

## Research Article

# On Matrix Projective Synchronization and Inverse Matrix Projective Synchronization for Different and Identical Dimensional Discrete-Time Chaotic Systems

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The problem of matrix projective synchronization (MPS) in discrete-time chaotic systems is investigated, and a new type of discrete chaos synchronization called inverse matrix projective synchronization (IMPS) is introduced. Sufficient conditions are derived for achieving MPS and IMPS between chaotic dynamical systems in discrete-time of different and identical dimensions. Based on new control schemes, Lyapunov stability theory, and stability theory of linear dynamical systems in discrete-time, some synchronization criteria are obtained. Numerical examples and simulations are used to illustrate the use of the proposed schemes.

## 1. Introduction

Over the past few decades, chaos synchronization has become an active research subject in nonlinear science and attracted much attention from many fields due its high-potential applications [1–4]. Many powerful methods have been reported to investigate some types of chaos (hyperchaotic) synchronization [5–10] and most of works on synchronization have been concentrated on continuous-time chaotic systems rather than discrete-time chaotic systems. Recently, more attention has been paid to the synchronization of chaos (hyperchaos) in discrete-time dynamical systems [11–19], due to its applications in secure communication and cryptology [20–22].

Up to now, many types of synchronization have been found in interesting chaotic systems in discrete-time such as projective synchronization [23], adaptive-function projective synchronization [24, 25], function-cascade synchronization [26], generalized synchronization [27, 28], lag synchronization [29], impulsive synchronization [30], hybrid synchronization [31], Q-S synchronization [32, 33], and full-state hybrid projective synchronization [34, 35]. Among all types of synchronization, projective synchronization (PS) has been extensively considered. In PS, drive and response systems

could be synchronized up to a scaling factor  $\alpha$ . When the scaling constant  $\alpha$  is generalized to constant matrix, a new synchronization type appears and is called matrix projective synchronization (MPS). Another interesting problem is the inverse case of MPS, that is, when each response system state synchronizes with a linear combination of drive system states. Obviously, complexity of the scaling factors, in MPS or in IMPS, can have important effect in applications.

In this paper, based on new design control method using Lyapunov stability theory, we would like to present constructive schemes to investigate two new synchronization types: matrix projective synchronization (MPS) and inverse matrix projective synchronization (IMPS) between chaotic dynamical systems in discrete-time. Numerical examples are given to illustrate the effectiveness of the proposed schemes. To be specific, we apply the MPS between discrete-time chaotic systems of different dimensions: the drive 3D Hénon-like map and the controlled 2D Fold map. Furthermore, for the proposed scheme of IMPS, we apply it to 3D generalized Hénon map and the controlled 3D Baier-Klein map.

The rest of this paper is arranged as follows. In Section 2, the problem of MPS between chaotic systems of different dimensions in discrete-time is investigated. In Section 3,

the IMPS type is proposed for  $n$ -dimensional chaotic systems in discrete-time. In Section 4, the proposed approaches are applied to two examples and simulations are used to verify the theoretical results derived in this paper. Finally, the paper is concluded in Section 5.

## 2. MPS between Different Dimensional Drive System and Response System

In this section, to study the problem of matrix projective synchronization (MPS), the following drive chaotic system is considered:

$$X(k+1) = f(X(k)), \quad (1)$$

where  $X(k) \in \mathbb{R}^n$  is the state vector of the drive system (1) and  $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

As a response system, we consider the following chaotic system:

$$Y(k+1) = BY(k) + g(Y(k)) + U, \quad (2)$$

where  $Y(k) \in \mathbb{R}^m$ ,  $B = [b_{ij}] \in \mathbb{R}^{m \times m}$ ,  $g: \mathbb{R}^m \rightarrow \mathbb{R}^m$ , and  $U = [u_i] \in \mathbb{R}^m$  are the state vector of the response system, the linear part of the response system, the nonlinear part of the response system, and a vector controller, respectively.

Now, we present the definition of matrix projective synchronization (MPS) between the drive system (1) and the response system (2).

*Definition 1.* The  $n$ -dimensional drive system (1) and the  $m$ -dimensional response system (2) are said to be matrix projective synchronized (MPS) if there exists a controller  $U = [u_i] \in \mathbb{R}^m$  and a given matrix  $\Lambda = [\Lambda_{ij}] \in \mathbb{R}^{m \times n}$  such that the synchronization error

$$e(k) = Y(k) - \Lambda X(k) \quad (3)$$

satisfies that

$$\lim_{k \rightarrow \infty} \|e(k)\| = 0. \quad (4)$$

Then, the error system between the drive system (1) and the response system (2) can be derived as

$$e(k+1) = BY(k) + g(Y(k)) - \Lambda f(X(k)) + U. \quad (5)$$

To achieve MPS between systems (1) and (2), we choose the vector controller  $U$  as follows:

$$U = -L_1 Y(k) - g(Y(k)) + \Lambda f(X(k)) + (L_1 - B) \Lambda X(k), \quad (6)$$

where  $L_1 \in \mathbb{R}^{m \times m}$  is an unknown control matrix to be determined.

**Theorem 2.** *The drive system (1) and the response system (2) are globally matrix projective synchronized under the controller law (6) if and only if  $L_1$  is chosen such that the eigenvalues of  $(B - L_1)$  lie inside the unit disk.*

*Proof.* By substituting (6) into (5), the error system can be described as

$$e(k+1) = (B - L_1) e(k). \quad (7)$$

Now the result follows immediately for the stability theory of autonomous linear discrete-time systems.  $\square$

The following result is a corollary of Theorem 2.

**Corollary 3.** *If  $L_1$  is chosen such that  $(B - L_1)^T (B - L_1) - I$  is a negative definite matrix, then the drive system (1) and the response system (2) are globally matrix projective synchronized under the control law (6).*

*Proof.* Although this result is a corollary of Theorem 2, we would like to present an independent proof. Anyhow, once again, by substituting (6) into (5), the error system can be described as

$$e(k+1) = (B - L_1) e(k). \quad (8)$$

To this end, consider a Lyapunov function in the form

$$V(e(k)) = e^T(k) e(k). \quad (9)$$

Then

$$\begin{aligned} \Delta V(e(k)) &= e^T(k+1) e(k+1) - e^T(k) e(k) \\ &= e^T(k) (B - L_1)^T (B - L_1) e(k) \\ &\quad - e^T(k) e(k) \\ &= e^T(k) [(B - L_1)^T (B - L_1) - I] e(k) \\ &< 0. \end{aligned} \quad (10)$$

Thus, from the Lyapunov stability theory, it is immediate that the zero solution of the error system (8) is globally asymptotically stable. Therefore, systems (1) and (2) are globally matrix projective synchronized.  $\square$

## 3. IMPS between $n$ -Dimensional Drive System and Response System

In this section, we investigate the problem of IMPS. The drive and the response chaotic systems are in the following forms:

$$X(k+1) = AX(k) + f(X(k)), \quad (11)$$

$$Y(k+1) = g(Y(k)) + U, \quad (12)$$

where  $X(k) \in \mathbb{R}^n$  and  $Y(k) \in \mathbb{R}^n$  are the states of the drive system (11) and the response system (12), respectively,  $A \in \mathbb{R}^{n \times n}$  is  $n \times n$  constant matrix,  $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a nonlinear function,  $g: \mathbb{R}^n \rightarrow \mathbb{R}^n$ , and  $U \in \mathbb{R}^n$  is a controller to be determined.

The definition of inverse matrix projective synchronization (IMPS) for the coupled drive-response chaotic systems given in (11) and (12) is given by the following.

*Definition 4.* The drive system (11) and the response system (12) are said to be inverse matrix projective synchronized (IMPS), if there exists a controller  $U = [u_i] \in \mathbb{R}^n$  and a given matrix  $\Lambda = [\Lambda_{ij}] \in \mathbb{R}^{n \times n}$  such that the synchronization error

$$e(k) = X(k) - \Lambda Y(k) \quad (13)$$

satisfies that

$$\lim_{k \rightarrow +\infty} \|e(k)\| = 0. \quad (14)$$

The error system between the drive system (11) and the response system (12) can be derived as

$$e(k+1) = AX(k) + f(X(k)) - \Lambda(g(Y(k)) + U). \quad (15)$$

To achieve IMPS between systems (11) and (12), we can choose the vector controller  $U$  as follows:

$$U = -g(Y(k)) + \Lambda^{-1} [f(X(k)) + L_2 X(k) + (A - L_2) \Lambda Y(k)], \quad (16)$$

where  $\Lambda^{-1}$  is the inverse of matrix  $\Lambda$  and  $L_2 \in \mathbb{R}^{n \times n}$  is an unknown control matrix to be determined.

With that in mind, we formulate the following result.

**Theorem 5.** *The drive system (11) and the response system (12) are globally inverse matrix projective synchronized under the control law (16) if and only if  $L_2$  is chosen such that all eigenvalues of  $A - L_2$  are strictly inside the unit disk.*

*Proof.* By substituting the control law (16) into (15), the error system can be described as

$$e(k+1) = (A - L_2)e(k). \quad (17)$$

Thus, by asymptotic stability of autonomous linear discrete-time systems, it is immediate that all solutions of error system (17) go to zero as  $k \rightarrow \infty$  if and only if all eigenvalues of  $A - L_2$  are strictly inside the unit disk. Therefore, systems (11) and (12) are globally inverse matrix projective synchronized.  $\square$

## 4. Simulation Examples

In this section, to illustrate the applicability of the theoretical synchronization results derived above, two examples are considered.

*4.1. Example 1: MPS between Hénon-Like Map and Fold Map.* Here, we consider the Hénon-like map as the drive system and the controlled Fold map as the response system. The Hénon-like map can be described as

$$\begin{aligned} x_1(k+1) &= 1 + x_3(k) - \alpha x_2^2(k), \\ x_2(k+1) &= 1 + \beta x_2(k) - \alpha x_1^2(k), \\ x_3(k+1) &= \beta x_1(k), \end{aligned} \quad (18)$$

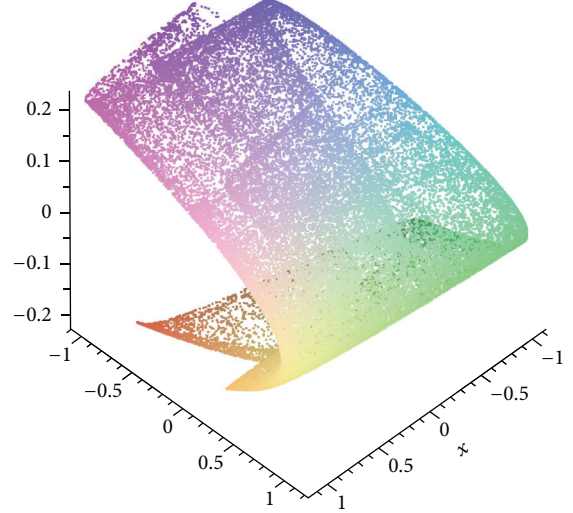


FIGURE 1: Chaotic attractor of the Hénon-like map when  $(\alpha, \beta) = (1.4, 0.2)$ .

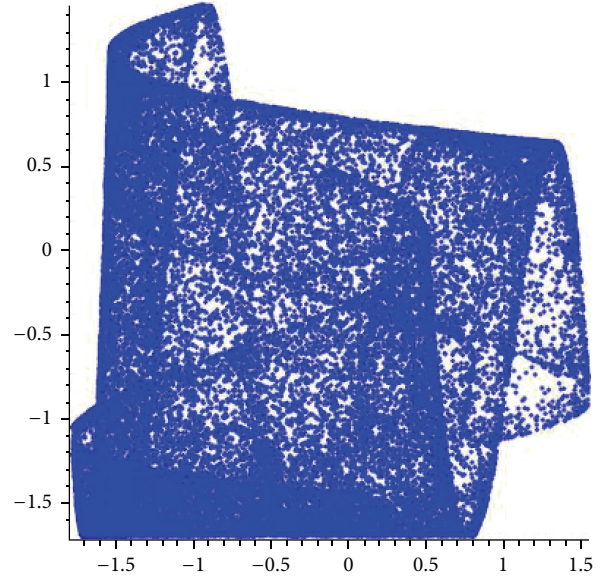


FIGURE 2: Chaotic attractor of the Fold map when  $(a, b) = (-0.1, -1.7)$ .

which has a chaotic attractor, for example, when  $(\alpha, \beta) = (1.4, 0.2)$  [32]. Using Maple 15.0, the chaotic attractor of the Hénon-like map for  $x_1(0) = x_2(0) = x_3(0) = 0$  is shown in Figure 1.

The controlled Fold map can be described as

$$\begin{aligned} y_1(k+1) &= y_2(k) + ay_1(k) + u_1, \\ y_2(k+1) &= b + y_1^2(k) + u_2, \end{aligned} \quad (19)$$

which has a chaotic attractor, for example, when  $(a, b) = (-0.1, -1.7)$  [33], where  $U = (u_1, u_2)^T$  is the vector controller. Using Maple 15.0, the chaotic attractor for  $y_1(0) = y_2(0) = 0$  of this map is shown in Figure 2.

In this example, the synchronization criterion presented in Section 3 is applied between systems (18) and (19). Then, quantities  $B$  and  $g(Y(k))$  are given by, respectively,

$$B = \begin{pmatrix} a & 1 \\ 0 & 0 \end{pmatrix}, \quad (20)$$

$$g(Y(k)) = \begin{pmatrix} 0 \\ b + y_1^2(k) \end{pmatrix};$$

then the scaling matrix  $\Lambda$  and the control matrix  $L_1$  are selected as, respectively,

$$\Lambda = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 0 & 2 \end{pmatrix}, \quad (21)$$

$$L_1 = \begin{pmatrix} a + \frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

According to the general control law given in Section 2 by (6), the vector controller can be designed as

$$u_1 = 1 - \left(\frac{1}{2} + a\right) y_1(k) - y_2(k) + \left(3\beta + \frac{1}{2}\right) x_1(k) + \frac{5}{2} x_3(k) - ax_2^2(k) \quad (22)$$

$$u_2 = (2 - b) - \frac{1}{2} y_2(k) + (2\beta + 1) x_1(k) + 3x_3(k) - y_1^2(k) - 2\alpha x_2^2(k).$$

Using simple calculations, we can show that  $(B - L_1)^T(B - L_1) - I$  is a negative definite matrix. In this case, since the scaling matrix  $\Lambda = [\Lambda_{ij}] \in \mathbb{R}^{2 \times 3}$  has no effect on the eigenvalues of  $(B - L_1)$ , one can adjust the scaling matrix arbitrarily during control without worrying about the control robustness. Therefore, in this case, systems (18) and (19) are matrix projective synchronized. Using the vector controller (22), the error functions can be written as

$$e_1(k+1) = -\frac{1}{2}e_1(k), \quad (23)$$

$$e_2(k+1) = -\frac{1}{2}e_2(k).$$

The error function evolution is shown in Figure 3.

**4.2. Example 2: IMPS between Generalized Hénon Map and Baier-Klein Map.** In this example, we apply the control scheme proposed in Section 3 to achieve IMPS between the drive 3D generalized Hénon map and the response Baier-Klein map. The 3D generalized Hénon map can be described as

$$x_1(k+1) = -\beta x_2(k),$$

$$x_2(k+1) = x_3(k) + 1 - \alpha x_2^2(k), \quad (24)$$

$$x_3(k+1) = \beta x_2(k) + x_1(k),$$

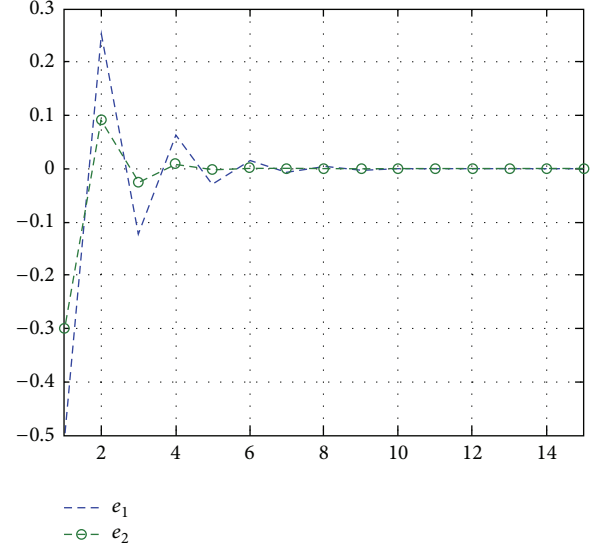


FIGURE 3: Time evolution of MPS errors between systems (18) and (19).

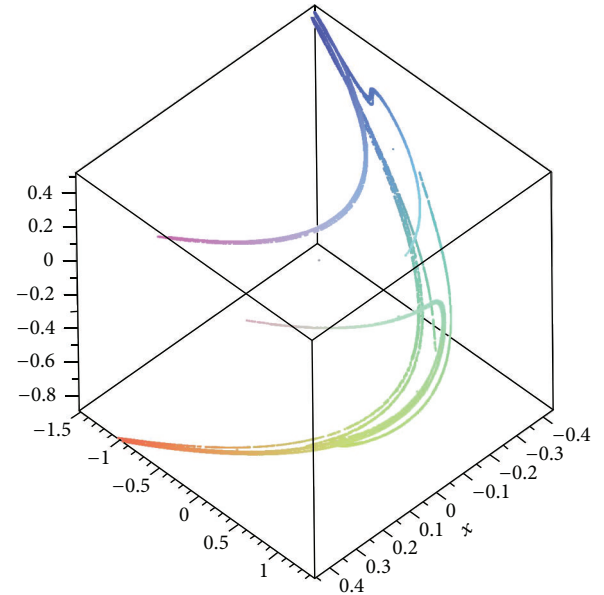


FIGURE 4: Chaotic attractor of the generalized Hénon map when  $(a, b) = (1.07, 0.3)$ .

which has a chaotic attractor, for example, when  $(a, b) = (1.07, 0.3)$  [32]. Using Maple 15.0, the chaotic attractor of the generalized Hénon map for  $x_1(0) = x_2(0) = x_3(0) = 0$  is shown in Figure 4.

The controlled Baier-Klein map [36] can be described as

$$y_1(k+1) = -0.1 y_3(k) - y_2^2(k) + 1.76 + u_1,$$

$$y_2(k+1) = y_1(k) + u_2, \quad (25)$$

$$y_3(k+1) = y_2(k) + u_3,$$

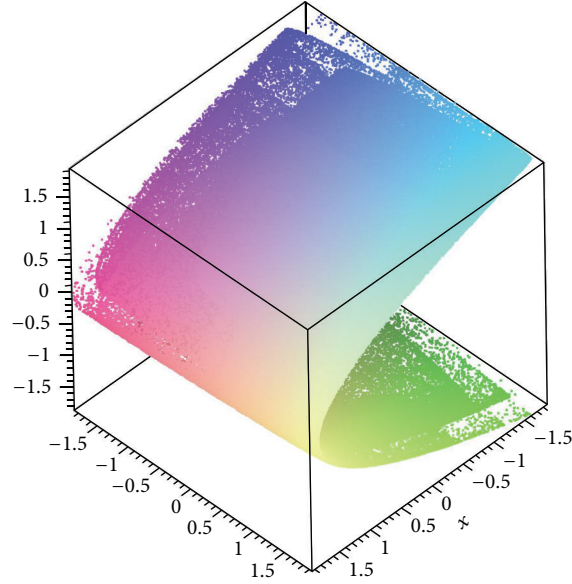


FIGURE 5: Chaotic attractor of the Baier-Klein map.

where  $U = (u_1, u_2, u_3)^T$  is the vector controller. Using Maple 15.0, the chaotic attractor of Baier-Klein map for  $y_1(0) = y_2(0) = y_3(0) = 0$  is shown in Figure 5.

According to our approach presented in Section 3, the quantities  $A$  and  $f(X(k))$  are given by, respectively,

$$A = \begin{pmatrix} 0 & -\beta & 0 \\ 0 & 0 & 1 \\ 1 & \beta & 0 \end{pmatrix}, \quad (26)$$

$$f(X(k)) = \begin{pmatrix} 0 \\ 1 - \alpha x_2^2(k) \\ 0 \end{pmatrix};$$

then the scaling matrix  $\Lambda$  and the control matrix  $L_2$  are chosen as, respectively,

$$\Lambda = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}, \quad (27)$$

$$L_2 = \begin{pmatrix} -0.1 & -\beta & 0 \\ 0 & 0.2 & 1 \\ 1 & \beta & 0.3 \end{pmatrix}.$$

In this case, by using the same formula of the control law given by (16), the vector controller can be constructed as follows:

$$u_1 = -1.76 + 0.1y_1(k) + 0.1y_3(k) - 0.2x_1(k) - 2\beta x_2(k) + y_2^2(k),$$

$$u_2 = 2 - 0.2y_2(k) - y_1(k) + 0.4x_2(k) - 2\alpha x_2^2(k),$$

$$u_3 = -0.3y_3(k) - y_2(k) + 2x_1(k) + 2\beta x_2(k) + 0.6x_3(k). \quad (28)$$

It is easy to show that all eigenvalues of  $A - L_2$  are strictly inside the unit disk. Therefore, in this case, systems (24) and (25) are inverse matrix projective synchronized. Using controllers (28), the error functions can be described as

$$e_1(k+1) = 0.1e_1(k),$$

$$e_2(k+1) = -0.2e_2(k), \quad (29)$$

$$e_3(k+1) = -0.3e_3(k).$$

The error function evolution is shown in Figure 6.

## 5. Conclusion

In this paper, the problems of matrix projective synchronization (MPS) and inverse matrix projective synchronization (IMPS) in different and identical dimensional discrete-time chaotic systems have been analyzed. Based on nonlinear controllers, Lyapunov stability theory, and stability theory of linear dynamical systems, some synchronization criteria have been obtained and new conditions have been derived for achieving MPS and IMPS. Firstly, to achieve MPS behavior between different dimensional systems, the derived control scheme was proposed by controlling the linear part of the response system. Secondly, to achieve IMPS between identical dimensional systems, the presented control method was proposed by controlling the linear part of the drive system. Numerical examples and simulations were used to verify the effectiveness of the proposed approaches.

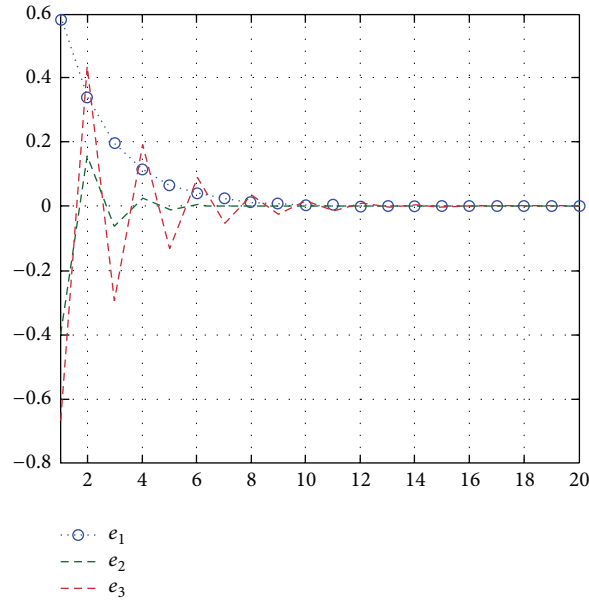


FIGURE 6: Time evolution of IMPS errors between systems (24) and (25).

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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