Hindawi Publishing Corporation
Journal of Control Science and Engineering
Volume 2015, Article ID 353712, 6 pages
http://dx.doi.org/10.1155/2015/353712

Research Article

Predictive Variable Gain Iterative Learning Control for PMSM

Huimin Xu,[1] Xuedong Zhang,[2] and Xiangjie Liu[1]

[1] *The State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China*
[2] *School of Energy and Power Engineering, North China Electric Power University, Baoding 071003, China*

Correspondence should be addressed to Huimin Xu; xuhuimin@126.com

Received 10 June 2015; Revised 2 September 2015; Accepted 11 October 2015

Academic Editor: Zoltan Szabo

A predictive variable gain strategy in iterative learning control (ILC) is introduced. Predictive variable gain iterative learning control is constructed to improve the performance of trajectory tracking. A scheme based on predictive variable gain iterative learning control for eliminating undesirable vibrations of PMSM system is proposed. The basic idea is that undesirable vibrations of PMSM system are eliminated from two aspects of iterative domain and time domain. The predictive method is utilized to determine the learning gain in the ILC algorithm. Compression mapping principle is used to prove the convergence of the algorithm. Simulation results demonstrate that the predictive variable gain is superior to constant gain and other variable gains.

1. Introduction

Due to their high efficiency, high power density and low noise, low loss, small size, and so forth, permanent magnet synchronous motor (PMSM) is used widely in various industrial fields. Furthermore, the applications of PMSM are expanding rapidly.

However, PMSM performance at low speed is bad because of the existence of torque ripple which deteriorates the accuracy and repeatability of PMSM and undermines potentially its suitability in precision electromechanical device. Thus, eliminating torque ripple is very important for improving PMSM performance.

Many control methods have been utilized to suppress the torque ripple. They include PID control scheme [1, 2], predictive control [3], adaptive fuzzy control [4], robust control [5], sliding-mode control [6], and so on. These control means improve the performance of PMSM system from different aspects [7], but applying conventional PID controller and modern control techniques mentioned in the above to deal with torque ripple cannot attain desired levels; moreover, some of them are too complex to employ in practice.

Because of the periodic feature of PMSM on some applications and the simplicity of iterative learning control, a large number of learning control schemes were developed to remove torque ripple in PMSM. Those learning schemes applied to permanent magnet synchronous motors can be divided into two categories according to the learning gain: the first class is fixed gain iterative learning control [8] and the other is variable gain iterative learning control. The second category has obvious advantages in instantaneous characteristics and robustness of system when compared with fixed gain iterative learning control, so designing a reasonable, objective, and effective algorithm in iterative learning controller to determine the value of learning gain at each moment is a vital important factor in solving the problem of instantaneous error growth which has aroused a strong interest of the researchers.

An iterative learning algorithm with a variable gain in iteration domain to remove measurement disturbances and guarantee that the tracking error converges to zero was developed by Zhang et al. [9], but time domain uncertainty is not considered; under this circumstance, Xu et al. [10] proposed a variable PID gain with iterative learning control scheme applied to nonlinear system to tracking the desired output; although this PID gain takes into account disturbances in both time domain and iterative domain, the choice of coefficients for PID gain is subjective.

Unlike [9, 10], we propose to use predictive control to determine the gain of iterative learning control during in both time index and iterative index. The main contributions of this method include three aspects: the first is superiority in control performance compared with constant gain and PID variable gain. The second is that this scheme can be applied not only to the linear system but also to the nonlinear system, so it has a wider application. Finally, although predictive control has been successful applied to the motor control [11–14], it is the first time, as far as the author known, that it is used to determine the gain of iterative learning control; the proposed scheme in this paper overcomes the subjectivity of the gain value choice.

2. Material and Methods

2.1. Process Description. We assume that the PMSM motion model is described as [15]

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{M}\left(F_m - \xi x_2 - f_{\text{cog}} - f_{\text{rel}} - f_{\text{fric}} + f_d\right), \\ y &= x_1; \end{aligned} \tag{1}$$

in the above, M is the load weight, ξ is a dampen coefficient, x_1, x_2 represent the position and velocity of mover, respectively, f_d is a motor load disturbance, f_{fric} is the friction, f_{cog} is a alveolar thrust ripple, f_{rel} is a reluctance thrust ripple, F_m is the reluctance motor electromagnetic force to remove part of the thrust fluctuations, and f_{fric} can be expressed as

$$f_{\text{fric}}(x_2) = \left[f_c + (f_s - f_c)\, e^{-|x_2/x_{2s}|^{\varepsilon}}\right] \operatorname{sgn}(x_2), \tag{2}$$

where f_s is the maximum static friction; f_c is a Coulomb friction, x_{2s} and ε are empirical parameters used to describe the Stribeck effect, and sgn() is a switching function; generally it is believed that f_{cog} is a periodic function which is described as

$$f_{\text{cog}}(x_1 + P) = f_{\text{cog}}(x_1). \tag{3}$$

P is polar distance. It is assumed that motor electromagnetic force $F_e = K_F(x_1)i_q$, where we assume that i_q is the q-axis mover current in the vector control mode:

$$K_F(x_1) = K_{F0} - K_{Fx}(x_1). \tag{4}$$

$K_{F0} > 0$ is an average thrust constant, K_{Fx} is reluctance thrust constant, and $F_m = K_{F0}i_q$:

$$f_{\text{rel}} = K_{Fx}(x_1)\, i_q. \tag{5}$$

f_{rel} is a periodic function to satisfy

$$K_{Fx}(x_1) = K_{Fx}(x_1 + P). \tag{6}$$

In (1), current i_q denotes the input signal, x_1, x_2 represent the states of system, and y denotes the output; generally, the exact input produces the desired output, but in practice, the torque ripple is unavoidable; as a result, the real output is not the desired.

In order to resolve above question, we develop a control scheme which merges the iterative learning and predictive control to eliminate the bad effect of the torque ripple.

The main target of this paper is to find a proper learning gain such that the output error is minimized, and arbitrary high precision output tracking is achieved.

2.2. Iterative Learning Control Law. In this section, we develop a scheme to get the appropriate learning gain by minimizing the performance function in the predictive control process.

For simplicity, we consider an open-loop iterative learning law. During the iterative index $k+1$, the learning update is given by

$$i_{qk+1} = \varphi i_{qk} + \phi e_k, \tag{7}$$

where ϕ is the learning control gain and e_k is the output error; that is,

$$e_k = y_{qd} - y_{qk}. \tag{8}$$

y_{qd} is a realizable desired output trajectory, i_{qk+1}, i_{qk} are the system inputs in the $k+1$st iterative and kth iterative, φ is a filter, and k is the iterative learning index.

2.3. Predictive Gain. In order to find out the appropriate learning gain by predictive control, we firstly set up the predictive model where the learning gain acts as a system input.

At sampled time t in the iterative index $k+1$, combining (1) with (5), we can write the equation for the PMSM:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{M}\left(\left(K_{F0}i_q - \xi x_2 - K_{Fx}\right) i_q - f_{\text{cog}} - f_{\text{fric}} + f_d\right), \\ y &= x_1. \end{aligned} \tag{9}$$

Inserting (9) into (7), we get

$$\begin{aligned} \dot{x}_{1,k} &= x_{2,k}, \\ \dot{x}_{2,k} &= \frac{1}{M}\Big(\left(K_{F0} - K_{Fx}\right)\left(\varphi i_{qk} + \phi e_k\right) - \xi x_{2,k} - f_{\text{cog}} \\ &\quad - \left[f_c + (f_s - f_c)\, e^{-|x_2/x_{2s}|^{\varepsilon}}\right] + f_d\Big), \\ y_k &= x_{1,k}. \end{aligned} \tag{10}$$

For the sake of getting an updating learning gain in next sample time $t+1$, we make use of predictive control method. The predictive model is given by

$$\begin{aligned} \dot{x}_{1,k} &= x_{2,k}, \\ \dot{x}_{2,k} &= \frac{1}{M}\left(K_{F0}\varphi i_{qk} - \xi x_{2,k}\right) + \frac{K_{F0}}{M} e_k \phi_k, \\ y_k &= x_{1,k} \end{aligned} \tag{11}$$

ϕ_k, y_k, $x_{1,k}$, and $x_{2,k}$ denote the input, output, and states of system in the kth iterative index, respectively; the system in the above is a nonlinear system.

Secondly, we turn the nonlinear PMSM model (11) decouples into a new linear system via the input-output feedback linearization scheme. According to the exact linear theory, calculating Lie derivative of the output variable, we get the relative degree which is equal to the number of the system state variables; as a result, (11) satisfies the exact linear condition and can be linearized as

$$\dot{x_k} = Ax_k + Bv_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_k, \qquad Y = Cx_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k, \tag{12}$$

where $x_k = \left[\begin{smallmatrix} x_{1k} \\ x_{2k} \end{smallmatrix}\right]$, $v_k = (1/M)(K_{F0}i_{qk} - \xi x_2 - \xi\phi_k)$, and v_k is a new control input. We can discretize (10):

$$x_k(t+1) = G_1 x_k(t) + G_2 v_k(t), \qquad y = Cx_k(t), \tag{13}$$

where $G_1 = e^{At}$, $G_2 = \int_0^T e^{At} dt$, $T = 0.1$ ms is the sampling period, and $\Delta v = v_d - v_k$, where v_d denotes desired input. We assume that the predictive horizon is H_p and the control horizon is H_v; Δv can be calculated by the following criterion:

$$V(t) = \sum_{i=0}^{H_p} \left\| y_{qd}(t+i \mid t) - y_{qk}(t+i \mid t) \right\|_L^2 + \sum_{i=0}^{H_v - 1} \left\| \Delta v(t+i \mid t) \right\|_R^2. \tag{14}$$

In the light of the least square formula, the expression of the Δv is obtained as

$$\Delta v(t) = \begin{bmatrix} S_L \Theta \\ S_R \end{bmatrix}^{-1} \begin{bmatrix} S_L \varepsilon(t) \\ 0 \end{bmatrix}, \tag{15}$$

where $\Theta = \left[\begin{smallmatrix} B & \cdots & 0 \\ AB+B & \cdots & 0 \\ \vdots & \dots & \vdots \end{smallmatrix}\right]$, $\Psi = \left[\begin{smallmatrix} A \\ A^2 \\ \vdots \end{smallmatrix}\right]$, $Y = \left[\begin{smallmatrix} B \\ AB+B \\ \vdots \end{smallmatrix}\right]$, $C\varepsilon(t) = C(x_d - \Psi x(t) - Yv(t-1))$, S_L and S_R are square roots of eigenvalues of matrix L and R, respectively; x_d is the desired state; according to the exact linear theory, we get

$$\phi_k = \frac{v - (1/M)\left(K_{F0} i_{qk} - \xi x_2\right)}{-\xi/M}. \tag{16}$$

The open-loop learning gain $\phi_k(t)$ is given by

$$\phi_k(t) = \phi_k(t-1) - \frac{M}{\xi}\Delta v(t). \tag{17}$$

Considering (5), we have

$$i_{qk+1}(t) = \varphi i_{qk}(t) + \left(\phi(t-1) - \frac{M}{\xi}\Delta v(t)\right) e_k(t). \tag{18}$$

It can be rewritten in the form

$$i_{qk+1}(t) = \varphi i_{qk}(t) + \left(\phi(t-1) - \frac{M}{\xi}\begin{bmatrix} S_L \Theta \\ S_R \end{bmatrix}^{-1} \begin{bmatrix} S_L \varepsilon \\ 0 \end{bmatrix}\right) e_k(t). \tag{19}$$

2.4. Convergence Analysis. In this section, we give the condition under which the system output error converges to zero.

Consider (1) and it can be written as

$$\dot{x} = f(t, x(t)) + g(t)u(t), \qquad y = h(t, x(t)), \tag{20}$$

where $\dot{x} = \left[\begin{smallmatrix} \dot{x}_1 \\ \dot{x}_2 \end{smallmatrix}\right]$, $g(t) = \left[\begin{smallmatrix} 0 \\ (K_{F0} - K_{Fx})(1/M) \end{smallmatrix}\right]$, $h(t, x(t)) = x_1$, $f = \left[\begin{smallmatrix} x_2 \\ (1/M)(-\xi x_2 - f_{\mathrm{cog}} - [f_c + (f_s - f_c)e^{-|x_2/x_{2s}|^{\varepsilon}}]\operatorname{sgn}(x_2) + f_d) \end{smallmatrix}\right]$, and $u(t) = i_q$.

We assume the following properties for system (20):

(1) The functions f, g, and h are assumed to satisfy the following conditions: $\forall x_1, x_2 \in R^n$

$$\begin{aligned} \|f(t, x_1) - f(t, x_1)\| &\le M_1 \|x_1 - x_2\|, \\ \|g(t, x_1) - g(t, x_1)\| &\le M_{11} \|x_1 - x_2\|, \\ \|h(t, x_1) - h(t, x_1)\| &\le M_{12} \|x_1 - x_2\| \end{aligned} \tag{21}$$

for all $t \in [0, T]$ and M_1, M_{11}, and M_{12} are constants.

(2) It is assumed that, at each iterative process, the initial state error sequence $\{\delta x_k(0)\}$, $k \ge 0$, converges to zero.

(3) For any realizable output trajectory and an appropriate initial condition, there exists a unique control input generating the trajectory for the plant.

Proof. Let system satisfy assumptions (1)–(3) and (7) be applied; define the state, input and the output errors as

$$\begin{aligned} \Delta x_k(t) &= x_d(t) - x_k(t), \\ \Delta y_k(t) &= y_d(t) - y_k(t), \\ \Delta u_k(t) &= u_d(t) - u_k(t), \\ f_1(t, x_k) &= f(t, x_d) - f(t, x_d - x_k). \end{aligned} \tag{22}$$

According to (20), we get

$$\|f_1\| \le M_1 \|x_k\|. \tag{23}$$

Then,

$$\begin{aligned} \Delta \dot{x_k}(t) &= f_1(t, \Delta x_k(t)) + g(t)\Delta u_k(t), \\ \Delta y_k(t) &= h(t, x_d(t)) - h(t, x_k(t)), \end{aligned}$$

$$
\begin{aligned}
& h_1(t, \Delta x_k(t)) = h(t, x_d(t)) - h(t, x_k(t)), \\
& \Delta u_{k+1}(t) \\
& \quad = u_d(t) - \varphi u_k(t) \\
& \qquad - \left(\phi(t-1) - \frac{M}{\xi} \begin{bmatrix} S_L\Theta \\ S_R \end{bmatrix}^{-1} \begin{bmatrix} S_L\varepsilon \\ 0 \end{bmatrix} \right) e_k(t) \\
& \quad = \varphi \Delta u_k(t) \\
& \qquad - \left(\phi(t-1) - \frac{M}{\xi} \begin{bmatrix} S_L\Theta \\ S_R \end{bmatrix}^{-1} \begin{bmatrix} S_L\varepsilon \\ 0 \end{bmatrix} \right) \Delta y_k(t) \\
& \qquad + (1-\varphi) u_d(t) \\
& \quad = \varphi \Delta u_k(t) \\
& \qquad - \left(\phi(t-1) - \frac{M}{\xi} \begin{bmatrix} S_L\Theta \\ S_R \end{bmatrix}^{-1} \begin{bmatrix} S_L\varepsilon \\ 0 \end{bmatrix} \right) h_1 \\
& \qquad + (1-\varphi) u_d(t).
\end{aligned} \tag{24}
$$

Define the operator

$$
\begin{aligned}
& \Upsilon_k : C_r[0,T] \longrightarrow C_r[0,T], \\
& \Upsilon_k(u(t)) \\
& \quad = \left(\phi(t-1) - \frac{M}{\xi} \begin{bmatrix} S_L\Theta \\ S_R \end{bmatrix}^{-1} \begin{bmatrix} S_L\varepsilon \\ 0 \end{bmatrix} \right) h_1 \\
& \qquad + (1-\varphi) u_d(t), \\
& \Delta u_{k+1}(t) = (\varphi I + \Upsilon_k) \Delta u_k(t), \\
& \Delta u_{k+1}(t) \\
& \quad = (\varphi I + \Upsilon_k)(\varphi I + \Upsilon_{k-1}) \cdots (\varphi I + \Upsilon_0) \Delta u_0(t).
\end{aligned} \tag{25}
$$

I is a unit matrix; we make an estimate for Υ_k:

$$
\begin{aligned}
& \|x(t)\| = \left\| x(0) + \int_0^t [f(s, x(s) + g(s) u(s))] \, ds \right\|, \\
& \|x(t)\| \\
& \quad \le \|x(0)\| + M_1 \int_0^t \|x(s)\| \, ds + M_{11} \int_0^t \|u(s)\| \, ds, \\
& \|x(t)\| \le M_3 \left(\|x(0)\| + \int_0^t \|u(s)\| \, ds \right), \\
& \|\Upsilon_k u(t)\| \le M_4 \left(\|x(0)\| + \int_0^t \|u(s)\| \, ds \right), \\
& \|\Upsilon_k \Delta u(t)\| \le M_4 \left(\|\Delta x(0)\| + \int_0^t \|\Delta u(s)\| \, ds \right).
\end{aligned} \tag{26}
$$

According to lemma 2 in [16], we have the convergence conditions $0 < \varphi < 1$, $M_4 > 1$, and M_3 and M_4 are constant. □

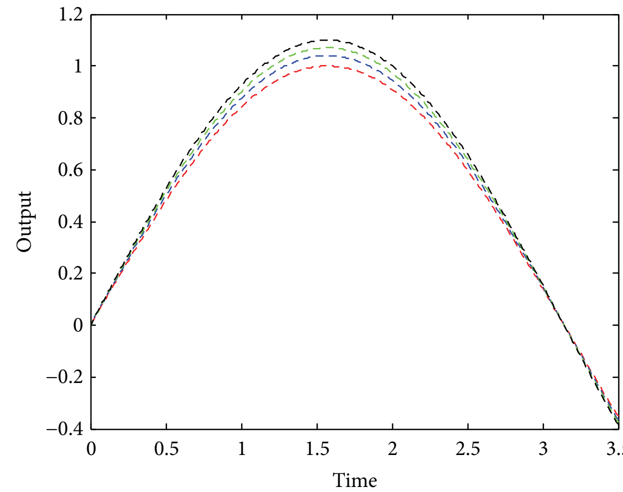

Figure 1: Desired and actual output.

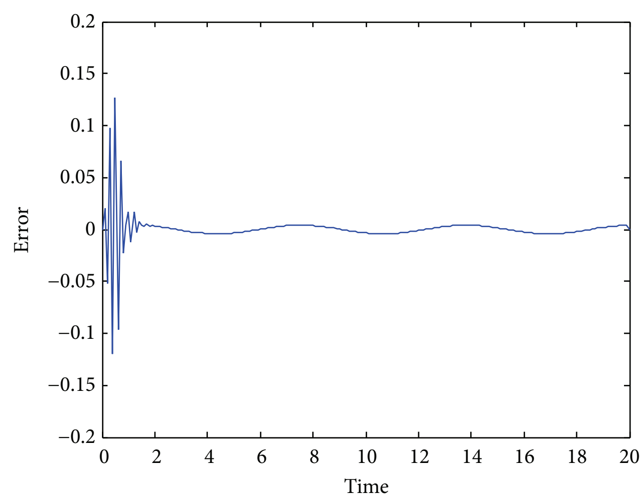

Figure 2: Error response in the 1st iterative.

2.5. Simulation. In this section, we will make a comparison between the iterative learning method based on the predicted variable gain and two kinds of efficient iterative learning control method which in recent years have been widely used for permanent magnet synchronous motors: constant gain of iterative learning control strategy and the variable PID gain iterative learning control strategy.

PMSM system preferences are as follows.

Load weight $M = 5$ kg, $l = 0.0333$ A/kg, damping coefficient is 20.99 Nm/min^{-1}, pole pitch is 60.9 mm, $F_m = (3/2)P_n\phi_f i_d$, $P_n = 2$, $\phi_f = 0.125$ wb, friction $f_s = f_c = 15$ N, control horizon $nc = 3$, and predictive horizon $np = 7$; assume that the initial state error is zero and the desired output is a sine wave and the cycle is π; amplitude is 5.

The simulation results are shown in Figures 1–7.

Figure 1 shows a comparison between the desired PMSM output and the actual PMSM output in the 1st, 2nd, and

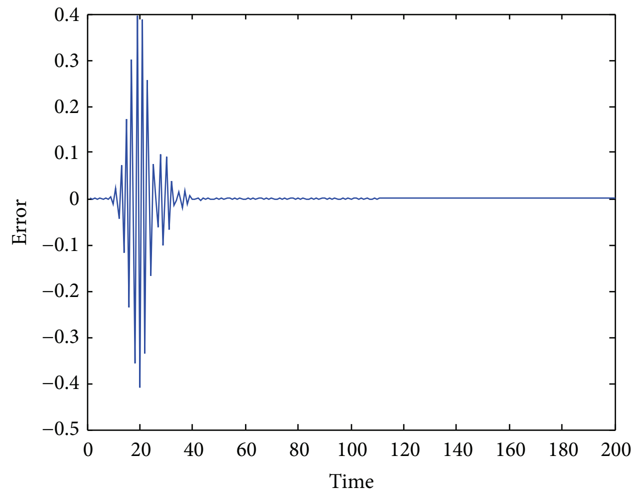

FIGURE 3: Error response in the 5th iterative.

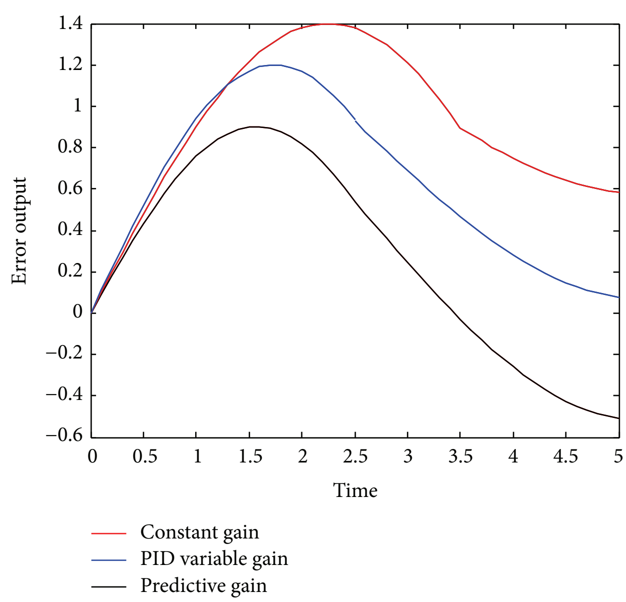

FIGURE 4: Errors of the constant gain, PID gain, and predictive gain in the 1st iterative.

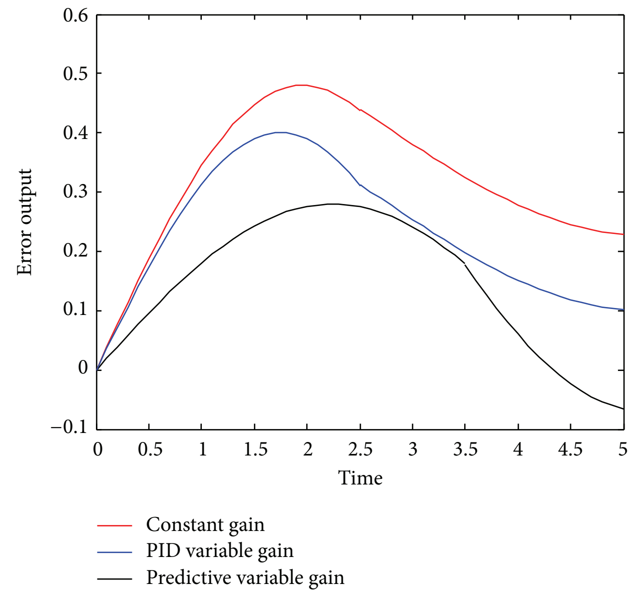

FIGURE 5: Errors of the constant gain, PID gain, and predictive gain in the 6th iterative.

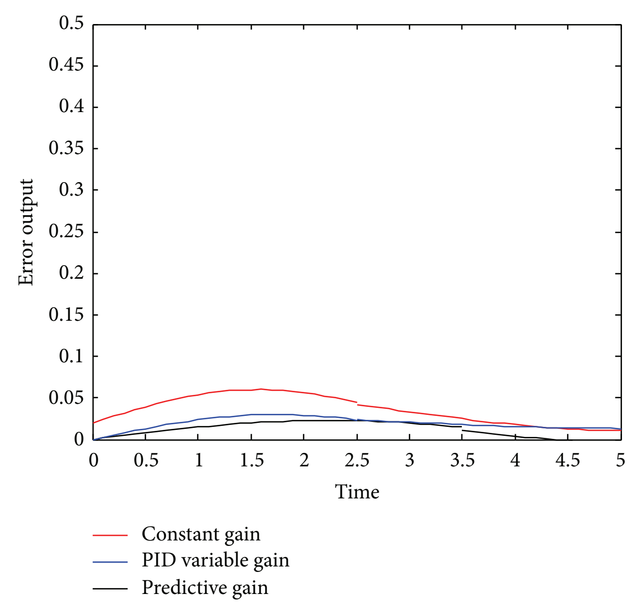

FIGURE 6: Errors of the constant gain, PID gain, and predictive gain in the 15th iterative.

3rd iteratives with the fixed gain iterative learning controller, although the actual system output will approach the desired output with the increase of the number of iterations, instantaneous vibration bandwidth is sometimes dramatic large which is in fact detrimental to system. Figures 2 and 3 give a more intuitive description about the instantaneous vibration growth which show that max vibration bandwidth increases from 0.158 mm to 1.4 mm just after four iteratives. These phenomena also give full explanation of the importance of predictive gain learning control.

Figures 4–6 show the actual system output in different iterative indices with constant gain learning controller, PID variable gain learning controller, and predictive gain learning controller, respectively. We can see from Figures 4–6 that the predictive gain is significantly better than the fixed gain and the PID gain. Firstly, the error vibration amplitude of predictive gain is less than the other two gains; secondly, error descent rate of predictive gain in time domain is also faster than fixed gain and PID gain iterative learning control.

For the sake of getting a more intuitive understanding for the advantages of the scheme proposed by this paper, Figure 7 is given. It can be seen that when the initial state error is zero, on the premise of the convergence conditions of iterative learning control, the predictive variable gain iterative learning has a higher convergent rate; meanwhile tracking accuracy also has been greatly improved.

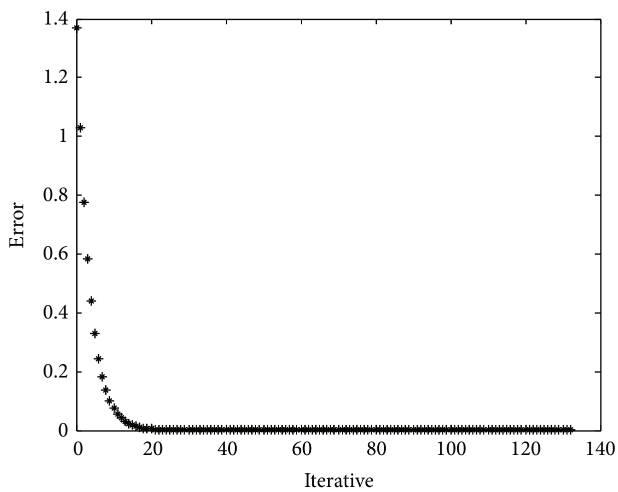

Figure 7: Error in iterative domain.

3. Conclusion

A variable gain iterative learning control strategy based on predictive control for PMSM has been developed. This method not only increases the iterative convergence rate, but also eliminates error in the time domain. The convergence of this technique is proved by the contractive operator theory.

Future work will consider the choosing of predictive horizon to get a better control performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This research was supported by the Central Universities Funds (9160215006).

References

[1] Q. Y. Xu, C. J. Zhang, L. Zhang, and C. Y. Wang, "Multiobjective optimization of PID controller of PMSM," *Journal of Control Science and Engineering*, vol. 2014, Article ID 471609, 9 pages, 2014.

[2] J. J. Xi, B. Sun, and H. Q. Zhao, "Adaptive PID controller based on single neuron for permanent magnet synchronous machine," *Electric Power Automation Equipment*, vol. 23, no. 10, pp. 59–61, 2003.

[3] A. M. Kasseml and A. A. Hassan, "Performance improvements of a permanent magnet synchronous machine via functional model predictive control," *Journal of Control Science and Engineering*, vol. 2012, Article ID 319708, 8 pages, 2012.

[4] C. Z. Zhou, D. C. Quach, N. X. Xiong et al., "An improved direct adaptive fuzzy controller of an uncertain PMSM for web-based e-service systems," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 1, pp. 58–71, 2015.

[5] M. C. Chou, C. M. Liaw, S. B. Chien, F. H. Shieh, J. R. Tsai, and H. C. Chang, "Robust current and torque controls for PMSM driven satellite reaction wheel," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 1, pp. 58–74, 2011.

[6] F. J. Lin, Y. C. Hung, and M. T. Tsai, "Fault-tolerant control for six-phase PMSM drive system via intelligent complementary sliding-mode control using TSKFNN-AMF," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 12, pp. 5747–5762, 2013.

[7] S. H. Li and H. Guo, "Fuzzy adaptive internal model control schemes for PMSM speed-regulation system," *IEEE Transactions on Industrial Informatics*, vol. 8, no. 4, pp. 767–779, 2013.

[8] B. Q. Li and H. Q. Lin, "PMSM periodical torque ripple minimization using iterative learning control," *Electric Machines and Control*, vol. 15, no. 9, pp. 51–55, 2011.

[9] H. W. Zhang, F. S. Yu, X. H. Bu, and F. Z. Wang, "Robust iterative learning control for permanent magnet linear motor," *Electric Machines and Control*, vol. 16, no. 6, pp. 81–86, 2012.

[10] M. Xu, H. Lin, and Z. Liu, "Iterative learning control law with variable learning gain," *Control Theory and Applications*, vol. 24, no. 5, pp. 856–860, 2006.

[11] X. B. Kong and X. J. Liu, "Effcient nonlinear model predictive control for permanent magnet synchronous motor," *Acta Automatica Sinica*, vol. 40, no. 9, pp. 1958–1966, 2014.

[12] X. B. Kong and X. J. Liu, "Nonlinear model predictive control for DFIG-based wind power generation," *Acta Automatica Sinica*, vol. 39, no. 5, pp. 636–643, 2013.

[13] X. J. Liu, "Present situation and prospect of model predictive control application in complex power industrial process," *Proceedings of the Chinese Society for Electrical Engineering*, vol. 33, no. 5, pp. 79–85, 2012.

[14] X. J. Liu, "Continuous-time nonlinear model predictive control with input/output linearization," *Control Theory and Applications*, vol. 29, no. 2, pp. 217–224, 2014.

[15] Y. X. Song, C. H. Wang, W. S. Yin, and P. F. Jia, "Adaptive-learning control for permanent magnet linear synchronous motors," *Proceedings of the Chinese Society of Electrical Engineering*, vol. 25, no. 20, pp. 151–156, 2005.

[16] J. X. Shou, Z. J. Zhang, and D. Y. Pi, "On the convergence of open-closed-loop D-type iterative learning control for nonlinear systems," in *Proceedings of the IEEE International Symposium on Intelligent Control*, pp. 963–967, IEEE, Houston, Tex, USA, October 2003.