Research Article

The Effect of Initial State Error for Nonlinear Systems with Delay via Iterative Learning Control

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An iterative learning control problem for nonlinear systems with delays is studied in detail in this paper. By introducing the λ-norm and being inspired by retarded Gronwall-like inequality, the novel sufficient conditions for robust convergence of the tracking error, whose initial states are not zero, with time delays are obtained. Finally, simulation example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Since iterative learning control is proposed by Arimoto et al. in 1984 (see [1]), this feedforward control approach has become a major research area in recent years. Iterative learning control, which belongs to the intelligent control methodology, is an approach for fully utilizing the previous control information and improving the transient performance of studied systems that is suitable for repetitive movements. Its goal is to achieve full range of tracking tasks on finite interval (see [2–14]).

In practice, the control problem of systems with delays has always been an interesting research, since time delay can be often encountered in a wide range, such as aircraft systems, turbojet engines, microwave oscillators, nuclear reactors, and chemical processes. The existence of time delay in a system may degrade the control performance and even at worst may become a source of instability. Stabilization problem of control systems with delay has received much attention for several decades and some research results have been reported in the literature (see [3, 11, 13, 15–23]). However, only a few results are available for nonlinear systems, combining with the iterative learning control items, with time delays [11, 24–26]. In this paper, under the case that the kth iterative state vector $x_k(t)$ is different from the $(k+1)$th iterative state vector $x_{k+1}(t)$, that is, $x_{k+1}(t) - x_k(t) \neq 0$, the iterative learning controller of nonlinear time-delayed systems is designed by using λ-norm and retarded Gronwall-like inequality.

Before ending this section, it is worth pointing out the main contributions of this paper as follows.

(1) The iterative learning control problem for nonlinear systems with delays is investigated. That is, we consider the system

$$\dot{x}_k(t) = f(t, x_k(t)) + g(t, x_k(t - \tau)) + u_k(t), \qquad (1)$$

which is different from the mentioned system

$$\dot{x}_k(t) = f(t, x_k(t)) + u_k(t) \qquad (2)$$

in past literature.

(2) Based on retarded Gronwall-like inequality and the convergence of tracking error $e_k(t) = y_d(t) - y_k(t)$, the practical output $y_k(t)$ is determinate by the previous iterative learning control information $x_k(t), u_k(t), \varphi_{k,h}(t)$.

2. Preliminaries

Throughout this paper, the 2-norm for the n-dimensional vector $x = (x_1, x_2, \ldots, x_n)^T$ is defined as $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$, while the λ-norm for a function is defined as $\| \cdot \|_\lambda = \sup_{t \in [0,T]} \{e^{-\lambda t} \| \cdot \|\}$, where the superscript T represents the transpose and $\lambda > 0$. I and 0 represent the identity matrix and a zero matrix, respectively.

Lemma 1 (see [10, 27]). *Consider*

$$\sup_{t\in[0,T]} \left(e^{-\lambda t} \int_0^t \|x(\tau)\| \, d\tau \right) \le \frac{1}{\lambda} \|x(t)\|_\lambda. \tag{3}$$

Lemma 2 (see [28] retarded Gronwall-like inequality). *Consider such an inequality*

$$u(t) \le a(t) + \sum_{i=1}^n \int_{b_i(t_0)}^{b_i(t)} f_i(t,s) \, w_i(u(s)) \, ds, \tag{4}$$

$$t_0 \le t < t_1,$$

and suppose that

   (1) *all w_i $(i = 1, 2, \ldots, n)$ are continuous and nondecreasing functions on $[0, +\infty)$ and are positive on $(0, +\infty)$ such that $w_1 \propto w_2 \propto \cdots \propto w_n$;*

   (2) *$a(t)$ is continuously differentiable in t and nonnegative on $[t_0, t_1)$, where t_0, t_1 are constants and $t_0 < t_1$;*

   (3) *all $b_i : [t_0, t_1) \to [t_0, t_1)$ $(i = 1, 2, \ldots, n)$ are continuously differentiable and nondecreasing such that $b_i(t) \le t$ on $[t_0, t_1)$;*

   (4) *all $f_i(t,s)$, $i = 1, 2, \ldots, n$, are continuous and nonnegative functions on $[t_0, t_1) \times [t_0, t_1)$.*

Take the notation $W_i(s, s_0) := \int_{s_0}^s (dz/w_i(z))$ for $s > 0$, where $s_0 > 0$ is a given constant. It is denoted by $W_i(s)$ simply when there is no confusion. If $u(t)$ is a continuous and nonnegative function on $[t_0, t)$ satisfying (4), then

$$u(t) \le W_n^{-1} \left[W_n(r_n(t)) + \int_{b_n(t_0)}^{b_n(t)} \max_{t_0 \le \tau < t} f_n(\tau, s) \, ds \right], \tag{5}$$

$$t_0 \le t \le T,$$

where $r_n(t)$ is determined recursively by

$$r_1(t) := a(t_0) + \int_{t_0}^t |a'(s)| \, ds,$$

$$r_{i+1}(t) := W_i^{-1} \left[W_i(r_i(t)) + \int_{b_n(t_0)}^{b_n(t)} \max_{t_0 \le \tau < t} f_n(\tau, s) \, ds \right], \tag{6}$$

$$i = 1, 2, \ldots, n-1;$$

$T < t_1$ and T is the largest number such that

$$W_i(r_i(T)) + \int_{b_n(t_0)}^{b_n(T)} \max_{t_0 \le \tau < T} f_n(\tau, s) \, ds \le \int_{s_0}^{+\infty} \frac{dz}{w_i(z)}, \tag{7}$$

$$i = 1, 2, \ldots, n.$$

3. Main Results

Consider the following system with time delay:

$$\dot{x}_k(t) = f(t, x_k(t)) + g(t, x_k(t - \tau)) + u_k(t),$$

$$u_{k+1}(t) = u_k(t) + Me_k(t)$$

$$\qquad - \psi_{k,h}(t)(x_{k+1}(0) - x_k(0)), \tag{8}$$

$$y_k(t) = Cx_k(t) + Du_k(t) - \varphi_{k,h}(t),$$

$$\varphi_{k+1,h}(t) = \varphi_{k,h}(t) - D\psi_{k,h}(t)(x_{k+1}(0) - x_k(0)),$$

where $x_k(t) \in R^n$, $y_k(t) \in R^n$, and $u_k(t) \in R^n$ are the state vector, output vector, and input vector, respectively. k is the number of iterations, $k \in \{1, 2, 3, \ldots\}$ and $t \in [0, T]$. $\int_0^t \psi_{k,h}(\theta) d\theta = 1$, $(t \ge h)$, M, C, D are real constant matrices; $e_k(t) = y_d(t) - y_k(t)$, $y_d(t)$ is a reference output.

Suppose that there exist the bounded constants $l_f > 0$ and $l_g > 0$ such that

$$\|f(t, x_{k+1}(t)) - f(t, x_k(t))\| \le l_f \|x_{k+1}(t) - x_k(t)\|,$$

$$\|g(t, x_{k+1}(t - \tau)) - g(t, x_k(t - \tau))\| \tag{9}$$

$$\le l_g \|x_{k+1}(t - \tau) - x_k(t - \tau)\|.$$

Theorem 3. *For system (8) and a given reference $y_d(t)$, if there exist matrices M, C, D and functions $\psi_{k,h}(t)$ and $\varphi_{k,h}(t)$ such that*

$$\|I - DM\| \le \rho < 1, \tag{10}$$

where ρ is a constant, then system (8) with the iterative learning control law can guarantee that $\|y_k(t) - y_d(t)\|$ is bounded but $y_k(t)$ cannot track $y_d(t)$ on $t \in [0, h]$ and $\lim_{k\to\infty} y_k(t) = y_d(t)$ on $t \in [h, T]$ for arbitrary initial state $x_k(0)$.

Proof. It is easy to know that, for any $t \in [h, T]$,

$$x_{k+1}(t) - x_k(t)$$

$$= \int_0^t (\dot{x}_{k+1}(\theta) - \dot{x}_k(\theta)) \, d\theta + x_{k+1}(0) - x_k(0)$$

$$= \int_0^t (f(\theta, x_{k+1}(\theta)) - f(\theta, x_k(\theta))) \, d\theta$$

$$\quad + \int_0^t (g(\theta, x_{k+1}(\theta - \tau)) - g(\theta, x_k(\theta - \tau))) \, d\theta$$

$$\quad + \int_0^t (u_{k+1}(\theta) - u_k(\theta)) \, d\theta + x_{k+1}(0) - x_k(0)$$

$$= \int_0^t (f(\theta, x_{k+1}(\theta)) - f(\theta, x_k(\theta))) \, d\theta$$

$$\quad + \int_0^t (g(\theta, x_{k+1}(\theta - \tau)) - g(\theta, x_k(\theta - \tau))) \, d\theta$$

$$\quad + \int_0^t Me_k(\theta) \, d\theta$$

$$- \left(x_{k+1}(0) - x_k(0)\right) \left(\int_0^t \psi_{k,h}(\theta) \, d\theta - 1\right)$$

$$= \int_0^t \left(f(\theta, x_{k+1}(\theta)) - f(\theta, x_k(\theta))\right) d\theta$$

$$+ \int_0^t \left(g(\theta, x_{k+1}(\theta - \tau)) - g(\theta, x_k(\theta - \tau))\right) d\theta$$

$$+ \int_0^t M e_k(\theta) \, d\theta. \tag{11}$$

So we obtain from condition (9)

$$\|x_{k+1}(t) - x_k(t)\| \leq l_f \int_0^t \|x_{k+1}(\theta) - x_k(\theta)\| \, d\theta$$

$$+ l_g \int_{0-\tau}^{t-\tau} \|x_{k+1}(\theta) - x_k(\theta)\| \, d\theta \tag{12}$$

$$+ \int_0^t \|M\| \|e_k(\theta)\| \, d\theta.$$

In this paper, we use Lemma 2. Taking $t_0 = 0$, $b_1(t) = t$, $b_2(t) = t - \tau$, $a(t) = \int_0^t \|M\| \|e_k(\theta)\| d\theta$, $W_1(s) = l_f \int_1^s (dz/z) = l_f \ln s$, and $W_2(s) = l_g \int_1^s (dz/z) = l_g \ln s$, then $r_1(t) = \int_0^t \|M\| \|e_k(\theta)\| d\theta$ and $r_2(t) = \exp[\ln(\int_0^t \|M\| \|e_k(\theta)\| d\theta) + \int_0^t (l_g/l_f) d\theta] = e^{(l_g/l_f)t} \cdot \int_0^t \|M\| \|e_k(\theta)\| d\theta$. So we have $\|x_{k+1}(t) - x_k(t)\| \leq \exp[\ln(e^{(l_g/l_f)t} \int_0^t \|M\| \|e_k(\theta)\| d\theta) + \int_{0-\tau}^{t-\tau} d\theta] = e^{(1+l_g/l_f)t} \cdot \int_0^t \|M\| \|e_k(\theta)\| d\theta$:

$$e_{k+1}(t) = e_k(t) + y_k(t) - y_{k+1}(t)$$

$$= e_k(t) + C x_k(t) + D u_k(t) - \varphi_{k,h}(t)$$

$$- C x_{k+1}(t) - D u_{k+1}(t) + \varphi_{k+1,h}(t)$$

$$= e_k(t) - C\left(x_{k+1}(t) - x_k(t)\right)$$

$$- D\left(u_{k+1}(t) - u_k(t)\right)$$

$$+ \left(\varphi_{k+1,h}(t) - \varphi_{k,h}(t)\right)$$

$$= e_k(t) - C\left(x_{k+1}(t) - x_k(t)\right) - D M e_k(t)$$

$$+ D \psi_{k,h}(t) \left(x_{k+1}(0) - x_k(0)\right)$$

$$+ \left(\varphi_{k+1,h}(t) - \varphi_{k,h}(t)\right)$$

$$= (I - DM) e_k(t) - C\left(x_{k+1}(t) - x_k(t)\right),$$

$$\|e_{k+1}(t)\| \leq \|I - DM\| \|e_k(t)\|$$

$$+ \|C\| \|x_{k+1}(t) - x_k(t)\|$$

$$\leq \|I - DM\| \|e_k(t)\| + \|C\| e^{(1+l_g/l_f)T}$$

$$\cdot \int_0^t \|M\| \|e_k(\theta)\| \, d\theta. \tag{13, 14}$$

Using Lemma 1 and multiplying both sides of the above inequality (14) by $e^{-\lambda t}$ and taking the λ-norm, we have

$$\|e_{k+1}(t)\|_\lambda$$

$$\leq \|I - DM\| \|e_k(t)\|_\lambda$$

$$+ \frac{\|C\| \|M\| e^{(1+l_g/l_f)T}}{\lambda} \|e_k(t)\|_\lambda \tag{15}$$

$$= \left(\|I - DM\| + \frac{\|C\| \|M\| e^{(1+l_g/l_f)T}}{\lambda}\right) \|e_k(t)\|_\lambda.$$

Thus, condition (10) can guarantee ($\|I - DM\| + \|C\| \|M\| e^{(1+l_g/l_f)T}/\lambda$) < 1 by selecting λ sufficiently large, so we have $\lim_{k \to \infty} \|e_k(t)\|_\lambda = 0$ for any $t \in [h, T]$. It follows from the equivalence of norms; we get that $\lim_{k \to \infty} \|e_k(t)\| = 0$.

For any $t \in [0, h]$,

$$x_{k+1}(t) - x_k(t)$$

$$= \int_0^t \left(f(\theta, x_{k+1}(\theta)) - f(\theta, x_k(\theta))\right) d\theta$$

$$+ \int_0^t \left(g(\theta, x_{k+1}(\theta - \tau)) - g(\theta, x_k(\theta - \tau))\right) d\theta$$

$$+ \int_0^t M e_k(\theta) \, d\theta$$

$$- \left(x_{k+1}(0) - x_k(0)\right) \left(\int_0^t \psi_{k,h}(\theta) \, d\theta - 1\right),$$

$$\|x_{k+1}(t) - x_k(t)\|$$

$$\leq l_f \int_0^t \|x_{k+1}(\theta) - x_k(\theta)\| \, d\theta \tag{16}$$

$$+ l_g \int_{0-\tau}^{t-\tau} \|x_{k+1}(\theta) - x_k(\theta)\| \, d\theta$$

$$+ \int_0^t \|M\| \|e_k(\theta)\| \, d\theta$$

$$+ \|x_{k+1}(0) - x_k(0)\| \left\|\int_0^t \psi_{k,h}(\theta) \, d\theta - 1\right\|$$

$$\leq l_f \int_0^t \|x_{k+1}(\theta) - x_k(\theta)\| \, d\theta$$

$$+ l_g \int_{0-\tau}^{t-\tau} \|x_{k+1}(\theta) - x_k(\theta)\| \, d\theta$$

$$+ \int_0^t \|M\| \|e_k(\theta)\| \, d\theta + \eta,$$

where $\eta = \sup_{t \in [0,h]} (\|x_{k+1}(0) - x_k(0)\| \|\int_0^t \psi_{k,h}(\theta) d\theta - 1\|)$.

It is easy to know that

$$\|x_{k+1}(t) - x_k(t)\|$$

$$\leq e^{(1+l_g/l_f)t} \left(\int_0^t \|M\| \|e_k(\theta)\| \, d\theta + \eta\right) \tag{17}$$

by using Lemma 2.

Then

$$\|e_{k+1}(t)\| \leq \|I - DM\| \|e_k(t)\|$$

$$+ \|C\| \|x_{k+1}(t) - x_k(t)\|$$

$$\leq \|I - DM\| \|e_k(t)\|$$

$$+ \|C\| e^{(1+l_g/l_f)T} \left(\int_0^t \|M\| \|e_k(\theta)\| d\theta \right) \quad (18)$$

$$+ \|C\| e^{(1+l_g/l_f)T} \eta.$$

Using Lemma 1 and multiplying both sides of the above inequality by $e^{-\lambda t}$ and taking the λ-norm, we have

$$\|e_{k+1}(t)\|_\lambda$$

$$\leq \left(\|I - DM\| + \frac{\|C\| \|M\| e^{(1+l_g/l_f)T}}{\lambda} \right) \|e_k(t)\|_\lambda + \eta \quad (19)$$

$$= \sigma \|e_k(t)\|_\lambda + \eta,$$

where $\sigma = \|I - DM\| + \|C\|\|M\|e^{(1+l_g/l_f)T}/\lambda$:

$$\|e_{k+1}(t)\|_\lambda + \frac{\eta}{\sigma - 1} \leq \sigma \left(\|e_k(t)\|_\lambda + \frac{\eta}{\sigma - 1} \right). \quad (20)$$

Imitating the above proof, the result $\lim_{k \to \infty} \|e_k(t)\|_\lambda = 0$ for any $t \in [h, T]$ is obtained by selecting λ sufficiently large. So it is true that $\|e_k(t)\|$ is bounded on $t \in [0, h]$. □

Remark 4. When the number of iterations $k \to \infty$ and $[h, T] \to [0, T]$, the tracking error satisfies that $\|e_k(t)\| \to 0$ on $t \in [0, T]$ for arbitrary initial state $x_k(0)$.

System (8) is

$$\dot{x}_k(t) = f(t, x_k(t)) + g(t, x_k(t - \tau(t))) + u_k(t),$$

$$u_{k+1}(t) = u_k(t) + M(t) e_k(t)$$

$$- \psi_{k,h}(t) (x_{k+1}(0) - x_k(0)), \quad (21)$$

$$y_k(t) = C(t) x_k(t) + D(t) u_k(t) - \varphi_{k,h}(t),$$

$$\varphi_{k+1,h}(t) = \varphi_{k,h}(t) - D(t) \psi_{k,h}(t) (x_{k+1}(0) - x_k(0)),$$

where the delay $\tau(t)$ satisfies $0 < \dot{\tau}(t) \leq \gamma < 1$.

Similar to the proof of Theorem 3, inequality (14) is written as

$$\|e_{k+1}(t)\| \leq \|I - D(t) M(t)\| \|e_k(t)\| + \|C(t)\|$$

$$\cdot \|M(t)\| e^{(1+(1-\gamma)l_g/l_f)T} \cdot \int_0^t \|e_k(\theta)\| d\theta,$$

$$\|e_{k+1}(t)\|_\lambda \leq \left(\|I - D(t) M(t)\| \right. \quad (22)$$

$$\left. + \frac{\|C(t)\| \|M(t)\| e^{(1+(1-\gamma)l_g/l_f)T}}{\lambda} \right) \|e_k(t)\|_\lambda.$$

Then we have the following.

Theorem 5. *For system (21) and a given reference $y_d(t)$, if there exist matrices $M(t)$, $C(t)$, and $D(t)$ and functions $\psi_{k,h}(t)$ and $\varphi_{k,h}(t)$ such that $\|C(t)\|\|M(t)\|$ is bounded and $\|I - D(t)M(t)\| \leq \rho < 1$, where ρ is a constant, then system (21) with the iterative learning control law can guarantee that $\|y_k(t) - y_d(t)\|$ is bounded but $y_k(t)$ cannot track $y_d(t)$ on $t \in [0, h]$ and $\lim_{k \to \infty} y_k(t) = y_d(t)$ on $t \in [h, T]$ for arbitrary initial state $x_k(0)$.*

When $y_k(t) = s(t, x_k(t), u_k(t)) - \varphi_{k,h}(t)$ and $s(t, x_k(t), u_k(t))$ satisfies

$$0 < \delta_1 I < s_u = \frac{\partial s(x_k(t), u_k(t))}{\partial u_k(t)} \leq \delta_2 I,$$

$$0 < \delta_3 I < s_x = \frac{\partial s(x_k(t), u_k(t))}{\partial x_k(t)} \leq \delta_4 I, \quad (23)$$

it is easy to obtain that

$$s(t, x_{k+1}(t), u_{k+1}(t)) - s(t, x_k(t), u_k(t))$$

$$= s_x(\zeta) (x_{k+1}(t) - x_k(t)) \quad (24)$$

$$+ s_u(\zeta) (u_{k+1}(t) - u_k(t)),$$

where $\zeta \in [x_k(t) + \omega(x_{k+1}(t) - x_k(t)), u_k(t) + \omega(u_{k+1}(t) - u_k(t))]$, $\omega \in (0, 1)$.

Consider the following system:

$$\dot{x}_k(t) = f(t, x_k(t)) + g(t, x_k(t - \tau)) + u_k(t),$$

$$u_{k+1}(t) = u_k(t) + M e_k(t)$$

$$- \psi_{k,h}(t) (x_{k+1}(0) - x_k(0)), \quad (25)$$

$$y_k(t) = s(t, x_k(t), u_k(t)) - \varphi_{k,h}(t),$$

$$\varphi_{k+1,h}(t) = \varphi_{k,h}(t) - s_u \psi_{k,h}(t) (x_{k+1}(0) - x_k(0)).$$

Similar to the proof of Theorem 3, we get

$$\|e_{k+1}(t)\|_\lambda$$

$$\leq \left(\|I - s_u M\| + \frac{\|s_x\| \|M\| e^{(1+l_g/l_f)T}}{\lambda} \right) \|e_k(t)\|_\lambda. \quad (26)$$

So we have the following result.

Theorem 6. *For system (25) and a given reference $y_d(t)$, if conditions (9) are true and there exist matrix M and functions $\psi_{k,h}(t)$ and $\varphi_{k,h}(t)$ such that $\int_0^t \psi_{k,h}(\theta)d\theta = 1$, $t > h$, $\|s_x\|$ is bounded, $\max(\|I - \delta_1 M\|, \|I - \delta_2 M\|) \leq \rho < 1$, where ρ is a constant, then system (25) with the iterative learning control law can guarantee that $\lim_{k \to \infty} y_k(t) = y_d(t)$ is bounded but $y_k(t)$ cannot track $y_d(t)$ on $t \in [0, h]$ and $\lim_{k \to \infty} y_k(t) = y_d(t)$ on $t \in [h, T]$ for arbitrary initial state $x_k(0)$.*

4. Numerical Example

For further illustration, we consider the following system:

$$\dot{x}_k(t) = 0.8 \cos^2(x_k(t))$$
$$- 0.5\left(\left|x_k(t-1)+1\right| - \left|x_k(t-1)-1\right|\right)$$
$$+ u_k(t),$$
$$u_{k+1}(t) = u_k(t) + 0.9 e_k(t) \tag{27}$$
$$- \psi_{k,h}(t)\left(x_{k+1}(0) - x_k(0)\right),$$
$$y_k(t) = \tanh\left(2x_k(t) + 0.9u_k(t)\right) - \varphi_{k,h}(t),$$
$$\varphi_{k+1}(t) = \varphi_k(t) - 0.9\psi_{k,h}(t)\left(x_{k+1}(0) - x_k(0)\right),$$

where $M = 0.9$ and

$$\psi_{k,h}(t) = \begin{cases} \dfrac{\pi}{2 \times 0.5} \cos\left(\dfrac{\pi}{2 \times 0.5}t\right), & t \in [0, 0.5), \\ 0, & t \in [0.5, 5], \end{cases} \tag{28}$$

taking the reference $y_d(t) = \sin t + 1$.

From the above numerical example, it can be easily proved that the conditions of Theorem 6 are satisfied.

5. Conclusion

In this paper, considering the iterative learning control problem for nonlinear systems with delays, the novel sufficient conditions for robust convergence of the tracking error have been addressed.

Competing Interests

The author declares that there are no competing interests.

Acknowledgments

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