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Research Article

Energy Loss of a Heavy Particle Near 3D Rotating Hairy Black Hole

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We consider rotating black hole in 3 dimensions with a scalar charge and discuss energy loss of heavy particle moving near the black hole horizon. We find that drag force was increased by scalar charge while it was decreased due to the rotation of black hole. We also study quasnormal modes.

1. Introduction

The lower dimensional theories may be used as toy models to study some fundamental ideas which yield to better understanding of higher dimensional theories, because they are easier to study [1]. Moreover, these are useful for application of AdS/CFT correspondence [2–5]. This paper is indeed an application of AdS/CFT correspondence to probe moving charged particle near the three-dimensional black holes which are recently introduced by Xu et al. [6, 7] where charged black holes with a scalar hair in $(2+1)$ dimensions and rotating hairy black hole in $(2+1)$ dimensions are constructed, respectively. Here, we are interested in the case of rotating black hole with a scalar hair in $(2+1)$ dimensions. Recently, a charged rotating hairy black hole in 3 dimensions corresponding to infinitesimal black hole parameters was constructed [8]. Also, thermodynamics of such systems is recently studied in [9, 10]. We consider this background in AdS side as a dual picture of a QCD model as CFT side.

In this paper, we would like to study the motion of a heavy charged particle near the black hole horizon and calculate the energy loss. The energy loss of moving heavy charged particle through a thermal medium is known as the drag force. One can consider a moving heavy particle (such as charm and bottom quarks) near the black hole horizon with the momentum P, mass m, and constant velocity v, which is influenced by an external force F. So, one can write the equation of motion as $\dot{P} = F - \zeta P$, where in the nonrelativistic motion $P = mv$, and in the relativistic motion $P = mv/\sqrt{1-v^2}$; also ζ is called the friction coefficient. In order to obtain drag force, one can consider two special cases. The first case is the constant momentum which yields to obtain $F = (\zeta m)v$ for the norelativistic case. In this case, the drag force coefficient (ζm) will be obtained. In the second case, the external force is zero, so one can find $P(t) = P(0)\exp(-\zeta t)$. In other words, by measuring the ratio $\dot{P}/P$ or $\dot{v}/v$, one can determine friction coefficient ζ without any dependence on mass m. These methods lead us to obtain the drag force for a moving heavy particle. The moving heavy particle in context of QCD has dual picture in the string theory in which an open string is attached to the D-brane and stretched to the horizon of the black hole. Therefore, we can apply AdS/CFT correspondence to probe a charged particle (such as a quark) moving through 3D hairy black hole background.

Similar studies are already performed in several backgrounds [11–22]. Most of them considered $\mathcal{N}=2$ and $\mathcal{N}=4$ super Yang-Mills plasma with asymptotically AdS geometries. Also [20] considered 4D Kerr-AdS black holes. All of the mentioned studies used AdS_5/CFT_4 correspondence. Now, we are going to consider the same problem in a rotating hairy 3D background and use AdS_3/CFT_2 correspondence [23–25].

This paper is organized as follows. In the next section, we review rotating hairy black hole in $(2+1)$ dimensions. In Section 3, we obtain equation of motion and in Section 4, we try to obtain solution and discuss about drag force. In Section 5, we give linear analysis and discuss quasinormal modes. Finally, in Section 6, we summarized our results.

2. Rotating Hairy Black Hole in (2 + 1) Dimensions

Rotating hairy black hole in $(2+1)$ dimensions is described by the following action:

$$S = \frac{1}{2}\int d^3x\sqrt{-g}\left[R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{R}{8}\phi^2 - 2V(\phi)\right], \quad (1)$$

which yields to the following line element [1]:

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\psi + \omega(r)\,dt)^2, \quad (2)$$

where

$$f(r) = 3\beta + \frac{2\beta B}{r} + \frac{(3r+2B)^2a^2}{r^4} + \frac{r^2}{l^2}, \quad (3)$$

where a is a rotation parameter related to the angular momentum of the solution and l is related to the cosmological constant via $\Lambda = -1/l^2$. β is integration constants depending on the black hole mass:

$$\beta = -\frac{M}{3}, \quad (4)$$

and scalar charge B is related to the scalar field as

$$\phi(r) = \pm\sqrt{\frac{8B}{r+B}}. \quad (5)$$

Also, one can obtain

$$\omega(r) = -\frac{(3r+2B)\,a}{r^3}, \qquad V(\phi) = \frac{2}{l^2} + \frac{1}{512}\left[\frac{1}{l^2} + \frac{\beta}{B^2}\right]\phi^6. \quad (6)$$

Ricci scalar of this model is given by

$$R = -\frac{6r^6 + 36Bl^2a^2r + 30l^2a^2B^2}{l^2r^6}. \quad (7)$$

We can see that Ricci scalar is singular at the origin.

Black hole horizon, which is obtained by $f(r) = 0$, may be written as follows:

$$r_h = \frac{4l}{2C}\left(1 + \sqrt{1 - \frac{BC}{3l^2}}\right), \quad (8)$$

where we defined

$$C \equiv \frac{2BM}{27a^2} - \frac{3l}{B}. \quad (9)$$

3. The Equations of Motion

The moving heavy particle near the black hole may be described by the following Nambu-Goto action:

$$S = -\frac{1}{2\pi\alpha'}\int d\tau\, d\sigma\sqrt{-G}, \quad (10)$$

where $T_0 = 1/2\pi\alpha'$ is the string tension. The coordinates τ and σ are corresponding to the string world-sheet. Also, G_{ab} is the induced metric on the string world-sheet with determinant G obtained as follows:

$$G = -1 - r^2f(r)\left(x'\right)^2 + \frac{r^2}{f(r)}(\dot{x})^2, \quad (11)$$

where we used static gauge in which $\tau = t$, $\sigma = r$, and the string only extends in one direction $x(r,t)$. Then, the equation of motion is obtained as follows:

$$\partial_r\left(\frac{r^2f(r)\,x'}{\sqrt{-G}}\right) - \frac{r^2}{f(r)}\partial_t\left(\frac{\dot{x}}{\sqrt{-G}}\right) = 0. \quad (12)$$

We should obtain canonical momentum densities associated with the string as follows:

$$\begin{aligned}
\pi_\psi^0 &= \frac{1}{2\pi\alpha'\sqrt{-G}}\frac{r^2}{f(r)}\dot{x},\\
\pi_r^0 &= -\frac{1}{2\pi\alpha'\sqrt{-G}}\frac{r^2}{f(r)}\dot{x}x',\\
\pi_t^0 &= -\frac{1}{2\pi\alpha'\sqrt{-G}}\left(1 + r^2f(r)\left(x'\right)^2\right),\\
\pi_\psi^1 &= \frac{1}{2\pi\alpha'\sqrt{-G}}r^2f(r)\,x',\\
\pi_r^1 &= -\frac{1}{2\pi\alpha'\sqrt{-G}}\left(1 - \frac{r^2}{f(r)}\dot{x}^2\right),\\
\pi_t^1 &= \frac{1}{2\pi\alpha'\sqrt{-G}}r^2f(r)\,\dot{x}x'.
\end{aligned} \quad (13)$$

The simplest solution of the equation of motion is static string described by x = constant with total energy of the form,

$$E = -\int_{r_h}^{r_m}\pi_t^0\,dr = \frac{1}{2\pi\alpha'}\left(r_h - r_m\right) = M_{\text{rest}}, \quad (14)$$

where r_m is an arbitrary location of D-brane. As we expected, the energy of static particle is interpreted as the remaining mass.

4. Time Dependent Solution

In the general case, we can assume that the particle moves with constant speed $\dot{x} = v$; in that case, the equation of motion (12) reduces to

$$\partial_r\left(\frac{r^2f(r)\,x'}{\sqrt{-G}}\right) = 0, \quad (15)$$

where

$$G = -1 - r^2 f(r) \left(x'\right)^2 + \frac{r^2}{f(r)} v^2. \quad (16)$$

Equation (15) gives the following expression:

$$\left(x'\right)^2 = \frac{C^2 \left(r^2 v^2 - f(r)\right)}{r^2 f(r)^2 \left(C^2 - r^2 f(r)\right)}, \quad (17)$$

where C is an integration constant which will be determined by using reality condition of $\sqrt{-G}$. Therefore, we yield to the following canonical momentum densities:

$$\pi_\psi^1 = -\frac{1}{2\pi\alpha'} C,$$
$$\pi_t^1 = \frac{1}{2\pi\alpha'} C v. \quad (18)$$

These give us loosing energy and momentum through an endpoint of string:

$$\frac{dP}{dt} = \pi_\psi^1 |_{r=r_h} = -\frac{1}{2\pi\alpha'} C,$$
$$\frac{dE}{dt} = \pi_t^1 |_{r=r_h} = \frac{1}{2\pi\alpha'} C v. \quad (19)$$

As we mentioned before, reality condition of $\sqrt{-G}$ gives us constant C. The expression $\sqrt{-G}$ is real for $r = r_c > r_h$. In the case of small v, one can obtain

$$r_c = r_h + \frac{r^2 v^2}{f(r)'} |_{r=r_h} + \mathcal{O}\left(v^4\right), \quad (20)$$

which yields to

$$C = v r_h^2 + \mathcal{O}\left(v^3\right). \quad (21)$$

Therefore, we can write drag force as follows:

$$\frac{dP}{dt} = -\frac{v r_h^2}{2\pi\alpha'} + \mathcal{O}\left(v^3\right). \quad (22)$$

We draw drag force in terms of velocity and in agreement with the previous works such as [11–22]; the value of drag force increased by v. In Figure 1, we can see behavior of drag force with rotation parameter and scalar charge. It is shown that the scalar charge increases the value of drag force but the increasing rotational parameter decreases the value of the drag force.

5. Linear Analysis

Motion of string yields to small perturbation after late time due to the drag force. In that case, the speed of particle is infinitesimal and one can write $G \approx -1$. Also, we assume that $x = e^{-\mu t}$, where μ is the friction coefficient. Therefore, one can rewrite the equation of motion as follows:

$$\frac{f(r)}{r^2} \partial_r \left(r^2 f(r) x'\right) = \mu^2 x. \quad (23)$$

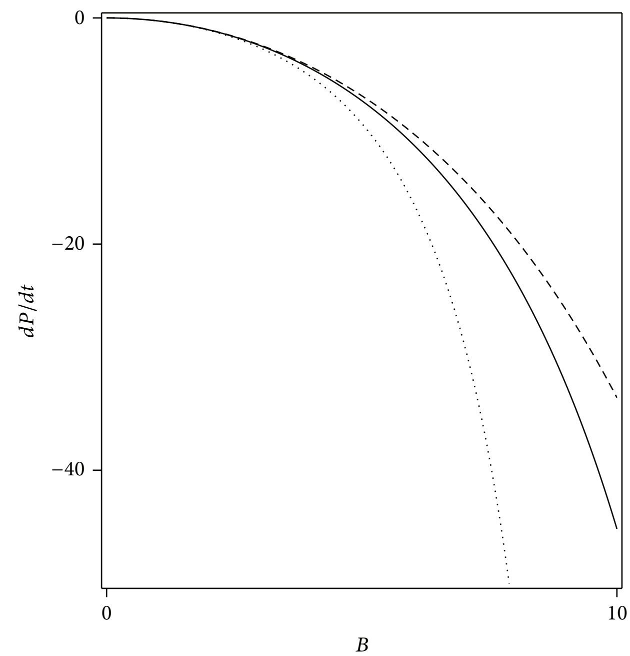

Figure 1: Drag force in terms of B for $M = 1$, $l = 1$, and $v = 0.1$; $a = 1.8$ (dotted line), $a = 3$ (solid line), and $a = 4.2$ (dashed line).

We assume outgoing boundary conditions near the black hole horizon and use the following approximation:

$$(4\pi T)^2 (r - r_h) \partial_r (r - r_h) x' = \mu^2 x, \quad (24)$$

which suggests the following solutions:

$$x = c(r - r_h)^{-\mu/4\pi T}, \quad (25)$$

where T is the black hole temperature. In the case of infinitesimal μ, we can use the following expansion:

$$x = x_0 + \mu^2 x_1 + \cdots. \quad (26)$$

Inserting this equation in the relation (24) gives $x_0 = \text{constant}$, and

$$x_1' = \frac{A}{r^2 f(r)} \int_{r_h}^{r_m} \frac{r^2}{f(r)} dr, \quad (27)$$

where A is a constant. Assuming near horizon limit enables us to obtain the following solution:

$$x_1 \approx \frac{A}{4\pi T r_h^2 (r - r_h)} \left(-r_m + \frac{r_h^2}{4\pi T} \ln (r - r_h)\right). \quad (28)$$

Comparing (25) and (27) gives the following quasinormal mode condition:

$$\mu = \frac{r_h^2}{r_m}. \quad (29)$$

It is interesting to note that these results recover drag force (22) for infinitesimal speed. In Figure 2, we can see behavior of μ with rotational parameter and scalar charge. We find that scalar charge increases the value of friction coefficient, but the effect of rotation decreases μ.

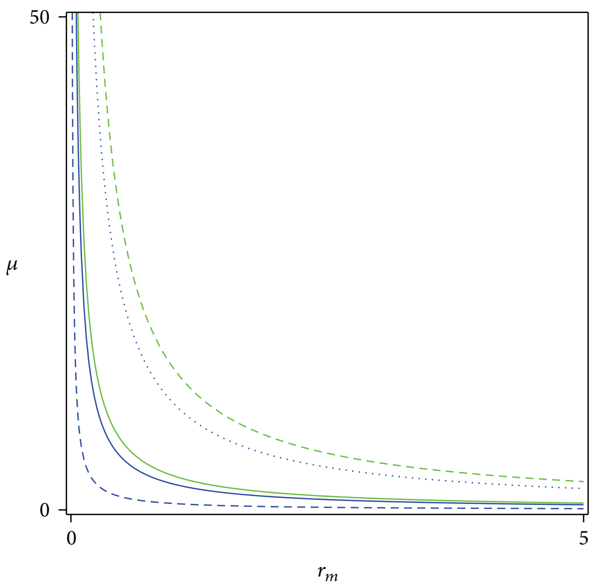

Figure 2: μ in terms of r_m: $B = 0.5$ and $a = 2$ (blue dashed line), $B = 1$ and $a = 2$ (blue solid line), $B = 2$ and $a = 2$ (blue dotted line), and $B = 1$ and $a = 0.2$ (green dashed line), and $B = 1$ and $a = 0.4$ (green solid line).

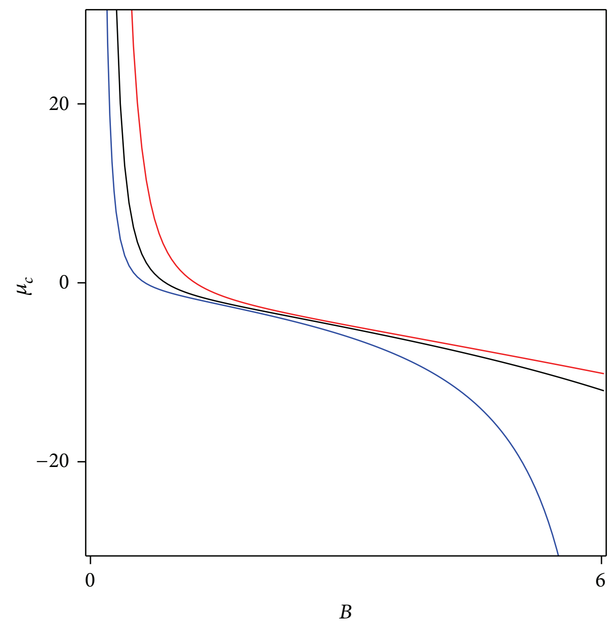

Figure 3: μ_c in terms of B for $M = 1$ and $l = 1$; $a = 1$ (blue line), $a = 2$ (black line), and $a = 4$ (red line).

5.1. Low Mass Limit. Low mass limit means that $r_m \to r_h$, and we use the following assumptions:

$$f(r) \approx 4\pi T (r - r_h),$$
$$r^2 = r_h^2 + 2r_h (r - r_h) + \cdots, \tag{30}$$

so, by using relation (23) we can write

$$x(r) = (r - r_h)^{-\mu/4\pi T} (1 + (r - r_h) A + \cdots). \tag{31}$$

Then, we can obtain constant A as follows:

$$A = \frac{\mu}{2\pi T r_h - \mu r_h}. \tag{32}$$

It tells that $\mu = 2\pi T$ yields to divergence; therefore we called this a critical behavior of the friction coefficient and found that

$$\mu_c = \frac{3r_h^6 + BMl^2 r_h^3 - 27a^2 l^2 r_h^2 - 54Ba^2 l^2 r_h - 24B^2 a^2 l^2}{r_h^5}. \tag{33}$$

Figure 3 shows behavior of critical friction coefficient with the black hole parameters.

6. Conclusions

In this paper, we considered rotating 3D black hole together with a scalar charge as a background where a charged particle moves with speed v and then calculated drag force. We used motivation of AdS/CFT correspondence and string theory method to study motion of charged particle. This is indeed in the context of AdS_3/CFT_2 where drag force on moving heavy particle is calculated. Numerically, we found that the scalar charge increases the value of drag force but rotational parameter decreases the value of the drag force. Therefore, in order to have the most free motion we need to increase a and decrease B. It means that a and B may cancel the effect of each other on the drag force. We can find critical values of scalar charge and rotational parameters in which the value of drag force will be infinite as

$$a_c = \sqrt{\frac{2M}{81l}} B_c. \tag{34}$$

Then, we studied quasinormal modes and obtained friction coefficient μ which was enhanced by the black hole charge and reduced by rotation. Quasinormal mode analysis also reproduced drag force at slow velocities. It is also possible to study dispersion relations which again reproduce the drag force which was obtained in (22). For the future work, we will consider charged rotating 3D hairy black hole and study drag force.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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