqi} W_{qi} \widetilde{I}_i\right)^T\right], \end{aligned} \tag{113}$$

where

$$\begin{aligned} \widetilde{H}_i &= \operatorname{diag}\{\sigma_{1i}, \sigma_{2i}, \ldots, \sigma_{mi}\}, \\ \widetilde{L}_i &= [K_i \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0], \\ \widetilde{I}_i &= [I \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0], \\ V_{pi} &= \operatorname{diag}\{\underbrace{0, 0, \ldots, 0}_{p-1}, 1, 0, \ldots, 0\}, \\ W_{qi} &= \operatorname{diag}\{\underbrace{0, 0, \ldots, 0}_{q-1}, 1, 0, \ldots, 0\}, \\ \widetilde{B}_i &= \left[E_i^T \widetilde{P}_i \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ E_i^T \widetilde{Z}_i \ \ h_m E_i^T \widetilde{T}_i\right]^T, \\ \widetilde{K}_{pqi} &= \left[K_i^T V_{pi}^T E_i^T \widetilde{P}_i \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ K_i^T V_{pi}^T E_i^T \widetilde{Z}_i \ \ h_m K_i^T V_{pi}^T E_i^T \widetilde{T}_i\right]^T; \end{aligned} \tag{114}$$

based on (6) and Lemma 12, for some $\epsilon > 0$, $Q_{qi} > 0$ ($q = 1, \ldots, n$) and $\emptyset_i = \operatorname{diag}\{\overline{\sigma}_{11i}, \overline{\sigma}_{12i}, \ldots, \overline{\sigma}_{1mi}\}$, it can be verified that

$$\begin{aligned} &\widetilde{B}_i \widetilde{H}_i \widetilde{L}_i + \left(\widetilde{B}_i \widetilde{H}_i \widetilde{L}_i\right)^T \le \epsilon \widetilde{B}_i \emptyset_i^2 \widetilde{B}_i^T + \epsilon^{-1} \widetilde{L}_i^T \widetilde{L}_i, \\ &\widetilde{K}_{pqi} W_{qi} \widetilde{I}_i + \left(\widetilde{K}_{pqi} W_{qi} \widetilde{I}_i\right)^T \\ &\quad \le \widetilde{K}_{pqi} W_{qi} Q_{qi}^{-1} W_{qi}^T \widetilde{K}_{pqi}^T + \widetilde{I}_i^T Q_{qi} \widetilde{I}_i, \\ &\sum_{p=1}^{m} \sum_{q=1}^{n} \sigma_{pqi} \widetilde{I}_i^T Q_{qi} \widetilde{I}_i \le m \sum_{q=1}^{n} \overline{\sigma}_{2qi} \widetilde{I}_i^T Q_{qi} \widetilde{I}_i, \end{aligned}$$

$$\sum_{p=1}^{m}\sum_{q=1}^{n}\sigma_{pqi}\widetilde{K}_{pqi}W_{qi}Q_{qi}^{-1}W_{qi}^{T}\widetilde{K}_{pqi}^{T} \leq \sum_{p=1}^{m}\sum_{q=1}^{n}\overline{\sigma}_{2qi}\widetilde{K}_{pqi}W_{qi}Q_{qi}^{-1}W_{qi}^{T}\widetilde{K}_{pqi}^{T}. \tag{115}$$

On the other hand, by Schur complement,

$$\begin{bmatrix} -cI & W_{qi} \\ W_{qi} & -Q_{qi} \end{bmatrix} < 0 \tag{116}$$

is equal to $W_{qi}Q_{qi}^{-1}W_{qi} < cI$. Then $W_{qi}Q_{qi}^{-1}W_{qi} < I$ holds. It can be proven that $\widetilde{K}_{pqi}W_{qi}Q_{qi}^{-1}W_{qi}^{T}\widetilde{K}_{pqi}^{T} \leq \widetilde{K}_{pqi}\widetilde{K}_{pqi}^{T}$. Hence, we have

$$\begin{aligned}
&\Omega_i + \widetilde{B}_i\widetilde{H}_i\widetilde{L}_i + \left(\widetilde{B}_i\widetilde{H}_i\widetilde{L}_i\right)^T \\
&\quad + \sum_{p=1}^{m}\sum_{q=1}^{n}\sigma_{pqi}\left[\widetilde{K}_{pqi}W_{qi}\widetilde{I}_i + \left(\widetilde{K}_{pqi}W_{qi}\widetilde{I}_i\right)^T\right] \\
&< \Omega_i + \epsilon\widetilde{B}_i\emptyset_i^2\widetilde{B}_i^T + \epsilon^{-1}\widetilde{L}_i^T\widetilde{L}_i + m\sum_{q=1}^{n}\overline{\sigma}_{2qi}\widetilde{I}_i^TQ_{qi}\widetilde{I}_i \\
&\quad + \sum_{p=1}^{m}\sum_{q=1}^{n}\overline{\sigma}_{2qi}\widetilde{K}_{pqi}\widetilde{K}_{pqi}^T.
\end{aligned} \tag{117}$$

Here we consider the norm-bounded uncertainties, and we set

$$\Omega_i = \Omega_{1i} + \Omega_{2i}, \tag{118}$$

where

$$\Omega_{1i} = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \widetilde{P}_iC_i & \widetilde{P}_iD_i - F_i^T\psi_2 & \widetilde{P}_iG_i & \Xi_{16} & h_m\left(A_i + E_iK_i\right)^T\widetilde{T}_i \\
* & \Xi_{22} & 0 & 0 & 0 & B_i^T\widetilde{Z}_i & h_mB_i^T\widetilde{T}_i \\
* & * & \Xi_{33} & 0 & 0 & C_i^T\widetilde{Z}_i & h_mC_i^T\widetilde{T}_i \\
* & * & * & -\psi_3 & 0 & D_i^T\widetilde{Z}_i & h_mD_i^T\widetilde{T}_i \\
* & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & G_i^T\widetilde{Z}_i & h_mG_i^T\widetilde{T}_i \\
* & * & * & * & * & -\widetilde{Z}_i & 0 \\
* & * & * & * & * & * & -h_m\widetilde{T}_i
\end{bmatrix}, \tag{119}$$

with

$$\begin{aligned}
\Xi_{11} &= -\alpha\widetilde{P}_i + \widetilde{P}_i\left(A_i + E_iK_i\right) + \left(A_i + E_iK_i\right)^T\widetilde{P}_i + \widetilde{Q}_i \\
&\quad + r_m\widetilde{M}_i - F_i^T\psi_1F_i + H_{1i} + H_{1i}^T + h_mX_{11i}, \\
\Xi_{12} &= \widetilde{P}_iB_i - H_{1i}^T + H_{2i} + h_mX_{12i}, \\
\Xi_{16} &= \left(A_i + E_iK_i\right)^T\widetilde{Z}_i, \\
\Xi_{22} &= -\left(1 - \widehat{h}\right)\widetilde{Q}_i - H_{2i} - H_{2i}^T + h_mX_{22i}, \\
\Xi_{33} &= -\left(1 - \widehat{\tau}\right)\widetilde{Z}_i, \\
\Omega_{2i} &= \begin{bmatrix} \widetilde{P}_iL_i \\ 0 \\ 0 \\ 0 \\ 0 \\ \widetilde{Z}_i^TL_i \\ h_m\widetilde{T}_i^TL_i \end{bmatrix}\Xi_i(t) \\
&\quad \cdot \begin{bmatrix} M_{1i} & M_{2i} & M_{3i} & 0 & M_{4i} & 0 & 0 \end{bmatrix} + \begin{bmatrix} M_{1i}^T \\ M_{2i}^T \\ M_{3i}^T \\ 0 \\ M_{4i}^T \\ 0 \\ 0 \end{bmatrix}\Xi_i^T(t) \\
&\quad \cdot \begin{bmatrix} L_i^T\widetilde{P}_i^{\,T} & 0 & 0 & 0 & 0 & L_i^T\widetilde{Z}_i & h_mL_i^T\widetilde{T}_i \end{bmatrix}.
\end{aligned} \tag{120}$$

By Lemma 11, there exists a scalar $\delta > 0$, such that

$$\Omega_{2i} \leq \delta\begin{bmatrix} \widetilde{P}_iL_i \\ 0 \\ 0 \\ 0 \\ 0 \\ \widetilde{Z}_iL_i \\ h_m\widetilde{T}_iL_i \end{bmatrix}\begin{bmatrix} L_i^T\widetilde{P}_i & 0 & 0 & 0 & 0 & L_i^T\widetilde{Z}_i & h_mL_i^T\widetilde{T}_i \end{bmatrix}$$

$$+\delta^{-1}\begin{bmatrix} M_{1i}^T \\ M_{2i}^T \\ M_{3i}^T \\ 0 \\ M_{4i}^T \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} M_{1i} & M_{2i} & M_{3i} & 0 & M_{4i} & 0 & 0 \end{bmatrix}. \tag{121}$$

Then pre- and postmultiplying (102) by $\operatorname{diag}\{\widetilde{P}_i, \widetilde{P}_i, \widetilde{P}_i, I, I, \widetilde{Z}_i, \widetilde{T}_i, I, I, I, \underbrace{I, \ldots, I}_{m}\}$, we have

$$\Pi_{2i} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \widetilde{P}_i C_i & \Psi_{14} & \widetilde{P}_i G_i & \Psi_{16} & \Psi_{17} & K_i^T & F_i^T & M_{1i}^T & \Delta_{1i} K_i \\ * & \Psi_{22} & 0 & 0 & 0 & B_i^T \widetilde{Z}_i & h_m B_i^T \widetilde{T}_i & 0 & 0 & M_{2i}^T & 0 \\ * & * & \Psi_{33} & 0 & 0 & C_i^T \widetilde{Z}_i & h_m C_i^T \widetilde{T}_i & 0 & 0 & M_{3i}^T & 0 \\ * & * & * & -\psi_3 & 0 & D_i^T \widetilde{Z}_i & h_m D_i^T \widetilde{T}_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\dfrac{\widetilde{M}_i}{r_m} & G_i^T \widetilde{Z}_i & h_m G_i^T \widetilde{T}_i & 0 & 0 & M_{4i}^T & 0 \\ * & * & * & * & * & \Psi_{66} & \Psi_{67} & 0 & 0 & 0 & \Delta_{2i} K_i \\ * & * & * & * & * & * & \Psi_{77} & 0 & 0 & 0 & \Delta_{3i} K_i \\ * & * & * & * & * & * & * & -\epsilon I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \psi_1^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\delta I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\sum_{q=1}^{n} \overline{\sigma}_{2qi} I \end{bmatrix} < 0, \tag{122}$$

where

$$\begin{aligned}
\Psi_{11} &= -\alpha \widetilde{P}_i + \widetilde{P}_i (A_i + E_i K_i) + (A_i + E_i K_i)^T \widetilde{P}_i + \widetilde{Q}_i \\
&\quad + r_m \widetilde{M}_i + H_{1i} + H_{1i}^T + h_m X_{11i} \\
&\quad + \epsilon \widetilde{P}_i E_i \emptyset_i^2 E_i^T \widetilde{P}_i + m \sum_{q=1}^{n} \overline{\sigma}_{2qi} Q_{qi} \\
&\quad + \delta \widetilde{P}_i L_i L_i^T \widetilde{P}_i, \\
\Psi_{12} &= \widetilde{P}_i B_i - H_{1i}^T + H_{2i} + h_m X_{12i}, \\
\Psi_{14} &= \widetilde{P}_i D_i - F_i^T \psi_2, \\
\Psi_{16} &= (A_i + E_i K_i)^T \widetilde{Z}_i + \epsilon \widetilde{P}_i E_i \emptyset_i^2 E_i^T \widetilde{Z}_i + \delta \widetilde{P}_i L_i L_i^T \widetilde{Z}_i, \\
\Psi_{17} &= h_m (A_i + E_i K_i)^T \widetilde{T}_i + \epsilon h_m \widetilde{P}_i E_i \emptyset_i^2 E_i^T \widetilde{T}_i \\
&\quad + h_m \delta \widetilde{P}_i L_i L_i^T \widetilde{T}_i, \\
\Psi_{22} &= -\left(1 - \widehat{h}\right) \widetilde{Q}_i - H_{2i} - H_{2i}^T + h_m X_{22i}, \\
\Psi_{33} &= -(1 - \widehat{\tau}) \widetilde{Z}_i, \\
\Psi_{66} &= -\widetilde{Z}_i + \epsilon \widetilde{Z}_i^T E_i \emptyset_i^2 E_i^T \widetilde{Z}_i + \delta \widetilde{Z}_i^T L_i L_i^T \widetilde{Z}_i, \\
\Psi_{67} &= \epsilon h_m \widetilde{Z}_i^T E_i \emptyset_i^2 E_i^T \widetilde{T}_i + h_m \delta \widetilde{Z}_i^T L_i L_i^T \widetilde{T}_i, \\
\Psi_{77} &= -h_m \widetilde{T}_i + \epsilon h_m^2 \widetilde{T}_i^T E_i \emptyset_i^2 E_i^T \widetilde{T}_i + h_m \delta \widetilde{T}_i^T L_i L_i^T \widetilde{T}_i, \\
\Delta_{1i} &= \left[\widetilde{P}_i E_i V_{1i} \;\; \widetilde{P}_i E_i V_{2i} \;\; \cdots \;\; \widetilde{P}_i E_i V_{mi}\right], \\
\Delta_{2i} &= \left[\widetilde{Z}_i E_i V_{1i} \;\; \widetilde{Z}_i E_i V_{2i} \;\; \cdots \;\; \widetilde{Z}_i E_i V_{mi}\right], \\
\Delta_{3i} &= \left[h_m \widetilde{T}_i E_i V_{1i} \;\; h_m \widetilde{T}_i E_i V_{2i} \;\; \cdots \;\; h_m \widetilde{T}_i E_i V_{mi}\right].
\end{aligned} \tag{123}$$

Based on above discussion, from $\Pi_{2i} < 0$, by Schur complement, we can conclude that $\Gamma_i < 0$. Similar to the proof of Theorem 15, we can obtain

$$\begin{aligned}
&\dot{V}(t) - \alpha V(t) - J(t) \\
&\quad \le X^T(t) \Gamma_i X(t) - \int_{t-h(t)}^{t} \vartheta^T(t,s) \Delta_i \vartheta(t,s) \, ds,
\end{aligned} \tag{124}$$

where

$$\begin{aligned} X(t) &= \left[x^T(t) \;\; x^T(t-h(t)) \;\; \dot{x}^T(t-\tau(t)) \;\; w^T(t) \;\; \int_{t-r(t)}^{t} x^T(s)\,ds\right]^T, \\ \vartheta(t,s) &= \left[x^T(t) \;\; x^T(t-h(t)) \;\; \dot{x}^T(s)\right]^T; \end{aligned} \tag{125}$$

Δ_i is given in (101). The following proof is similar to that of Theorem 15; it is omitted here. □

Remark 18. The concept of extended dissipative could be employed to lots of other systems, for example, the T-S fuzzy systems [34–37], which shows the effectiveness of the powerful tool.

4. Numerical Example

In this section, we present an example to illustrate the effectiveness of the controller design method.

Example 1. Consider system (1) with two subsystems with parameters as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}, \\ E_1 &= \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}, \\ G_1 &= \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix}, \\ F_1 &= \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ M_{11} = M_{21} = M_{31} = M_{41} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}, \\ E_2 &= \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}, \\ G_2 &= \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}, \\ L_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ M_{12} = M_{22} = M_{32} = M_{42} &= \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \\ \hat{h} &= 0.01, \\ \hat{\tau} &= 0.01, \\ \alpha &= 0.01, \\ h_m &= 0.5, \\ r_m &= 0.5, \\ \delta &= 0.5, \\ \epsilon &= 0.5. \end{aligned} \tag{126}$$

Case 1. When $\Delta K_i(t)$ satisfies additive form (4), we set

$$\begin{aligned} J_{11} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ J_{21} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ J_{12} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ J_{22} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}. \end{aligned} \tag{127}$$

Table 1: Matrices for each case.

Analysis	Ψ_1	Ψ_2	Ψ_3	Ψ_4
$L_2 - L_\infty$ performance	0	0	$\gamma^2 I$	I
H_∞ performance	$-I$	0	$\gamma^2 I$	0
Passivity	0	I	γ	0
Dissipativity	$-I$	I	$\begin{bmatrix} 1 & 0 \\ 0.3 & 1 \end{bmatrix} - \beta * I$	0

Table 2: Optimized variable for each case.

$L_2 - L_\infty$ performance	H_∞ performance	Passivity	Dissipativity
$\gamma_{1\min}^2 = 1 * 10^{-7}$	$\gamma_{1\min}^2 = 1 * 10^{-7}$	$\gamma_{1\min} = 1 * 10^{-7}$	$\beta_{1\max} = 1.9999999$

Table 3: Controller gain of the additive form controller uncertainty for each case.

Subsystem	1	2
$L_2 - L_\infty$ performance	$K_1 = 10^3 * \begin{bmatrix} -8.5600 & -0.2148 \\ -0.1787 & -9.6241 \end{bmatrix}$	$K_2 = 10^4 * \begin{bmatrix} -1.4313 & -0.1736 \\ -0.1853 & -0.9176 \end{bmatrix}$
H_∞ performance	$K_1 = 10^4 * \begin{bmatrix} -1.3762 & -0.8518 \\ -0.5214 & -5.3445 \end{bmatrix}$	$K_2 = 10^4 * \begin{bmatrix} -4.8652 & -0.9760 \\ -1.4654 & -1.9109 \end{bmatrix}$
Passivity	$K_1 = 10^8 * \begin{bmatrix} -0.3767 & -0.1899 \\ -0.1414 & -1.5588 \end{bmatrix}$	$K_2 = 10^8 * \begin{bmatrix} -1.4961 & -0.4558 \\ -0.7175 & -0.5430 \end{bmatrix}$
Dissipativity	$K_1 = 10^8 * \begin{bmatrix} -0.2581 & -0.1677 \\ -0.1381 & -1.2602 \end{bmatrix}$	$K_2 = 10^8 * \begin{bmatrix} -1.0597 & -0.4132 \\ -0.5249 & -0.4398 \end{bmatrix}$

Case 2. When $\Delta K_i(t)$ satisfies multiplicative form (5), we choose

$$\begin{gathered} \overline{\sigma}_{111} = 0.2, \\ \overline{\sigma}_{121} = 0.2, \\ \overline{\sigma}_{211} = 0.4, \\ \overline{\sigma}_{221} = 0.2 \\ (m = 2,\ n = 2), \\ \overline{\sigma}_{112} = 0.2, \\ \overline{\sigma}_{122} = 0.4, \\ \overline{\sigma}_{212} = 0.2, \\ \overline{\sigma}_{222} = 0.2 \\ (m = 2,\ n = 2). \end{gathered} \tag{128}$$

Furthermore, just as the discussion in Remark 6, we choose the values for the extended dissipative parameters in Table 1.

Then, solve the LMIs from (47) to (50) in Theorem 15, and we can get the results of optimized variables of four performances in Table 2.

Furthermore, solve the LMIs presented in Theorems 16 and 17, and we can obtain the controller gain for the additive form controller uncertainty and the multiplicative form controller uncertainty in Tables 3 and 4, respectively.

5. Conclusion

In this paper, we have investigated the problem of finite-time extended dissipative analysis and nonfragile control of switched neutral system with unknown time-varying disturbance. The average dwell-time approach is utilized for finite-time boundedness and extended dissipative performance analysis; controllers are designed to guarantee that the system is finite-time bounded and satisfies the extended dissipative performance. Based on extended dissipative performance, we can solve H_∞, $L_2 - L_\infty$, Passivity, and (Q, S, R)-dissipativity performance in a unified framework. All the results are given in terms of linear matrix inequalities (LMIs), and numerical examples are provided to show the effectiveness of the proposed method. In our future research, the nonfragile control and extended dissipative performance will be extended to more complex systems, such as Markovian jump delayed systems, sliding control systems, and T-S fuzzy systems, which deserve further study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Table 4: Controller gain of the multiplicative form controller uncertainty for each case.

Subsystem	1	2
$L_2 - L_\infty$ performance	$K_1 = 10^{-4} * \begin{bmatrix} -0.8602 & -0.2082 \\ -0.2037 & -0.5895 \end{bmatrix}$	$K_2 = 10^{-3} * \begin{bmatrix} -0.1000 & -0.1023 \\ -0.1017 & -0.1090 \end{bmatrix}$
H_∞ performance	$K_1 = 10^{-3} * \begin{bmatrix} -0.1922 & -0.2047 \\ -0.0870 & -0.4234 \end{bmatrix}$	$K_2 = 10^{-3} * \begin{bmatrix} -0.4842 & -0.2812 \\ -0.5055 & -0.3274 \end{bmatrix}$
Passivity	$K_1 = \begin{bmatrix} -0.2875 & -0.2582 \\ -0.1017 & -0.7210 \end{bmatrix}$	$K_2 = \begin{bmatrix} -1.2292 & -0.5381 \\ -1.2374 & -0.5245 \end{bmatrix}$
Dissipativity	$K_1 = \begin{bmatrix} -0.2711 & -0.4428 \\ -0.1380 & -1.1896 \end{bmatrix}$	$K_2 = \begin{bmatrix} -2.5753 & -0.8837 \\ -2.4237 & -0.9115 \end{bmatrix}$

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