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Research Article

Static Output Feedback Control for Discrete-Time Switched Systems via Improved Path-Following Method

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This paper focuses on the problems of static output feedback control and H_∞ controller design for discrete-time switched systems. Based on piecewise quadratic Lyapunov functions and a new linearization method, new sufficient conditions for system stability and H_∞ controller design are obtained. Then, an improved path-following algorithm is built to solve the problems. Finally, the merits and effectiveness of the proposed method are shown by two numerical examples.

1. Introduction

Recently, there has been an increasing interest in the study of switched systems because a wide class of nonlinear systems are naturally written as switched systems [1]. Moreover, many other types of nonlinear systems can also be modeled as switched systems approximately [2]. Switched systems are a particular class of hybrid systems which are bounded together by a switching rule. Such systems can be used to describe a wide range of physical and engineering systems in practice [3].

There are a large number of literatures about the stability analysis and design of switched systems during the last few years [4–9]. For discrete-time systems, several sufficient conditions have been presented based on different Lyapunov functions [6, 7, 10, 11], which are different in the conservative level and in the numerical difficulties. Previous work has concentrated on the output feedback controller design methods for switched systems based on piecewise quadratic Lyapunov function [12, 13]. Lyapunov-based controller synthesis is formulated as a biconvex optimization problem which is nonconvex, NP-hard, and very expensive to solve globally [14]. Although there exist some results in solving this problem that the corresponding conditions can be determined by checking a set of linear matrix inequalities (LMIs) [12, 13], most of them are very restrictive.

Path-following method, which is an effective method for solving the biconvex optimization problem, was proposed by Hassibi et al. [15] and employed to solve mixed H_2/H_∞ control [16, 17] and other control problems [18]. An improved path-following method [19] has enhanced the convergence and the performance of the algorithm. As a step-by-step method, implying linearization approach at its key step, it gradually shows enormous potential in solving control problems.

In this paper, the problem of static output feedback (SOF) control for discrete-time switched systems is studied. Based on piecewise quadratic Lyapunov functions [12, 13] and a new linearization method [19], the piecewise quadratic stability conditions are linearized around some points. As a result, less conservative conditions for system stability are derived. The problems of H_∞ control design can be readily treated as well. Then, based on an improved path-following method, an iterative algorithm is built. Finally, two examples are given to show the merits and effectiveness of our work.

This paper is organized as follows. Section 2 is the problem formulation and preliminaries. Section 3 gives the SOF controller design for switched systems. Section 4 extends the method to H_∞ SOF control. Section 5 provides two numerical examples to show the merits and effectiveness of the results and Section 6 concludes this paper.

Notation. R^n denotes the n-dimensional Euclidean space; the superscripts -1 and T denote the matrix inverse and transpose, respectively; $X > 0$ ($X \geq 0$) means that X is positive definite (positive semi-definite); $\|\cdot\|$ is the spectral norm; the star $*$ denotes the symmetric term in a matrix; $l(X, Y) = XY + X^T Y^T$.

2. Problem Formulation and Preliminaries

Consider the discrete-time switched system:

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k), \\ y(k) &= C_i x(k), \\ &\text{for } x \in S_i,\ i = 1, 2, \ldots, l, \end{aligned} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$, and $y(k) \in R^p$ are the state, the control input, and the measured output, respectively; $S_i \subseteq R^n$, $i = 1, 2, \ldots, l$, denotes a partition of the output space into a number of closed polyhedral regions. For future use, define a set Ω to represent all possible switches from one region to itself or another region; that is

$$\begin{aligned} \Omega = \{(i, j) : i, j = 1, 2, \ldots, l, \text{ s.t. } x(k) \in S_i,\ x(k+1) \\ \in S_j\}. \end{aligned} \tag{2}$$

The set Ω can be determined by the reachability analysis for mixed logical dynamical systems. The system is allowed to switch arbitrarily between subsystems.

We study the problem of designing a static output feedback controller:

$$u(k) = F_i y(k), \quad i = 1, 2, \ldots, l, \tag{3}$$

where $F_i \in R^{m \times p}$ such that the closed-loop switched system

$$x(k+1) = (A_i + B_i F_i C_i) x(k), \quad i = 1, 2, \ldots, l \tag{4}$$

is stable.

The following lemmas give the stability condition of closed-loop systems (4) and the new linearization method proposed in [19].

Lemma 1 (see [12]). *If there exist matrices $P_i = P_i^T > 0$ ($\forall i = 1, 2, \ldots, l$), such that the positive definite function $V(x) = x^T P_i x$ ($\forall x \in S_i$), satisfies $V(x(k+1)) - V(x(k)) < 0$, that is,*

$$(A_i + B_i F_i C_i)^T P_j (A_i + B_i F_i C_i) - P_i < 0, \quad \forall i, j \in \Omega, \tag{5}$$

then the closed-loop switched systems (4) are exponentially stable.

Lemma 2 (see [19]). *If there exists a fixed point (M, N) such that the LMI*

$$\begin{pmatrix} l(M + \Delta A, N) + l(M, \Delta P) & * \\ \Delta A^T + \Delta P & -2I \end{pmatrix} < 0 \tag{6}$$

holds for some $(\Delta A, \Delta P)$, then the point $(A = M + \Delta A, P = N + \Delta P)$ is a feasible solution to the bilinear matrix inequality (BMI)

$$l(A, P) < 0. \tag{7}$$

3. SOF Controller Design

In this section, based on a piecewise quadratic Lyapunov function and the new linearization method, we will give new sufficient conditions for solving this problem.

Theorem 3. *If there exist points $(F_i(k), P_i(k))$, $i = 1, 2, \ldots, l$, such that the following inequalities:*

$$\begin{gathered} \begin{pmatrix} -(P_i(k) + \Delta P_i) & * & * \\ (P_j(k) + \Delta P_j) A_{ci}(k) + P_j(k) B_i \Delta F_i C_i & -(P_j(k) + \Delta P_j) & * \\ B_i \Delta F_i C_i & \Delta P_j & -2I \end{pmatrix} < 0, \\ P_i(k) + \Delta P_i > 0,\ \forall (i, j) \in \Omega, \end{gathered} \tag{8}$$

hold for some $(\Delta F_i, \Delta P_i)$, where $A_{ci}(k) = A_i + B_i F_i(k) C_i$, then the points $(P_i = P_i(k) + \Delta P_i, F_i = F_i(k) + \Delta F_i)$ are feasible solutions to inequalities (5).

Proof. By Schur complement, (5) are equivalent to

$$\begin{pmatrix} -P_i & (A_i + B_i F_i C_i)^T P_j \\ P_j (A_i + B_i F_i C_i) & -P_j \end{pmatrix} < 0. \tag{9}$$

Write $F_i = F_i(k) + \Delta F_i$, $P_i = P_i(k) + \Delta P_i$, and $A_{ci}(k) = A_i + B_i F_i(k) C_i$, where $F_i(k)$ and $P_i(k)$ are fixed matrices. The left side of inequality (9) is expanded around $(F_i(k), P_i(k))$ as

$$\begin{aligned} &\begin{pmatrix} -P_i & (A_i + B_i F_i C_i)^T P_j \\ P_j (A_i + B_i F_i C_i) & -P_j \end{pmatrix} \\ &= \begin{pmatrix} -(P_i(k) + \Delta P_i) & * \\ (P_j(k) + \Delta P_j)(A_i + B_i (F_i(k) + \Delta F_i) C_i) & -(P_j(k) + \Delta P_j) \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} -(P_i(k)+\Delta P_i) & * \\ (P_j(k)+\Delta P_j)A_{ci}(k)+P_j(k)B_i\Delta F_i C_i & -(P_j(k)+\Delta P_j) \end{pmatrix} + l\left((B_i\Delta F_i C_i \;\; 0)^T, (0 \;\; \Delta P_j)\right) < 0; \tag{10}$$

that is

$$l\left((B_i\Delta F_i C_i \;\; 0)^T, (0 \;\; \Delta P_j)\right) < \begin{pmatrix} P_i(k)+\Delta P_i & * \\ -(P_j(k)+\Delta P_j)A_{ci}(k)-P_j(k)B_i\Delta F_i C_i & P_j(k)+\Delta P_j \end{pmatrix}. \tag{11}$$

Thus, by Lemma 2, inequalities (8) hold.

With Theorem 3, the nonlinear SOF optimization problem OP1

$$\begin{aligned} &\min \quad \alpha \\ &\text{subject to} \quad P_i > 0, \\ &\begin{pmatrix} -P_i - \alpha P_i & (A_i + B_i F_i C_i)^T P_j \\ P_j (A_i + B_i F_i C_i) & -P_j \end{pmatrix} < 0 \end{aligned} \tag{12}$$

can be replaced by solving the following linear optimization problem OP1$'$:

$$\begin{aligned} &\min \quad \alpha \\ &\text{subject to} \quad P_i(k) + \Delta P_i > 0, \\ &\begin{pmatrix} \beta P_i(k) & \Delta P_i \\ \Delta P_i & \beta P_i(k) \end{pmatrix} > 0, \\ &\begin{pmatrix} -(P_i(k)+\Delta P_i) - \alpha P_i(k) & * & * \\ (P_j(k)+\Delta P_j)A_{ci}(k) + P_j(k)B_i\Delta F_i C_i & -(P_j(k)+\Delta P_j) & * \\ B_i\Delta F_i C_i & \Delta P_j & -2I \end{pmatrix} < 0. \end{aligned} \tag{13}$$

□

Remark 4. In OP1$'$, additional inequalities $\begin{pmatrix} \beta P_i(k) & \Delta P_i \\ \Delta P_i & \beta P_i(k) \end{pmatrix} > 0$ are added. This inequalities are the variation of inequalities $\|\Delta P_i\| < \beta \|P_i(k)\|$. Due to our new linearization method, the prescribed scalar β should be small [19]. Otherwise, the conservation will increase.

4. Extension to H_∞ Control

Consider the discrete-time switched system:

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + B_{wi}\omega(k), \\ z(k) &= C_{zi} x(k) + D_{ui} u(k), \\ y(k) &= C_i x(k), \\ &\text{for } x \in S_i, \; i = 1, 2, \ldots, l, \end{aligned} \tag{14}$$

where $x(k) \in R^n, u(k) \in R^m, y(k) \in R^p, z(k) \in R^q$, and $\omega(k) \in R^r$ are the state, the control input, the measured output, the control output, and the disturbance input, respectively; $S_i \subseteq R^n$, $i = 1, 2, \ldots, l$, denotes a partition of the output space into a number of closed polyhedral regions. Let Ω be the set of all possible switches from one region to itself or another region; that is,

$$\Omega = \left\{(i, j) : i, j = 1, 2, \ldots, l \text{ s.t. } x(k) \in S_i, \; x(k+1) \in S_j\right\}. \tag{15}$$

With the controller (3), the closed-loop system of system (14) becomes

$$\begin{aligned} x(k+1) &= (A_i + B_i F_i C_i) x(k) + B_{wi}\omega(k), \\ z(k) &= (C_{zi} + D_{ui} F_i C_i) x(k), \\ &\qquad i = 1, 2, \ldots, l. \end{aligned} \tag{16}$$

In this section, new sufficient conditions for SOF control design for the switched system (14) in the H_∞ framework will be present. Given a scalar $\gamma > 0$, assuming $x(0) = 0$, the exogenous signal ω is attenuated by γ if for each integer $N > 0$ and for every $\omega \in L_2([0, N], R^r)$

$$\sum_{k=0}^{N} \|z(k)\|^2 < \gamma^2 \sum_{k=0}^{N} \|\omega(k)\|^2. \tag{17}$$

The H_∞ performance of the closed-loop system (16) proposed by Cuzzola and Morari in [13] is reviewed in the next lemma.

Lemma 5 (see [13]). *Consider the switched system (14), if there exists a function $V(x) = x^T P_i x$, $\forall x \in S_i$ with $P_i = P_i^T > 0$ satisfying the following inequality:*

$$V(x(k+1)) - V(x(k)) < \gamma^2 \|\omega(k)\|^2 - \|z(k)\|^2, \quad \forall k; \tag{18}$$

then the closed-loop switched system (16) is exponentially stable with H_∞ performance γ.

Obviously, inequality (18) is equivalent to the following inequalities:

$$\begin{aligned}&(A_i+B_iF_iC_i)^T P_j (A_i+B_iF_iC_i)\\&\quad+(C_{zi}+D_{ui}F_iC_i)^T(C_{zi}+D_{ui}F_iC_i)\\&\quad+(A_i+B_iF_iC_i)^T P_jB_{wi}\Upsilon^{-1}B_{wi}^TP_j(A_i+B_iF_iC_i)\\&\quad-P_i<0,\quad \forall(i,j)\in\Omega,\end{aligned} \tag{19}$$

where $\Upsilon=\gamma^2 I-B_{wi}^TP_jB_{wi}>0$.

Now, the sufficient conditions to obtain SOF control gains with H_∞ performance are given in the following theorem.

Theorem 6. *If there exist points $(F_i(k),P_i(k))$, $i=1,2,\ldots,l$, such that the LMIs*

$$\begin{pmatrix}-(P_i(k)+\Delta P_i) & * & * & * & *\\ (P_j(k)+\Delta P_j)A_{ci}(k)+P_j(k)B_i\Delta F_iC_i & -(P_j(k)+\Delta P_j) & & & *\\ B_{wi}^T(P_j(k)+\Delta P_j)A_{ci}(k)+B_{\omega i}^TP_j(k)B_i\Delta F_iC_i & & -\Upsilon(k)+B_{wi}^T\Delta P_jB_{wi} & & *\\ C_{zi}+D_{ui}(F_i(k)+\Delta F_i)C_i & & & -I & \\ B_i\Delta F_iC_i & \Delta P_j & \Delta P_jB_{wi} & -2I & \end{pmatrix}<0, \tag{20}$$

$$\Upsilon(k)-B_{wi}^T\Delta P_jB_{wi}>0,\quad P_i(k)+\Delta P_i>0,\quad \forall(i,j)\in\Omega$$

hold for some $(\Delta F_i,\Delta P_i)$, where $A_{ci}(k)=A_i+B_iF_i(k)C_i$ and $\Upsilon(k)=\gamma^2I-B_{wi}^TP_j(k)B_{wi}>0$, then the points $(P_i=P_i(k)+\Delta P_i, F_i=F_i(k)+\Delta F_i)$ are feasible solutions to inequalities (19).

Proof. By Schur complement, inequalities (19) are equivalent to

$$\begin{pmatrix}-P_i & * & * & *\\ P_j(A_i+B_iF_iC_i) & -P_j & & \\ B_{wi}^TP_j(A_i+B_iF_iC_i) & & -\Upsilon & \\ C_{zi}+D_{ui}F_iC_i & & & -I\end{pmatrix}<0. \tag{21}$$

Write $F_i=F_i(k)+\Delta F_i$, $P_i=P_i(k)+\Delta P_i$, and $A_{ci}(k)=A_i+B_iF_i(k)C_i$, where $F_i(k)$ and $P_i(k)$ are fixed matrices. The left side of inequality (21) is expanded around $(F_i(k),P_i(k))$ as

$$\begin{aligned}&\begin{pmatrix}-(P_i(k)+\Delta P_i) & * & * & *\\ (P_j(k)+\Delta P_j)(A_i+B_i(F_i(k)+\Delta F_i)C_i) & -(P_j(k)+\Delta P_j) & & \\ B_{wi}^T(P_j(k)+\Delta P_j)(A_i+B_i(F_i(k)+\Delta F_i)C_i) & & -\Upsilon(k)+B_{wi}^T\Delta P_jB_{wi} & \\ C_{zi}+D_{ui}(F_i(k)+\Delta F_i)C_i & & & -I\end{pmatrix}\\&=\begin{pmatrix}-(P_i(k)+\Delta P_i) & * & * & *\\ (P_j(k)+\Delta P_j)A_{ci}(k)+P_j(k)B_i\Delta F_iC_i & -(P_j(k)+\Delta P_j) & & \\ B_{wi}^T(P_j(k)+\Delta P_j)A_{ci}(k)+B_{wi}^TP_j(k)B_i\Delta F_iC_i & & -\Upsilon(k)+B_{wi}^T\Delta P_jB_{wi} & \\ C_{zi}+D_{ui}(F_i(k)+\Delta F_i)C_i & & & -I\end{pmatrix}\\&\quad+l\left((B_i\Delta F_iC_i\ \ 0\ \ 0\ \ 0)^T,(0\ \ \Delta P_j\ \ \Delta P_jB_{wi}\ \ 0)\right).\end{aligned} \tag{22}$$

That is, the following inequalities hold:

$$
\begin{aligned}
& l\left(\left(B_i\Delta F_iC_i \;\; 0 \;\; 0 \;\; 0\right)^T, \left(0 \;\; \Delta P_j \;\; \Delta P_jB_{wi} \;\; 0\right)\right) \\
& < \begin{pmatrix} P_i(k)+\Delta P_i & * & * & * \\ -\left(P_j(k)+\Delta P_j\right)A_{ci}(k)-P_j(k)B_i\Delta F_iC_i & P_j(k)+\Delta P_j & & \\ -B_{wi}^T\left(P_j(k)+\Delta P_j\right)A_{ci}(k)-B_{wi}^TP_j(k)B_i\Delta F_iC_i & & \Upsilon(k)-B_{wi}^T\Delta P_jB_{wi} & \\ -C_{zi}-D_{ui}\left(F_i(k)+\Delta F_i\right)C_i & & & I \end{pmatrix}.
\end{aligned} \tag{23}
$$

Thus, by Lemma 2, inequalities (20) hold.

With Theorem 6, the nonlinear optimization problem OP2 for solving SOF control with H_∞ performance

$$
\begin{aligned}
\min \quad & \alpha \\
\text{subject to} \quad & P_i > 0, \\
& \Upsilon = \gamma^2 I - B_{wi}^TP_jB_{wi} > 0 \\
& \begin{pmatrix} -P_i-\alpha P_i & * & * & * \\ P_j\left(A_i+B_iF_iC_i\right) & -P_j & & \\ B_{wi}^TP_j\left(A_i+B_iF_iC_i\right) & & -\Upsilon & \\ C_{zi}+D_{ui}F_iC_i & & & -I \end{pmatrix} \\
& < 0, \\
& \qquad \forall (i,j)\in\Omega
\end{aligned} \tag{24}
$$

can be replaced by solving the following linear optimization problem OP2$'$:

$$
\begin{aligned}
\min \quad & \alpha \\
\text{subject to} \quad & P_i(k)+\Delta P_i > 0, \\
& \Upsilon(k) - B_{wi}^T\Delta P_jB_{wi} > 0, \\
& \begin{pmatrix} \beta P_i(k) & \Delta P_i \\ \Delta P_i & \beta P_i(k) \end{pmatrix} > 0, \\
& \begin{pmatrix} -\left(P_i(k)+\Delta P_i\right)-\alpha P_i(k) & * & * & * & * \\ \left(P_j(k)+\Delta P_j\right)A_{ci}(k)+P_j(k)B_i\Delta F_iC_i & -\left(P_j(k)+\Delta P_j\right) & & & * \\ B_{\omega i}^T\left(P_j(k)+\Delta P_j\right)A_{ci}(k)+B_{\omega i}^TP_j(k)B_i\Delta F_iC_i & & -\Upsilon(k)+B_{wi}^T\Delta P_jB_{wi} & & * \\ C_{zi}+D_{ui}\left(F_i(k)+\Delta F_i\right)C_i & & & -I & \\ B_i\Delta F_iC_i & \Delta P_j & \Delta P_jB_{wi} & & -2I \end{pmatrix} \\
& < 0, \\
& \qquad \forall (i,j)\in\Omega.
\end{aligned} \tag{25}
$$

□

Based on Theorems 3 and 6, an iterative algorithm to solve stabilization and H_∞ control via static output feedback for discrete-time switched systems is established.

Algorithm 7. Consider the following.

Step 1 (initialization step). At initial, we need to obtain initial values of F_i and P_i.

Firstly, let $P_i = I$. Then, solve the optimization problem OP1 (or OP2 instead for H_∞ case) with respect to F_i and α.

If $\alpha < 0$, stop; else let $F_i(0) = F_i$, $P_i(0) = P_i$, $A_{ci}(0) = A_i + B_iF_i(0)C_i$, and $\alpha(0) = \alpha$, and set $k = 1$.

Step 2 (small perturbation step). Set $\beta = \beta_0$, where $\beta_0 > 0$ is a prescribed small value. Solve LMI optimization problem

OP1$'$ (or OP2$'$ instead for H_∞ case) with respect to ΔF_i, ΔP_i, and α.

Step 3 (update step). Let $F_i(k) = F_i(k-1)+\Delta F_i$, $P_i(k) = P_i(k-1)+\Delta P_i$, $\alpha(k) = \alpha$, and $A_{ci}(k) = A_{ci}(k-1)+B_i\Delta F_i C_i$. For fixed $F_i(k)$, compute new $P_i(k)$ by solving OP1 (or OP2 instead for H_∞ case), and then compute new $F_i(k)$ and $\alpha(k)$ by solving OP1 (or OP2 instead for H_∞ case).

If $\alpha(k) < 0$, stop; else if the relative improvement in α is more than a desired accuracy, set $k = k + 1$, and go to Step 2. Else, set $k = 1$, and let $F_i(0) = F_i(k)$, $P_i(0) = P_i(k)$, $A_{ci}(0) = A_{ci}(k)$, and $\alpha(0) = \alpha(k)$.

Step 4 (wide perturbation step). Set $\beta = \beta \times 2$. Solve LMI optimization problem OP1$'$ (or OP2$'$ instead for H_∞ case) with respect to ΔF_i, ΔP_i, and α.

Step 5 (update step). Let $F_i(k) = F_i(k-1)+\Delta F_i$, $P_i(k) = P_i(k-1)+\Delta P_i$, $\alpha(k) = \alpha$, and $A_{ci}(k) = A_{ci}(k-1)+B_i\Delta F_i C_i$. For fixed $F_i(k)$, compute new $P_i(k)$ by solving OP1 (or OP2 instead for H_∞ case), and then compute new $F_i(k)$ and $\alpha(k)$ by solving OP1 (or OP2 instead for H_∞ case).

If $\alpha(k) < 0$, stop; else if the relative improvement in α is more than a desired accuracy, set $k = 1$, and let $F_i(0) = F_i(k)$, $P_i(0) = P_i(k)$, $A_{ci}(0) = A_{ci}(k)$, and $\alpha(0) = \alpha(k)$, and go to Step 2. Else if the relative improvement in α is inferior to the desired accuracy and $k < k_0$, where $k_0 > 0$ is a prescribed integer, set $k = k + 1$, and go to Step 4. Else, stop.

Remark 8. The wide perturbation step is a crucial step in improved path-following method. The purpose of this step is to broaden the search scope during each iteration so that the algorithm has the opportunity to escape form the local optimum. However, the enlarged search scope may cause nonconvergence. So the iteration number of wide perturbation step should not be too large. As long as the objective function α has been improved significantly, the wide perturbation step will be replaced by a small perturbation step immediately.

5. Numerical Examples

In this section, two examples are given to show the effectiveness of our method. Example 1 is with respect to the SOF control problem for switched systems. Example 2 is concerning the H_∞ controller design problem for switched systems.

Example 1. Consider system (1) with the following parameters:

$$A_1 = \begin{pmatrix} 1 & 0.3 & 2 \\ 1 & 0 & 1 \\ 0.3 & 0.6 & 0.6 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -0.5871 & -0.8441 & -0.0092 \\ -0.6865 & -0.5090 & -0.8561 \\ 0.0974 & 0.4523 & -0.2280 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0.1089 & 0.2458 & -0.9035 \\ 0.3998 & -0.9213 & -0.4161 \\ 0.6745 & -0.5750 & 0.7138 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0.1930 & -0.4204 \\ -0.7359 & 0.0346 \\ 0.5073 & -0.9077 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} -0.4164 & 0.0244 \\ 0.8297 & -0.4366 \\ -0.0900 & -0.8416 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix},$$

$$C_3 = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}. \tag{26}$$

It cannot be stabilized by the method in [12, 20]. However, using our method, set $\beta_0 = 0.2$; after iterations, output feedback stabilizing controller matrices are computed to be

$$F_1 = \begin{pmatrix} -0.7454 \\ -0.8806 \end{pmatrix},$$

$$F_2 = \begin{pmatrix} -0.8899 \\ -0.2598 \end{pmatrix},$$

$$F_3 = \begin{pmatrix} 0.5736 \\ 0.4941 \end{pmatrix},$$

$$P_1 = \begin{pmatrix} 11359.9012 & -13835.4590 & 3078.1446 \\ -13835.4590 & 45253.4272 & -20668.2151 \\ 3078.1446 & -20668.2151 & 36198.7545 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 7437.9005 & -6354.0674 & -141.5298 \\ -6354.0674 & 34828.7955 & -8170.2774 \\ -141.5298 & -8170.2774 & 16785.3283 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 12867.1197 & -18832.8739 & 5382.2570 \\ -18832.8739 & 43961.3298 & -14881.7520 \\ 5382.2570 & -14881.7520 & 23348.2946 \end{pmatrix}. \tag{27}$$

Example 2. Consider a system with the following parameters:

$$A_1 = \begin{pmatrix} -0.5871 & -0.8441 & -0.0092 \\ -0.6865 & -0.5090 & -0.8561 \\ 0.0974 & 0.4523 & -0.2280 \end{pmatrix},$$
$$A_2 = \begin{pmatrix} 0.1089 & 0.2458 & -0.9035 \\ 0.3998 & -0.9213 & -0.4161 \\ 0.6745 & -0.5750 & 0.7138 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} 0.1930 & -0.4204 \\ -0.7359 & 0.0346 \\ 0.5073 & -0.9077 \end{pmatrix},$$
$$B_2 = \begin{pmatrix} -0.4164 & 0.0244 \\ 0.8297 & -0.4366 \\ -0.0900 & -0.8416 \end{pmatrix},$$
$$C_1 = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix},$$
$$C_2 = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix},$$
$$B_{w1} = B_{w2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$
$$C_{z1} = C_{z2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$D_{u1} = D_{u2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}. \tag{28}$$

Using our method, set $\beta_0 = 0.2$; after iterations, output feedback control matrices are computed to be

$$F_1 = \begin{pmatrix} -0.9811 \\ -0.6662 \end{pmatrix},$$
$$F_2 = \begin{pmatrix} 0.4284 \\ -0.2097 \end{pmatrix}. \tag{29}$$

In this case,

$$P_1 = \begin{pmatrix} 5.3254 & -2.9578 & 1.2843 \\ -2.9578 & 12.6330 & -2.3378 \\ 1.2843 & -2.3378 & 2.8444 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 6.0132 & -3.0439 & 3.3919 \\ -3.0439 & 6.3742 & -3.3469 \\ 3.3919 & -3.3469 & 9.2144 \end{pmatrix} \tag{30}$$

and the closed-loop system has the H_∞ performance $\gamma = 4.7328$. The result is better than the solution solved by the method in [12] which gives $\gamma = 5.6853$, which is inferior compared to our result $\gamma = 4.7328$ and

$$F_1 = \begin{pmatrix} -1.0832 \\ -0.5259 \end{pmatrix},$$
$$F_2 = \begin{pmatrix} 0.3563 \\ -0.1241 \end{pmatrix}. \tag{31}$$

6. Conclusion

This paper studies the problems of static output feedback control and H_∞ controller synthesis for discrete-time switched systems. Based on piecewise quadratic Lyapunov functions and a new linearization method, new sufficient conditions for system stability and H_∞ controller design are obtained. Then, an improved path-following algorithm is built to solve the problems. Finally, the merits and effectiveness of the proposed method are shown by two numerical examples. Compared to the existing methods, the proposed method is less conservative.

Important future research work will be applying the results to some real-world systems. How to reduce the design conservatism is an important research topic that deserves further investigation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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