

# Conditional Variables Double Sampling Plan for Weibull Distributed Lifetimes under Sudden Death Testing

S. Balamurali and J. Subramani

**Abstract**--- In this paper, we propose a conditional sampling plan called conditional double sampling plan for lot acceptance of parts whose life time follows a Weibull distribution with known shape parameter under sudden death testing. A table is also developed for the selection and application of optimal parameters of the proposed plan for specified two points on the operating characteristic curve namely the acceptable reliability level and the limiting reliability level along with the producer and consumer's risks. The optimization problem is formulated as a nonlinear programming where the objective function to be minimized is the average group number and the constraints are related to lot acceptance probabilities at acceptable reliability level and limiting reliability level under the operating characteristic curve.

**Keywords**--- Acceptable Reliability Level, Average Group number, Conditional Sampling Plans, Limiting Reliability Level, Lot Acceptance, Operating Characteristic Curve, Weibull Distribution

## I. INTRODUCTION

ACCEPTANCE sampling is one of the major areas of statistical quality control. Acceptance sampling is the methodology that deals with procedures by which decision to accept or reject the lot based on the results of the inspection of samples. Acceptance sampling prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of given quality. In other words, acceptance sampling yields quality assurance. Implementation of acceptance sampling in industries through the operation of sampling plan yields quality assurance. Use of acceptance sampling is essential to secure ISO certification which gives a passport for larger exports.

One of the major areas of acceptance sampling is variables sampling plans. The main advantage of a variables sampling plan is that it provides better operating characteristics as compared to an attributes sampling plan. In other words, a

variables sampling plan may require smaller sample size than an attributes sampling plan at the specified quality level. Although the attributes sampling incurs lower cost to conduct an experiment, the variables sampling plan with a smaller sample size may lead to the reduced total experiment cost particularly when the inspection is destructive. The variables sampling plan is based on the measurement data, which usually provides more information about the items under inspection than simply labeling the items as conforming or non-conforming.

Several authors have investigated the efficiency of the variables sampling plans and subsequently developed certain efficient variables sampling plans. Pearn and Wu [11] pointed out that a well-designed sampling plan can minimize the gap between the required and the actual supplied product quality. Hamaker [8] developed the designing methodology of the variables single sampling plan and Sommers [13] proposed a two-point method for determining parameters of a variables double sampling plan. Pearn and Wu [11, 12] considered the design of variables single sampling plan based on the process capability indices, where the plan parameters are obtained by deriving the exact sampling distributions. Variables repetitive group sampling (VRGS) plans were proposed by Balamurali et al. [2] and Balamurali and Jun [3], where they were demonstrated to outperform the single and the double sampling plans in terms of the average sample number. Jun et al. [9] developed single and double variables sampling plans under sudden death testing based on Weibull distribution. Govindaraju and Balamurali [7] extended the concept of chain sampling to variables inspection for a normally distributed quality characteristics. Balamurali and Jun [4] developed multiple-dependent sampling plan for variables inspection. Balamurali and Jun [5] developed a designing methodology to determine the parameters of tightened-normal-tightened (TNT) sampling scheme under variables sampling.

Hence, this paper proposes the designing methodology for determining the parameters of a conditional variables double sampling plan for Weibull distributed life times under sudden death testing. Quick switching system for normally distributed quality characteristics having lower and upper specification limits. The optimal parameters of the proposed plan can be determined for specified two points namely acceptable reliability level (ARL) and lot tolerance reliability level (LTRL) along with the respective risks on the operating characteristic (OC) curve approach.

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## II. SUDDEN DEATH TESTING

In the reliability sampling, two types of censoring namely time censoring (or type I censoring) and failure censoring (or type II censoring) are applied to reduce the test time of an experiment. Sudden death testing is also frequently applied to life testing problems to reduce the testing time. The total number of items to be tested say  $n$  items is divided into equal sized groups according to the available number of experimental testers (For further details one can refer (Pauscal and Meeker [10] and Vlcek et al. [14]). Thus there are  $r$  items in each of  $g$  groups so that  $n=rg$ . The items in each group are tested simultaneously and identically on different testers. The testing of first group of items is run until the first failure occurs. At this juncture, surviving items are suspended and removed from testing. Another equal set of  $r$  items is tested next until the first failure is observed. This process is continued until at least one failure is observed from each of  $g$  groups. At the end,  $g$  failures are observed from  $g$  groups and  $(r-1)g$  items are suspended and removed from the testing. Balasooriya [6] developed reliability sampling plans for this type of testing for two parameter exponential distribution. Wu et al. [15] use the terminology ‘limited failure-censored life test’ for this type of testing and they have also analyzed the average testing time.

## III. CONDITIONAL DOUBLE SAMPLING PLAN

The concept of conditional double sampling was developed by Baker and Brobst [1]. The conditional double sampling plan has OC curves identical to comparable double sampling plan, but is operationally different since the results of the second sample, if required, are obtained from a related lot rather than the current lot. The operation of conditional double sampling plan is as follows.

- From the lot  $i$ , draw a random sample of size  $n=rg$  and allocate  $r$  parts to each of  $g$  groups.
- Perform sudden death testing and observe  $Y_j$ , the time to the first failure from the  $j^{\text{th}}$  group ( $j=1,2,\dots,g$ ).

Then calculate the quantity  $v_i = \sum_{j=1}^g Y_j^m$ .

- Accept the lot if  $v_i \geq k_a L^m$  and reject the lot if  $v_i < k_r L^m$ . If  $k_r L^m \leq v_i < k_a L^m$  then defer the decision until the results of the lot  $(i+1)$  are obtained.
- From the lot  $(i+1)$ , draw a random sample of size  $n=rg$  and allocate  $r$  parts to each of  $g$  groups.
- Perform sudden death testing and observe  $Y_j$ , the time to the first failure from the  $j^{\text{th}}$  group ( $j=1,2,\dots,g$ ).

Then calculate the quantity  $v_{i+1} = \sum_{j=1}^g Y_j^m$ .

- Accept the lot if  $v_i + v_{i+1} \geq k_t L^m$  and otherwise reject the lot.

## IV. CONDITIONAL DOUBLE SAMPLING PLAN UNDER WEIBULL DISTRIBUTION

Suppose that the life time of a product follows a Weibull distribution with shape parameter  $m$ , and a scale parameter  $\delta$  and the cumulative distribution is given by

$$F(x) = 1 - \exp(-(\delta x)^m), \quad x \geq 0 \quad (1)$$

Assuming that there is a lower specification limit  $L$  regarding the lifetime of the product is given and the shape parameter  $m$  is known, the fraction nonconforming or unreliability is determined by

$$p = P(X < L) = F(L) \quad (2)$$

where  $X$  is the life time of a product which follows Weibull distribution. If  $p$  is given then  $\delta L$  is obtained from (2) through

$$w = (\delta L)^m = -\ln(1 - p) \quad (3)$$

Based on the operating procedure of the proposed conditional double sampling plan, it is to be observed that the acceptance criteria do not involve the scale parameter of the Weibull distribution. The probability of acceptance of the lot based on the sample results of the  $i^{\text{th}}$  lot is given by

$$P_{ai}(p) = 1 - G_{2g_1}(2rk_a w) \quad (4)$$

Where  $w$  is given in (3),  $G_\varphi$  is the distribution function of a Chi-square random variable with  $\varphi$  degrees of freedom. The probability of acceptance of the lot based on the sample results of the  $(i+1)^{\text{st}}$  lot is given by

$$P_{a(i+1)}(p) = P\{k_r L^m \leq v_i \leq k_a L^m, v_i + v_{i+1} \geq k_t L^m | p\} \quad (5)$$

Since  $2\delta^m r v_i$  and  $2\delta^m r v_{i+1}$  are independent and follow Chi-square distribution with  $2g_1$  and  $2g_2$  degrees of freedom respectively. Now equation (5) reduces to

$$P_{a(i+1)}(p) = \int (1 - G_{2g_2}(2rk_t w - x)) \frac{(x/2)^{g_1-1}}{\Gamma(g_1)} \left(\frac{1}{2}\right) e^{-\frac{x}{2}} dx \quad (6)$$

Then the probability of acceptance based on conditional double sampling plan is

$$L(p) = P_{ai}(p) + P_{a(i+1)}(p) \quad (7)$$

To determine the optimal parameters of the proposed conditional double sampling plan namely  $g_1$ ,  $g_2$ ,  $k_a$ ,  $k_r$  and  $k_t$  the two points on the OC curve approach can be utilized. The probability of acceptance should be atleast  $(1-\alpha)$  at the acceptable reliability level ( $p_1$ ) and the probability of acceptance should be  $\beta$  at the lot tolerance reliability level ( $p_2$ ). To obtain the optimal parameters we need to solve the following two equations.

$$L(p_1) \geq 1 - \alpha$$

$$L(p_2) \leq \beta \quad (8)$$

Where  $p_1$  is the acceptable reliability level,  $p_2$  is the lot tolerance reliability level,  $\alpha$  is the producer’s risk and  $\beta$  is called the consumer’s risk. By solving the equation (8), we can get the optimal parameters of the proposed plan and are

tabulated in Table 1. In this table, we have assumed that  $k_i=k_a$  in order to reduce the number of parameters.

Table 1: Optimal Parameters of Contional Double Sampling Plan

p <sub>1</sub>	p <sub>2</sub>	Optimal Parameters		
		g	rk <sub>a</sub>	rk <sub>r</sub>
0.001	0.002	18.70	12195.0	999.9
	0.004	5.10	2022.8	99.8
	0.005	3.90	2000.0	100.0
	0.01	2.10	404.9	0.0
	0.05	1.00	300.0	5.0
	0.10	0.80	200.0	5.0
0.005	0.01	18.60	2417.3	500.0
	0.015	7.70	757.5	2.7
	0.02	5.10	398.4	0.3
	0.025	3.90	256.5	0.1
	0.05	2.10	78.5	0.0
	0.25	0.90	26.2	0.0
0.01	0.02	18.50	1195.1	100.1
	0.04	5.00	195.6	1.4
	0.05	3.80	125.5	0.0
	0.10	2.10	37.8	0.0
	0.15	1.60	23.9	0.0
	0.30	1.10	8.9	5.0
0.05	0.10	17.40	217.6	50.0
	0.20	4.60	33.4	0.0
	0.25	3.50	20.7	0.0
	0.30	2.80	14.3	0.0
	0.50	1.80	5.5	0.4
0.10	0.20	16.10	95.8	10.0
	0.40	4.00	13.2	0.0
	0.50	3.00	7.6	0.0

Example: Suppose that one wants to find a conditional double sampling for given conditions of  $p_1=0.01$ ,  $p_2=0.04$ ,  $\alpha=0.05$  and  $\beta=0.10$ , Table 1 gives the optimal parameters of the conditional double sampling as  $g=5.00$ ,  $rk_a=195.6$  and  $rk_r=1.4$ . For the case of  $r=10$ , we get the acceptance criterion as  $k_a=19.56$  and  $k_i=0.14$ . For this example, the proposed plan can be operated as follows.

- a. From the lot  $i$ , draw a random sample of size  $n=rg=50$  and allocate 10 parts to each of 5 groups.
- Perform sudden death testing and observe  $Y_j$ , the time to the first failure from the  $j^{\text{th}}$  group ( $j=1,2,\dots,5$ ). Then calculate the quantity  $v_i = \sum_{j=1}^5 Y_j^m$ .
- Accept the lot if  $v_i \geq 19.56L^m$  and reject the lot if  $v_i < 0.14L^m$ . If  $0.14L^m \leq v_i < 19.56L^m$  then defer the decision until the results of the lot ( $i+1$ ) are obtained.
- From the lot ( $i+1$ ), draw a random sample of size  $n=rg=50$  and allocate 10 parts to each of 5 groups.

- Perform sudden death testing and observe  $Y_j$ , the time to the first failure from the  $j^{\text{th}}$  group ( $j=1,2,\dots,5$ ). Then calculate the quantity  $v_{i+1} = \sum_{j=1}^5 Y_j^m$ .
- Accept the lot if  $v_i + v_{i+1} \geq 19.56L^m$  and otherwise reject the lot.

V. CONCLUSIONS

In this paper, a conditional double sampling plan has been proposed for variables inspection for deciding whether the lot of manufactured products to be accepted or not under sudden death testing. The sudden death testing can be applied to reduce testing time. Tables have also been developed and examples have also been given for implementation of the proposed plan.

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