Research Article

Phase Transition of the Higher Dimensional Charged Gauss-Bonnet Black Hole in de Sitter Spacetime

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We study the phase transition of charged Gauss-Bonnet-de Sitter (GB-dS) black hole. For black holes in de Sitter spacetime, there is not only black hole horizon, but also cosmological horizon. The thermodynamic quantities on both horizons satisfy the first law of the black hole thermodynamics, respectively; moreover, there are additional connections between them. Using the effective temperature approach, we obtained the effective thermodynamic quantities of charged GB-dS black hole. According to Ehrenfest classification, we calculate some response functions and plot their figures, from which one can see that the spacetime undergoes a second-order phase transition at the critical point. It is shown that the critical values of effective temperature and pressure decrease with the increase of the value of GB parameter α.

1. Introduction

Due to the existence of Hawking radiation and the entropy, black holes may display themselves like thermodynamic systems [1–4]. Phase transitions and critical phenomena are important characteristics of ordinary thermodynamic system. Thus, the natural question to ask is whether there also exists phase transition in the black hole thermodynamics. The pioneer work of Hawking and Page gives us an definitive answer [5]. Recently, the phase transitions of black holes in asymptotically anti-de Sitter (AdS) spacetime have received considerable attention [6–17]. In particular, by considering the cosmological constant as thermodynamic pressure $P = -\Lambda/8\pi = (d-1)(d-2)/16\pi l^2$, one can introduce an extended phase space, in which the P-V criticality has been extensively discussed [18–35]. It is shown that many black holes in AdS spacetime exhibit similar critical behaviors to those of van der Waals liquid-gas system.

The astronomical observations show that our universe is probably an asymptotically de Sitter (dS) one. It raises the

interest on black holes in dS spacetime. There are many works on the thermal properties of black holes in de Sitter (dS) spacetime [36–48]. As is well known, there are multihorizons for black holes in dS spacetime. On the black hole event horizon and the cosmological horizon, there are different temperatures, which prevent the black hole thermodynamic system in equilibrium. It should be noted that there are common parameters M, Q, and Λ in the thermodynamic quantities corresponding to the black hole horizon to the cosmological horizon. Thus, these thermodynamic quantities are not independent. Taking into account their connections is relevant to the understanding of thermal properties of dS spacetime.

Higher derivative curvature terms occur in many occasions, such as in the semiclassically quantum gravity and in the effective low-energy action of superstring theories. Among the many theories of gravity with higher derivative curvature terms, due to the special features, the Gauss-Bonnet (GB) gravity has attracted much interest. The thermodynamic properties and phase structures of GB-AdS black

hole have been briefly discussed in [49]. In [26–29], the critical phenomena and phase transition of the charged GB-AdS black hole have been studied extensively.

In this paper, we study the thermal properties of charged GB-dS black hole after considering the connections between the black hole horizon and the cosmological horizon. Using the effective equilibrium temperature approach, we calculate some effective thermodynamic quantities, from which we can analyze whether there exists phase transition for charged GB-dS black hole.

The paper is arranged as follows: in Section 2, we simply introduce the charged GB-dS black hole. In Section 3, we will calculate the effective thermodynamic quantities. In Section 4, we derive the heat capacity at constant pressure, the volume expansivity β, and the isothermal compressibility κ and plot some curves to demonstrate the phase transition. We will make some concluding remarks in Section 5 (we use the units $G_d = \hbar = c = k_B = 1$).

2. Charged GB-dS Black Hole

The action of the d-dimensional Einstein-Gauss-Bonnet-Maxwell-dS theory has the form

$$S = \frac{1}{16\pi} \int d^d x \sqrt{-g} \left[R - 2\Lambda + \alpha \left(R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \right) - 4\pi F_{\mu\nu} F^{\mu\nu} \right], \tag{1}$$

where α is the GB coupling constant, the cosmological constant is $\Lambda = (d-1)(d-2)/2l^2$ for dS spacetime, and $F_{\mu\nu}$ is the Maxwell field strength. The d-dimensional static charged GB-dS black hole solution for the action is described by

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2 d\phi^2 + \cos^2\theta d\Omega_{d-4}^2 \right), \tag{2}$$

with the metric function given by [26, 27, 33]

$$f(r) = 1 + \frac{r^2}{2\widetilde{\alpha}} \left(1 - \sqrt{1 + \frac{64\pi\widetilde{\alpha}M}{(d-2)\Sigma_k r^{d-1}} - \frac{2\widetilde{\alpha}Q^2}{(d-2)(d-3)r^{2d-4}} + \frac{8\widetilde{\alpha}\Lambda}{(d-1)(d-2)}} \right), \tag{3}$$

where $\widetilde{\alpha} = (d-3)(d-4)\alpha$, Σ_k is the area of a unite $(d-2)$-dimensional sphere, M is the black hole mass, and Q is related to the charge of the black hole. Horizons occur where $f(r) = 0$, the largest root is the cosmological horizon $r = r_c$, and the next root is the black hole event horizon $r = r_+$. When $M = Q = 0$, solution (2) reduces to the pure GB-dS space with only one cosmological horizon at $r_c = l_c$.

The equations $f(r_+) = 0$ and $f(r_c) = 0$ are rearranged to br as follows:

$$M = \frac{(d-2)\Sigma_k r_+^{d-3}}{16\pi} \left(1 + \frac{\widetilde{\alpha}}{r_+^2} \right) - \frac{\Sigma_k r_+^{d-1}\Lambda}{8\pi(d-1)} + \frac{\Sigma_k Q^2}{32\pi(d-3) r_+^{d-3}},$$

$$M = \frac{(d-2)\Sigma_k r_c^{d-3}}{16\pi} \left(1 + \frac{\widetilde{\alpha}}{r_c^2} \right) - \frac{\Sigma_k r_c^{d-1}\Lambda}{8\pi(d-1)} + \frac{\Sigma_k Q^2}{32\pi(d-3) r_c^{d-3}}, \tag{4}$$

from which one can derive

$$M = \frac{(d-2)\Sigma_k r_+^{d-3} r_c^{d-3} \left(r_c^2 - r_+^2 \right)}{16\pi \left(r_c^{d-1} - r_+^{d-1} \right)} \left(1 + \frac{\widetilde{\alpha}\left(r_c^2 + r_+^2 \right)}{r_c^2 r_+^2} \right) + \frac{\Sigma_k Q^2 \left(r_c^{2d-4} - r_+^{2d-4} \right)}{32\pi(d-3) r_c^{d-3} r_+^{d-3} \left(r_c^{d-1} - r_+^{d-1} \right)}, \tag{5}$$

by eliminating the Λ, and

$$\Lambda = \frac{(d-1)(d-2)}{2\left(r_c^{d-1} - r_+^{d-1} \right)} \left(r_c^{d-3} - r_+^{d-3} + \widetilde{\alpha}\left(r_c^{d-5} - r_+^{d-5} \right) \right) - \frac{(d-1)Q^2 \left(r_c^{d-3} - r_+^{d-3} \right)}{4(d-3) r_c^{d-3} r_+^{d-3} \left(r_c^{d-1} - r_+^{d-1} \right)} \tag{6}$$

by eliminating M.

The surface gravities of black hole horizon and the cosmological horizon are

$$\kappa_+ = \left. \frac{f'(r)}{2} \right|_{r=r_+}$$

$$= -\frac{1}{r_+} + \frac{(d-1) r_c^{d-3} r_+ \left(r_c^2 - r_+^2 \right)}{2\left(r_+^2 + 2\widetilde{\alpha} \right)\left(r_c^{d-1} - r_+^{d-1} \right)} \left(1 + \frac{\widetilde{\alpha}\left(r_c^2 + r_+^2 \right)}{r_c^2 r_+^2} \right)$$

$$- \frac{Q^2}{4(d-3)\left(r_+^2 + 2\widetilde{\alpha} \right) r_+^{2d-7}}$$

$$\cdot \left(2 - \frac{(d-1)\left(r_c^{2d-4} - r_+^{2d-4} \right)}{(d-2) r_c^{d-3}\left(r_c^{d-1} - r_+^{d-1} \right)} \right),$$

$$\kappa_c = \left. \frac{f'(r)}{2} \right|_{r=r_c}$$

$$= -\frac{1}{r_c} + \frac{(d-1) r_+^{d-3} r_c \left(r_c^2 - r_+^2 \right)}{2\left(r_c^2 + 2\widetilde{\alpha} \right)\left(r_c^{d-1} - r_+^{d-1} \right)} \left(1 + \frac{\widetilde{\alpha}\left(r_c^2 + r_+^2 \right)}{r_c^2 r_+^2} \right)$$

$$- \frac{Q^2}{4(d-3)(r_c^2 + 2\tilde{\alpha}) r_c^{2d-7}}$$

$$\cdot \left(2 - \frac{(d-1)\left(r_c^{2d-4} - r_+^{2d-4}\right)}{(d-2) r_+^{d-3} \left(r_c^{d-1} - r_+^{d-1}\right)} \right). \tag{7}$$

The thermodynamic quantities for the two horizons satisfy the first law of black hole thermodynamics [34, 37]

$$\delta M = \frac{\kappa_+}{2\pi} \delta S_+ + \Phi_+ \delta Q + V_+ \delta P + \widetilde{V}_+ d\tilde{\alpha}, \tag{8}$$

$$\delta M = \frac{\kappa_c}{2\pi} \delta S_c + \Phi_c \delta Q + V_c \delta P + \widetilde{V}_c d\tilde{\alpha}, \tag{9}$$

where

$$
\begin{aligned}
S_+ &= \frac{\Sigma_k r_+^{(d-2)}}{4} \left(1 + \frac{2(d-2)\tilde{\alpha}}{(d-4) r_+^2} \right), \\
S_c &= \frac{\Sigma_k r_c^{(d-2)}}{4} \left(1 + \frac{2(d-2)\tilde{\alpha}}{(d-4) r_c^2} \right), \\
\Phi_+ &= \frac{\Sigma_k Q}{16\pi (d-3) r_+^{d-3}}, \\
\Phi_c &= \frac{\Sigma_k Q}{16\pi (d-3) r_c^{d-3}}, \\
V_+ &= \frac{\Sigma_k r_+^{d-1}}{d-1}, \\
V_c &= \frac{\Sigma_k r_c^{d-1}}{d-1}, \\
\widetilde{V}_+ &= \frac{(d-2)}{16\pi} r_+^{d-5}, \\
\widetilde{V}_c &= \frac{(d-2)}{16\pi} r_c^{d-5}, \\
P &= -\frac{\Lambda}{8\pi}.
\end{aligned}
\tag{10}
$$

3. Effective Thermodynamic Quantities of Charged GB Black Hole

We have given the thermodynamic quantities without considering the relationship between the black hole horizon and the cosmological horizon. Because there are four variables M, Q, Λ, and $\tilde{\alpha}$ in the spacetime, the thermodynamic quantities corresponding to the black hole horizon and the cosmological horizon are functions of these variables. Through the four variables, there are some connections between the thermodynamic quantities corresponding to the black hole horizon and the ones corresponding to the cosmological horizon. When the thermodynamic property of charged de Sitter spacetime is studied, we must consider the relationship with the two horizons. Recently, by studying Hawking radiation of de Sitter spacetime, [50–52] obtained that the outgoing rate of the

charged de Sitter spacetime which radiates particles with energy ω is

$$\Gamma = e^{\Delta S_+ + \Delta S_c}, \tag{11}$$

where ΔS_+ and ΔS_c are Bekenstein-Hawking entropy difference corresponding to the black hole horizon and the cosmological horizon after charged de Sitter spacetime radiates particles with energy ω. Therefore, the thermodynamic entropy of the charged de Sitter spacetime is the sum of the black hole horizon entropy and the cosmological horizon entropy:

$$S = S_+ + S_c. \tag{12}$$

Recently, thermodynamic volume of charged dS black hole is given by [34, 37]

$$V = V_c - V_+. \tag{13}$$

From (9), one can obtain

$$
\begin{aligned}
dS_+ &= \frac{\Sigma_k (d-2)}{4} r_+^{d-5} \left(r_+^2 + 2\tilde{\alpha}\right) dr_+, \\
dS_c &= \frac{\Sigma_k (d-2)}{4} r_c^{d-5} \left(r_c^2 + 2\tilde{\alpha}\right) dr_c.
\end{aligned}
\tag{14}
$$

Substituting (12) and (13) into (14), we obtain

$$
\begin{aligned}
dS_+ &= \frac{r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) dS}{r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) + r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right)} \\
&\quad - \frac{(d-2)\left(r_+^2 + 2\tilde{\alpha}\right)\left(r_c^2 + 2\tilde{\alpha}\right) dV}{4 \left[r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) + r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right) \right]}, \\
dS_c &= \frac{r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right) dS}{r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) + r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right)} \\
&\quad + \frac{(d-2)\left(r_+^2 + 2\tilde{\alpha}\right)\left(r_c^2 + 2\tilde{\alpha}\right) dV}{4 \left[r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) + r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right) \right]}.
\end{aligned}
\tag{15}
$$

Substituting (15) into (8) and (9), we get

$$dM = T_{\text{eff}} dS - P_{\text{eff}} dV + \Phi_{\text{eff}} dQ + \widetilde{V}_{\text{eff}} d\tilde{\alpha}, \tag{16}$$

where the effective temperature T_{eff}, effective electric potential Φ_{eff}, and effective pressure P_{eff} are, respectively,

$$
\begin{aligned}
T_{\text{eff}} &= \frac{\kappa_+ V_c r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) - \kappa_c V_+ r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right)}{2\pi \left(V_c - V_+\right) \left[r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) + r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right) \right]}, \\
\Phi_{\text{eff}} &= \frac{\Phi_+ V_c - \Phi_c V_+}{V_c - V_+}, \\
P_{\text{eff}} &= \frac{(d-2)\left(r_+^2 + 2\tilde{\alpha}\right)\left(r_c^2 + 2\tilde{\alpha}\right)}{8\pi \left[r_c^3 \left(r_+^2 + 2\tilde{\alpha}\right) + r_+^3 \left(r_c^2 + 2\tilde{\alpha}\right) \right]} \frac{\left(\kappa_+ V_c + \kappa_c V_+\right)}{\left(V_c - V_+\right)}, \\
\widetilde{V}_{\text{eff}} &= \frac{V_c \widetilde{V}_+ - V_+ \widetilde{V}_c}{V_c - V_+}.
\end{aligned}
\tag{17}
$$

Substituting (7) and (10) into (17), we obtain

$$
\begin{aligned}
T_{\mathrm{eff}} \\
= \frac{x\left[(d-1)\left(1-x^2\right)\left(1+x^{d-1}\right)-2\left(1-x^{d+1}\right)\right]}{4\pi r_c\left(1-x^{d-1}\right)\left[\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)\right]} \\
+ \frac{\widetilde{\alpha}\left[(d-1)\left(1-x^2\right)\left(1+x^{d-1}\right)\left(1+x^2\right)-4\left(1-x^{d+3}\right)\right]}{4\pi r_c^3 x\left(1-x^{d-1}\right)\left[\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)\right]} \\
+ \frac{Q^2}{8\pi(d-3)(d-2)} \\
\cdot \frac{(d-1)\left(1+x^{d-1}\right)\left(1-x^{2d-4}\right)-2(d-2)\left(1-x^{3d-5}\right)}{r_c^{2d-5}x^{2d-7}\left(1-x^{d-1}\right)\left[\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)\right]},
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
P_{\mathrm{eff}} \\
= -\frac{(d-2)\left(x^2+2\widetilde{\alpha}/r_c^2\right)\left(1+2\widetilde{\alpha}/r_c^2\right)\left(1+x^d\right)}{4\pi r_c^2 x\left(1-x^{d-1}\right)\left[\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)\right]} \\
+ \frac{(d-2)(d-1)\left(1-x^2\right)x}{8\pi r_c^2\left(1-x^{d-1}\right)^2} \\
\cdot \left(\frac{\left(1+x^{2d-3}\right)+2\widetilde{\alpha}\left(1+x^{2d-5}\right)/r_c^2}{\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)}\right) \\
+ \frac{\widetilde{\alpha}(d-2)(d-1)\left(1-x^2\right)\left(1+x^2\right)}{8\pi r_c^4\left(1-x^{d-1}\right)^2 x} \\
\cdot \left(\frac{\left(1+x^{2d-3}\right)+2\widetilde{\alpha}\left(1+x^{2d-5}\right)/r_c^2}{\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)}\right) \\
- \frac{Q^2(d-2)}{8\pi(d-3)r_c^{2d-4}x^{2d-7}\left(1-x^{d-1}\right)} \\
\cdot \left(\frac{\left(1+x^{3d-6}\right)+2\widetilde{\alpha}\left(1+x^{3d-8}\right)/r_c^2}{\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)}\right) \\
+ \frac{Q^2(d-1)\left(1-x^{2d-4}\right)}{16\pi(d-3)r_c^{2d-4}x^{2d-7}\left(1-x^{d-1}\right)^2} \\
\cdot \left(\frac{\left(1+x^{2d-3}\right)+2\widetilde{\alpha}\left(1+x^{2d-5}\right)/r_c^2}{\left(x^2+2\widetilde{\alpha}/r_c^2\right)+x^3\left(1+2\widetilde{\alpha}/r_c^2\right)}\right),
\end{aligned}
\tag{19}
$$

$$
\Phi_{\mathrm{eff}} = \frac{\Sigma_k Q\left(1-x^{2d-4}\right)}{16\pi(d-3)r_c^{d-3}x^{d-3}\left(1-x^{d-1}\right)},
\tag{20}
$$

where $x := r_+/r_c$ and $0 < x < 1$.

In the case of neglecting the connections between the two horizons, the black hole horizon and the cosmological horizon are two independent thermodynamic systems. Due to the different horizon temperatures, the spacetime cannot
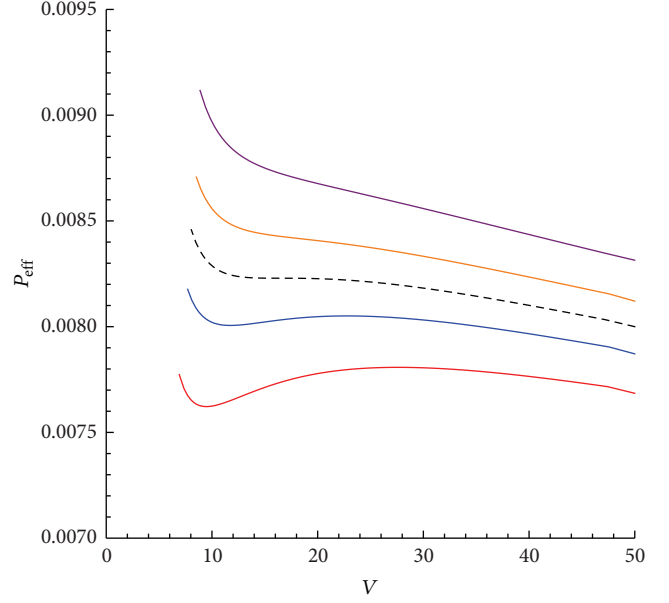

FIGURE 1: P_{eff}-V (here V is the thermodynamic volume) diagram of charged GB-dS black hole with $Q = 1$, $d = 5$, $\widetilde{\alpha} = 0.1$, and $T_c = 0.028628$.

be in thermal equilibrium. After considering the connections between both horizons, from (18), one can see that only one effective temperature T_{eff} left.

4. Phase Transition in GB-dS Black Hole Spacetime

Recently, there are many works on the phase transition of black hole thermodynamic system. One can also compare the critical behaviors of van der Waals liquid-gas system and black hole system. Nevertheless, in dS spacetime, there are the cosmological horizon and the black hole horizon. Generally, equilibrium cannot be achieved unless for extremal black hole. In this section, we employe the effective thermodynamic quantities to study the phase transition and critical behaviors of charged GB-dS black hole. This may avoid the apprehension of equilibrium of the black hole system.

Compared with the van der Waals equation,

$$
\left(P + \frac{a}{v^2}\right)\left(v - \widetilde{b}\right) = kT.
\tag{21}
$$

Here, $v = V/N$ is the specific volume of the fluid, P is its pressure, T is its temperature, and k is the Boltzmann constant. a is a measure for the attraction between the particles, and $\widetilde{b}$ is the average volume excluded from v by a particle. Similar to [53], for the GB-dS black hole, we can set the specific volume as

$$
v = r_c(1-x).
\tag{22}
$$

According to (19), one can employ the two equations

$$
\left(\frac{\partial P_{\mathrm{eff}}}{\partial x}\right)_{T_{\mathrm{eff}}} = 0, \qquad \left(\frac{\partial^2 P_{\mathrm{eff}}}{\partial x^2}\right)_{T_{\mathrm{eff}}} = 0,
\tag{23}
$$

TABLE 1: Numerical solutions for T^c_{eff}, P^c_{eff} with fixed $Q = 1$ and spacetime dimension $d = 5, 6, 7$ and GB parameter $\tilde{\alpha} = 0.1, 0.01, 0.001$, respectively.

| | T^c_{eff} | | | P^c_{eff} | | |
	$d = 5$	$d = 6$	$d = 7$	$d = 5$	$d = 6$	$d = 7$
$\tilde{\alpha} = 0.1$	0.028628	0.040562	0.049202	0.008229	0.018877	0.031920
$\tilde{\alpha} = 0.01$	0.033651	0.045736	0.054251	0.010356	0.021471	0.034484
$\tilde{\alpha} = 0.001$	0.034297	0.046385	0.054889	0.010632	0.021802	0.034836
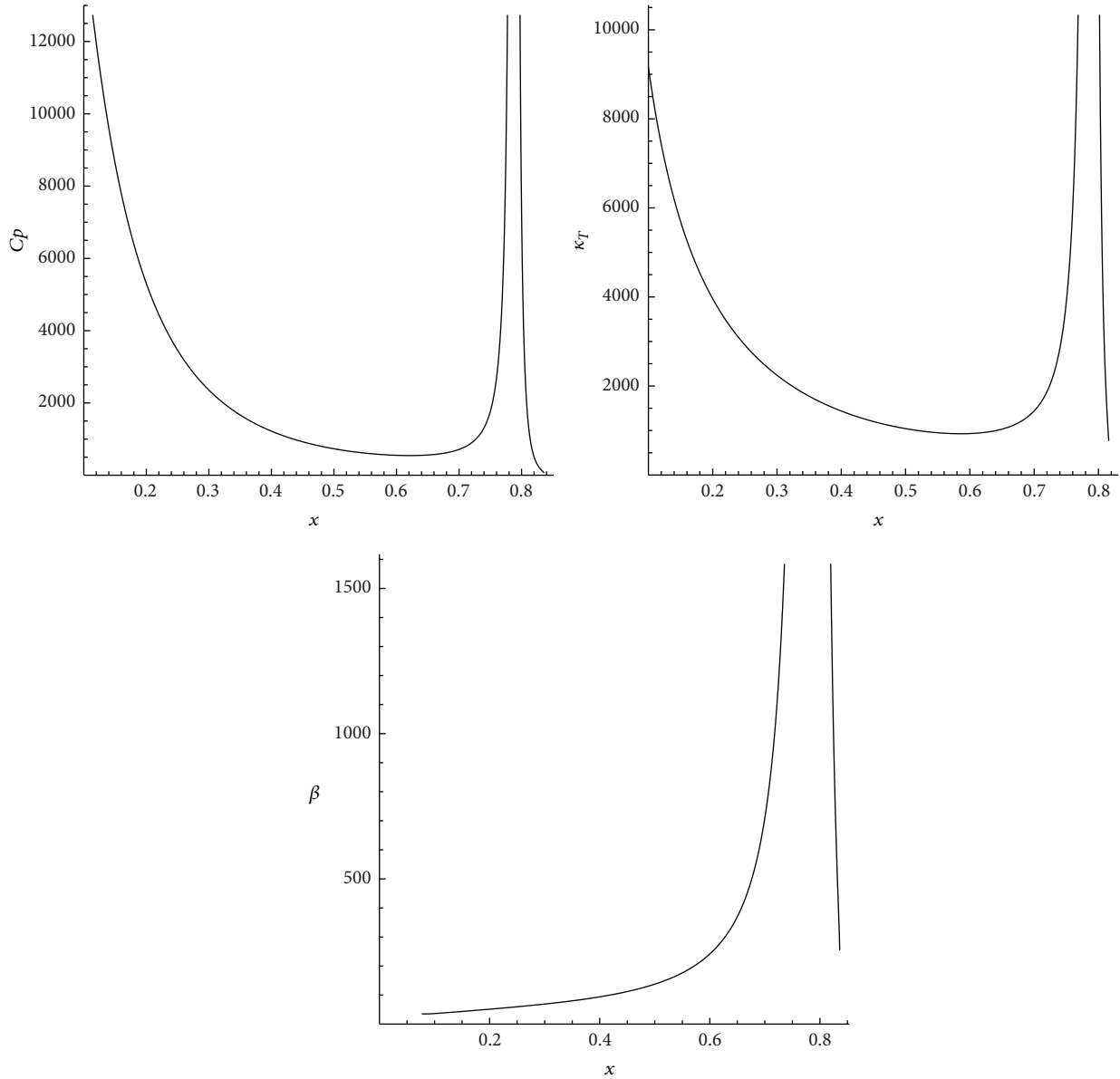
FIGURE 2: C_P-x, κ_T-x and β-x curves for charged GB-dS black hole with $Q = 1$, $d = 5$, and $\tilde{\alpha} = 0.1$.

to calculate the critical temperature, critical pressure, and the critical specific volume. We can ascertain the critical values (r^0_c, x^0) for given Q, d, and $\tilde{\alpha}$. According to these values, we can obtain the critical temperature, critical electric potential, critical pressure, and critical volume. As it is shown in Table 1, the critical temperature and critical pressure increase with

the dimension of spacetime for fixed Q and $\tilde{\alpha}$. However, they decrease with the increase of the GB parameter $\tilde{\alpha}$ for fixed spacetime dimension and electric charge Q.

In order to describe the relation of P_{eff} and V in the vicinity of critical temperature, we plot the curves of P_{eff}-V at different temperatures, as shown in Figure 1. When
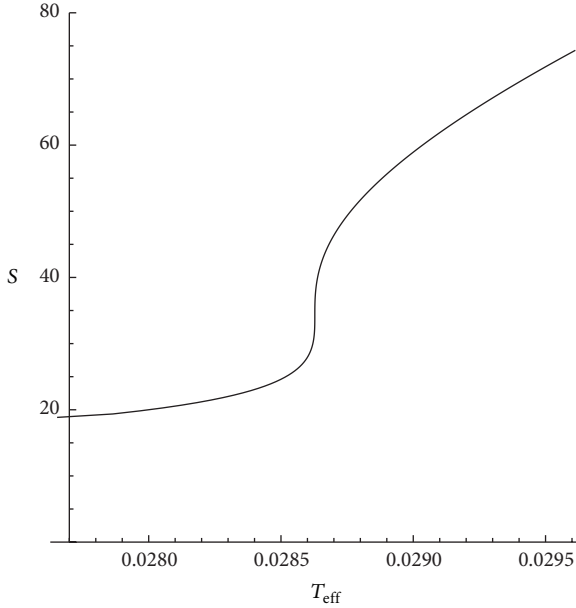
FIGURE 3: S-T_{eff} curves for charged GB-dS black hole with $Q = 1$, $d = 5$, and $\tilde{\alpha} = 0.1$.

the effective temperature $T_{\mathrm{eff}} > T_{\mathrm{eff}}^c$, the stable condition $(\partial P/\partial V)_{T_{\mathrm{eff}}} < 0$ can be satisfied. In the case of $T_{\mathrm{eff}} < T_{\mathrm{eff}}^c$, there exists a portion in the curve, where $(\partial P/\partial V)_{T_{\mathrm{eff}}} > 0$; thus, in these parts, the system is unstable. So, phase transition may occur only at $T_{\mathrm{eff}} = T_{\mathrm{eff}}^c$.

According to Ehrenfest classification, when the chemical potential and its first derivative are continuous, whereas the second derivative of chemical potential is discontinuous, this kind of phase transition is called the second-order phase transition. For van der Waals equation, at the critical point, no latent and sudden change in volume happen between liquid phase and gas phase. According to Ehrenfest classification, this kind of phase transition is continuous.

To understand the types of the phase transition for the GB-dS black hole, we should calculate the specific heat at constant pressure C_P, the volume expansivity β, and the isothermal compressibility κ:

$$C_P = T_{\mathrm{eff}} \left(\frac{\partial S}{\partial T_{\mathrm{eff}}} \right)_{P_{\mathrm{eff}}} = T_{\mathrm{eff}} \left(\frac{\partial S}{\partial x} \frac{\partial x}{\partial T_{\mathrm{eff}}} \right)_{P_{\mathrm{eff}}},$$

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T_{\mathrm{eff}}} \right)_{P_{\mathrm{eff}}} = \frac{1}{v} \left(\frac{\partial v}{\partial x} \frac{\partial x}{\partial T_{\mathrm{eff}}} \right)_{P_{\mathrm{eff}}}, \quad (24)$$

$$\kappa_T = -\frac{1}{v} \left(\frac{\partial v}{\partial P_{\mathrm{eff}}} \right)_{T_{\mathrm{eff}}} = -\frac{1}{v} \left(\frac{\partial v}{\partial x} \frac{\partial x}{\partial P_{\mathrm{eff}}} \right)_{T_{\mathrm{eff}}},$$

where v is the specific volume.

We also depict the curves of C_P-x, κ_T-x and β-x in Figure 2, respectively. From these curves, we find that, for the charged GB-dS black hole, there exists an infinite peak in these curves. Moreover, in Figure 3, one can see that the curves of S-T_{eff} are continuous at the critical point. According to Ehrenfest, the phase transition of the charged GB-dS black

hole should be the second-order one, which is similar to the RN-(A)dS black hole.

5. Discussion and Conclusion

In this paper, we adopted Ehrenfests classification to study the phase transition of the charged GB-dS black hole. After introducing the connection between the thermodynamic quantities corresponding to the black hole horizon and the cosmological horizon, we give the effective thermodynamic quantities, such as effective pressure P_{eff}, effective temperature T_{eff} of the black hole. From the equation of the state described by the effective thermodynamic quantities, we study the P-V criticality. As illustrated in Figure 1, it exhibits a similar phase transition to van der Waals equation.

By treating the effective cosmological constant as the thermodynamic pressure, in the extended phase space, we completely follow the Ehrenfest classification to explore the type of the phase transition of the charged GB-dS black hole. It is shown that the entropy is a continuous function of temperature, however, the heat capacity C_P, the isothermal compressibility κ_T, and the expansion coefficient β are all divergent at the critical point. This means this kind of phase transition for the charged GB-dS black hole is second order.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.
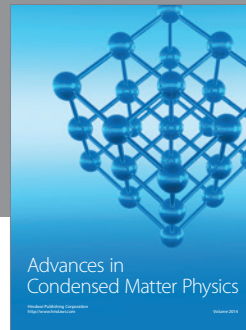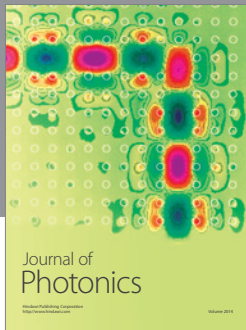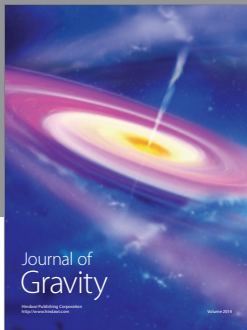
Acknowledgments

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