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Research Article

Impulsive Cluster Anticonsensus of Discrete Multiagent Linear Dynamic Systems

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Cluster anticonsensus is another important type consensus of multiagent systems. In this paper, we investigate the problem of impulsive cluster anticonsensus of discrete multiagent linear dynamic systems. Firstly, an impulsive protocol is designed to achieve the cluster anticonsensus. Then sufficient conditions are given to guarantee the cluster anticonsensus of the discrete multiagent linear dynamic system based on the Q-theory. Numerical simulation shows the effectiveness of our theoretical results.

1. Introduction

Recently, the consensus problem of multiagent systems has been intensively studied in the literature [1–4]. In [1, 2], a systematical framework of consensus problem in networks of agents was investigated. The problem of information consensus among multiple agents in the presence of limited and unreliable information exchange with dynamically changing interaction topologies was considered in [3]. In [4], the authors considered the consensus problem for multiagent systems, in which all agents have an identical linear dynamic mode that can be of any order. On the other hand, cluster anticonsensus is another important type consensus of multiagent systems. When the multiagent systems achieve cluster anticonsensus, the nodes in the same group achieve consensus with each other, but there is no consensus between nodes in different groups. Very recently the signless Laplacian has attracted the attention of researchers. Several papers on the signless Laplacian spectrum have been reported since 2005 and a new spectral theory of graphs which is called the Q-theory is developing by many researchers [5, 6]. To the best of our knowledge, however, there are very few results on cluster anticonsensus of multiagent systems, which motivates this study.

Impulsive control is widely used in various applications, such as ecosystems, financial systems, mechanical systems with impacts, and orbital transfer of satellite [7–12]. Very recently, impulsive control protocol for multiagent systems has received much attention [13–19]. In [14], the authors introduced impulsive control protocols for multiagent linear continuous dynamic systems. The convergence analysis of the impulsive control protocol for the networks with fixed and switching topologies is presented, respectively. The proposed impulsive control protocol is only applied to the multiagent system at certain discrete instants, which is different from continuous control protocol [1, 2, 4]. In [15], the authors investigated the problem of impulsive synchronization of networked multiagent systems, where each agent can be modeled as an identical nonlinear dynamical system.

In this paper, we investigate the problem of impulsive cluster anticonsensus of discrete multiagent linear dynamic systems. The main contribution of our paper includes (1) an impulsive control protocol is introduced to seek the cluster anticonsensus of discrete multiagent linear dynamic systems and (2) a new type consensus, that is, cluster anticonsensus is studied.

This paper is organized as follows. In Section 2, we provide some results in the Q-theory. In Section 3, we formulate the cluster anticonsensus problem for discrete multiagent linear dynamic systems and introduce the impulsive control protocol. The convergence analysis of the cluster anticonsensus problem is discussed in Section 4. In Section 5, numerical simulation is included to show the effectiveness of our theoretical results. Some conclusions are drawn in Section 6.

Notation 1. Throughout this paper, the superscripts "-1" and "T" stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; let $\mathbb{R}_+ = [0, \infty)$, $\mathbb{N} = \{0, 1, 2, \ldots\}$, $\mathbb{N}_+ = \{1, 2, \ldots\}$; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; for real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix X-Y is positive semidefinite (resp., positive definite); $I_n \in \mathbb{R}^{n\times n}$ is an identity matrix; $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) denotes the smallest (largest) eigenvalue of P. For a vector $x \in \mathbb{R}^n$, let $\|x\|$ denote the Euclidean vector norm, that is, $\|x\| = \sqrt{x^T x}$, and for $A \in R^{n\times n}$, let $||A||$ indicate the norm of A induced by the Euclidean vector norm, that is, $||A|| = \sqrt{\lambda_{\max}(A^T A)}$. The Kronecker product of two matrices $A = [a_{ij}] \in \mathbb{R}^{m\times n}$ and $B = [b_{ij}] \in \mathbb{R}^{p\times q}$ is denoted by $A \otimes B$. For more properties of the Kronecker product the reader is referred to [20].

2. Preliminaries

In this section, we provide some results in the Q-theory [4–6, 21].

An undirected graph G of order N consists of a vertex set $V = \{1, 2, \ldots, N\}$ and an edge set $E = \{(i, j) : i, j \in V\} \subset V \times V$. The set of neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in V : (i, j) \in E, j \neq i\}$. A path between each distinct vertices i and j is meant a sequence of distinct edges of G of the form $(i, k_1), (k_1, k_2), \ldots, (k_l, j)$. A cycle is a path such that the start vertex and end vertex are the same. If there is a path between any two vertices of a graph G, then G is connected, otherwise disconnected. A graph G is a bipartite graph if $V(G)$ can be partitioned into two disjoint subsets U and W, called partite sets, such that every edge of G joins a vertex of U and a vertex of W. A graph is bipartite if and only if it does not contain an odd cycle.

A weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $a_{ii} = 0$ and $a_{ij} = a_{ji} \geq 0$, $i \neq j$. $a_{ij} > 0$ if and only if there is an edge between vertex i and vertex j. For an unweighted graph G, A is a 0-1 matrix. The out-degree of vertex i is defined as follows $\deg_{\text{out}}(i) = \sum_{j=1}^{n} a_{ij}$. Let D be the diagonal matrix with the out-degree of each vertex along the diagonal and call it the degree matrix of G. The signless Laplacian matrix of the weighted graph is defined as $Q_G = D + A$. For an unweighted graph G, $Q_G = [q_{ij}]_{N\times N}$, where

$$q_{ij} = \begin{cases} |\mathcal{N}_i|, & i = j, \\ 1, & j \in \mathcal{N}_i, \\ 0, & \text{otherwise}, \end{cases} \tag{2.1}$$

here $|\mathcal{N}_i|$ denotes the cardinality of the set $\mathcal{N}_i$.

Let G be an undirected graph on N vertices, having m edges. Let R be its vertex edge incidence matrix which is an $N \times m$ matrix such that $r_{ij} = 1$ if the vertex i and edge e_j are incident and 0 otherwise. The following relations are well-known: $RR^T = D + A = Q$. Thus the signless Laplacian is a positive semi-definite matrix, that is, all its eigenvalues are non-negative. Let G be a graph with Q-eigenvalues $q_1, q_2, \ldots, q_N$ $(q_1 \le q_2 \le \cdots \le q_N)$.

Lemma 2.1. *Let Q be the singless Laplacian matrix of an undirected graph G with N vertices, and $q_1 \le q_2 \le \cdots \le q_N$ be the eigenvalues of Q. Suppose that the graph G is bipartite. Let U and W be two subsets of graph G. Define $\xi \in \mathbb{R}^N$, $\xi(i) = 1$, $i \in U$, $\xi(j) = -1$, $j \in W$ and $e_i \in \mathbb{R}^n$, $e_i(i) = 1$, $e_i(j) = 0$, $j \ne i$, $i, j = 1, \ldots, n$. Then*

(1) *if G is connected, then $q_1 = 0$ is the algebraically simple eigenvalue of Q and the corresponding eigenvector is ξ;*

(2) *if 0 is the simple eigenvalue of Q, then it is an n multiplicity eigenvalue of $Q \otimes I_n$ and the corresponding eigenvectors are $\xi \otimes e_i$, $i = 1, 2, \cdots, n$.*

3. Problem Formulation

Here we consider a system consisting of N agents indexed by $i = 1, 2, \ldots, N$. The dynamics of each agent is

$$x^i(t+1) = Ax^i(t), \quad t \ge t_0 \ge 0, \; i = 1, 2, \ldots, N, \tag{3.1}$$

where $x^i(t) = (x_1^i(t), x_2^i(t), \ldots, x_n^i(t))^T \in \mathbb{R}^n$ is the state of agent i at time t, $t \in \mathbb{N}$, $A \in \mathbb{R}^{n\times n}$ is a constant matrix. We assume that $\|A\| \ne 0$.

The following impulsive control protocol is applied to agent i

$$u^i(t) = \Delta x^i(t) = x^i(t^+) - x^i(t^-) = -B_k \sum_{j\in\mathcal{N}_i(t^-)} \left(x^j(t^-) + x^i(t^-)\right), \quad t = t_k, \; k \in \mathbb{N}_+, \tag{3.2}$$

where $B_k \in \mathbb{R}^{n\times n}$, $k \in \mathbb{N}_+$, are impulsive matrices to be determined later, $\mathcal{N}_i(t^-)$ is the set of neighbors of agent i at time t^-, the discrete instants $t_k \in \mathbb{N}_+$ satisfy $0 \le t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$, and $\lim_{k\to+\infty} t_k = +\infty$. When $t = t_k$, $k \in \mathbb{N}_+$, t^+ and t^- denote the instant just after t and just before t, respectively, which is also considered in [8, 10]. Without loss of generality, we assume that $x^i(t^-) = x^i(t)$, $t = t_k$, $k \in \mathbb{N}_+$.

Under the impulsive control protocol (3.2), the dynamics of agent i satisfies the following equations

$$\begin{aligned} x^i(t+1) &= Ax^i(t), \quad t \neq t_k, \\ \Delta x^i(t) &= -B_k \sum_{j \in \mathcal{N}_i(t)} \left(x^j(t) + x^i(t) \right), \quad t = t_k, \ k \in \mathbb{N}_+. \end{aligned} \tag{3.3}$$

Definition 3.1. For the system (3.1), the cluster anticonsensus is said to be achieved under the impulsive control protocol (3.2) if

$$\begin{gathered} \lim_{i_1, i_2 \in U, t \to +\infty} \left\| x^{i_1}(t) - x^{i_2}(t) \right\| = 0, \qquad \lim_{j_1, j_2 \in W, t \to +\infty} \left\| x^{j_1}(t) - x^{j_2}(t) \right\| = 0, \\ \lim_{i \in U, j \in W, t \to +\infty} \left\| x^i(t) + x^j(t) \right\| = 0, \end{gathered} \tag{3.4}$$

where U and W are two nonempty subsets of $V(G)$ and $U \cap W = \emptyset$, $U \cup W = V(G)$.

Consider the following discrete impulsive system

$$\begin{aligned} x(t+1) &= Ax(t), \quad t \neq t_k, \\ \Delta x(t) = x(t^+) - x(t^-) &= B_k x(t), \quad t = t_k, \ k \in \mathbb{N}_+, \end{aligned} \tag{3.5}$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$, $B_k \in \mathbb{R}^{n \times n}$. Then the solution $x(t; t_0, x_0)$ satisfies

$$x(t; t_0, x_0) = A^{(t-t_k)} \prod_{i=1}^{k} (I_n + B_i) A^{(t_i - t_{i-1})} x_0, \tag{3.6}$$

where $t_k < t \leq t_{k+1}$, $k \in \mathbb{N}_+$.

The representation (3.6) implies the following.

Lemma 3.2. *All solutions of the system* (3.5) *are asymptotically stable if the conditions (H1) and (H2) hold,*

(H1) $0 < \theta_1 \leq t_k - t_{k-1} \leq \theta_2 < \infty$, $k \in \mathbb{N}_+$,

(H2) $\|\Xi_k\| \leq r < 1$, $k \in \mathbb{N}_+$,

where $\Xi_k = (I_n + B_k) A^{(t_k - t_{k-1})}$, $k \in \mathbb{N}_+$.

4. Main Results

In this section, we provide convergence analysis of the cluster anticonsensus problem of discrete multiagent linear dynamic systems.

Let $x(t) = (x^1(t), x^2(t), \ldots, x^N(t))^T$, then the system (3.3) can be described as

$$\begin{aligned} x(t+1) &= (I_N \otimes A) x(t), \quad t \neq t_k, \\ \Delta x(t) &= (I_N \otimes B_k)(-Q \otimes I_n) x(t), \quad t = t_k, \ k \in \mathbb{N}_+. \end{aligned} \tag{4.1}$$

Since Q is symmetric, there is an orthogonal matrix $Y \in \mathbb{R}^{N\times N}$ such that

$$YQY^T = \Lambda = \mathrm{diag}(q_1, q_2, \ldots, q_N), \tag{4.2}$$

where $\{q_1, q_2, \ldots, q_N\} = \chi(Q)$ is the spectrum of Q.

Let

$$\widetilde{x}(t) = (Y \otimes I_n)x(t). \tag{4.3}$$

Using the properties of Kronecker product, we have when $t \neq t_k$, $k \in \mathbb{N}_+$,

$$\widetilde{x}(t+1) = (Y \otimes I_n)x(t+1) = (Y \otimes I_n)(I_N \otimes A)(Y \otimes I_n)^{-1}\widetilde{x}(t) = (I_N \otimes A)\widetilde{x}(t), \tag{4.4}$$

and when $t = t_k$, $k \in \mathbb{N}_+$,

$$\Delta\widetilde{x}(t) = (Y \otimes I_n)\Delta x(t) = (Y \otimes I_n)(I_N \otimes B_k)(-Q \otimes I_n)(Y \otimes I_n)^{-1}\widetilde{x}(t) = (-\Lambda \otimes B_k)\widetilde{x}(t). \tag{4.5}$$

Thus (4.1) becomes

$$\begin{aligned} \widetilde{x}(t+1) &= (I_N \otimes A)\widetilde{x}(t), \quad t \neq t_k, \\ \Delta\widetilde{x}(t) &= (-\Lambda \otimes B_k)\widetilde{x}(t), \quad t = t_k,\ k \in \mathbb{N}_+. \end{aligned} \tag{4.6}$$

Therefore

$$\begin{aligned} \widetilde{x}^i(t+1) &= A\widetilde{x}^i(t), \quad t \neq t_k, \\ \Delta\widetilde{x}^i(t) &= -q_i B_k \widetilde{x}^i(t), \quad t = t_k,\ k \in \mathbb{N}_+,\ i = 1, 2, \ldots, N. \end{aligned} \tag{4.7}$$

Theorem 4.1. *Consider the system* (3.1). *Assume that the graph G of the network is connected and bipartite. If there exist discrete instants t_k and impulsive matrices B_k such that the conditions (H3) and (H4) hold, then the cluster anticonsensus is achieved under the impulsive control protocol* (3.2)*,*

(H3) $0 < \theta_1 \leq t_k - t_{k-1} \leq \theta_2 < \infty$, $k \in \mathbb{N}_+$,

(H4) $\|V_{i_k}\| \leq r < 1$, $i = 2, \ldots, N$, $k \in \mathbb{N}_+$,

where $V_{i_k} = (I_n - q_i B_k)A^{(t_k - t_{k-1})}$, $i = 2, \ldots, N$, $k \in \mathbb{N}_+$.

Proof. Since the graph G is connected and bipartite, by Lemma 2.1, $q_1 = 0$ is the algebraically simple eigenvalue of Q. All the other eigenvalues of Q are positive. Then we have $0 = q_1 < q_2 \leq \cdots \leq q_N$.

By Lemma 3.2, it follows that if there exist discrete instants t_k and impulsive matrices B_k such that the conditions (H3) and (H4) hold, then the system (4.7) is asymptotically stable, that is, $\widetilde{x}^i(t) \to 0$, $t \to +\infty$, $i = 2, \ldots, N$.

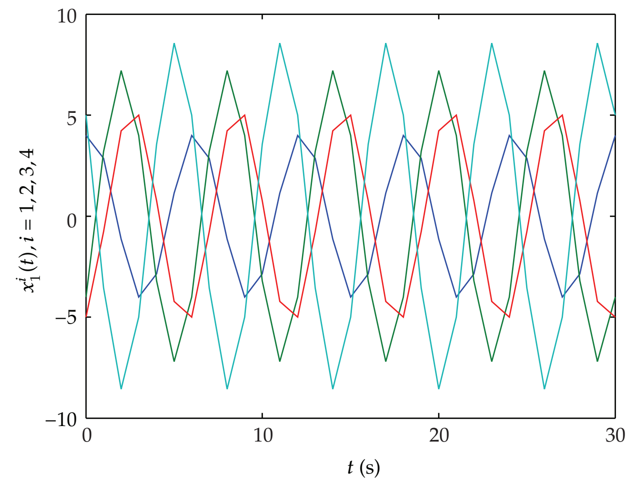

Figure 1: The time histories of $x_1^i(t)$, $i = 1,2,3,4$.

It can be verified that

$$\begin{aligned}(Q \otimes I_n)x(t) &= (Y \otimes I_n)^{-1}(Y \otimes I_n)(Q \otimes I_n)\left(Y^{-1} \otimes I_n\right)\tilde{x}(t)\\ &= (Y \otimes I_n)^{-1}(\Lambda \otimes I_n)\tilde{x}(t)\\ &= (Y \otimes I_n)^{-1}\begin{bmatrix} 0 \\ q_2\tilde{x}^2(t) \\ \vdots \\ q_N\tilde{x}^N(t)\end{bmatrix}.\end{aligned} \tag{4.8}$$

Hence $(Q \otimes I_n)x(t) \to 0$, $t \to +\infty$. Since the graph G is connected and bipartite, by Lemma 2.1, 0 is the eigenvalue of $Q \otimes I_n$ with multiplicity n. The n linearly independent eigenvectors associated with the eigenvalue 0 of $Q \otimes I_n$ are $\xi \otimes e_i$, $i = 1,2,\cdots,n$. Therefore $x \to \xi \otimes s$, $t \to +\infty$, where $s = \sum_{i=1}^{n} \gamma_i e_i \in \mathbb{R}^n$, $\gamma_i \in \mathbb{R}$, $i = 1,2,\ldots,n$. Thus the system (3.1) achieves the cluster anticonsensus under the impulsive control protocol (3.2). This completes the proof. □

Remark 4.2. For simplicity, we usually choose the equidistant impulsive interval $\Delta t_k = t_{k+1} - t_k = \Delta$, $k \in \mathbb{N}_+$. The impulsive matrices B_k, $k \in \mathbb{N}_+$, are chosen as pI_n, $p \in \mathbb{R}$. If $\|(1 - q_i p)A^{\Delta}\| \le r < 1$, $i = 2,\ldots,N$, then the conditions (H3) and (H4) are satisfied.

5. Simulations

Consider the following multiagent discrete linear dynamic system

$$x^i(t+1) = Ax^i(t), \qquad x^i(t) \in \mathbb{R}^3,\ i = 1,2,3,4, \tag{5.1}$$

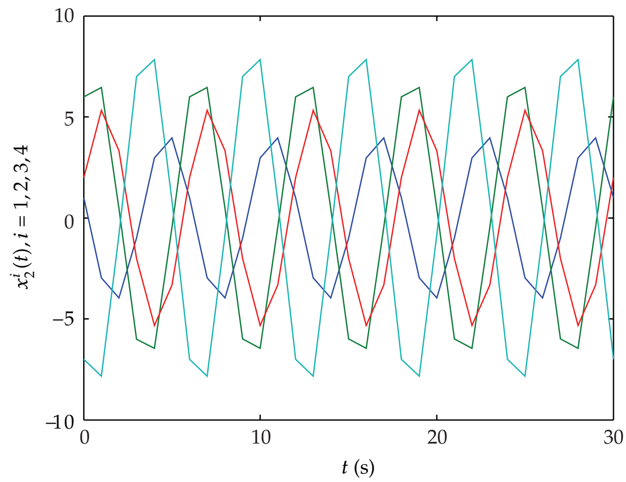

Figure 2: The time histories of $x_2^i(t)$, $i = 1, 2, 3, 4$.

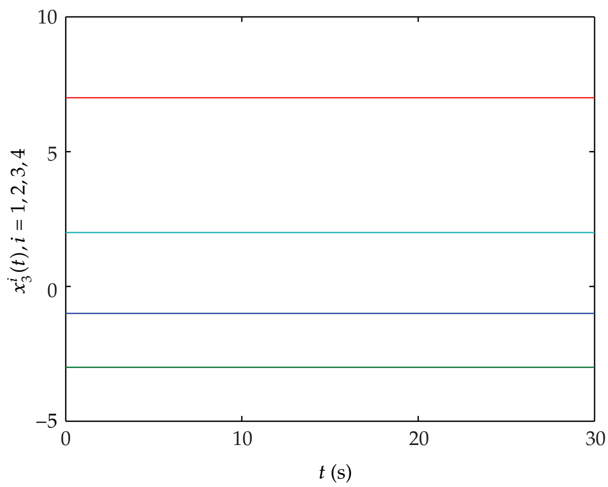

Figure 3: The time histories of $x_3^i(t)$, $i = 1, 2, 3, 4$.

where $A = \begin{bmatrix} 1/2 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Simulation results for the system (5.1) are shown in Figures 1, 2, and 3. From Figures 1–3, we know that the system cannot achieve the cluster anticonsensus without the impulsive control protocols.

The control input of agent i is designed as (3.2). The graph is considered as a simple path on four vertices and the singless Laplacian matrix of graph G_1 is $Q_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. For simplicity, the impulsive matrices B_k, $k \in \mathbb{N}_+$, are chosen as $0.43 * I_3$. Choose the equidistant impulsive interval $\Delta t_k = t_k - t_{k-1} = 2$, $k \in \mathbb{N}_+$. It is easy to check that $\|V_{2_k}\| = 0.7481 < 1$, $\|V_{3_k}\| = 0.1400 < 1$, $\|V_{4_k}\| = 0.4681 < 1$, where $q_2 = 0.5858$, $q_3 = 2$, $q_4 = 4$. The initial values are chosen as $x^1(0) = [4 \;\; 1 \;\; -1]^T$, $x^2(0) = [-4 \;\; 6 \;\; -3]^T$, $x^3(0) = [-5 \;\; 2 \;\; 7]^T$, $x^4(0) = [5 \;\; -7 \;\; 2]^T$.

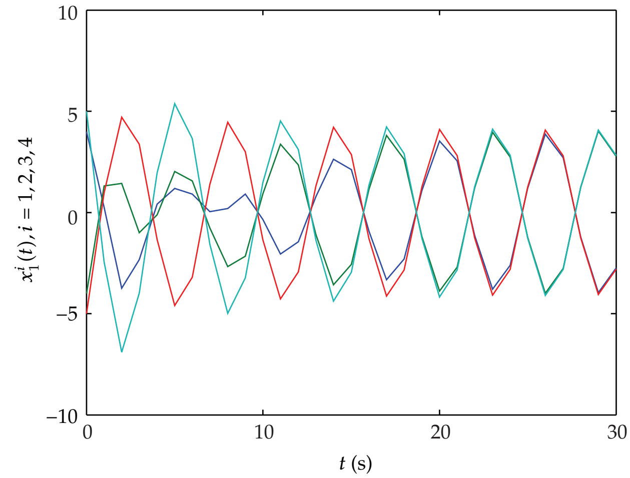

Figure 4: The time histories of $x_1^i(t)$, $i = 1, 2, 3, 4$ for G_1.

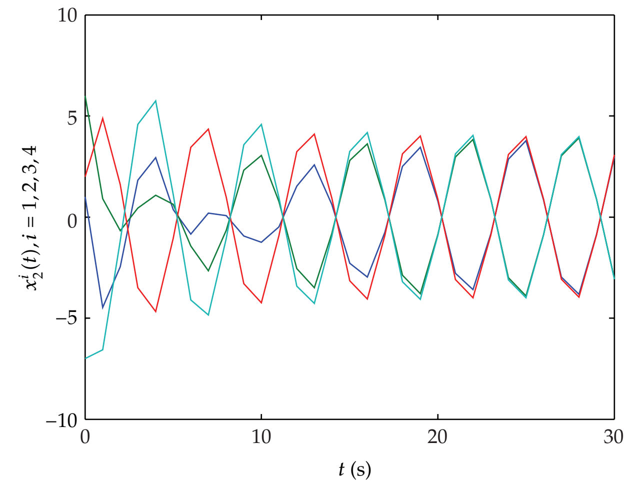

Figure 5: The time histories of $x_2^i(t)$, $i = 1, 2, 3, 4$ for G_1.

Simulation results for G_1 are shown in Figures 4, 5, and 6. The simulation results show that the cluster anticonsensus of the multiagent discrete linear dynamic system is achieved by the impulsive control protocol. Besides, let $\overline{x}(t) = (x^1(t) - x^2(t) + x^3(t) - x^4(t))/4$, then $\lim_{t\to+\infty}\|x^i(t) - \overline{x}(t)\| = 0$, $i = 1, 3$ and $\lim_{t\to+\infty}\|x^i(t) + \overline{x}(t)\| = 0$, $i = 2, 4$.

If the graph is considered as a simple circle on four vertices, then the singless Laplacian matrix of graph G_2 is $Q_2 = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$. The other parameters are chosen as above. Then the conditions (H3) and (H4) in Theorem 4.1 are also satisfied. Simulation results for G_2 are shown in Figures 7, 8, and 9. From Figures 4–9, it can be seen that the speed of convergence of cluster anticonsensus algorithms is closely related with the network structure.

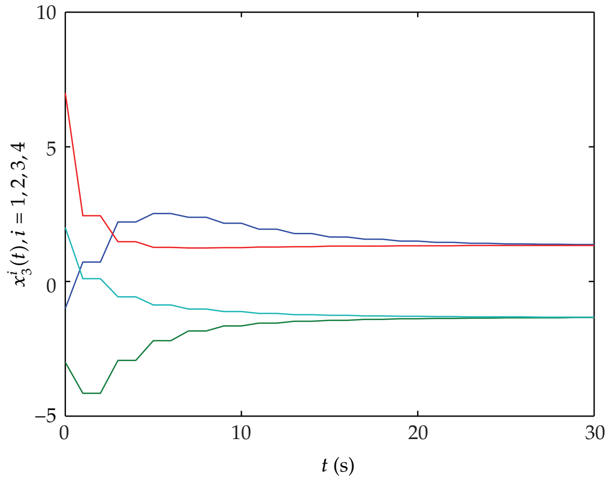

Figure 6: The time histories of $x_3^i(t)$, $i = 1, 2, 3, 4$ for G_1.

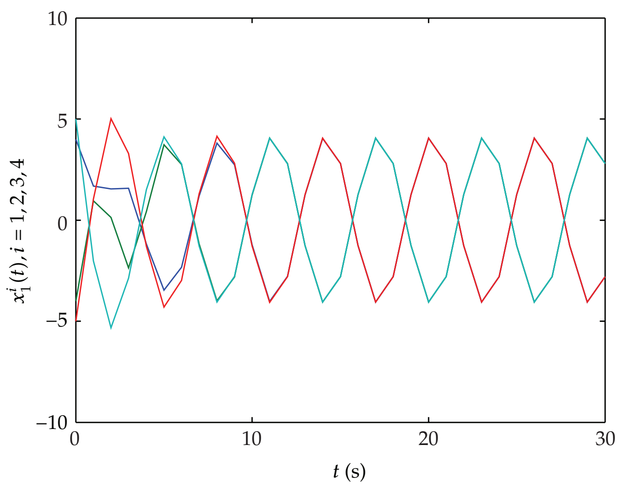

Figure 7: The time histories of $x_1^i(t)$, $i = 1, 2, 3, 4$ for G_2.

6. Conclusions

In this paper, we have introduced impulsive control protocols for discrete multiagent linear dynamic systems. The convergence analysis of the cluster anticonsensus is presented. When the multiagent systems achieve cluster anticonsensus, the nodes in the same group achieve consensus with each other, but there is no consensus between nodes in different groups. In our future, we will consider the impulsive cluster anticonsensus problem of multiagent nonlinear dynamic systems.

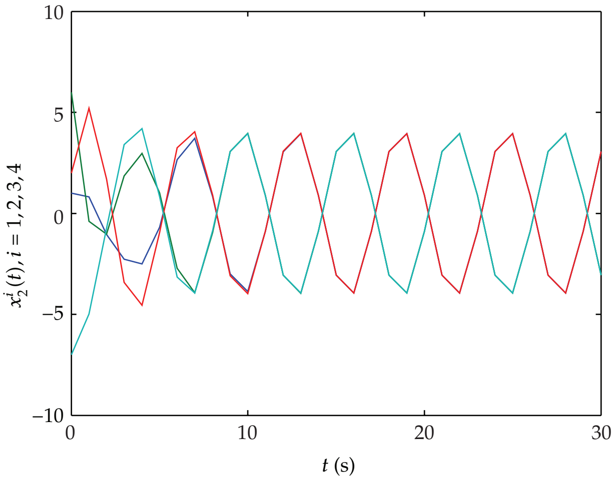

Figure 8: The time histories of $x_2^i(t)$, $i = 1, 2, 3, 4$ for G_2.

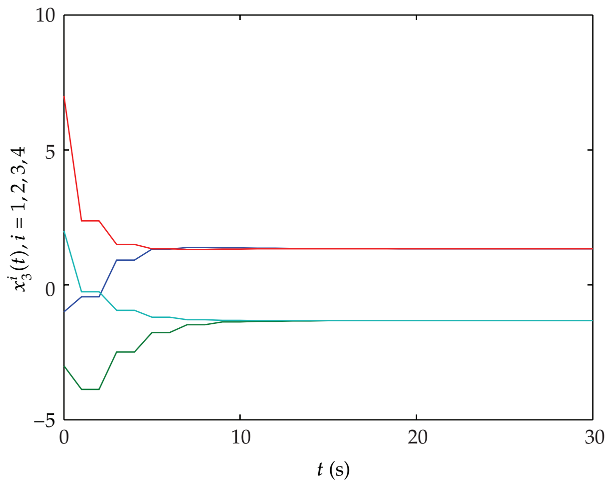

Figure 9: The time histories of $x_3^i(t)$, $i = 1, 2, 3, 4$ for G_2.

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