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Research Article

Continuous Finite-Time Terminal Sliding Mode IDA-PBC Design for PMSM with the Port-Controlled Hamiltonian Model

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Finite-time control scheme for speed regulation of permanent magnet synchronous motor (PMSM) is investigated under the port-controlled Hamiltonian (PCH), terminal sliding mode (TSM), and fast TSM stabilization theories. The desired equilibrium is assigned to the PCH structure model of PMSM by maximum torque per ampere (MTPA) principle, and the desired Hamiltonian function of state error is constructed in the form of fractional power structure as TSM and fast TSM, respectively. Finite-time TSM and fast TSM controllers are designed via interconnection and damping assignment passivity-based control (IDA-PBC) methodology, respectively, and the finite-time stability of the desired equilibrium point is also achieved under the PCH framework. Simulation results validate the improved performance of the presented scheme.

1. Introduction

Recently, permanent magnet synchronous motors (PMSM) have become increasingly popular in high-performance AC drive applications because of their advantages over many other kinds of motors, such as induction motors and DC motors [1]. These high dynamical performances include high power density, torque-to-inertia ratio, and efficiency. However, the speed regulation of PMSM is a nonlinear control problem because of the strong coupling between the motor speed and the electrical quantities. Therefore, it is a challenging task to design a controller with high-performance of speed regulation.

Various nonlinear control theories have been investigated for the speed control of PMSM, such as sliding mode control [2], backstepping control [3], and predictive control [4]. Compared to these nonlinear controls, the energy-shaping control can be implemented with a clear physical interpretation when a dynamical system is viewed as an energy transformation mechanism [5–8], for example, the port-controlled Hamiltonian (PCH) system [9]. A significant case of PCH is the port-controlled Hamiltonian system with dissipation (PCHD) expressed by its interconnection and damping structure with Hamiltonian energy function [10]. These explicit structural pieces of information naturally lead to the interconnection and damping assignment passivity-based control (IDA-PBC) framework [11–13]. The essence of IDA-PBC approach is to find a controller and an interconnection pattern such that the closed-loop system is preserved in a desired PCHD structure. The procedure sets up an easy way of stability analysis with the desired Hamiltonian energy function as the Lyapunov function. The speed regulation of PMSM has been investigated with IDA-PBC [14–16].

All these approaches share the common property of asymptotic stability with infinite convergent time. In contrast, the finite-time controller possesses not only fast convergence to the equilibrium states in finite-time but also stronger robustness and disturbance attenuation properties [17]. However, the finite-time control design is a challenging problem because such a controller leads to a non-Lipschitzian dynamics [18]. Finite-time control can be obtained in either a discontinuous or continuous manner. In practical implementations, the infinite fast switching of discontinuous control can lead to chattering behavior which may damage the actuators and excite unmodeled high-frequency dynamics. Continuous finite-time control can be obtained by fractional power feedback as the formalism of terminal sliding mode (TSM), resulting in a finite-time convergent closed-loop differential equation with fractional power. Higher order sliding mode control is also a kind of continuous finite-time control through using the discontinuous control in the higher order derivative of the control instead of the actual control. Furthermore, the speed regulation of PMSM has been investigated with the continuous finite-time controls [19, 20].

Based on a finite-time stability criterion and the energy-shaping plus damping injection technique, the finite-time stabilization problem is investigated for the PCH systems [21, 22]. According to the PCH structure model of PMSM, a kind of finite-time TSM feedback realization is proposed to achieve the rapid speed regulation of PMSM in this paper. The desired equilibrium is assigned by maximum torque per ampere (MTPA) principle, and the desired Hamiltonian function of state error is shaped in the form of fractional power structure. Finite-time TSM control is designed via IDA-PBC methodology.

The remaining part of the paper is organized as follows. Some background on the PCHD formalism and finite-time TSM stabilization principle is firstly introduced in Section 2. The problem is formulated in Section 3. These techniques are then applied to the speed regulation problem of PMSM in the form of PCH model, and the finite-time stability of equilibrium point is also given in Section 4. The closed-loop performance is evaluated via simulations in Section 5. Finally, some conclusions are presented in Section 6.

2. Preliminaries

2.1. Port-Controlled Hamiltonian with Dissipation (PCHD) Systems. PCHD system is a geometrically defined class of systems with an internal interconnection structure, a Hamiltonian function defined as the total stored energy, and a resistive structure. Energy dissipation is included in the framework of PCH systems by terminating some of the ports by resistive elements. PCHD systems are a class of passive systems, which have attracted the attention of many researchers lately, in particular for stabilization objectives. A PCHD system is defined as

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x) u, \\ y &= g^T(x) \frac{\partial H}{\partial x}(x), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u, y \in R^m$ representing the input and output vector, respectively, are conjugated variables whose product represents the system environment power exchange, $H(x) : R^n \rightarrow R$ is the stored energy function, $J(x) = -J^T(x) : R^n \rightarrow R^{n\times n}$ captures the power-conserving interconnection structure, $R(x) = R^T(x) \geq 0 : R^n \rightarrow R^{n\times n}$ is the dissipation matrix, and $g(x) : R^n \rightarrow R^{n\times m}$ is the port matrix of the input u on the system and also the output from the system via its transpose. In this case, the energy-balancing property takes the following form:

$$\frac{dH}{dt}(x) = u^T(t) y(t) - \frac{\partial^T H}{\partial x}(x) R(x) \frac{\partial H}{\partial x}(x) \leq u^T(t) y(t), \tag{2}$$

showing passivity if the Hamiltonian H is bounded from below.

2.2. IDA-PBC Principle of PCHD Systems. The interconnection structural properties of PCHD systems can be exploited for the control design. While preserving the PCHD form, IDA-PBC technique can assign the desired interconnection matrix J_d and dissipative matrix R_d of the closed-loop and shape a new closed-loop Hamiltonian energy function H_d with a stable desired equilibrium point. Consider the following:

$$\dot{x} = [J_d(x) - R_d(x)] \frac{\partial H_d}{\partial x}(x). \tag{3}$$

Proposition 1. *For system* (1), *given a desired equilibrium* x^*, *define the stabilization error as* $\tilde{x} = x - x^*$ *and assign a closed-loop energy function* $H_d > 0$ *and* $H_d = 0$; *the purpose of IDA-PBC is to find the new structure matrices* J_a, *damping matrix* R_a, *a vector function* $K(x)$, *and a feedback control* $u = \beta(x)$ *such that*

$$\begin{aligned} &[J(x) + J_a(x) - (R(x) - R_a(x))] K(x) \\ &\quad = -[J_a(x) - R_a(x)] \frac{\partial H}{\partial x}(x) + g(x)\beta(x), \end{aligned} \tag{4}$$

with the following properties.

(i) *Structure preservation:*

$$\begin{aligned} J_d(x) &= J(x) + J_a(x) = -J_d^T(x), \\ R_d(x) &= R(x) + R_a(x) = R_d^T(x) > 0. \end{aligned} \tag{5}$$

(ii) *Integrability:* $K(x)$ *is the gradient of a scalar function. That is,*

$$\frac{\partial K}{\partial x}(x) = \frac{\partial^T K}{\partial x}(x). \tag{6}$$

(iii) *Equilibrium assignment: at the equilibrium* x^*, $K(x)$ *satisfies*

$$K(x^*) = -\frac{\partial H}{\partial x}(x^*). \tag{7}$$

(iv) *Lyapunov stability: at the equilibrium* x^*, *the Jacobian of* $K(x)$ *satisfies the following bound:*

$$\frac{\partial K}{\partial x}(x^*) > -\frac{\partial^2 H}{\partial x^2}(x^*). \tag{8}$$

Under these conditions, the closed-loop system will be a PCHD system with dissipation of the form (3) *where*

$$H_d(x) = H(x) + H_a(x), \quad \frac{\partial H_a}{\partial x}(x) = K(x); \tag{9}$$

the desired equilibrium x^* is asymptotically stable.

2.3. Finite-Time Stability of TSM. The TSM and fast TSM concepts are based on a class of nonlinear differential equations described by the following first order dynamics, respectively:

$$\begin{aligned} s &= \dot{x} + \beta x^{\gamma} = 0, \quad \beta > 0,\ 0 < \gamma < 1, \\ s &= \dot{x} + \alpha x + \beta x^{\gamma} = 0, \quad \alpha, \beta > 0,\ 0 < \gamma < 1. \end{aligned} \tag{10}$$

Lemma 2. *The equilibrium point $x = 0$ of the continuous non-Lipschitz differential equations* (10) *is globally finite-time stable; that is, for any initial condition $x(0) = x_0$, the system state converges to $x = 0$ in finite-time:*

$$T(x_0) = \frac{1}{\beta(1-\gamma)}|x_0|^{1-\gamma}, \tag{11}$$

$$T(x_0) = \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha x_0^{1-\gamma} + \beta}{\beta}, \tag{12}$$

respectively, and stay there forever, that is, $x = 0$ for $t > T(x_0)$.

A Lyapunov-type theorem has been developed for finite-time stability [17, 18].

Lemma 3. *Consider the nonlinear system described in* (1)*; suppose that there is a C^1 function $V(x)$ defined in a neighborhood $D \subset R^n$ of the origin, such that $V(x) > 0$ on D and*

$$\begin{aligned} \dot{V}(x) + \beta V^{\gamma}(x) &\le 0, \\ \dot{V}(x) + \alpha V(x) + \beta V^{\gamma}(x) &\le 0 \end{aligned} \tag{13}$$

along the trajectory on D. Then, the origin of the system is finite-time stable. Moreover, the settling time, depending on the initial state $x(0) = x_0$, is given by

$$\begin{aligned} T(x_0) &\le \frac{1}{\beta(1-\gamma)} V^{1-\gamma}(x_0), \\ T(x_0) &\le \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha x_0^{1-\gamma} + \beta}{\beta}, \end{aligned} \tag{14}$$

for x_0 in some open neighborhood of the origin, respectively. If $D = R^n$ and $V(x)$ is also radially unbounded, the origin is globally finite-time stable.

2.4. Finite-Time Stability of PCHD System. Based on the finite-time convergence principle of TSM and FTSM, the corresponding finite-time stability of PCHD system can be summarized as follows [21, 22].

Lemma 4. *Consider the PCHD system* (3) *with $J_d(x) = -J_d^T(x)$ and $R_d(x) = R_d^T(x) > 0$,*

$$\begin{aligned} H_d(x) &= \sum_{i=1}^{n} \beta_i x_i^{\gamma_i}, \\ H_d(x) &= \frac{1}{2}\sum_{i=1}^{n} \alpha_i x_i^2 + \sum_{i=1}^{n} \beta_i x_i^{\gamma_i}, \end{aligned} \tag{15}$$

where $\alpha_i, \beta_i > 0$ and $1 < \gamma_i < 2$; then, the equilibrium $x = 0$ is globally finite-time stable.

3. Problem Formulation

The model of the PMSM can be described in a synchronously rotating d-q reference frame as

$$\begin{aligned} L_d \frac{di_d}{dt} &= u_d - R_s i_d + n_p L_q i_q \omega, \\ L_q \frac{di_q}{dt} &= u_q - R_s i_q - n_p L_d i_d \omega - n_p \phi \omega, \\ J \frac{d\omega}{dt} &= \tau - \tau_L = n_p \phi i_q - \tau_L, \end{aligned} \tag{16}$$

where $D = \operatorname{diag}\{L_d \ \ L_q \ \ J\}$, $L_d = L_q$, n_p is the number of pole pairs, ω is the mechanical angular speed of the rotor, J is the moment of inertia, L_d and L_q are d-axis and q-axis stator inductances, respectively, R is the stator resistance per phase, τ and τ_L are the electromagnetic and load torque, respectively, and ϕ is the rotor flux linking the stator.

The PMSM model (16) can be written as a PCHD system as follows:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x) u, \\ y &= g^T(x) \frac{\partial H}{\partial x}(x), \end{aligned} \tag{17}$$

where the state, input, and output are defined as follows, respectively:

$$\begin{gathered} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & J \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} = D \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} \\ u = \begin{bmatrix} u_d & u_q & -\tau_L \end{bmatrix}^T, \qquad y = \begin{bmatrix} i_d & i_q & \omega \end{bmatrix}^T, \end{gathered} \tag{18}$$

and the interconnection, dissipative, weight, and Hamiltonian matrix are defined as follows, respectively:

$$\begin{gathered} J(x) = \begin{bmatrix} 0 & 0 & n_p x_2 \\ 0 & 0 & -n_p(x_1 + \phi) \\ -n_p x_2 & n_p(x_1 + \phi) & 0 \end{bmatrix}, \\ R = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ H(x) = \frac{1}{2}\left[\frac{1}{L_d}x_1^2 + \frac{1}{L_q}x_2^2 + \frac{1}{J}x_3^2\right]. \end{gathered} \tag{19}$$

According to the principle of maximum torque per ampere, the desired equilibrium can be acquired as

$$x^* = \begin{bmatrix} x_1^* & x_2^* & x_3^* \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{L_q \tau_L}{n_p \phi} & J\omega^* \end{bmatrix}, \tag{20}$$

where τ_L and ω^* are the known load torque and references speed, respectively.

The control objective is to design a state-feedback control $u = u(x)$ such that the closed-loop dynamics is a PCHD form (3) with the strict local minimum at the desired equilibrium x^* as (20), and the equilibrium is finite-time stable.

4. Finite-Time TSM Controller Design via IDA-PBC

According to the desired equilibrium (20), the desired Hamiltonian function is chosen as

$$H_d(x) = \frac{1}{\gamma+1}\left(\frac{1}{L_d}(x_1 - x_1^*)^{\gamma+1} + \frac{1}{L_q}(x_2 - x_2^*)^{\gamma+1} + \frac{1}{J}(x_3 - x_3^*)^{\gamma+1}\right), \quad 0 < \gamma < 1; \tag{21}$$

IDA-PBC approach first designs the desired structure of interconnection and damping matrices named as IDA. Then, we derive a PDE parameterized by the chosen matrices whose solutions characterize all the energy functions that can be assigned. Finally, from this family of solutions, we choose one that satisfies the minimum requirement and computes the control. More precisely, the final objective of IDA-PBC is to find a static state-feedback control such that the closed-loop dynamics is a desired PCH system with dissipation of the form (3).

We choose

$$J_a(x) = \begin{bmatrix} 0 & -J_{12} & J_{13} \\ J_{12} & 0 & -J_{23} \\ -J_{13} & J_{23} & 0 \end{bmatrix}, \qquad R_a = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{22}$$

where J_{12}, J_{13}, J_{23}, r_1, r_2 are the interconnection and damping parameters to be designed and $r_1, r_2 > 0$. According to the standard procedure of IDA-PBC [12], we have

$$\begin{aligned} J_d(x) &= J(x) + J_a(x) \\ &= \begin{bmatrix} 0 & -J_{12} & J_{13} + n_p x_2 \\ J_{12} & 0 & -J_{23} - n_p(x_1 + \phi) \\ -J_{13} - n_p x_2 & J_{23} + n_p(x_1 + \phi) & 0 \end{bmatrix}, \end{aligned}$$

$$R_d(x) = R(x) + R_a(x) = \begin{bmatrix} R_s + r_1 & 0 & 0 \\ 0 & R_s + r_2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} &J_d(x) - R_d(x) \\ &= \begin{bmatrix} -(R_s + r_1) & -J_{12} & J_{13} + n_p x_2 \\ J_{12} & -(R_s + r_2) & -J_{23} - n_p(x_1 + \phi) \\ -J_{13} - n_p x_2 & J_{23} + n_p(x_1 + \phi) & 0 \end{bmatrix}, \end{aligned}$$

$$J_a(x) - R_a = \begin{bmatrix} -r_1 & -J_{12} & J_{13} \\ J_{12} & -r_2 & -J_{23} \\ -J_{13} & J_{23} & 0 \end{bmatrix},$$

$$\begin{aligned} K(x) &= \frac{\partial H_d}{\partial x} - \frac{\partial H}{\partial x} \\ &= \left[\frac{1}{L_d}(x_1 - x_1^*)^\gamma, \frac{1}{L_q}(x_2 - x_2^*)^\gamma, \frac{1}{J}(x_3 - x_3^*)^\gamma\right]^T \\ &\quad - \left[\frac{1}{L_d}x_1, \frac{1}{L_q}x_2, \frac{1}{J}x_3\right]^T \\ &= \left[\frac{1}{L_d}\left[(x_1 - x_1^*)^\gamma - x_1\right], \frac{1}{L_q}\left[(x_2 - x_2^*)^\gamma - x_2\right],\right. \\ &\quad \left.\frac{1}{J}\left[(x_3 - x_3^*)^\gamma - x_3\right]\right]^T. \end{aligned} \tag{23}$$

Through solving the following matched equation,

$$\begin{aligned} &\left[J_d(x) - R_d(x)\right] K(x) \\ &\quad = -\left[J_a(x) - R_a(x)\right] D^{-1} x + g(x) u(x). \end{aligned} \tag{24}$$

The controller $u(x) = \begin{bmatrix} u_d & u_q & -\tau_L \end{bmatrix}^T$ can be obtained as

$$\begin{aligned} u_d = &-r_1 \frac{x_1}{L_d} - \frac{J_{12} x_2}{L_q} + \frac{J_{13} x_3}{J} - \frac{R_s + r_1}{L_d} \\ &\times \left[(x_1 - x_1^*)^\gamma - x_1\right] - \frac{J_{12}}{L_q}\left[(x_2 - x_2^*)^\gamma - x_2\right] \\ &+ \frac{J_{13} + n_p x_2}{J}\left[(x_3 - x_3^*)^\gamma - x_3\right], \\ u_q = &\frac{J_{12} x_1}{L_d} - \frac{r_2 x_2}{L_q} - \frac{J_{23} x_3}{J} + \frac{J_{12}}{L_d}\left[(x_1 - x_1^*)^\gamma - x_1\right] \\ &- \frac{R_s + r_2}{L_q}\left[(x_2 - x_2^*)^\gamma - x_2\right] \\ &- \frac{J_{23} + n_p(x_1 + \phi)}{J}\left[(x_3 - x_3^*)^\gamma - x_3\right] \\ = &\frac{J_{12} x_1}{L_d} - \frac{r_2 x_2}{L_q} - \frac{J_{23} x_3}{J} + \frac{J_{12}}{L_d}\left[(x_1 - x_1^*)^\gamma - x_1\right] \\ &- \frac{R_s + r_2}{L_q}\left[(x_2 - x_2^*)^\gamma - x_2\right] \\ &- \frac{J_{23} + n_p(x_1 + \phi)}{J}(x_3 - x_3^*)^\gamma \\ &+ \frac{J_{23} x_3}{J} + \frac{n_p(x_1 + \phi) x_3}{J} \end{aligned}$$

$$
\begin{aligned}
&= \frac{J_{12}x_1}{L_d} - \frac{r_2 x_2}{L_q} + \frac{J_{12}}{L_d}\left[(x_1 - x_1^*)^\gamma - x_1\right] \\
&\quad - \frac{R_s + r_2}{L_q}\left[(x_2 - x_2^*)^\gamma - x_2\right] \\
&\quad - \frac{J_{23} + n_p(x_1 + \phi)}{J}(x_3 - x_3^*)^\gamma \\
&\quad + \frac{n_p(x_1 + \phi)x_3}{J}, \\
-\tau_L &= -\frac{J_{13}}{L_d}x_1 + \frac{J_{23}}{L_q}x_2 - \frac{J_{13} + n_p x_2}{L_d} \\
&\quad \times \left((x_1 - x_1^*)^\gamma - x_1\right) + \frac{J_{23} + n_p(x_1 + \phi)}{L_q} \\
&\quad \times \left((x_2 - x_2^*)^\gamma - x_2\right) \\
&= -\tau_L - \frac{J_{13} + n_p x_2}{L_d}(x_1 - x_1^*)^\gamma + \frac{J_{23}}{L_q}(x_2 - x_2^*)^\gamma \\
&\quad + \frac{n_p(x_1 + \phi)}{L_q}(x_2 - x_2^*)^\gamma - \frac{n_p\phi}{L_q}(x_2 - x_2^*) \\
&= -\tau_L - \frac{J_{13} + n_p x_2}{L_d}(x_1 - x_1^*)^\gamma \\
&\quad + \left(\frac{J_{23}}{L_q} + \frac{n_p(x_1 + \phi)}{L_q} - \frac{n_p\phi}{L_q}(x_2 - x_2^*)^{1-\gamma}\right) \\
&\quad \times (x_2 - x_2^*)^\gamma.
\end{aligned} \tag{25}
$$

In this paper, we choose the free parameters J_{12}, J_{13}, J_{23} as

$$
\begin{aligned}
J_{12} &= 0, \qquad J_{13} = -n_p x_2, \\
J_{23} &= -n_p(x_1 + \phi) + n_p\phi(x_2 - x_2^*)^{1-\gamma}.
\end{aligned} \tag{26}
$$

Then, the final finite-time TSM controller $u(x)$ becomes

$$
\begin{aligned}
u_d &= -r_1 i_d - n_p L_q i_q \omega - (R_s + r_1)\left[L_d^{\gamma-1} i_d{}^\gamma - i_d\right], \\
u_q &= -r_2 i_q + n_p \omega(L_d i_d + \phi) - \frac{R_s + r_2}{L_q} \\
&\quad \times \left[\left(L_q i_q - \frac{L_q \tau_L}{n_p\phi}\right)^\gamma - L_q i_q\right] \\
&\quad - \frac{n_p\phi\left(L_q i_q - (L_q\tau_L/n_p\phi)\right)^{1-\gamma}}{J}(J\omega - J\omega^*)^\gamma \\
-\tau_L &= -\tau_L.
\end{aligned} \tag{27}
$$

By applying this control to the PMSM system (16), the desired closed-loop PCHD system (3) can be reached with the desired hamiltonian function (21). With Lemma 4, the finite-time stability of the equilibrium (20) can be obtained.

Remark 5. Actually, the desired hamiltonian function (21) can be rewritten as

$$
\begin{aligned}
H_d(x) &= \frac{1}{\gamma + 1} \\
&\quad \times \left(\frac{1}{L_d}\left((x_1 - x_1^*)^2\right)^{(\gamma+1)/2} + \frac{1}{L_q}\left((x_2 - x_2^*)^2\right)^{(\gamma+1)/2}\right. \\
&\quad \left. + \frac{1}{J}\left((x_3 - x_3^*)^2\right)^{(\gamma+1)/2}\right).
\end{aligned} \tag{28}
$$

Because of $0 < (\gamma + 1)/2 < 1$, according to Theorem 1 in [21], the closed-loop system will converge to the desired equilibrium (20) in finite-time.

Remark 6. In the controller (27), all the fractional powers of the state variables are positive; therefore, the singularity problem with conventional TSM controller is avoided.

Remark 7. When $\gamma = 1$, the controller (27) is reduced to the conventional IDA-PBC design as in [14, 16]:

$$
\begin{aligned}
u_d &= -r_1 i_d - n_p L_q i_q \omega \\
u_q &= -r_2\left(i_q - \frac{\tau^*}{n_p\phi}\right) + n_p\omega L_d i_d + \frac{R_s}{n_p\phi}\tau^* + n_p\phi\omega^*.
\end{aligned} \tag{29}
$$

Remark 8. If we choose the desired hamiltonian function as

$$
\begin{aligned}
H_d(x) &= \frac{1}{2}\left(\frac{1}{L_d}(x_1 - x_1^*)^2 + \frac{1}{L_q}(x_2 - x_2^*)^2\right. \\
&\quad \left. + \frac{1}{J}(x_3 - x_3^*)^2\right) \\
&\quad + \frac{1}{\gamma + 1}\left(\frac{1}{L_d}(x_1 - x_1^*)^{\gamma+1} + \frac{1}{L_q}(x_2 - x_2^*)^{\gamma+1}\right. \\
&\quad \left. + \frac{1}{J}(x_3 - x_3^*)^{\gamma+1}\right),
\end{aligned} \tag{30}
$$

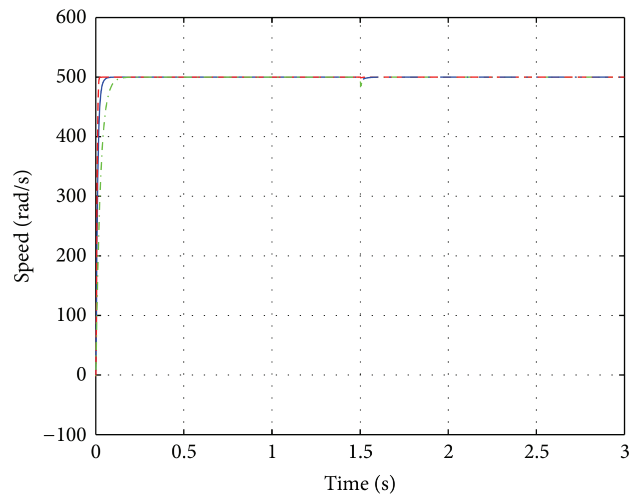

FIGURE 1: The speed response with different control strategies.

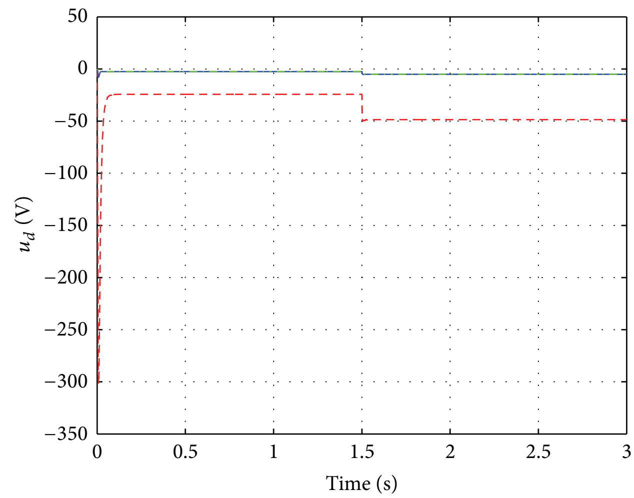

FIGURE 2: The d-axis control voltage.

following the similar IDA-PBC procedure as before, we can design the fast TSM controller as

$$u_d = -r_1 i_d - n_p L_q i_q \omega - \frac{R_s + r_1}{L_d} (L_d i_d)^\gamma$$

$$u_q = -\frac{r_2 x_2}{L_q} - \frac{R_s + r_2}{L_q} \left[(x_2 - x_2^*)^\gamma - x_2^* \right] + \frac{n_p x_1 x_3}{J} + \frac{n_p \phi x_3 - n_p \phi (x_2 - x_2^*)^{1-\gamma} (x_3 - x_3^*)^\gamma + n_p \phi (x_2 - x_2^*)^{1-\gamma} x_3^*}{J \left(1 + (x_2 - x_2^*)^{1-\gamma}\right)}. \quad (31)$$

When $\gamma = 1$, the previous controller is also reduced to the conventional IDA-PBC as

$$u_d = -(R_s + 2r_1) i_d - n_p L_q i_q \omega$$

$$u_q = -(R_s + 2r_2) \left(i_q - \frac{\tau^*}{n_p \phi} \right) + n_p \omega L_d i_d + \frac{R_s}{n_p \phi} \tau^* + n_p \phi \omega^*. \quad (32)$$

5. Simulation Results

In this section, the speed regulation performance of the PMSM based on the proposed controllers is investigated. The system parameters are $R_s = 2.875\,\Omega$, $\tau_L = 1\,\text{N}\cdot\text{m}$, $J = 0.00085\,\text{kg}\cdot\text{m}^2$, $n_p = 4$, $\phi = 0.175\,\text{wb}$, and $L_d = L_q = 0.0085\,\text{H}$. The reference speed is 500 rad/s. The initial load torque is $1\,\text{N}\cdot\text{m}$ and becomes $2\,\text{N}\cdot\text{m}$ at $t = 1.5$ s.

The simulation results are shown in Figures 1, 2, 3, 4, 5, and 6. Figure 1 illustrates the speed response with different control strategies: fast TSM control (31) (red dash line), TSM control (27) (blue real line), and conventional IDA-PBC (29) (green dash dot line). The comparison results demonstrate that the proposed control strategies have superior performance on rapid response and disturbance rejection with respect to the conventional IDA-PBC. The speed regulation can reach the reference speed in finite-time 0.02 s and 0.12 s for fast TSM control and TSM control, respectively, and the conventional IDA-PBC only obtains the asymptotical convergence to the reference speed. When the load perturbance happens at $t = 1.5$ s, the maximum speed variations are 499, 494, and 486 rad/s for the three controllers, respectively. The speed tracking performance is improved greatly as shown in Figure 1. This is because the control current, voltage, and torque response can rapidly change to deal with reference speed and load torque variation with the proposed control strategies as shown in Figures 2–6.

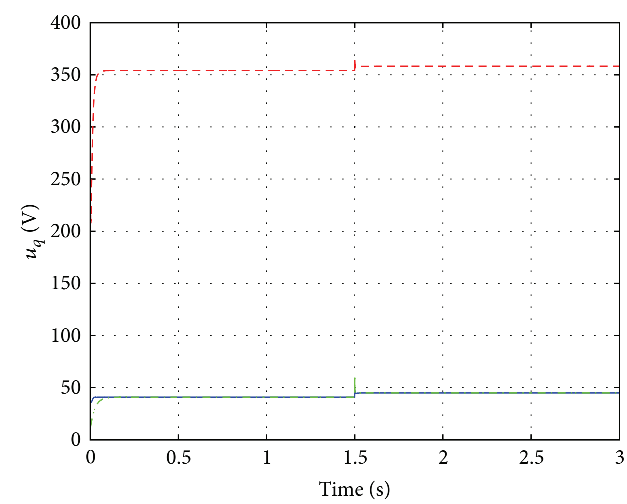

FIGURE 3: The q-axis control voltage.

6. Conclusions

This paper studies the finite-time speed regulating problem of PMSM with TSM and PCHD theories. A fractional power form controller is designed with finite-time convergence to

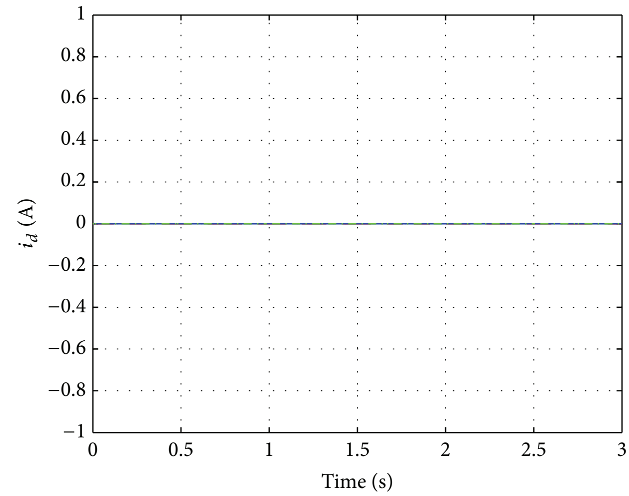

FIGURE 4: The d-axis control current.

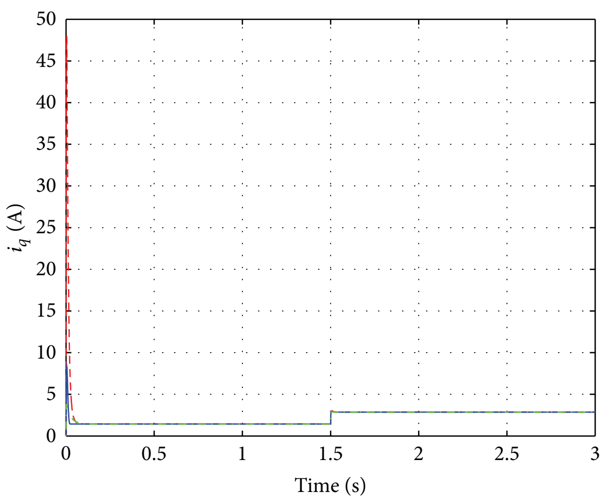

FIGURE 5: The q-axis control current.

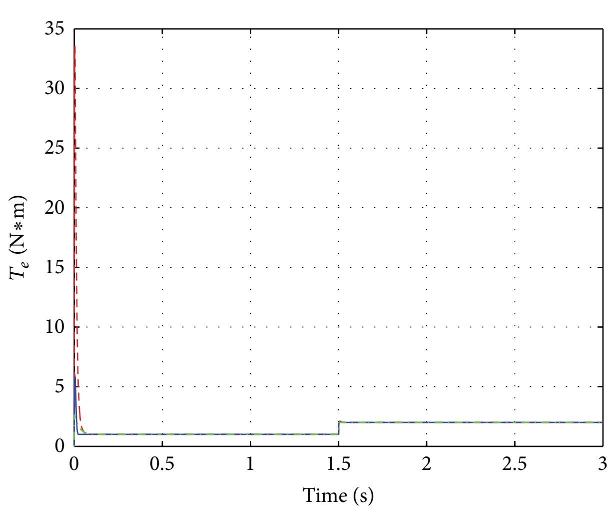

FIGURE 6: The control torque.

the desired equilibrium. By comparing finite-time stable system with asymptotically stable system, the simulation results show that finite-time control method has faster convergence, better robustness, and antidisturbance, which improves the permanent magnet synchronous motor vector control system of performance. Further research will conduct the finite-time observer and finite-time disturbance observer and combine them with the current results.

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