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Research Article

Generalized Wavelet Fisher's Information of $1/f^{\alpha}$ Signals

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This paper defines the generalized wavelet Fisher information of parameter q. This information measure is obtained by generalizing the time-domain definition of Fisher's information of Furuichi to the wavelet domain and allows to quantify smoothness and correlation, among other signals characteristics. Closed-form expressions of generalized wavelet Fisher information for $1/f^{\alpha}$ signals are determined and a detailed discussion of their properties, characteristics and their relationship with wavelet q-Fisher information are given. Information planes of $1/f$ signals Fisher information are obtained and, based on these, potential applications are highlighted. Finally, generalized wavelet Fisher information is applied to the problem of detecting and locating weak structural breaks in stationary $1/f$ signals, particularly for fractional Gaussian noise series. It is shown that by using a joint Fisher/F-Statistic procedure, significant improvements in time and accuracy are achieved in comparison with the sole application of the F-statistic.

1. Introduction

$1/f^{\alpha}$ signals are used as models of several phenomena in a diversity of fields of science. From physics [1] to computer networking [2] and even in psychology [3], $1/f^{\alpha}$ processes govern the observed characteristics of the phenomena within these fields. The parameter α, which appears in several equations defining these processes, plays an important role since the behaviour and characteristics of the signals is dependent upon the values of α. Consequently, the estimation of α is of importance since the estimated α not only describes the phenomena, but also allows to propose actions in accordance with the relation of α and the process it describes. For instance, in computer networking, a value of $\alpha > 0$ gives rise to increased delays and packet losses in the network. In response, network analysts may increase network resources or modify network node algorithmics with the purpose of reducing the observed delays and losses and enhance network performance. This type of problem is also observed in other fields and, as a consequence, the efficient and robust estimation of α plays a role of major importance [4, 5]. In the literature, several methodologies have been proposed for estimating α but it is recognized that no technique can provide robust and efficient estimates for the variety of conditions observed in measured data [5]. Modern techniques, which may consist of combinations of single techniques, attempt to estimate α under these conditions; however, they have shown limited performance. Current techniques utilizing signal processing techniques [5] and information theory approaches offer new possibilities of obtaining robust and efficient estimates. Information theory techniques have indeed been used to study the nature, structure, and complexity of many physical phenomena [6–9]. Fisher information, for instance, was used in [10] to study electroencephalogram (EEG) signals and later in [11] for characterizing nonlinear systems. Moreover, Fisher information in wavelet domain was used in [12] to detect weak structural breaks in $1/f^{\alpha}$ signals. Tsallis entropies, on the other hand, have been used for studying a variety of signal characteristics [13] and its wavelet counterparts are currently being used from structural damage identification in [14], signal classification in [15], and for detecting structural breaks in the mean in pure-power law (PPL) signals [16] among others [17, 18]. In general, with the use of wavelet based information tools, significant improvements can be achieved in the overall analysis and estimation of $1/f^{\alpha}$ signals. This paper extends the generalized Fisher information proposed by Furuichi in [19, 20] to the wavelet domain and then obtains a closed-form expression of this quantifier for $1/f$ signals. The relationship between generalized wavelet Fisher's information and wavelet q-Fisher information is discussed in detail since both quantifiers measure similar characteristics and are dependent upon q. Furthermore, the construction of information planes for $1/f^{\alpha}$ signals Fisher information allows to identify analysis/estimation applications. Finally, the application of this technique to the problem of detecting and locating structural breaks within $1/f^{\alpha}$ signals is given along with statistical analyses of their results. Since the family of $1/f$ signals is of importance in physics, biology, physiology, medicine, and so forth, the results presented in this paper are relevant for enhancing the analyses and estimation procedures within these fields. The rest of the paper is structured as follows. In Section 2, a brief review of the properties, definitions, types, and open issues regarding $1/f^{\alpha}$ signals is provided. Also, some important results of the wavelet analysis of these signals are also briefly outlined. Section 3, defines the generalized wavelet Fisher's information concept, derives a closed-form expression of this quantifier for the family of $1/f^{\alpha}$ signals, and studies the behaviour of the information planes of $1/f^{\alpha}$ signals's Fisher information. A comparison with the wavelet q-Fisher information is discussed in detail in this section. Section 4 highlights the potential applications of generalized wavelet Fisher's information for the class of $1/f^{\alpha}$ signals and proposes a technique to detect and locate structural breaks in the mean within these processes. Section 5 presents the results of the level-shift detection capabilities of generalized wavelet Fisher's information on synthesized fractional Gaussian noise (fGn) signals and finally; Section 6 draws the conclusions of the paper.

2. Signals with $1/f^{\alpha}$ Behaviour

$1/f^{\alpha}$ signals are ubiquitous in science and engineering and model phenomena as diverse as DNA sequences [21, 22], heart-beat time series [23, 24], mood [25], self-steem [26], and so forth. An important feature of $1/f$ signals is their power-law behaviour of their power spectral density (PSD); that is, as,

$$S(f) \sim c_f |f|^{-\alpha}, \quad f \in (f_a, f_b), \tag{1}$$

where c_f is a constant, $\alpha \in \mathbb{R}$ is the scaling parameter, and f_a, f_b represent the power-law scaling interval [27]. Depending upon α, f_a, f_b, several well-known processes are obtained; for example, when $f_b > f_a \to 0$ and $0 < \alpha < 1$, long-memory signals are obtained. Moreover, $1/f$ signals are self-similar in the sense that their distributional properties are invariant under dilations in time and space. $1/f$ signals may be regarded as stationary if $\alpha < 1$ and nonstationary if $\alpha > 1$ [4, 28]. The well-known fractional Brownian motion (fBm), a Gaussian nonstationary self-similar signal with parameter H whose autocovariance given by

$$\mathbb{E}B_H(t)\,B_H(s) = \frac{\sigma^2}{2}\left\{|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right\} \tag{2}$$

with $H \in (0,1)$, has a spectral density given by

$$S_{\text{fBm}}(f) \sim c\,|f|^{-(2H+1)}, \quad f \longrightarrow 0, \tag{3}$$

and thus it can be regarded as a $1/f^{\alpha}$ signal with parameter $\alpha = 2H + 1$. Fractional Gaussian noise (fGn), the first difference of fBm is a stationary Gaussian self-similar signal with PSD given by [27]

$$S_{\text{fGn}} = 4\sigma_X^2 c_H \sin^2(\pi f) \sum_{j=-\infty}^{\infty} \frac{1}{|f+j|^{2H+1}}, \quad |f| < \frac{1}{2}, \tag{4}$$

and $H \in (0,1)$. As $f \to 0$, $S_{\text{fGn}} \sim c|f|^{-2H+1}$ and therefore is a $1/f$ signal. Many other $1/f^{\alpha}$ signals exist and the interested reader is referred to [4, 27, 28] for further information.

2.1. Wavelet Representations of $1/f^{\alpha}$ Signals. Wavelets and wavelet transforms have been extensively used for the analysis and estimation of $1/f$ signals [8, 9, 29–31]. In this paper, the wavelet spectrum [31] of $1/f$ signals is of interest since it allows constructing distributions from which information measures can be computed. For $1/f$ signals, the wavelet spectrum is computed as

$$\mathbb{E}d_X^2(j,k) = \int_{-\infty}^{\infty} S_X\left(2^{-j} f\right) |\Psi(f)|^2\, df, \tag{5}$$

where $\Psi(f) = \int \psi(t) e^{-j2\pi f t} dt$ is the Fourier integral of the mother wavelet $\psi_o(t)$, $S_X(\cdot)$ is the PSD of the process X_t, $\mathbb{E}$ is the expectation operator, and $d_X(j,k)$ is the discrete wavelet transform of the process X_t at time k and wavelet scale j [29–31]. Using the well-known PSD of $1/f^{\alpha}$ signals, (5) now becomes

$$\mathbb{E}d_X^2(j,k) = C2^{j\alpha}, \tag{6}$$

where C is a constant. For a more detailed discussion of the wavelet analysis and synthesis of scaling signals, please refer to [27–31].

3. Generalized Wavelet Fisher's Information Measure

The first uses of Fisher information were in the context of statistical estimation [32, 33]. Recently, however, Fisher information has been used to characterize nonstationarity in complex processes such as EEGs and other nonlinear signals [10, 11]. Fisher information quantifies the spreading of a probability distribution in the sense that is high for flat densities and low for narrow ones. Fisher information is also sensitive to local discontinuities in the density as reported in [34]. Furthermore, Fisher's information also measures the oscillatory degree of a function and their smoothness character. In this context, Fisher's information is large for smooth signals and small for highly oscillatory ones. In the same way Tsallis q-entropies generalize Shannon entropy, generalizations of Fisher information have been proposed in the literature. Plastino and coworkers defined the q-Fisher information in [6]; later, this measure was extended to the wavelet domain in [7]. Lutwak et al. defined the (p, λ) Fisher information in [35, 36] and later Furuichi defined the generalized Fisher information in [19, 20]. The wavelet domain generalization of Plastino's (wavelet q-Fisher information) was used to detect and locate weak structural breaks in $1/f$ signals using a joint wavelet and Bai & Perron technique [7]. Significant improvements were found specifically for long time series with LRD. This paper presents a wavelet-domain extension to the time-domain generalization of Fisher's information of Furuichi [19]. To study in more detail the properties of generalized Fisher's information for $1/f$ signals, a closed-form expression is first obtained. In addition, an in-depth discussion of their relationship with the wavelet q-Fisher information is given. The generalized Fisher information of a probability density $f(x)$ is computed as [19, 20]

$$\mathcal{I}_q(x) = \mathbb{E}_q\left\{s_q(X)^2\right\}, \tag{7}$$

where $\mathbb{E}_q$ is the q-expectation operator defined as $\mathbb{E}_q[g(X)] \equiv \int_{-\infty}^{+\infty} f(x)^q g(x)dx$ and $s_q(x)$ is the q-score function which is defined as the derivative of the q-logarithm of a density; that is, as

$$s_q(x) \equiv \frac{d\ln_q f(x)}{dx}, \tag{8}$$

where $\ln_q(x) \equiv (x^{1-q} - 1)/(1 - q)$ is the q-logarithm function. For discrete distributions, p_j, the generalized Fisher information is computed using a discrete-time version of (7), which is given by

$$\mathcal{I}_q = \sum_j \left\{p_j^q \left\{\ln_q\left(p_{j+1}\right) - \ln_q\left(p_j\right)\right\}^2\right\}, \tag{9}$$

for some $q \in \mathbb{R}$ and $\sum_j p_j = 1$. When p_j is substituted by the relative wavelet energy (RWE), (9) results in the generalized wavelet Fisher information of parameter q which measures the spreading of a distribution, the oscillatory nature of signals, among others. Unlike standard wavelet Fisher information measure, generalized wavelet Fisher information provides increased flexibility (with q) and the possibility to adapt the analyses to the characteristics of the data under study.

3.1. Generalized Wavelet Fisher's Information of $1/f^\alpha$ *Signals.* Generalized wavelet Fisher's information is obtained by substituting the RWE in (9). Recall that the RWE of $1/f^\alpha$ signals is obtained by means of the wavelet spectrum or wavelet variance of these signals. For stationary $1/f$ signals, the wavelet spectrum is $\mathbb{E}d_X^2(j,k) \sim 2^{j\alpha}C(\psi,\alpha)$, where $C(\psi,\alpha)$ is a constant depending on the mother wavelet and α. The RWE of $1/f^\alpha$ signals is thus given by [7–9, 15, 16],

$$\text{RWE}_j = 2^{(j-1)\alpha}\frac{1-2^\alpha}{1-2^{\alpha M}}, \tag{10}$$

where M and j represent the (logarithmic) length of the signal and the wavelet scale, respectively. The generalized wavelet Fisher information of parameter q is, thus,

$$\begin{aligned}\mathcal{I}_q &= \left\{\frac{2^{\alpha(1-q)}-1}{1-q}\right\}^2 \left\{\frac{1-2^\alpha}{1-2^{\alpha M}}\right\}^{2-q} \left\{\frac{1-2^{\alpha(2-q)(M-1)}}{1-2^{\alpha(2-q)}}\right\} \\ &= \left(\ln_q\left\{2^\alpha\right\}\right)^2 \left\{\frac{1-2^\alpha}{1-2^{\alpha M}}\right\}^{2-q} \left\{\frac{1-2^{\alpha(2-q)(M-1)}}{1-2^{\alpha(2-q)}}\right\}.\end{aligned} \tag{11}$$

Equations (11) are employed for the construction of information planes which permit to study in more detail the characteristics and behaviour of Fisher information for $1/f$ signals. Figure 1 displays $1/f^\alpha$ signals Fisher information for $\alpha \in (-2, 4)$, $M \in (8, 16)$, and negative q. It is interesting to note that under these conditions, the Fisher information planes display two different states. First state corresponds to Fisher information values of zero ($-\infty < \alpha < \alpha_L$) and the second to nonzero exponentially increasing Fisher information values ($\alpha > \alpha_L$). The exponential increase of Fisher information starts in α_L and it is entirely controlled by q. Observe that by setting $\alpha_L = 1$, the two states lie in the intervals $(-\infty, 1)$ and $(1, \infty)$ and a maping of zero Fisher information values to stationary signals and of nonzero values to nonstationary signals is made. In the same way, by setting $\alpha_L = 3$, a maping of zero Fisher values to nonstationary fBms is made and a maping of nonzero exponentially increasing Fisher values to extended fBms is performed. The case $\alpha_L = 1$ allows to distinguish among the stationary and nonstationary families of $1/f$ signals while the case $\alpha_L = 3$ permits distinguishing among nonstationary ones. Parameter q controls the boundary α_L; decreasing q further shifts the value of α_L to the right. Figure 2 displays $1/f^\alpha$ signals Fisher information for $q > 0$. As in Figure 1, the planes of Figure 2 were obtained for $M \in (8, 16)$ and $\alpha \in (-2, 4)$. For the particular case $q \in (0, 1)$, the Fisher information are exponentially decreasing in the interval $\alpha \in (-\infty, 0)$ and converging to zero for $\alpha > 0$. As q increases, the information planes converge to zero more quickly. Notice that, for $q \gg 1$, a two-state behaviour is observed as well. In contrast to the $q < 0$ case, the $q \gg 1$ displays an exponentially decreasing behaviour for the first state and zero values for the second one. Similar applications as those for negative q can also be attached to the $q \gg 1$ case. Generalized wavelet Fisher information therefore allows characterizing the complexities of $1/f^\alpha$ signals. Fisher information for $q < 0$ ($q \gg 1$) are zero (exponentially decreasing) for random $1/f$ signals and exponentially increasing (converging to zero) for smooth correlated signals. The length of the intervals on which the Fisher information is zero (exponentially decreasing) and exponentially increasing (converging to zero) is controlled by the value of parameter q. Because of the similarities in

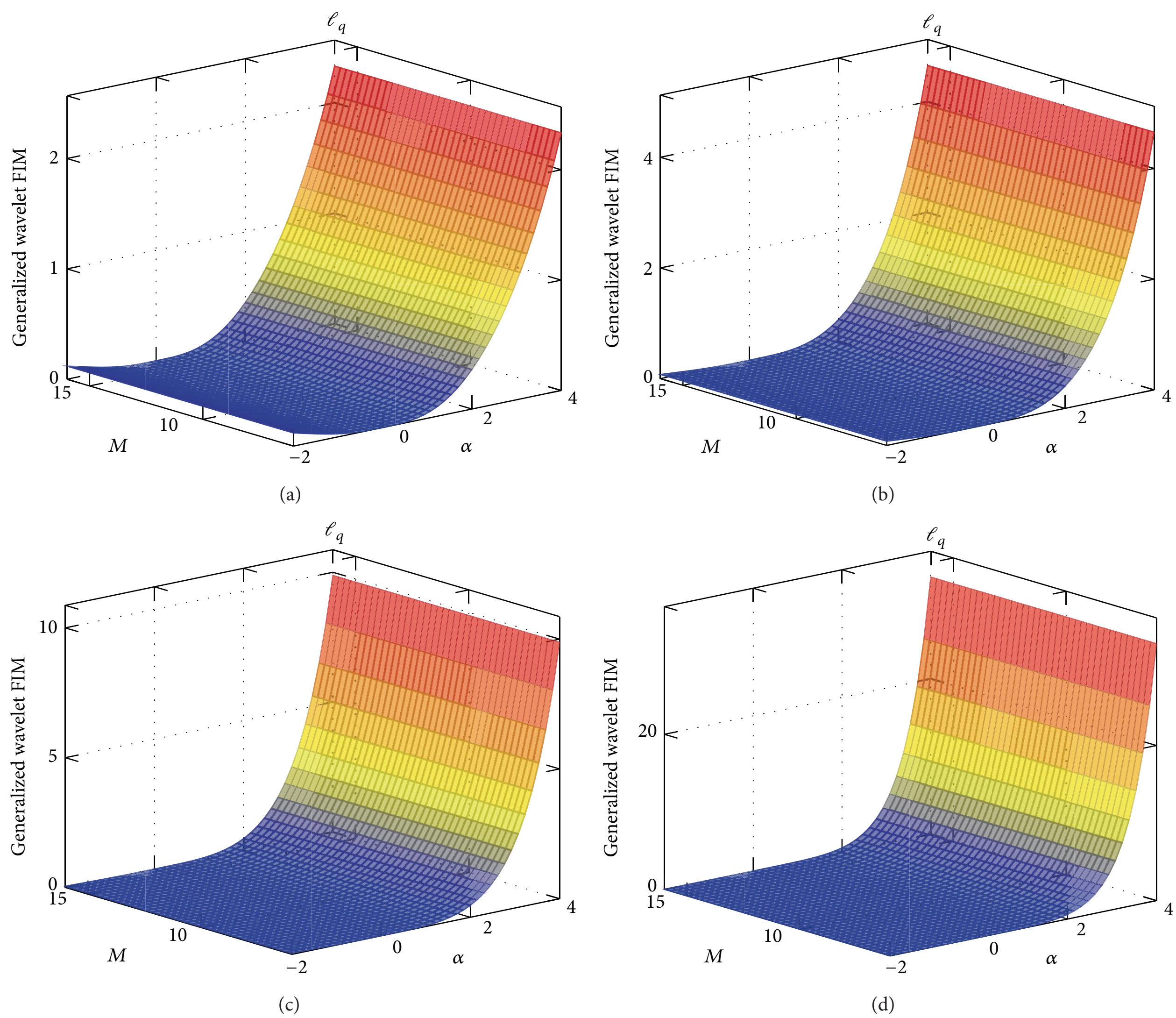

FIGURE 1: Generalized wavelet Fisher's information for $1/f^{\alpha}$ signals. Top left with $q = -0.8$, top right with $q = -1.2$, bottom plot with $q = -1.6$, and bottom right with parameter $q = -2.2$.

the information planes with those of the wavelet q-Fisher information, an interesting question is how generalized wavelet Fisher information is related with wavelet q-Fisher information. In the following, we elaborate further on this question and discuss the similarities and/or differences between both information measures in more detail.

3.2. Relation with Wavelet q-Fisher Information. This section explores the relationship that exists between generalized wavelet Fisher's information and wavelet q-Fisher information. In principle, both quantifiers can be used to measure smoothness, complexity, and correlation within a signal or system [7]. For $q \to 1$, it is straightforward to derive that generalized wavelet Fisher information is given by the relation

$$\mathcal{J}_1 = \left(\ln 2^{\alpha}\right)^2 \left\{\frac{1 - 2^{\alpha(M-1)}}{1 - 2^{\alpha M}}\right\}. \tag{12}$$

On the other hand, wavelet q-Fisher's information (for $q \to 1$) is given by the following equation [7, 12]:

$$\mathcal{I} = \left(2^{\alpha} - 1\right)^2 \left\{\frac{1 - 2^{\alpha(M-1)}}{1 - 2^{\alpha M}}\right\}. \tag{13}$$

Observe the similarity that exists between (12) and (13). Notice that both equations differ only in the term that multiplies expression $(1-2^{\alpha(M-1)})/(1-2^{\alpha M})$. The exact relationship between these two quantifiers is obtained by observing the fact that, for $\alpha \in \mathbb{R}$ and $2^{\alpha} > 0$,

$$\ln\left(2^{\alpha}\right) \leq \left(2^{\alpha} - 1\right). \tag{14}$$

On the other hand, for $\alpha < 0$, $\ln(2^{\alpha})$ is negative, which gives rise to

$$\ln\left(2^{\alpha}\right)^2 > \left(2^{\alpha} - 1\right)^2. \tag{15}$$

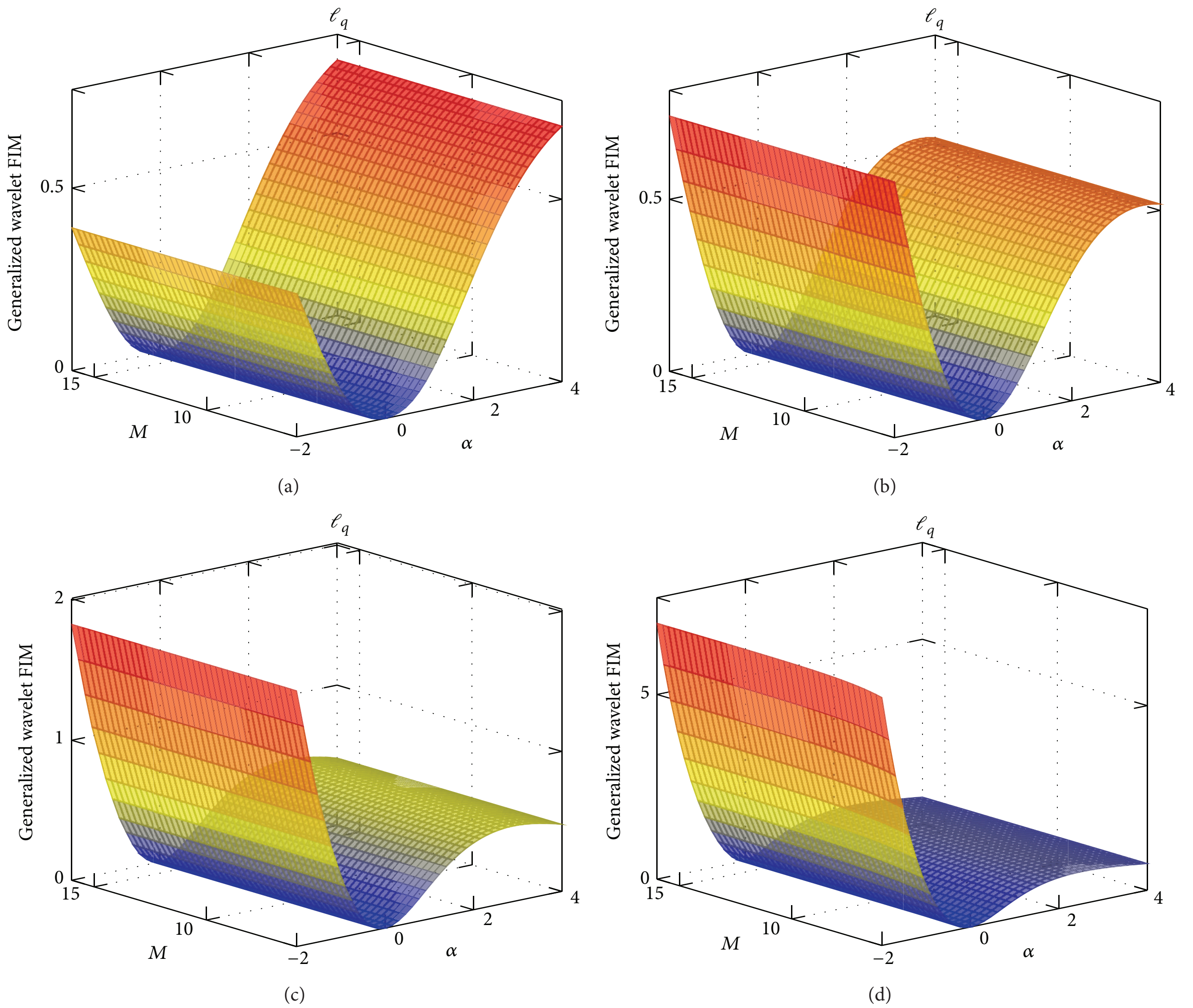

FIGURE 2: Generalized wavelet Fisher's information for $1/f^{\alpha}$ signals. Top left with $q = 0.1$, top right with $q = 0.5$, bottom plot with $q = 0.99$, and bottom right with parameter $q = 1.5$.

The above results allow to establish the following exact relationship between these two quantifiers; that is,

$$\begin{aligned} \mathcal{J}_1 &> \mathcal{J} \quad \text{for } \alpha < 0, \\ \mathcal{J}_1 &< \mathcal{J} \quad \text{for } \alpha > 0, \\ \mathcal{J}_1 &= \mathcal{J} \quad \text{for } \alpha = 0. \end{aligned} \tag{16}$$

Figure 3 displays this mathematical relationship between these two quantifiers. Notice that both Fisher information intersect at $\alpha = 0$ which means that, for $q \rightarrow 1$, they are equal only when they analyze random white noise. For the rest of Fisher information values, they seem to have opposite behaviour. Standard wavelet Fisher information is higher than Furuichi's wavelet Fisher information for $\alpha > 0$ but it is less than when $\alpha < 0$. As can be noted in Figure 3, the analysis of $1/f^{\alpha}$ signals with standard versions of generalized Fisher information is limited and thus the attention is now turned to the case $q \neq 1$. The relationship between these two quantifiers for the case $q \neq 1$ is obtained by first recalling that, for $q \in \mathbb{R}$, the following holds [20]:

$$\ln_q(x) \leq 1. \tag{17}$$

Based on this, it is observed that the following relationship is satisfied:

$$\begin{aligned} \ln_q(2^{\alpha}) &> 2^{\alpha} - 1 \quad \text{for } q < 0 \\ \ln_q(2^{\alpha}) &\leq 2^{\alpha} - 1 \quad \text{for } q > 0. \end{aligned} \tag{18}$$

Thus, for $q < 0$, $\ln_q^2(2^{\alpha}) < (2^{\alpha} - 1)^2$ for the case $\alpha < 0$ and for the case $\alpha > 0$, $\ln_q^2(2^{\alpha}) > (2^{\alpha} - 1)^2$. For nonnegative q, $\ln_q^2(2^{\alpha}) > (2^{\alpha} - 1)^2$ holds for $\alpha < 0$ and for the case $\alpha > 0$, $\ln_q^2(2^{\alpha}) < (2^{\alpha} - 1)^2$. Although some differences are observed between these two quantifiers, both Fisher information are

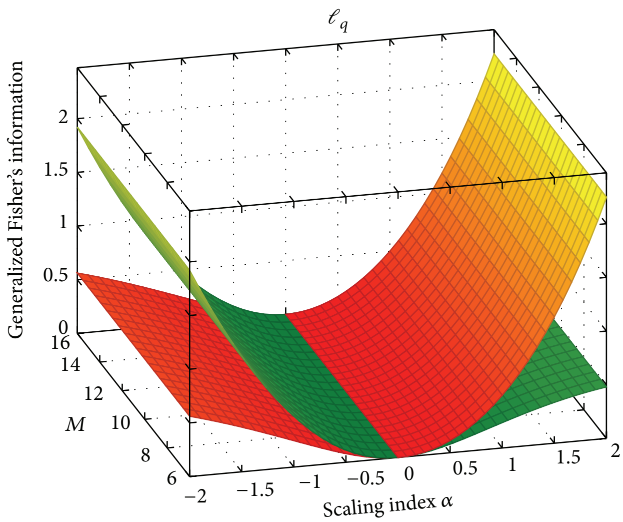

Figure 3: Comparison of standard wavelet Fisher's information (red-yellow plane) with generalized wavelet Fisher's information for $q \to 1$ (green plane).

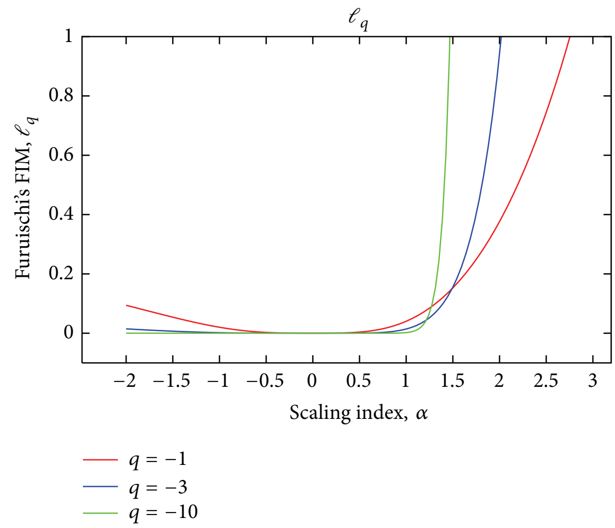

Figure 4: Generalized wavelet Fisher's information for several values of the parameter q (q negative). As q becomes more negative, the exponential increase for the nonstationary signals is emphasized.

equal at $\alpha = 0$ and some similar behaviour are observed for different values of the parameter q. The above results allow us to obtain an exact relationship between generalized wavelet Fisher information of parameter q and wavelet q-Fisher information in the following way:

$$\mathcal{J}_q \le 2^{\alpha(1-q/2)+2} \left\{\sinh^2_{1-\nu_1}(u_2)\right\} \left\{\frac{\sinh^q_{1-\nu_2/(M-1)}(u_2)}{\sinh_{1-\nu_1}(u_1)}\right\} \times \left\{\frac{\sinh_{1-\nu_1}(u_1)}{\sinh^q_{1-\nu_2}(u_2)}\right\} \left\{\frac{P_{\text{num}}}{P_{\text{den}}}\right\} = \mathcal{I}_q, \tag{19}$$

where P_{num} and P_{den} are given by the following polynomial expressions:

$$\begin{aligned} P_{\text{num}} &= 2\cosh_{1-\nu_1/(M-2)}(u_1(M-2)) \\ &\quad + 2\cosh_{1-\nu_1/(M-4)}(u_1(M-4)) \\ &\quad + 2\cosh_{1-\nu_1/(M-6)}(u_1(M-6)) + \cdots, \\ P_{\text{den}} &= 2\cosh_{1-\nu_2/(M-1)}(u_2(M-1)) \\ &\quad + 2\cosh_{1-\nu_2/(M-3)}(u_2(M-3)) \\ &\quad + 2\cosh_{1-\nu_2/(M-5)}(u_2(M-5)) + \cdots \end{aligned} \tag{20}$$

with $u_1 = \alpha q \ln_q(2)/2$, $u_2 = qu_1$, $\nu_1 = 2(1-q)/(\alpha q)$, and $\nu_2 = q\nu_1$ [7].

4. Applications

Figure 4 displays the behaviour of $1/f^{\alpha}$ signals Fisher information for $-2 < \alpha < 3$ and values of $q = \{-1, -3, -10\}$. Note that as q decreases, the exponential increase of Fisher information is higher. This behaviour can be used to apply generalized wavelet Fisher information to a variety of phenomena. First, generalized Fisher information can be used to detect nonstationary behaviour within stationary signals. In addition, it can also be used to extend the analyses of standard Fisher information in EEG and nonlinear signals in [10, 11]. Another application can also be in the classification of scaling signals as stationary or nonstationary. The classification as stationary or nonstationary has been recognized as fundamental for $1/f$ signal analysis and estimation. Other application area is on the detection and location of weak structural breaks within $1/f$ signals. In this context, weak level-shift detection and location can be mapped to the problem of detecting the transition from zero to exponentially increasing Fisher information. As a matter of fact, as would be explained in the next section, level-shift detection in the context of generalized wavelet Fisher information is simply the problem of detecting and locating impulses which indicate the presence and location of level-shifts. Next section focuses on the detection and location of weak level-shifts embedded within fGn signals, a particular class of $1/f$ signals.

5. Detection of Level-Shifts with Fisher Information

The detection of weak level-shifts embedded in random signals is of importance in many disciplines of science since they indicate the presence of some irregularity in the phenomena. Level-shifts can indicate the presence of a change in an economical policy [37], the presence of a malignant tumor in a magnetic resonace image (MRI) signal [38], and so forth. Although important and widely studied, level-shift detection is still complex and time-consuming, specially for highly correlated waveforms [39]. In the following, the results of the joint application of the generalized wavelet Fisher information and the F-statistic for the detection and location of weak level-shifts are discussed. The wavelet Fisher information is used as a preprocessing technique and the F-statistic as a location and detection methodology. The purpose of using this joint technique is for enhancing the detection/location and to reduce the time required to get an estimate as well. The detection/location procedure is started by computing the generalized wavelet Fisher information in sliding windows of length W and sliding factor Δ. With this, the time evolution of generalized wavelet Fisher information, denoted by $\mathcal{J}_m$, is obtained by applying (9) to subsets of the data of the form:

$$X(m; W, \Delta) = X(t_k)\,\Pi\left(\frac{t - m\Delta}{W} - \frac{1}{2}\right), \qquad (21)$$

for various m, where $m = 0, 1, 2, 3, \ldots, m_{\max}$ and $\Pi(\cdot)$ is the well-known rectangular function. A plot of $W + m\Delta$ versus $\mathcal{J}_m$ for $m = 0, 1, 2, 3, \ldots, m_{\max}$ represents the time evolution of generalized wavelet Fisher information. The work of Stoev et al. [31] demonstrated that a sudden jump in the structure of a self-similar fGn signal caused its theoretical Hurst parameter in the wavelet domain to be $H > 1$. In view of this, the jump will cause the Fisher information of the sudden jump to change from zero Fisher information to highly increasing Fisher information; therefore, the time evolution plot of the generalized Fisher information will show the presence of a weak level-shift in the form of an impulse-like signal. Therefore, an impulse shaped function in the generalized wavelet Fisher information indicates the presence of a level-shift in the signal under study. It is important to note that by the use of the generalized wavelet Fisher information the objective is twofold. First is to highlight further the detection of the weak level-shift and secondly to decrease the time required for obtaining a detection and location estimate. In order to achieve a detection and location estimate, the F-statistic is used.

5.1. Level-Shift Detection in fGn Signals. Figure 5 shows the generalized wavelet Fisher information of a fGn signal with Hurst parameter $H = 0.6$ and a single level-shift at time $t = 4096$, that is, located in the middle of the signal. The level shift is small and its magnitude is of $\sqrt{\sigma^2}/2$, where σ^2 is the signal variance. Generalized Fisher information in the wavelet domain is computed with $q = -1$ using windows of $W = 512$ and $\Delta = 256$. Note that the resulting time-evolution of generalized wavelet Fisher information displays an impulse-shaped form at $t \approx 4096$ indicating that a level-shift is located around that point. Increasing the window length W increases accuracy and long windows are preferred whenever $H \rightarrow 1$. Decreasing q further increases the Fisher information values and as a consequence increases the detection/location process maintaining the form of the graph, that is, as an impulse shaped form. The generalized wavelet Fisher information, therefore, enhances the detection procedure and outputs a small length signal. Note that level-shift detection can even be done by eye. In order to get a better location and detection, the F-statistic is used. Using the F-statistic, the resulting location estimate is 4096, that is, the exact place where the level-shift is located.

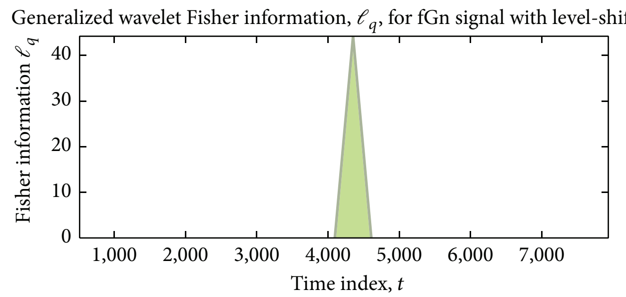

Figure 5: Level shift detection of generalized Wavelet Fisher information, $\mathcal{J}_q$. The fGn signal is of length $N = 8192$, $H = 0.6$, and is analyzed in windows of $W = 512$, $\Delta = 256$. Note that the level shift located in $\tau_{\text{ls}} = 4096$ is captured by the impulse of Fisher information.

5.2. Results for Fractional Gaussian Noise Signals. Figure 6 displays the results of the detection and location estimates of a single level-shift within fGn signals of parameter H using the F-statistic alone and various versions of the joint Fisher/F-statistic technique. The level-shifts were set at $t = 4096$ and for each value of H, 50 fGn signals with a single level-shift were generated. Top left plot shows the results for the F-statistic. Note that small variations in the estimates are observed for $H \leq 0.6$ while for $H > 0.6$ the variations increase correlation in the $1/f^{\alpha}$ signal. Top right graph of Figure 6 displays the joint application of generalized wavelet Fisher information as a preprocessing tool and the F-statistic as a detection/location technique. It is observed that no variantion is observed when $H < 0.6$ and when $H > 0.6$ the variation is constant although the average estimates are 4096. The variance is higher than those observed for the F-statistic for $H = 0.7$ and $H = 0.8$. This same behaviour is observed for the case $q = -20$ and $W = 1024$ (bottom left plot) and it is only when $W = 2048$ that the joint technique outperforms the results of the F-statistic.

6. Conclusions

In this paper, generalized wavelet Fisher information of parameter q was defined as a wavelet-domain extension of the Furuichi's generalized Fisher information. Closed-form formulas for the generalized wavelet Fisher information were obtained for the class of $1/f^{\alpha}$ signals and those expressions allowed to construct $1/f$ signal Fisher information planes and compare their properties with those of wavelet q-Fisher information. $1/f$ signal Fisher information planes describe the characteristics and properties of generalized wavelet Fisher information for $1/f$ signals and permitted to propose several applications for the analysis and estimation of these signals. An application of particular interest was the detection and location of weak level-shifts within fGn signals and it was demonstrated, using a detailed statistical study, that generalized wavelet Fisher information in conjunction with the F-statistic detects and locates weak level shifts in these

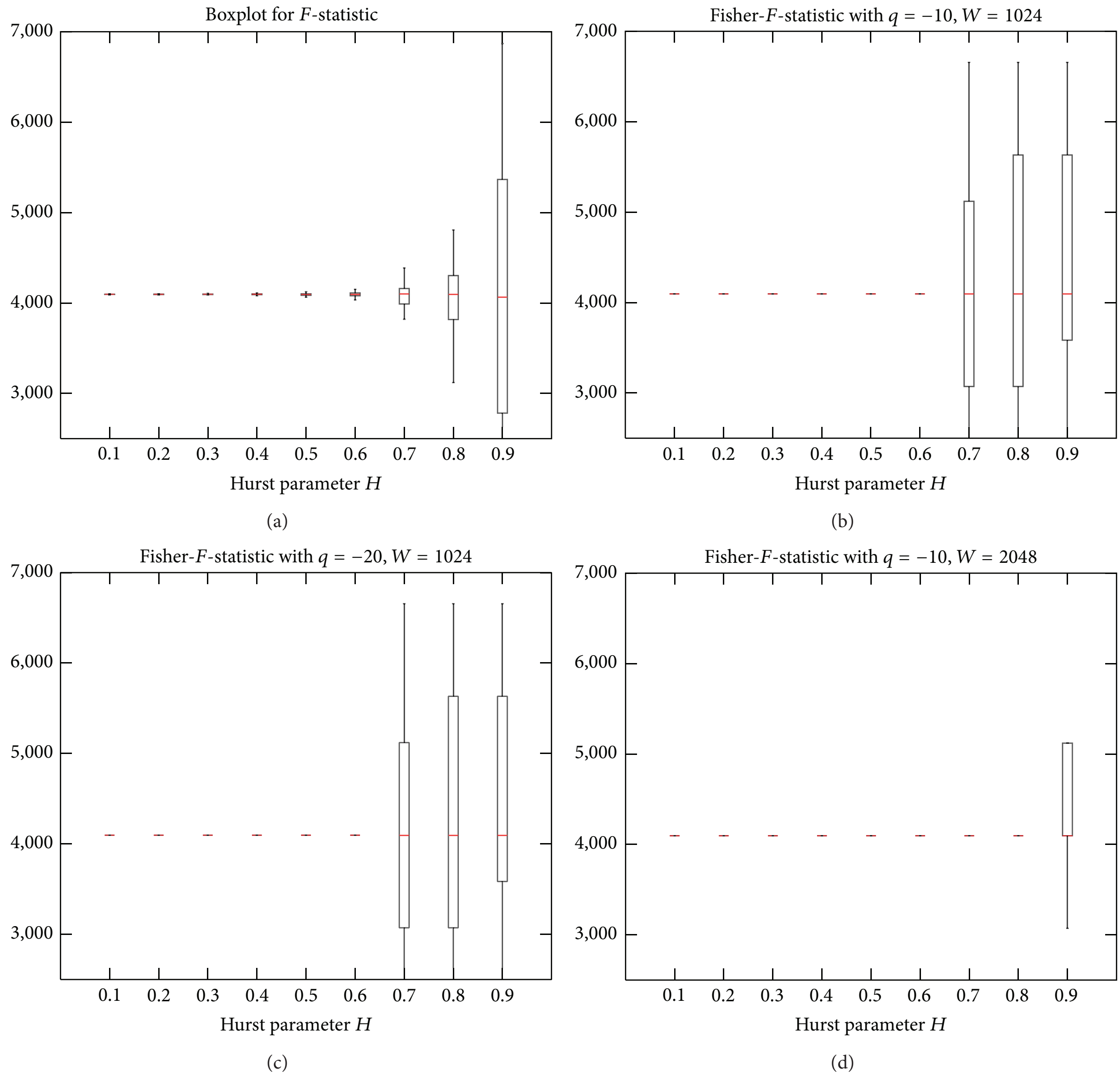

Figure 6: Boxplots that represent the detection capabilities of the F-statistic (top left plot) and several version of the joint Fisher/F-statistic procedure for fractional Gaussian noise signals. Top right plot displays the results for the joint technique using $W = 1024$ and $q = -10$, bottom left plot shows the results for the joint technique with $W = 1024$ and $q = -20$ and finally bottom right plot presents the results for the joint technique with $W = 2048$ and $q = -10$. The level shift was located at $t = 4096$.

signals. By appropiately selecting the value of q and the length W of the anlalysing window, the proposed technique can outperform the results obtained by using the F-statistic alone.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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