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Research Article

Stability Analysis and H_∞ Model Reduction for Switched Discrete-Time Time-Delay Systems

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This paper is concerned with the problem of exponential stability and H_∞ model reduction of a class of switched discrete-time systems with state time-varying delay. Some subsystems can be unstable. Based on the average dwell time technique and Lyapunov-Krasovskii functional (LKF) approach, sufficient conditions for exponential stability with H_∞ performance of such systems are derived in terms of linear matrix inequalities (LMIs). For the high-order systems, sufficient conditions for the existence of reduced-order model are derived in terms of LMIs. Moreover, the error system is guaranteed to be exponentially stable and an H_∞ error performance is guaranteed. Numerical examples are also given to demonstrate the effectiveness and reduced conservatism of the obtained results.

1. Introduction

Switched systems belong to a special class of hybrid control systems, which comprises a collection of subsystems described by dynamics differential or difference equations, together with a switching law that specifies the switching rule among the subsystems. Due to the theoretical development as well as practical applications, analysis and synthesis of switched systems have recently gained considerable attention [1–4].

Furthermore, the time-delay phenomenon is frequently encountered in a variety of industrial and engineering systems [5–7], for instance, chemical process, long distance transmission line, communication networks, and so forth. Moreover, time-delay is a predominant source of the poor performance and instability. In the last two decades, there has been increasing interest in the stability analysis for the systems; see, for example, [8, 9] and the references cited there in. For switched delay systems, due to the impact of time-delays, the behavior of switched delay systems is usually much more complicated than that of switched systems or delay systems [10, 11].

The average dwell time (ADT) technique [12] and multiple Lyapunov function approach [13] are two powerful and effective tools for studying the problems of stability for switched systems under controlled switching. By applying ADT scheme, the disturbance attenuation properties of time-controlled switched systems are investigated in [14]. In [15], the exponential stability and L_2-gain of switched delay systems are studied by using ADT approach. Furthermore, based on ADT method, in [16–18] the stability of switched systems with stable and unstable subsystems co existing was considered. Using the ADT scheme, switching design for exponential stability was proposed in [19] for a class of switched discrete-time constant time-delay systems. By using the multiple Lyapunov function approach and ADT technique, the literature [20] studied the problem of state feedback stabilization of a class of discrete-time switched singular systems with time-varying state delay under asynchronous switching. However, many free weighing matrices were introduced, which made the stability result complicated. In [11], the problem of stabilization and robust H_∞ control via ADT method switching for discrete switched system with time-delay was considered. However, the procedures given in [11] could not be applied to the case of asynchronous switching or the case of switching delay systems with stable and unstable subsystems co existing. This motivates the present study.

On another research front line, it is well known that mathematical modeling of physical systems often results in complex high-order models. However, this causes the great difficulties in analysis and synthesis of the systems. Therefore, in practical applications it is desirable to replace high-order models with reduced-order ones for reducing the computational complexities in some given criteria without incurring much loss of performance or information. The purpose of model reduction is to obtain a lower-order system which approximates a high-order system according to certain criterion. Recently, much attention has been focused on the model reduction problem [21–25]. Many important results have been reported, which involve various efficient approaches, such as the balanced truncation method [24], the optimal Hanker norm reduction method [25], the cone complementarily linearization method [26], and sequential linear programming matrix method [27]. In terms of LMIs with inverse constraints or other non convex conditions, the model reduction of the discrete-time context has been investigated in [28, 29]. However, it is difficult to obtain the numerical solutions. In [30], the existence conditions for H_∞ model reduction for discrete-time uncertain switched systems are derived in terms of strict LMIs by using switched Lyapunov function method. However, time delays are not taken into account. In the literature [31], a novel idea to approximate the original time-delay system by a reduced time-delay model has been proposed recently. However, the unstable subsystems are not taken into account.

Motivated by the preceding discussion, the main contributions of this paper are highlighted as follows. The problem of exponential stability and H_∞ model reduction for a class of switched linear discrete-time systems with time-varying delay have been investigated. To lessen the computation complexity and to reduce the conservatism, new discrete LKF are constructed and the delay interval is divided into two unequal subintervals by the delay decomposition method. The switching law is given by ADT scheme, such that even if one or more subsystem is unstable the overall switched system still can be stable. For the high-order systems, sufficient conditions for the existence of the desired reduced-order model are derived in terms of strict LMIs, which can be easily solved by using MATLAB LMI control toolbox. Finally, numerical examples are given to show the effectiveness of the proposed methods.

The remainder of this paper is structured as follows. In Section 2, the problem formulation and some preliminaries are introduced. In Section 3, the main results are presented on the exponential stability of switched discrete-time systems with time-varying delay. In Section 4, the main results on the H_∞ model reduction for the high-order systems are presented. Numerical examples are given in Section 5. The last section concludes the work.

Notations. We use standard notations throughout the paper. $\lambda_{\min}(M)$ ($\lambda_{\max}(M)$) stands for the minimal (maximum) eigenvalue of M. M^T is the transpose of the matrix M. The relation $M > N$ ($M < N$) means that the matrix $M - N$ is positive (negative) definite. $\|x\|$ denotes the Euclidian-norm of the vector $x \in R^n$. R^n represents the n-dimensional real Euclidean space. $R^{n\times m}$ is the set of all real $n \times m$ matrices. diag$\{\cdots\}$ stands for a block-diagonal matrix. In symmetric block matrices or long matrix expressions, we use an asterisk "$*$" to represent a term that is induced by symmetry. I denotes the identity matrix.

2. Problem Description and Preliminaries

Consider a class of switched linear discrete-time systems with time-varying state delay of the form

$$\begin{aligned} x(k+1) &= A_i x(k) + A_{di} x(k - d(k)) + B_i u(k), \\ y(k) &= C_i x(k) + C_{di} x(k - d(k)) + D_i u(k), \\ x(\theta) &= \phi(\theta), \quad \theta = -h, -h+1, \ldots, 1, \end{aligned} \tag{1}$$

where $x(k) \in R^n$ denotes the system state, $y(k) \in R^m$ is the measured output, $u(k) \in R^p$ is the disturbance input vector which belongs to $l_2[0,\infty)$. $\phi(\theta) \in R^n$ is a vector-valued initial function. The switching signal σ (denoting $\sigma(k)$ for simplicity) : $[0,\infty) \rightarrow \overline{N} = \{1, 2, \ldots, T\}$ is a piecewise constant function and depends on time. $\sigma = i$ means that the ith subsystem is activated. T is the number of subsystems. The system matrices $A_i, A_{di}, B_i, C_i, C_{di}$, and D_i are a set of known real matrices with appropriate dimensions.

For a given finite positive integer $h > 0$, $d(k)$ is time-varying delay and satisfies the following condition

$$0 \le d(k) \le h, \quad \forall k \in N^+. \tag{2}$$

To facilitate theoretical development, we introduce the following definitions and lemmas.

Definition 1 (see [19]). The system (1) with disturbance input $u(k) = 0$ is said to be exponentially stable if there exist a switching function $\sigma(\cdot)$ and positive number c such that every solution $x(k,\phi)$ of the system satisfies

$$\|x(k)\| \le c\lambda^{k-k_0} \|\phi\|_s, \quad \forall k \ge k_0, \tag{3}$$

for any initial conditions $(k_0, \phi) \in R^+ \times C^n$. $c > 0$ is the decay coefficient, $0 < \lambda \le 1$ is the decay rate, and $\|\phi\|_s = \sup\{\|\phi(l)\|, l = k_0 - h, k_0 - h + 1, \ldots, k_0\}$.

Definition 2 (see [11]). Consider the system (1) with the following conditions.

(1) With $u(k) = 0$, the system (1) is exponentially stable with convergence rate $\lambda > 0$.

(2) The H_∞ performance $\|y(k)\|_2 < \gamma\|u(k)\|_2$ is guaranteed for all nonzero $u(k) \in L_2[0,\infty)$ and a prescribed $\kappa > 0$ under the zero condition.

In the above conditions, the system (1) is exponentially stabilizable with H_∞ performance γ and convergence rate λ.

Here γ characterizes the disturbance attenuation performance. The smaller the γ is, the better the performance is.

Definition 3 (see [12]). For a switching signal $\sigma(k)$ and any $T_2 > k > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching of $\sigma(k)$ over (T_1, T_2). If for any given $N_0 \geq 1$ and $T_a > 0$, we have $N_\sigma(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_a$, then T_a and N_0 are called the ADT and the chatter bound, respectively.

Lemma 4 (see [9]). *For any matrix $R = R^T > 0$, integers $a \leq b$, vector function $\xi(k) : \{-b, -b+1, \ldots, -a\} \to R^n$, then*

$$(a-b)\sum_{s=k-b}^{k-a-1} z^T(s) R z(s) \leq \xi^T(k) \begin{bmatrix} -R & R \\ & -R \end{bmatrix} \xi(k). \tag{4}$$

Here

$$\begin{gathered} z(k) = x(k+1) - x(k), \\ \xi^T(k) = \left[x^T(k-a) \;\; x^T(k-b)\right]. \end{gathered} \tag{5}$$

Lemma 5 (Schur complement [32]). *Let M, P, Q be given matrices such that $Q > 0$. Then*

$$\begin{bmatrix} P & M \\ * & -Q \end{bmatrix} < 0 \Longleftrightarrow P + MQ^{-1}M^T < 0. \tag{6}$$

The aim of this paper is to find a class of time-based switching signals for the discrete-time switched time-delay systems (1), whose subsystem is not necessarily stable, to guarantee the system to be exponentially stable. For a high-order system, we are interested in constructing a reduced-order switched system to approximate the system.

3. Stability Analysis

With the preliminaries given in the previous section we are ready to state the exponential stability and H_∞ performance of switched systems (1). To obtain the exponential stability of switched systems (1), we construct following discrete LKF:

$$V_i(k) = V_{i1}(k) + V_{i2}(k) + V_{i3}(k), \quad \forall i \in \overline{N}. \tag{7}$$

Here

$$\begin{aligned} V_{i1}(k) &= x^T(k) P_i x(k), \\ V_{i2}(k) &= \sum_{s=k-\vartheta}^{k-1} (1+\alpha_i)^{k-1-s} x^T(s) Q_{i1} x(s) \\ &\quad + \sum_{s=k-h}^{k-\vartheta-1} (1+\alpha_i)^{k-1-s} x^T(s) Q_{i2} x(s) \\ &\quad + \sum_{s=k-d(k)}^{k-1} (1+\alpha_i)^{k-1-s} x^T(s) Q_{i3} x(s), \\ V_{i3}(k) &= \sum_{\theta=-\vartheta}^{-1} \sum_{s=k+\theta}^{k-1} (1+\alpha_i)^{k-1-s} z^T(s) R_{i1} z(s) \\ &\quad + \sum_{\theta=-h}^{-\vartheta-1} \sum_{s=k+\theta}^{k-1} (1+\alpha_i)^{k-1-s} z^T(s) R_{i2} z(s) \\ &\quad + \sum_{\theta=-d(k)}^{-1} \sum_{s=k+\theta}^{k-1} (1+\alpha_i)^{k-1-s} z^T(s) R_{i3} z(s), \end{aligned} \tag{8}$$

where P_i, Q_{im}, R_{im} $(i \in \overline{N},\ m = 1, 2, 3)$ are symmetric positive definite matrices with appropriate dimensions, $z(k) = x(k+1) - x(k)$, and integer $\vartheta \in (0, h)$ and α_i are given constants.

Remark 6. In order to derive less conservative criteria than the existing ones, the delay interval $[0, h]$ is divided into two unequal subintervals: $[0, \vartheta]$ and $[\vartheta, h]$, where $\vartheta \in (0, h)$ is a tuning parameter. The information about $x(t - \vartheta)$ can be taken into account. This plays a vital role in deriving less conservative results. Thus, for any $k \in Z^+$, we have $d(k) \in [0, \vartheta]$ or $d(k) \in [\vartheta, h]$.

Firstly, we will provide a decay estimation of the system LKF in (7) along the state trajectory of switched system (1) without disturbance input (i.e., $u(k) = 0$).

Lemma 7. *Given constants $-1 < \alpha_i \leq 0$, $h > 0$ and $\vartheta \in (0, h)$, if there exist some symmetric positive definite matrices P_i, Q_{im}, R_{im} $(i \in \overline{N},\ m = 1, 2, 3)$ such that the following LMIs hold:*

$$\begin{bmatrix} \Psi_{i11} & \Psi_{i12} \\ & \Psi_{i22} \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} \Phi_{i11} & \Phi_{i12} \\ & \Phi_{i22} \end{bmatrix} < 0, \tag{10}$$

where

$$\Psi_{i11} = \begin{bmatrix} \psi_{11}^i & \psi_{12}^i & 0 & 0 \\ & \psi_{22}^i & \psi_{23}^i & 0 \\ & * & \psi_{33}^i & \psi_{34}^i \\ & * & * & \psi_{44}^i \end{bmatrix},$$

$$\Phi_{i11} = \begin{bmatrix} \psi_{11}^i & 0 & \phi_{13}^i & 0 \\ & \phi_{22}^i & \phi_{23}^i & \phi_{24}^i \\ & * & \phi_{33}^i & 0 \\ & * & * & \psi_{44}^i \end{bmatrix},$$

$$\Psi_{i12} = \begin{bmatrix} A_i^T P_i & (A_i - I)^T W_{1i}^T \\ A_{di}^T P_i & A_{di}^T W_{1i}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Phi_{i12} = \begin{bmatrix} A_i^T P_i & (A_i - I)^T W_{2i}^T \\ A_{di}^T P_i & A_{di}^T W_{2i}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Psi_{i22} = \operatorname{diag}\{-P_i \;\; -W_{1i}\}, \qquad \Phi_{i22} = \operatorname{diag}\{-P_i \;\; -W_{2i}\},$$

$$\psi_{11}^i = -(1+\alpha_i) P_i - \frac{(1+\alpha_i)^\vartheta}{\vartheta}(R_{i1} + R_{i3}) + Q_{i1} + Q_{i3},$$

$$\psi_{12}^i = \frac{(1+\alpha_i)^\vartheta}{\vartheta}(R_{i1} + R_{i3}),$$

$$\psi_{22}^i = -(1+\alpha_i)^\vartheta Q_{i3} - \frac{(1+\alpha_i)^\vartheta}{\vartheta}(2R_{i1} + R_{i3}),$$

$$\psi_{23}^{i} = \frac{(1+\alpha_i)^{\vartheta}}{\vartheta} R_{i1},$$

$$\psi_{33}^{i} = (1+\alpha_i)^{\vartheta}(Q_{i2} - Q_{i1}) - \frac{(1+\alpha_i)^{h}}{h-\vartheta} R_{i2} - \frac{(1+\alpha_i)^{\vartheta}}{\vartheta} R_{i1},$$

$$\psi_{34}^{i} = \frac{(1+\alpha_i)^{h}}{h-\vartheta} R_{i2},$$

$$\psi_{44}^{i} = -(1+\alpha_i)^{h} Q_{i2} - \frac{(1+\alpha_i)^{h}}{h-\vartheta} R_{i2},$$

$$\phi_{13}^{i} = \frac{(1+\alpha_i)^{\vartheta}}{\vartheta}(R_{i1} + R_{i3}),$$

$$\phi_{22}^{i} = -(1+\alpha_i)^{h} Q_{i3} - \frac{(1+\alpha_i)^{h}}{h-\vartheta}(2R_{i2} + R_{i3}),$$

$$\phi_{23}^{i} = \frac{(1+\alpha_i)^{h}}{h-\vartheta}(R_{i2} + R_{i3}),$$

$$\phi_{24}^{i} = \frac{(1+\alpha_i)^{h}}{h-\vartheta} R_{i2},$$

$$\phi_{33} = -\frac{(1+\alpha_i)^{h}}{h-\vartheta}(R_{i2} + R_{i3}) - \frac{(1+\alpha_i)^{\vartheta}}{\vartheta}(R_{i1} + R_{i3}) - (1+\alpha_i)^{\vartheta}(Q_{i1} - Q_{i2}),$$

$$W_{1i} = (h-\vartheta) R_{i2} + \vartheta R_{i1} + \vartheta R_{i3},$$

$$W_{2i} = (h-\vartheta) R_{i2} + \vartheta R_{i1} + h R_{i3}. \tag{11}$$

Then, by means of LKF (7)*, along the trajectory of the systems* (1) *without disturbance input, one has*

$$\Delta V_i(k) = V_i(k+1) - V_i(k) \le \alpha_i V_i(k). \tag{12}$$

Proof. Let us choose the system LKF (7). Define

$$V_i(k+1) - (1+\alpha_i) V_i(k) = \sum_{m=1}^{3} \widetilde{\Delta} V_{im}(k), \tag{13}$$

where

$$\widetilde{\Delta} V_{im}(k) = V_{im}(k+1) - (1+\alpha_i) V_{im}(k). \tag{14}$$

Therefore, the following equality holds along the solution of (1):

$$\widetilde{\Delta} V_{i1}(k) = x^T(k+1) P_i x(k+1) - (1+\alpha_i) x^T(k) P_i x(k), \tag{15}$$

$$\begin{aligned} \widetilde{\Delta} V_{i2}(k) = {} & x^T(k)(Q_{i1} + Q_{i3}) x(k) \\ & - (1+\alpha_i)^{h} x^T(k-h) Q_{i2} x(k-h) \\ & - (1+\alpha_i)^{d(k)} x^T(k-d(k)) Q_{i3} x(k-d(k)) \\ & - (1+\alpha_i)^{\vartheta} x^T(k-\vartheta)(Q_{i1} - Q_{i2}) x(k-\vartheta), \end{aligned} \tag{16}$$

$$\begin{aligned} \widetilde{\Delta} V_{i3}(k) = {} & z^T(k)((h-\vartheta) R_{i2} + \vartheta R_{i1} + d(k) R_{i3}) z(k) \\ & - \sum_{s=k-\vartheta}^{k-1} (1+\alpha_i)^{k-s} z^T(s) R_{i1} z(s) \\ & - \sum_{s=k-h}^{k-\vartheta-1} (1+\alpha_i)^{k-s} z^T(s) R_{i2} z(s) \\ & - \sum_{s=k-d(k)}^{k-1} (1+\alpha_i)^{k-s} z^T(s) R_{i3} z(s). \end{aligned} \tag{17}$$

For any $k \in Z^{+}$, we have $d(k) \in [0, \vartheta]$ or $d(k) \in [\vartheta, h]$.

(1) If $d(k) \in [0, \vartheta]$, it gets

$$\begin{aligned} & - \sum_{s=k-\vartheta}^{k-1} (1+\alpha_i)^{k-s} z^T(s) R_{i1} z(s) \\ & = - \sum_{s=k-\vartheta}^{k-1-d(k)} (1+\alpha_i)^{k-s} z^T(s) R_{i1} z(s) \\ & \quad - \sum_{s=k-d(k)}^{k-1} (1+\alpha_i)^{k-s} z^T(s) R_{i1} z(s). \end{aligned} \tag{18}$$

So (17) could be

$$\begin{aligned} \widetilde{\Delta} V_{i3}(k) \le {} & z^T(k)((h-\vartheta) R_{i2} + \vartheta R_{i1} + \vartheta R_{i3}) z(k) \\ & - \sum_{s=k-h}^{k-\vartheta-1} (1+\alpha_i)^{k-s} z^T(s) R_{i2} z(s) \\ & - \sum_{s=k-\tau(k)}^{k-1} (1+\alpha_i)^{k-s} z^T(s)(R_{i1} + R_{i3}) z(s) \\ & - \sum_{s=k-\vartheta}^{k-1-\tau(k)} (1+\alpha_i)^{k-s} z^T(s) R_{i1} z(s). \end{aligned} \tag{19}$$

From Lemma 4, we have

$$-\sum_{s=k-h}^{k-\vartheta-1}(1+\alpha_i)^{k-s}z^T(s)R_{i2}z(s) \leq \frac{(1+\alpha_i)^h}{h-\vartheta}\xi_1^T(t)\begin{bmatrix}-R_{i2} & R_{i2}\\ & -R_{i2}\end{bmatrix}\xi_1(t), \tag{20}$$

$$\begin{aligned}-\sum_{s=k-d(k)}^{k-1}&(1+\alpha_i)^{k-s}z^T(s)(R_{i1}+R_{i3})z(s)\\ &\leq \frac{(1+\alpha_i)^{\tau(k)}}{\tau(k)}\xi_2^T(k)\begin{bmatrix}-R_{i1}-R_{i3} & R_{i1}+R_{i3}\\ & -R_{i1}-R_{i3}\end{bmatrix}\xi_2(k)\\ &\leq \frac{(1+\alpha_i)^{\vartheta}}{\vartheta}\xi_2^T(k)\begin{bmatrix}-R_{i1}-R_{i3} & R_{i1}+R_{i3}\\ & -R_{i1}-R_{i3}\end{bmatrix}\xi_2(k),\end{aligned} \tag{21}$$

$$\begin{aligned}-\sum_{s=k-\vartheta}^{k-1-d(k)}&(1+\alpha_i)^{k-s}z^T(s)R_{i1}z(s)\\ &\leq \frac{(1+\alpha_i)^{\vartheta}}{\vartheta-\tau(k)}\xi_3^T(k)\begin{bmatrix}-R_{i1} & R_{i1}\\ & -R_{i1}\end{bmatrix}\xi_3(k)\\ &\leq \frac{(1+\alpha_i)^{\vartheta}}{\vartheta}\xi_3^T(k)\begin{bmatrix}-R_{i1} & R_{i1}\\ & -R_{i1}\end{bmatrix}\xi_3(k),\end{aligned} \tag{22}$$

where

$$\begin{aligned}\xi_1^T(k)&=\left[x^T(k-\vartheta)\;\; x^T(k-h)\right],\\ \xi_2^T(k)&=\left[x^T(k)\;\; x^T(k-d(k))\right],\\ \xi_3^T(k)&=\left[x^T(k-d(k))\;\; x^T(k-\vartheta)\right].\end{aligned} \tag{23}$$

Combining (13)–(22), it yields

$$\begin{aligned}V_i(k+1)&-(1+\alpha_i)V_i(k)\\ &\leq \xi^T(k)\Psi_{i11}\xi(k)\\ &\quad+x^T(k+1)P_ix(k+1)+z^T(k)W_{1i}z(k),\end{aligned} \tag{24}$$

where

$$\xi^T(k)=\left[x^T(k)\;\; x^T(k-d(k))\;\; x^T(k-\vartheta)\;\; x^T(k-h)\right]. \tag{25}$$

Multiplying (9) both from left and right by diag $\{0\;\;0\;\;0\;\;0\;\;P_i^{-1}\;\;W_i^{-T}\}$, by *Schur Complement*, further, considering (24), one can infer that (12) holds.

(2) If $d(k)\in[\vartheta,h]$, it gets

$$\begin{aligned}-\sum_{s=k-d(k)}^{k-1}&(1+\alpha_i)^{k-s}z^T(s)R_{i3}z(s)\\ &=-\sum_{s=k-\vartheta}^{k-1}(1+\alpha_i)^{k-s}z^T(s)R_{i3}z(s)\\ &\quad-\sum_{s=k-d(k)}^{k-\vartheta-1}(1+\alpha_i)^{k-s}z^T(s)R_{i3}z(s).\end{aligned} \tag{26}$$

One obtains

$$\begin{aligned}\widetilde{\Delta}V_{i3}(k)&\leq z^T(k)((h-\vartheta)R_{i2}+\vartheta R_{i1}+hR_{i3})z(k)\\ &\quad-\sum_{s=k-\vartheta}^{k-1}(1+\alpha_i)^{k-s}z^T(s)(R_{i1}+R_{i3})z(s)\\ &\quad-\sum_{s=k-d(k)}^{k-\vartheta-1}(1+\alpha_i)^{k-s}z^T(s)(R_{i2}+R_{i3})z(s)\\ &\quad-\sum_{s=k-h}^{k-d(k)-1}(1+\alpha_i)^{k-s}z^T(s)R_{i2}z(s).\end{aligned} \tag{27}$$

Similarly, it is easy to get that

$$\begin{aligned}V_i(k+1)&-(1+\alpha_i)V_i(k)\\ &\leq \xi^T(k)\Phi_{i11}\xi(k)\\ &\quad+x^T(k+1)P_ix(k+1)+z^T(k)W_{2i}z(k).\end{aligned} \tag{28}$$

If (10) holds, by *Schur Complement*, then we have (12). This completes the proof. □

Remark 8. Our LKF does not include free-weighing matrices as in previous investigations, and this may lead to reduce the computational complexity and get less conservation results.

Remark 9. In order to get less conservative results, the delay interval $[0,h]$ can be divided into much more subintervals. However, when the number of dipartite numbers increases, the matrix formulation becomes more complex and the time-consuming grows bigger.

Now we have the following theorem.

Theorem 10. *If there exist some constants* $-1<\alpha_i<0$ *and positive definite symmetric matrices* P_i, Q_{im}, R_{im} $(i\in\overline{N}, m=1,2,3)$ *and* $\mu\geq 1$ *such that* (9), (10), *and the following inequalities hold:*

$$P_i\leq\mu P_j,\quad Q_{im}\leq\mu Q_{jm},\quad R_{im}\leq\mu R_{jm},\quad \forall i,j\in\overline{N}. \tag{29}$$

Then, the switched system (1) *with* $u(k)=0$ *and ADT satisfies* $\tau_a>-\ln\mu/\ln\alpha$ *which is exponentially stable.*

Proof. By Lemma 7, we have

$$\Delta V_i(k)=V_i(k+1)-V_i(k)\leq\alpha_iV_i(k),\quad \forall i\in\overline{N}. \tag{30}$$

Therefore,

$$V_i(k_0+n) \le (\alpha_i+1)^n V_i(k_0). \quad (31)$$

There exists $\mu_i \ge 1$ $(i \in \overline{N})$, such that

$$V_i(k) \le \mu_i V_j(k), \quad \forall i, j \in \overline{N}. \quad (32)$$

We let $\tau_1, \ldots, \tau_{N_{\sigma(k_0,k_0+k)}}$ denote the switching times of σ in (k_0, k_0+k), and let $N_{\sigma(k_0,k+k_0)}$ be the switching number of σ in $(k_0, k+k_0)$, by (31) and (32), one obtains

$$V_{\sigma(k+k_0)}(k+k_0) \le \mu_{\sigma(\tau_1)} \cdots \mu_{\sigma(\tau_{N_{\sigma(k_0,k_0+k)}})} \left(\alpha_{\sigma(k+k_0)}+1\right)^{m_1} \cdots \left(\alpha_{\sigma(k_0)}+1\right)^{m_{N_{\sigma(k_0,k+k_0)}}} V_{\sigma(k_0)}(k_0), \quad (33)$$

where $m_1 + \cdots + m_{N_{\sigma(k_0,k+k_0)}} = k$.

By $-1 < \alpha_i < 0$, for all $i \in \overline{N}$, we know that there exists $\alpha \triangleq \max_{i\in\overline{N}}\{\alpha_i+1\} \in (0,1)$. Let $\mu = \max_{i\in\overline{N}}\{\mu_i\}$; from (33), one obtains

$$V_i(k+k_0) \le \alpha^k \mu^{N_\sigma} V_j(k_0) = \alpha^{k+N_\sigma(\ln\mu/\ln\alpha)} V_j(k_0). \quad (34)$$

By Definition 2, for any $k_0 < k$, it follows that

$$V_i(k) \le \alpha^{k+N_\sigma(\ln\mu/\ln\alpha)} V_j(k_0) \le \alpha^{k(1+(\ln\mu/T_a\ln\alpha))} V_j(k_0). \quad (35)$$

By the system LKF (7), there always exist two positive constants c_1, c_2 such that

$$c_1\|x(k)\|^2 \le V_i(k), \qquad V_i(k_0) \le c_2\|x(k_0)\|_s^2, \quad (36)$$

where

$$c_1 = \min_{i\in\overline{N}}\{\lambda_{\min}(P_i)\},$$

$$c_2 = \max_{i\in\overline{N}}\left\{\lambda_{\max}(P_i) + \sum_{m=1}^{3}\left(\lambda_{\max}(Q_{im}) + \lambda_{\max}(R_{im})\right)\right\}. \quad (37)$$

Therefore,

$$\|x(k)\|^2 \le \frac{c_2}{c_1}\alpha^{k(1+(\ln\mu/T_a\ln\alpha))}\|x(k_0)\|_s^2. \quad (38)$$

If the average dwell time τ_a satisfies $\tau_a > -\ln\mu/\ln\alpha$ for $\mu \ge 1$, then the switched system (1) with $u(k) = 0$ is exponentially stable with $\lambda = \alpha^{1/2} = \max_{i\in\overline{N}}\{(\alpha_i+1)^{1/2}\} \in (0,1)$ stability degree. □

Remark 11. The case $\alpha = 0$ implies the asymptotic stability.

The following theorem provides exponential stability analysis with H_∞ performance of the system (1).

Theorem 12. *For given constants $\gamma > 0$, $\lambda > 0$ and $-1 < \alpha_i < 0$, if there exist positive definite symmetric matrices P_i, Q_{im}, R_{im} $(i \in \overline{N}, m = 1,2,3)$ and $\mu \ge 1$ such that (29) and the following LMIs hold:*

$$\begin{bmatrix} \Psi_{i11} & 0 & \Psi_{i13} \\ & -\gamma^2 I & \Psi_{i23} \\ & * & \Psi_{i33} \end{bmatrix} < 0, \quad (39)$$

$$\begin{bmatrix} \Phi_{i11} & 0 & \Phi_{i13} \\ & -\gamma^2 I & \Phi_{i23} \\ & * & \Phi_{i33} \end{bmatrix} < 0, \quad (40)$$

where

$$\Psi_{i13} = \begin{bmatrix} A_i^T P_i & (A_i - I)^T W_{1i}^T & C_i^T \\ A_{di}^T P_i & A_{di}^T W_{1i}^T & C_{di}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{i13} = \begin{bmatrix} A_i^T P_i & (A_i - I)^T W_{2i}^T & C_i^T \\ A_{di}^T P_i & A_{di}^T W_{2i}^T & C_{di}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{i33} = \operatorname{diag}\{-P_i \;\; -W_{1i} \;\; -I\},$$

$$\Phi_{i33} = \operatorname{diag}\{-P_i \;\; -W_{2i} \;\; -I\},$$

$$\Psi_{i23} = \begin{bmatrix} B_i^T P_i & B_i^T W_{1i}^T & D_i^T \end{bmatrix}, \qquad \Phi_{i23} = \begin{bmatrix} B_i^T P_i & B_i^T W_{2i}^T & D_i^T \end{bmatrix}. \quad (41)$$

Then, the system (1) with average dwell time satisfies $\tau_a > -\ln\mu/\ln\alpha$ which is globally exponentially stable with convergence rate λ and H_∞ performance γ.

Proof. Choose the LKF (7); the result is carried out by using the techniques employed for proving Lemma 7 and Theorem 10. If $d(k) \in [0, \vartheta]$, by (24), we have

$$\begin{aligned} &V_i(k+1) - \alpha V_i(k) + y^T(k)y(k) - \gamma^2 u^T(k)u(k) \\ &\quad \le x^T(k+1)P_i x(k+1) \\ &\qquad + z^T(k)W_i z(k) + y^T(k)y(k) \\ &\qquad + \zeta_1^T(k)\begin{bmatrix} \Psi_{i11} & 0 \\ 0 & -\gamma^2 \end{bmatrix}\zeta_1(k), \end{aligned} \quad (42)$$

where

$$\zeta_1^T(k) = \begin{bmatrix} \xi^T(k) & u^T(k) \end{bmatrix}. \quad (43)$$

If $d(k) \in [\vartheta, h]$, by (28), we have

$$\begin{aligned} &V_i(k+1) - \alpha V_i(k) + y^T(k)y(k) - \gamma^2 u^T(k)u(k) \\ &\quad \le x^T(k+1)P_i x(k+1) + z^T(k)W_i z(k) + y^T(k)y(k) \\ &\qquad + \zeta_1^T(k)\begin{bmatrix} \Phi_{i11} & 0 \\ 0 & -\gamma^2 \end{bmatrix}\zeta_1(k). \end{aligned} \quad (44)$$

Combining (39) and (40), by *Schur Complement*, one can obtain

$$V_i(k+1) - \alpha V_i(k) + y^T(k)\,y(k) - \gamma^2 u^T(k)\,u(k) \le 0. \tag{45}$$

Let

$$J(k) = y^T(k)\,y(k) - \gamma^2 u^T(k)\,u(k); \tag{46}$$

we have

$$V_i(k+1) \le \alpha V_i(k) - J(k). \tag{47}$$

By Definition 2 and Theorem 10, it is sufficient to show that $\sum_{k=0}^{\infty} J(k) < 0$ for any nonzero $u(k)$. Combining (35) and (47), it can be shown that

$$V_\sigma(k) \le \alpha^k \mu^{N_{\sigma(0,k)}} V_\sigma(0) - \sum_{s=0}^{k-1} \alpha^{k-s-1} \mu^{N_{\sigma(s,k)}} J(s). \tag{48}$$

Under the zero initial condition, we have

$$V(0) = 0, \quad V(\infty) \ge 0. \tag{49}$$

Combining (48), we have

$$\sum_{s=0}^{k-1} \alpha^{k-s-1} \mu^{N_{\sigma(s,k)}} J(s) = \sum_{s=0}^{k-1} \alpha^{-1} e^{\ln\alpha + \ln\mu/\tau_a} J(s) \le 0. \tag{50}$$

Now, we consider

$$\sum_{k=1}^{\infty}\sum_{s=0}^{k-1} \alpha^{-1} e^{\ln\alpha + \ln\mu/\tau_a} J(s). \tag{51}$$

Exchanging the double-sum region, by $\tau_a > -\ln\mu/\ln\alpha$ and $\alpha \in (0,1)$, one can easily get

$$\begin{aligned}
&\sum_{k=1}^{\infty}\sum_{s=0}^{k-1} \alpha^{-1} e^{\ln\alpha + \ln\mu/\tau_a} J(s) \\
&\quad = \sum_{s=0}^{\infty} J(s) \sum_{k=s+1}^{\infty} \alpha^{-1} e^{\ln\alpha + \ln\mu/\tau_a} \\
&\quad = \frac{e^{\ln\alpha + \ln\mu/\tau_a}\alpha^{-1}}{1 - e^{\ln\alpha + \ln\mu/\tau_a}} \sum_{s=1}^{\infty} J(s) \le 0,
\end{aligned} \tag{52}$$

which means that $\sum_{s=1}^{\infty} J(s) \le 0$. Then, by Definition 2, the system (1) with average dwell time satisfies $\tau_a > -\ln\mu/\ln\alpha$ which is globally exponentially stable with convergence rate λ and H_∞ performance γ. This completes the proof. □

If there exist some unstable subsystems in the switched system (1) with $u(k) = 0$, in this case, we need to estimate the growth rate of the system LKF in (7) along the state trajectory of switched system (1). And the corresponding $\alpha_j > 0$ ($j \in \overline{N}$). By using the techniques employed for proving Lemma 7, one can easily obtain the following Lemma.

Lemma 13. *Given constants $\alpha_j > 0$, $h > 0$ and $\vartheta \in (0, h)$, if there exist some symmetric positive definite matrices P_j, Q_{jm}, R_{jm} ($j \in \overline{N}$, $m = 1, 2, 3$) such that the following LMIs hold:*

$$\begin{gathered}
\begin{bmatrix} \overline{\Psi}_{j11} & \Psi_{j12} \\ & \Psi_{j22} \end{bmatrix} < 0, \\
\begin{bmatrix} \overline{\Phi}_{j11} & \Phi_{j12} \\ & \Phi_{j22} \end{bmatrix} < 0,
\end{gathered} \tag{53}$$

where

$$\begin{gathered}
\overline{\Psi}_{j11} = \begin{bmatrix} \overline{\psi}_{11}^j & \overline{\psi}_{12}^j & 0 & 0 \\ & \overline{\psi}_{22}^j & \overline{\psi}_{23}^j & 0 \\ & * & \overline{\psi}_{33}^j & \overline{\psi}_{34}^j \\ & * & * & \overline{\psi}_{44}^j \end{bmatrix}, \\
\overline{\Phi}_{j11} = \begin{bmatrix} \overline{\psi}_{11}^j & 0 & \overline{\phi}_{13}^j & 0 \\ & \overline{\phi}_{22}^j & \overline{\phi}_{23}^j & \overline{\phi}_{24}^j \\ & * & \overline{\phi}_{33}^j & 0 \\ & * & * & \overline{\psi}_{44}^j \end{bmatrix}, \\
\overline{\psi}_{11}^j = -(1+\alpha_j) P_j + Q_{j1} + Q_{j3} - \frac{1}{\vartheta}(R_{j1} + R_{j3}), \\
\overline{\psi}_{12}^j = \frac{1}{\vartheta}(R_{j1} + R_{j3}), \qquad \overline{\psi}_{22}^j = -Q_{j3} - \frac{1}{\vartheta}(2R_{j1} + R_{j3}), \\
\overline{\psi}_{23}^j = \frac{1}{\vartheta} R_{j1}, \qquad \overline{\psi}_{34}^j = \frac{(1+\alpha_j)^\vartheta}{h-\vartheta} R_{j2}, \\
\overline{\psi}_{33}^j = (1+\alpha_j)^\vartheta (Q_{j2} - Q_{j1}) - \frac{(1+\alpha_j)^\vartheta}{h-\vartheta} R_{j2} - \frac{1}{\vartheta} R_{j1}, \\
\overline{\psi}_{44}^j = -(1+\alpha_j)^h Q_{j2} - \frac{(1+\alpha_j)^\vartheta}{h-\vartheta} R_{i2}, \qquad \overline{\phi}_{11}^j = \overline{\psi}_{11}^j, \\
\overline{\phi}_{13}^j = \frac{(1+\alpha_j)}{\vartheta}(R_{j1} + R_{j3}), \\
\overline{\phi}_{22}^j = -(1+\alpha_j)^\vartheta Q_{j3} - \frac{(1+\alpha_j)^\vartheta}{h-\vartheta}(2R_{j2} + R_{j3}), \\
\overline{\phi}_{23}^j = \frac{(1+\alpha_j)^\vartheta}{h-\vartheta}(R_{j2} + R_{j3}), \qquad \overline{\phi}_{24}^j = \frac{(1+\alpha_j)^\vartheta}{h-\vartheta} R_{j2}, \\
\overline{\phi}_{33}^j = -\frac{(1+\alpha_j)^\vartheta}{h-\vartheta}(R_{j2} + R_{j3}) \\
\quad - \frac{(1+\alpha_j)}{\vartheta}(R_{j1} + R_{j3}) - (1+\alpha_j)^\vartheta (Q_{j1} - Q_{j2}).
\end{gathered} \tag{54}$$

Then, by means of LKF (7)*, along the trajectory of the systems* (1) *without disturbance input, one has*

$$\Delta V_j(k) = V_j(k+1) - V_j(k) \le \alpha_j V_j(k). \tag{55}$$

Remark 14. The proof of Lemma 13 is similar to that of Lemma 7 and is thus omitted here. Based on Lemmas 7 and 13, one can easily design the stabilizing switching law to guarantee the system (1) with $u(k) = 0$ to be exponentially stable, although some subsystems are unstable.

Without loss of generality, we can assume that $\overline{N}_u = \{j_1, j_2, \ldots, j_s\}$ is the set of all unstable subsystems and $\overline{N}_s = \{i_{s+1}, i_{s+2}, \ldots, i_p\}$ is the set of all stable subsystems. For simplicity, the LKF (7) is defined as $V_i(\alpha_i, k) \triangleq V_i(k)$. Choose the LKF $V_i(\alpha_i, k)$ $(-1 < \alpha_i < 0,\ i \in \overline{N}_s)$ for the stable subsystem and choose the LKF $V_j(\alpha_j, k)$ $(\alpha_j > 0,\ j \in \overline{N}_u)$ for the unstable subsystem. Then, we have the following conclusion.

Theorem 15. *If there exist some constants* $-1 < \alpha_i < 0$, $\alpha_j > 0$ $(j \neq i,\ i \in \overline{N}_s,\ j \in \overline{N}_u)$ *and positive definite symmetric matrices* P_i, Q_{im}, R_{im}, P_j, Q_{jm}, R_{jm} $(m = 1, 2, 3)$ *and* $\mu \geq 1$ *such that Lemmas 7 and 13 and the following LMIs hold:*

$$P_l \leq \mu P_s, \quad Q_{lm} \leq \mu Q_{sm}, \quad R_{lm} \leq \mu R_{sm}, \quad \forall l, s \in \overline{N}. \tag{56}$$

Then, the switched system (1) *with* $u(k) = 0$ *and the average dwell time satisfies* $\tau_a > \ln \mu / -\kappa$, $T^{\alpha}_{k_0,n+k_0} / T^{\beta}_{k_0,n+k_0} \geq (\ln \beta - \kappa)/(-\ln \alpha + \kappa)$, $\kappa \in (\ln \alpha, 0)$ *which is exponentially stable.*

Proof. Consider the following LKF candidate:

$$V_{\sigma(k)}(k) = \begin{cases} V_i(\alpha_i, k), & \sigma(k) = i \in \overline{N}_s, \\ V_j(\alpha_j, k), & \sigma(k) = j \in \overline{N}_u. \end{cases} \tag{57}$$

By Lemmas 7 and 13, we have

$$V_{\sigma(k+1)}(k+1) \leq (\alpha_{\sigma(k+1)} + 1) V_{\sigma(k+1)}(k). \tag{58}$$

Let $T^{\alpha}_{k_0,n+k_0}$ be the total activity time in which all subsystems satisfied $0 > \alpha_i > -1$ on the interval $(k_0, n + k_0)$ and $T^{\beta}_{k_0,n+k_0} \triangleq n - T^{\alpha}_{k_0,n+k_0}$ the total activity time in which all subsystems satisfied $\alpha_j > 0$ on the interval $(k_0, n + k_0)$. By using the techniques employed for proving Theorem 10, combining (56) and (58), we derive that

$$\begin{aligned} & V_{\sigma(n+k_0)}(n+k_0) \\ & \leq \mu^{N_{\sigma(n+k_0)}} \alpha^{T^{\alpha}_{k_0,n+k_0}} \beta^{T^{\beta}_{k_0,n+k_0}} V_{\sigma(k_0)}(k_0) \\ & = e^{T^{\alpha}_{k_0,n+k_0} \ln \alpha + T^{\beta}_{k_0,n+k_0} \ln \beta + N_{\sigma(k_0,n+k_0)} \ln \mu} V_{\sigma(k_0)}(k_0), \end{aligned} \tag{59}$$

where

$$\alpha \triangleq \max_{i \in \overline{N}_s} \{\alpha_i + 1\} \in (0, 1), \qquad \beta \triangleq \max_{j \in \overline{N}_u} \{\alpha_j + 1\} > 1. \tag{60}$$

By $T^{\alpha}_{k_0,n+k_0} / T^{\beta}_{k_0,n+k_0} \geq (\ln \beta - \kappa)/(-\ln \alpha + \kappa)$, $\kappa \in (\ln \alpha, 0)$, one obtains

$$T^{\alpha}_{k_0,n+k_0} \ln \alpha + T^{\beta}_{k_0,n+k_0} \ln \beta \leq \kappa n. \tag{61}$$

So we have

$$V_{\sigma(n+k_0)}(n+k_0) \leq e^{\kappa n + N_{\sigma(k_0,n+k_0)} \ln \mu} V_{\sigma(k_0)}(k_0). \tag{62}$$

By Definition 2, for any $n + k_0 > k_0$, it follows that

$$\begin{aligned} V_{\sigma(n+k_0)}(n+k_0) & \leq e^{\kappa n + N_{\sigma(k_0,n+k_0)} \ln \mu} V_{\sigma(k_0)}(k_0) \\ & \leq e^{n(\kappa + (\ln \mu / \tau_a))} V_{\sigma(k_0)}(k_0). \end{aligned} \tag{63}$$

By $\tau_a > \ln \mu / -\kappa$, we have $\lim_{k \to \infty} V_{\sigma}(k) = 0$. Moreover, the overall system is exponentially stable. This completes the proof. □

Remark 16. From the proof of Theorem 15, one can see that the obtained exponential stability for the switched system (1) with $u(k) = 0$ is exponential stable with $e^{-1/2}$ stability degree. In order to get a free decay rate, we can replace the condition $\tau_a > \ln \mu / -\kappa$, $T^{\alpha}_{k_0,n+k_0} / T^{\beta}_{k_0,n+k_0} \geq (\ln \beta - \kappa)/(-\ln \alpha + \kappa)$, $\kappa \in (\ln \alpha, 0)$ by $\tau_a > \log^{\mu}_{\epsilon} / -\kappa$, $T^{\alpha}_{k_0,n+k_0} / T^{\beta}_{k_0,n+k_0} \geq (\log^{\beta}_{\epsilon} - \kappa)/(-\log^{\alpha}_{\epsilon} + \kappa)$, $\kappa \in (\log^{\alpha}_{\epsilon}, 0)$, $\epsilon > 1$; then the switched system (1) with $u(k) = 0$ is exponentially stable with $\epsilon^{-1/2}$ stability degree.

Theorem 17. *For given constants* $\gamma > 0$, $-1 < \alpha_i < 0$, $\alpha_j > 0$ $(j \neq i,\ i \in \overline{N}_s,\ j \in \overline{N}_u)$, *if there exist positive definite symmetric matrices* P_i, Q_{im}, R_{im}, P_j, Q_{jm}, R_{jm} $(m = 1, 2, 3)$ *and* $\mu \geq 1$ *such that* (56), (39), (40), *and the following LMIs hold:*

$$\begin{aligned} & \begin{bmatrix} \overline{\Psi}_{j11} & 0 & \Psi_{j13} \\ & -\gamma^2 I & \Psi_{j23} \\ & * & \Psi_{j33} \end{bmatrix} 0, \\ & \begin{bmatrix} \overline{\Phi}_{j11} & 0 & \Phi_{j13} \\ & -\gamma^2 I & \Phi_{j23} \\ & * & \Phi_{j33} \end{bmatrix} < 0, \end{aligned} \tag{64}$$

and $T^{\alpha}_{k_0,n+k_0} / T^{\beta}_{k_0,n+k_0} \geq (\ln \beta - \kappa)/(-\ln \alpha + \kappa)$, $\kappa \in (\ln \alpha, 0)$, *and the average dwell time satisfies* $\tau_a > \ln \mu / -\kappa$; *then the switched system* (1) *is exponentially stable and with* H_∞ *performance* γ.

Remark 18. The proof of Theorem 17 is similar to that of Theorems 12 and 15 and is thus omitted here.

4. H_∞ Model Reduction

In this section, we will approximate system (1) by a reduced-order switched system described by

$$\begin{aligned} \widehat{x}(k+1) &= A_{ri} \widehat{x}(k) + A_{rdi} \widehat{x}(k - d(k)) + B_{ri} u(k), \\ \widehat{y}(k) &= C_{ri} \widehat{x}(k) + C_{rdi} \widehat{x}(k - d(k)) + D_{ri} u(k), \end{aligned} \tag{65}$$

where $\widehat{x}(k) \in R^q$ is the state vector of the reduced-order system with $q < n$ and $\widehat{y}(k) \in R^m$ is the output of reduced-order system. A_{ri}, A_{rdi}, C_{ri}, C_{rdi}, B_{ri}, and D_{ri} are the matrices with compatible dimensions to be determined. The system (65) is assumed to be switched synchronously by switching signal $\sigma(k)$ in system (1).

Augmenting the model of system (1) to include the states of (65), we can obtain the error system as follows:

$$\tilde{x}(k+1) = \widetilde{A}_i\tilde{x}(k) + \widetilde{A}_{di}\tilde{x}(k-d(k)) + \widetilde{B}_i u(k),$$
$$\tilde{e}(k) = \widetilde{C}_i\tilde{x}(k) + \widetilde{C}_{di}\tilde{x}(k-d(k)) + \widetilde{D}_i u(k). \quad (66)$$

Here

$$\widetilde{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_{ri} \end{bmatrix}, \qquad \widetilde{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & A_{rdi} \end{bmatrix},$$
$$\widetilde{B}_i = \begin{bmatrix} B_i \\ B_{ri} \end{bmatrix}, \qquad \tilde{x}(k) = \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}, \quad (67)$$
$$\widetilde{C}_i = [C_i \;\; -C_{ri}], \qquad \widetilde{C}_{di} = [C_{di} \;\; -C_{rdi}],$$
$$\widetilde{D}_i = D_i - D_{rdi}, \qquad \tilde{e}(k) = y(k) - \hat{y}(k).$$

The following theorem gives a sufficient condition for the existence of an admissible H_∞ reduced-order model (65) for system (1).

Theorem 19. *Given constants $0 < \alpha < 1$, $\gamma > 0$, $\mu \geq 1$, $h > 0$, and ϑ $(0 < \vartheta < h)$, if there exist some symmetric positive definite matrices $\widetilde{P}_i, \widetilde{Q}_{im}, \widetilde{R}_{im}$ $(m = 1, 2, 3)$ and matrices X_i, Y_i, L_i, H_i, F_i $(i \in \overline{N})$ such that the following LMIs hold*

$$\begin{bmatrix} \Pi_{i1} & \Pi_{i2} \\ & \Pi_{i3} \end{bmatrix} < 0, \quad (68)$$

$$\begin{bmatrix} \overline{\Pi}_{i1} & \Pi_{i2} \\ & \overline{\Pi}_{i3} \end{bmatrix} < 0, \quad (69)$$

$$\widetilde{P}_i \leq \mu\widetilde{P}_j, \quad \widetilde{Q}_{im} \leq \mu\widetilde{Q}_{jm}, \quad \widetilde{R}_{im} \leq \mu\widetilde{R}_{jm}, \quad \forall i, j \in \overline{N}. \quad (70)$$

Then system (66) with the average dwell time τ_a satisfies $\tau_a > -\ln\mu/\ln\alpha$ which is exponentially stable with an H_∞ norm bound γ.

Here

$$\Pi_{i1} = \begin{bmatrix} \varphi_{11}^i & \varphi_{12}^i & 0 & 0 & 0 \\ & \varphi_{22}^i & \varphi_{23}^i & 0 & 0 \\ & * & \varphi_{33}^i & \varphi_{34}^i & 0 \\ & * & * & \varphi_{44}^i & 0 \\ & * & * & * & \varphi_{55}^i \end{bmatrix},$$

$$\overline{\Pi}_{i1} = \begin{bmatrix} \varphi_{11}^i & 0 & \overline{\varphi}_{13}^i & 0 & 0 \\ & \overline{\varphi}_{22}^i & \overline{\varphi}_{23}^i & \overline{\varphi}_{24}^i & 0 \\ & * & \overline{\varphi}_{33}^i & 0 & 0 \\ & * & * & \varphi_{44}^i & 0 \\ & * & * & * & \varphi_{55}^i \end{bmatrix},$$

$$\Pi_{i2} = \begin{bmatrix} \varphi_{i16}^T & \varphi_{i17}^T & \varphi_{i18}^T \\ \varphi_{i26}^T & \varphi_{i27}^T & \varphi_{i28}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \varphi_{i56}^T & \varphi_{i57}^T & \varphi_{i58}^T \end{bmatrix},$$

$$\Pi_{i3} = \operatorname{diag}\left\{\widetilde{P}_i - 2\widetilde{U}_i \;\; \widetilde{W}_i - 2\widetilde{U}_i \;\; -I\right\},$$

$$\overline{\Pi}_{i3} = \operatorname{diag}\left\{\widetilde{P}_i - 2\widetilde{U}_i \;\; \widehat{W} - 2\widetilde{U}_i \;\; -I\right\},$$

$$\varphi_{11}^i = \widetilde{Q}_{i1} + \widetilde{Q}_{i3} - \alpha\widetilde{P} - \frac{\alpha^\vartheta}{\vartheta}\left(\widetilde{R}_{i1} + \widetilde{R}_{i3}\right),$$

$$\varphi_{12}^i = \frac{\alpha^\vartheta}{\vartheta}\left(\widetilde{R}_{i1} + \widetilde{R}_{i3}\right), \qquad \varphi_{22}^i = -\alpha^\vartheta\widetilde{Q}_{i3} - \frac{\alpha^\vartheta}{\vartheta}\left(2\widetilde{R}_{i1} + \widetilde{R}_{i3}\right),$$

$$\varphi_{23}^i = \frac{\alpha^\vartheta}{\vartheta}\widetilde{R}_{i1}, \qquad \varphi_{34}^i = \frac{\alpha^h}{h-\vartheta}\widetilde{R}_{i2},$$

$$\varphi_{33}^i = \alpha^\vartheta\left(\widetilde{Q}_{i2} - \widetilde{Q}_{i1}\right) - \frac{\alpha^h}{h-\vartheta}\widetilde{R}_{i2} - \frac{\alpha^\vartheta}{\vartheta}\widetilde{R}_{i1},$$

$$\varphi_{44}^i = -\alpha^h\widetilde{Q}_{i2} - \frac{\alpha^h}{h-\vartheta}\widetilde{R}_{i2},$$

$$\varphi_{55}^i = -\gamma^2 I, \qquad \overline{\varphi}_{13}^i = \varphi_{12}^i,$$

$$\overline{\varphi}_{22}^i = -\alpha^h\widetilde{Q}_{i3} - \frac{\alpha^h}{h-\vartheta}\left(2\widetilde{R}_{i2} + \widetilde{R}_{i3}\right),$$

$$\overline{\varphi}_{23}^i = \frac{\alpha^h}{h-\vartheta}\left(\widetilde{R}_{i2} + \widetilde{R}_{i3}\right), \qquad \overline{\varphi}_{24}^i = \varphi_{34}^i,$$

$$\overline{\varphi}_{33} = \alpha^\vartheta\left(\widetilde{Q}_{i2} - \widetilde{Q}_{i1}\right) - \frac{\alpha^\vartheta}{\vartheta}\left(\widetilde{R}_{i1} + \widetilde{R}_{i3}\right) - \frac{\alpha^h}{h-\vartheta}\left(\widetilde{R}_{i2} + \widetilde{R}_{i3}\right),$$

$$\widetilde{W}_i = (h-\vartheta)\widetilde{R}_{i2} + \vartheta\widetilde{R}_{i1} + \vartheta\widetilde{R}_{i3},$$

$$\widehat{W}_i = (h-\vartheta)\widetilde{R}_{i2} + \vartheta\widetilde{R}_{i1} + h\widetilde{R}_{i3},$$

$$\varphi_{i16}^T = \begin{bmatrix} A_i^T X_i^T & A_i^T E^T Y_i \\ 0 & L_i^T \end{bmatrix},$$

$$\varphi_{i17}^T = \begin{bmatrix} A_i^T X_i^T - X_i^T & A_i^T E^T Y_i - E^T Y \\ 0 & L_i^T - Y_i^T \end{bmatrix},$$

$$\varphi_{i18}^T = \begin{bmatrix} C_i^T \\ -C_{ri}^T \end{bmatrix},$$

$$\varphi_{i26}^T = \varphi_{i27}^T = \begin{bmatrix} A_{id}^T X_i^T & A_{id}^T E^T Y_i \\ 0 & H_i^T \end{bmatrix},$$

$$\varphi_{i28}^T = \begin{bmatrix} C_{di}^T \\ -C_{rdi}^T \end{bmatrix},$$

$$\varphi_{i56} = \varphi_{i57} = \begin{bmatrix} X_i B_i \\ F_i + Y_i^T E B_i \end{bmatrix},$$

$$\varphi_{i58} = D_i - D_{rdi}. \quad (71)$$

Furthermore, if a feasible solution to the above LMIs (68), (69), and (70) exists, then the system matrices of an admissible H_∞ reduced-order model in the form of (65) are given by

$$A_{ri} = Y_i^{-1}L_i, \qquad A_{rdi} = Y_i^{-1}H_i, \qquad B_{ri} = Y_i^{-1}F_i. \quad (72)$$

Proof. Consider the following LKF for the switched system (66):

$$V_i(k) = V_{i1}(k) + V_{i2}(k) + V_{i3}(k). \tag{73}$$

Here

$$\begin{aligned} V_{i1}(k) &= \tilde{x}^T(k)\tilde{P}_i\tilde{x}(k), \\ V_{i2}(k) &= \sum_{s=k-\vartheta}^{k-1}\alpha^{k-1-s}\tilde{x}^T(s)\tilde{Q}_{i1}\tilde{x}(s) \\ &\quad + \sum_{s=k-h}^{k-\vartheta-1}\alpha^{k-1-s}\tilde{x}^T(s)\tilde{Q}_{i2}\tilde{x}(s) \\ &\quad + \sum_{s=k-d(k)}^{k-1}\alpha^{k-1-s}\tilde{x}^T(s)\tilde{Q}_{i3}\tilde{x}(s), \\ V_{i3}(k) &= \sum_{\theta=-\vartheta}^{-1}\sum_{s=k+\theta}^{k-1}\alpha^{k-1-s}\tilde{z}^T(s)\tilde{R}_{i1}\tilde{z}(s) \\ &\quad + \sum_{\theta=-h}^{-\vartheta-1}\sum_{s=k+\theta}^{k-1}\alpha^{k-1-s}\tilde{z}^T(s)\tilde{R}_{i2}\tilde{z}(s) \\ &\quad + \sum_{\theta=-d(k)}^{-1}\sum_{s=k+\theta}^{k-1}\alpha^{k-1-s}\tilde{z}^T(s)\tilde{R}_{i3}\tilde{z}(s), \end{aligned} \tag{74}$$

where $\tilde{z}(k) = \tilde{x}(k+1) - \tilde{x}(k)$ and $\tilde{P}_i, \tilde{Q}_{im}, \tilde{R}_{im}$ ($i \in \overline{N}$, $m = 1, 2, 3$) are symmetric positive definite matrices with appropriate dimensions; integer ϑ and α are given constants.

By using the techniques employed for proving Lemma 7, one can easily obtain the result. Calculate the difference of $V_i(k)$ in (73) along the state trajectory of system (66).

(1) If $d(k) \in [0, \vartheta]$, it gets

$$\begin{aligned} &V_i(k+1) - \alpha V_i(k) + \tilde{e}^T(k)\tilde{e}(k) - \gamma^2 u^T(k)u(k) \\ &\quad \le \tilde{\xi}^T(k)\Pi_{i1}\tilde{\xi}(k) \\ &\qquad + \tilde{x}^T(k+1)\tilde{P}_i\tilde{x}(k+1) \\ &\qquad + z^T(k)\widetilde{W}_i z(k) + \tilde{e}^T(k)\tilde{e}(k), \end{aligned} \tag{75}$$

where

$$\begin{aligned} &\tilde{\xi}^T(k) \\ &\quad = \left[\tilde{x}^T(k) \;\; \tilde{x}^T(k-d(k)) \;\; \tilde{x}^T(k-\vartheta) \;\; \tilde{x}^T(k-h) \;\; u^T(k)\right]. \end{aligned} \tag{76}$$

For any appropriately dimensioned matrices $\tilde{P}_i > 0$ and nonsingular matrices $\tilde{U}_i$, we have

$$\left(\tilde{P}_i - \tilde{U}_i\right)^T\tilde{P}_i^{-1}\left(\tilde{P}_i - \tilde{U}_i\right) \ge 0. \tag{77}$$

Thus

$$-\tilde{U}_i^T\tilde{P}_i^{-1}\tilde{U}_i \le \tilde{P}_i - 2\tilde{U}_i. \tag{78}$$

If (68) holds, we have

$$\begin{bmatrix} \Pi_{i1} & \Pi_{i2} \\ & \Theta_{i3} \end{bmatrix} < 0, \tag{79}$$

where

$$\Theta_{i3} = \operatorname{diag}\left\{-\tilde{U}_i^T\tilde{P}_i^{-1}\tilde{U}_i \;\; \tilde{U}_i^T\widetilde{W}_i^{-1}\tilde{U}_i \;\; -I\right\}. \tag{80}$$

Let

$$\begin{aligned} &\tilde{U}_i = \begin{bmatrix} X_i & 0 \\ Y_i^T E & Y_i \end{bmatrix}, \qquad E = \begin{bmatrix} I & 0 \end{bmatrix}, \\ &Y_i A_{ri} = L_i, \qquad Y_i A_{rdi} = H_i, \qquad Y_i B_{ri} = F_i. \end{aligned} \tag{81}$$

Multiplying (79) both from left and right by $\operatorname{diag}\{0\;0\;0\;0\;0\;\tilde{U}_i^{-T}\;\tilde{U}_i^{-T}\;-I\}$, by Schur Complement, further, considering (75), one can infer

$$V_i(k+1) - \alpha V_i(k) + \tilde{e}^T(k)\tilde{e}(k) - \gamma^2 u^T(k)u(k) \le 0. \tag{82}$$

Similarly, for the case of $d(k) \in [\vartheta, h]$, the fact that (69) holds means that (82) is true. Set

$$\Gamma(k) = \tilde{e}^T(k)\tilde{e}(k) - \gamma^2 u^T(k)u(k), \tag{83}$$

we have

$$V_i(k+1) \le \alpha V_i(k) - \Gamma(k). \tag{84}$$

Let $N_{\sigma(k_0,k)}$ be the number of switching times in (k_0, k). From (84) and (70), we can obtain

$$\begin{aligned} V_i(k+k_0) &\le \alpha^k\mu^{N_{\sigma(k_0,k)}}V_i(k_0) \\ &\quad - \sum_{s=k_0}^{k-1}\alpha^{k-s-1}\mu^{N_{\sigma(s,k)}}\Gamma(s) \\ &\le \alpha^{k+N_{\sigma(k_0,k)}(\ln\mu/\ln\alpha)}V_j(k_0) \\ &\quad - \sum_{s=k_0}^{k-1}\alpha^{k-s-1+N_{\sigma(s,k)}(\ln\mu/\ln\alpha)}\mu^{N_{\sigma(s,k)}}\Gamma(s). \end{aligned} \tag{85}$$

Assume the zero disturbances input $u(k) = 0$ to the state equation of system (66). By Definition 2, for any $k_0 < k$, it follows that

$$V_i(k) \le \alpha^{k+N_\sigma(\ln\mu/\ln\alpha)}V_j(k_0) \le \alpha^{k(1+(\ln\mu/\tau_a\ln\alpha))}V_j(k_0). \tag{86}$$

From $\tau_a > -\ln\mu/\ln\alpha$, one obtains $\lim_{k\to\infty}V_i(k) = 0$. There exist $c_n > 0$, $n = 1, 2$, such that

$$c_1\|\tilde{x}(k)\|^2 \le V_i(k), \qquad V_i(k_0) \le c_2\|\tilde{x}(k_0)\|_s^2. \tag{87}$$

Here

$$\begin{aligned} &\|\tilde{x}(k)\|_s = \max_{\theta=-h,\dots,0}\|\tilde{x}(k+\theta)\|, \qquad c_1 = \lambda_{\min}(P_i), \\ &c_2 = \lambda_{\max}(P_i) + \sum_{k=1}^{3}\left(\lambda_{\max}(Q_{ik}) + \lambda_{\max}(R_{ik})\right). \end{aligned} \tag{88}$$

Therefore

$$\|\tilde{x}(k)\|^2 \le \frac{c_2}{c_1}\alpha^{k(1+(\ln\mu/\tau_a\ln\alpha))}\|\tilde{x}(k_0)\|_s^2. \tag{89}$$

If the average dwell time τ_a satisfies $\tau_a > -\ln\mu/\ln\alpha$, then the switched system (66) is exponentially stable with $\lambda = \alpha^{1/2}$ stability degree. For any nonzero $u(k) \in l_2[0,\infty)$, under zero initial condition, combining (68), (69), (70), (85), and (89), one can easily obtain

$$J = \sum_{k=0}^{\infty}\left[\tilde{e}^T(k)\,\tilde{e}(k) - \gamma^2 u^T(k)\,u(k)\right] \le 0. \tag{90}$$

Therefore $\|\tilde{e}(k)\|_2 \le \gamma\|u(k)\|_2$. This completes the proof. □

Remark 20. Recently, authors in [30, 31] have studied the problem of model reduction for discrete-time switched systems. In those papers, time delays are not taken into account. However, in most of the cases in engineering problems, there always exist unknown time-varying delays; moreover, the case of stable and unstable subsystems co exists. Motivated by this, in this paper, we discussed the problem of H_∞ model reduction for switched linear discrete-time systems with time-varying delays via delay decomposition approach [10–12]. Accordingly, numerical results are given for time-varying delay cases.

If there exist some unstable subsystems in the switched system (1), we have the following conclusion.

Theorem 21. *Given constants* $0 < \alpha < 1$, $\beta > 1$, $\gamma > 0$, $\mu \ge 1$, $h > 0$, *and* ϑ $(0 < \vartheta < h)$, *if there exist some symmetric positive definite matrices* $\tilde{P}_i, \tilde{Q}_{im}, \tilde{R}_{im}$ $(m = 1,2,3)$ *and matrices* X_i, Y_i, L_i, H_i, F_i $(i \in \overline{N})$ *such that* (68), (69), (70), *and the following LMIs hold:*

$$\begin{bmatrix}\tilde{\Pi}_{i1} & \Pi_{i2}\\ & \Pi_{i3}\end{bmatrix} < 0,$$
$$\begin{bmatrix}\widehat{\Pi}_{i1} & \Pi_{i2}\\ & \Pi_{i3}\end{bmatrix} < 0. \tag{91}$$

And $T^{\alpha}_{k_0,n+k_0}/T^{\beta}_{k_0,n+k_0} \ge (\ln\beta - \kappa)/(-\ln\alpha + \kappa)$, $\kappa \in (\ln\alpha, 0)$; *then system* (66) *with the average dwell time* τ_a *satisfies* $\tau_a > -\ln\mu/\ln\alpha$ *which is exponentially stable with an* H_∞ *norm bound* γ. *Furthermore, if a feasible solution to the above LMIs* (68), (69), (70), *and* (91) *exists, then the system matrices of an admissible* H_∞ *reduced-order model in the form of* (65) *are given by* (72).

Here,

$$\tilde{\Pi}_{i1} = \begin{bmatrix}\tilde{\varphi}^i_{11} & \tilde{\varphi}^i_{12} & 0 & 0 & 0\\ & \tilde{\varphi}^i_{22} & \tilde{\varphi}^i_{23} & 0 & 0\\ & * & \tilde{\varphi}^i_{33} & \tilde{\varphi}^i_{34} & 0\\ & * & * & \tilde{\varphi}^i_{44} & 0\\ & * & * & * & \tilde{\varphi}^i_{55}\end{bmatrix},$$

$$\widehat{\Pi}_{i1} = \begin{bmatrix}\tilde{\varphi}^i_{11} & 0 & \hat{\varphi}^i_{13} & 0 & 0\\ & \hat{\varphi}^i_{22} & \hat{\varphi}^i_{23} & \hat{\varphi}^i_{24} & 0\\ & * & \hat{\varphi}^i_{33} & 0 & 0\\ & * & * & \tilde{\varphi}^i_{44} & 0\\ & * & * & * & \tilde{\varphi}^i_{55}\end{bmatrix},$$

$$\tilde{\varphi}^i_{11} = \tilde{Q}_{i1} + \tilde{Q}_{i3} - \beta\tilde{P} - \frac{1}{\vartheta}\left(\tilde{R}_{i1} + \tilde{R}_{i3}\right),$$

$$\tilde{\varphi}^i_{12} = \frac{1}{\vartheta}\left(\tilde{R}_{i1} + \tilde{R}_{i3}\right), \qquad \tilde{\varphi}^i_{22} = -\tilde{Q}_{i3} - \frac{1}{\vartheta}\left(2\tilde{R}_{i1} + \tilde{R}_{i3}\right),$$

$$\tilde{\varphi}^i_{23} = \frac{1}{\vartheta}\tilde{R}_{i1}, \qquad \tilde{\varphi}^i_{33} = \beta^{\vartheta}\left(\tilde{Q}_{i2} - \tilde{Q}_{i1}\right) - \frac{\beta^{\vartheta}}{h-\vartheta}\tilde{R}_{i2} - \frac{1}{\vartheta}\tilde{R}_{i1},$$

$$\tilde{\varphi}^i_{34} = \frac{\beta^{\vartheta}}{h-\vartheta}\tilde{R}_{i2}, \qquad \tilde{\varphi}^i_{44} = -\beta^{h}\tilde{Q}_{i2} - \frac{\beta^{\vartheta}}{h-\vartheta}\tilde{R}_{i2},$$

$$\tilde{\varphi}^i_{55} = -\gamma^2 I, \qquad \hat{\varphi}^i_{13} = \frac{\beta}{\vartheta}\left(\tilde{R}_{i1} + \tilde{R}_{i3}\right),$$

$$\hat{\varphi}^i_{22} = -\beta^{\vartheta}\tilde{Q}_{i3} - \frac{\beta^{\vartheta}}{h-\vartheta}\left(2\tilde{R}_{i2} + \tilde{R}_{i3}\right),$$

$$\hat{\varphi}^i_{23} = \frac{\beta^{\vartheta}}{h-\vartheta}\left(\tilde{R}_{i2} + \tilde{R}_{i3}\right), \qquad \hat{\varphi}^i_{24} = \frac{\beta^{\vartheta}}{h-\vartheta}\left(\tilde{R}_{i2}\right),$$

$$\hat{\varphi}^i_{33} = \beta^{\vartheta}\left(\tilde{Q}_{i2} - \tilde{Q}_{i1}\right) - \frac{\beta}{\vartheta}\left(\tilde{R}_{i1} + \tilde{R}_{i3}\right) - \frac{\beta^{\vartheta}}{h-\vartheta}\left(\tilde{R}_{i2} + \tilde{R}_{i3}\right). \tag{92}$$

Remark 22. The proof of Theorem 21 is carried out by using the techniques employed in the previous section and is thus omitted here.

5. Examples

In this section, we consider some numerical examples to illustrate the benefits of our results.

Example 1 (see [20]). Consider the discrete-time switched system (1) with $u(k) = 0$ and the following parameters:

$$A_1 = \begin{bmatrix}0 & 0.3\\ -0.2 & 0.1\end{bmatrix}, \qquad A_{d1} = \begin{bmatrix}0 & 0.1\\ 0 & 0.2\end{bmatrix},$$
$$A_2 = \begin{bmatrix}0 & 0.3\\ -0.2 & -0.1\end{bmatrix}, \qquad A_{d2} = \begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix}. \tag{93}$$

For this system, we choose $\mu = 1.1$ and $\lambda = 0.931$. Applying Theorem 10, by solving the LMIs (9) and (10) and (29), we can obtain the allowable delay upper bound $h = 20$. It is reported, with decay rate $\lambda = 0.931$, that the upper bound h can be obtained as 14 in [19] and 16 in [20]. Therefore, the result in this brief can indeed provide larger delay bounds than the results in [19, 20]. This supports the effectiveness of the proposed idea in Theorem 10 in reducing the conservatism of stability criteria.

Example 2. Consider the discrete-time switched system (1) with $u(k) = 0$ and parameters as follows:

$$A_1 = \begin{bmatrix}0 & 0.3\\ -0.2 & 0.1\end{bmatrix}, \qquad A_{d1} = \begin{bmatrix}0 & 0.1\\ 0 & 0.2\end{bmatrix},$$
$$A_2 = \begin{bmatrix}0 & 0.3\\ -0.2 & -0.1\end{bmatrix}, \qquad A_{d2} = \begin{bmatrix}1.3 & 0.1\\ 0 & 0.9\end{bmatrix}. \tag{94}$$

It is easy to check that the $A_2 + A_{d2}$ is unstable. In this case, we need to find a class of switching signals to guarantee

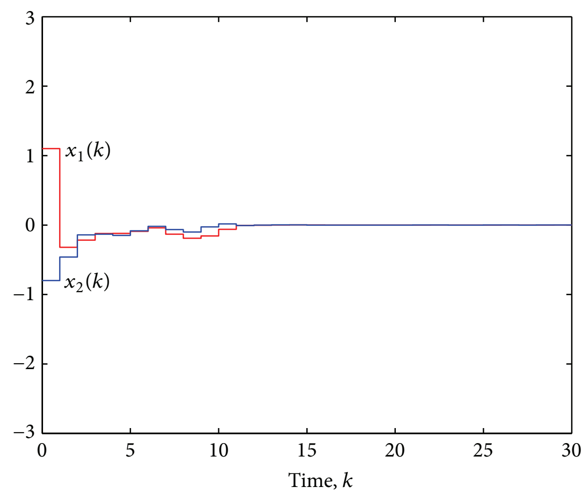

Figure 1: The state response.

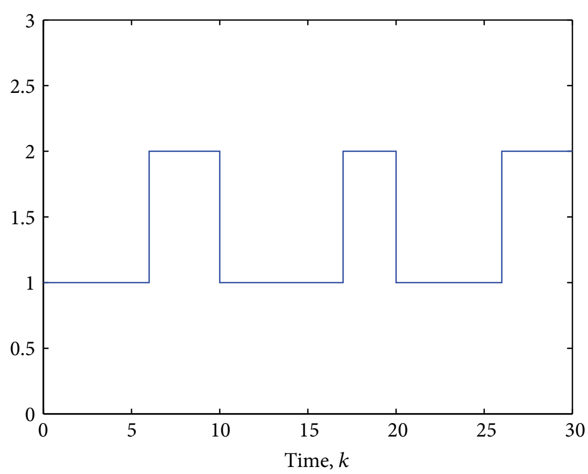

Figure 2: Switching law.

the overall switched system to be exponentially stable. Set $d(k) = [|3\sin(k\pi/6)|]$ and $\alpha = 0.5329$, according to Theorem 15 and by solving the LMIs (9), (10), (53), and (29), set $\vartheta = 1$; we have $\mu = 2.4$ and $\beta = 2.01$. Choosing $\gamma' = -0.18$, we have $T^{\alpha}_{k_0,n+k_0}/T^{\beta}_{k_0,n+k_0} \geq (\ln\beta - \gamma')/(-\ln\alpha + \gamma') = 1.953$ and $\tau_a > \ln\mu/-\gamma' = 4.9$. The simulation result of the switched system is shown in Figure 1, where the initial condition $\phi(\theta) = [1.1\ \ -0.8]^T$ and the switching law is shown in Figure 2. It can be seen from Figure 1 that the designed switching signals are effective although one subsystem is unstable. However, the results in [20] cannot find any feasible solution to guarantee the exponential stability of system (1).

Example 3 (see [31]). Consider the system (1) with parameters as follows:

$$
\begin{gathered}
A_1 = \begin{bmatrix} 0.13 & 0.22 & -0.13 & 0.08 \\ 0.05 & -0.03 & 0.19 & 0.06 \\ -0.07 & -0.05 & -0.04 & -0.12 \\ -0.17 & 0.21 & 0.03 & 0.28 \end{bmatrix}, \\
A_2 = \begin{bmatrix} 0.11 & 0.22 & -0.13 & 0.08 \\ 0.05 & -0.03 & 0.15 & 0.06 \\ -0.07 & -0.03 & -0.04 & -0.12 \\ -0.17 & 0.21 & 0.03 & 0.2 \end{bmatrix}, \\
A_{d1} = A_{d2} = \begin{bmatrix} 0.02 & 0.01 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.02 & 0.01 \\ 0 & 0 & 0 & 0.02 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 0.19 & -0.18 & 0.16 & -0.08 \end{bmatrix}^T, \\
B_2 = \begin{bmatrix} 0.23 & -0.13 & 0.16 & -0.04 \end{bmatrix}^T, \\
C_1 = C_2 = \begin{bmatrix} 1.2 & 0.5 & 0.03 & 0.28 \end{bmatrix}, \\
C_{d1} = C_{d2} = \begin{bmatrix} 0.02 & 0.05 & 0.01 & 0.09 \end{bmatrix}, \\
D_1 = D_2 = 0.1.
\end{gathered} \tag{95}
$$

When the decay rate α is fixed, the maximum value of the time-delay h and the minimum value of the performance index γ can computed by solving the LMIs (68)–(70) procedure in Theorem 19, which is listed in Table 1 via different methods. Here, we choose $\mu = 1.001$. Assume that decay rate $\alpha = 0.9$; we can compute the maximum value of allowed delay $h = 42$ and the minimum value of the performance index $\gamma = 1.67$. From ADT $\tau_a > -\ln\mu/\ln\alpha$, we have $\tau_a > 0.0095$. When $h = 2$ and $\alpha = 0.9$, we can compute the minimum value of performance index $\gamma = 0.53$. On the other hand, assume that maximum allowed delay $h = 2$ and performance index $\gamma = 2$; we can compute the minimum value of the decay rate $\alpha = 0.59$ and $\tau_a > 0.0019$.

Let $\alpha = 0.9$; here, we are interested in designing a q-order ($q < 4$) system (65) and choose the ADT $\tau_a = 2$ switching signals such that the model error system (66) is exponentially stable with H_∞ norm bound $\gamma = 2$. By solving the corresponding LMIs (68)–(70) procedure in Theorem 19. For comparison with [31], we set the delay $d(k) = 2$, and the following reduced-order models can be given.

Third Order Model

$$
A_{r1} = \begin{bmatrix} 0.2753 & 0.0282 & -0.0033 \\ 0.0097 & 0.2507 & -0.0033 \\ -0.0045 & -0.0124 & 0.2569 \end{bmatrix},
$$

$$
A_{r2} = \begin{bmatrix} 0.2799 & 0.0259 & -0.0058 \\ 0.0074 & 0.2581 & -0.0025 \\ -0.0051 & -0.01 & 0.2611 \end{bmatrix},
$$

$$
A_{rd1} = \begin{bmatrix} -0.005 & 0.0069 & -0.0023 \\ 0.0037 & -0.0046 & 0.003 \\ -0.0011 & 0.0002 & -0.0006 \end{bmatrix},
$$

$$
A_{rd2} = \begin{bmatrix} -0.001 & 0.0066 & -0.0025 \\ 0.0039 & -0.0044 & 0.0033 \\ -0.0018 & 0.001 & -0.0024 \end{bmatrix},
$$

Table 1: Comparison of parameters via different methods.

	α	γ	h	τ_a
[31]	0.9	2	2	>1.7305
Theorem 19	0.9	1.67	42	>0.0095
Theorem 19	0.9	0.53	2	>0.0095
Theorem 19	0.59	2	2	>0.0019
Theorem 19	0.6	1.8	2	>0.002

$$\begin{gathered} B_{r1} = \begin{bmatrix} -0.171 & 0.1795 & -0.111 \end{bmatrix}^T, \\ B_{r2} = \begin{bmatrix} -0.191 & 0.148 & -0.1285 \end{bmatrix}^T, \\ C_{r1} = \begin{bmatrix} -0.3016 & -0.1328 & -0.0149 \end{bmatrix}, \\ C_{r2} = \begin{bmatrix} -0.2987 & -0.1265 & -0.0173 \end{bmatrix}, \\ D_{r1} = -0.1754, \\ C_{rd1} = \begin{bmatrix} -0.0314 & -0.0047 & -0.0182 \end{bmatrix}, \\ C_{rd2} = \begin{bmatrix} -0.0361 & -0.0011 & -0.0199 \end{bmatrix}, \\ D_{r2} = -0.2396. \end{gathered} \tag{96}$$

Second Order Model

$$\begin{gathered} A_{r1} = \begin{bmatrix} 0.2419 & 0.0355 \\ 0.015 & 0.2141 \end{bmatrix}, \qquad A_{rd1} = \begin{bmatrix} -0.0028 & 0.0088 \\ 0.0052 & -0.007 \end{bmatrix}, \\ B_{r1} = \begin{bmatrix} -0.1528 \\ 0.1617 \end{bmatrix}, \qquad C_{r1} = \begin{bmatrix} -0.3109 \\ -0.1453 \end{bmatrix}^T, \\ A_{r2} = \begin{bmatrix} 0.2382 & 0.0324 \\ 0.0147 & 0.2183 \end{bmatrix}, \qquad A_{rd2} = \begin{bmatrix} -0.0023 & 0.0076 \\ 0.0046 & -0.006 \end{bmatrix}, \\ B_{r2} = \begin{bmatrix} -0.1667 \\ 0.1203 \end{bmatrix}, \qquad C_{r2} = \begin{bmatrix} -0.3076 \\ -0.1362 \end{bmatrix}^T, \\ C_{rd1} = \begin{bmatrix} -0.0488 & 0.0034 \end{bmatrix}, \qquad D_{r1} = -0.2422, \\ C_{rd2} = \begin{bmatrix} -0.05 & 0.0057 \end{bmatrix}, \qquad D_{r2} = -0.3605. \end{gathered} \tag{97}$$

First Order Model

$$\begin{gathered} A_{r1} = 0.2528, \qquad A_{rd1} = -0.0057, \qquad B_{r1} = -0.1498, \\ C_{r1} = -0.2769, \qquad C_{rd1} = 0.0301, \qquad D_{r1} = -0.1792, \\ A_{r2} = 0.2606, \qquad A_{rd2} = -0.005, \qquad B_{r2} = -0.1787, \\ C_{r2} = -0.2851, \qquad C_{rd2} = -0.04, \qquad D_{r2} = -0.2624. \end{gathered} \tag{98}$$

To illustrate the model reduction performances of the obtained reduced-order models, let the initial condition be zero; the exogenous input is given as $u(k) = 1.8\exp(-0.4k)$. The output errors between the original system and the corresponding three reduced models obtained in this paper (shown by the blue line) and the literature [31] (shown by the red line) are displayed in Figures 3, 4, and 5. The switching signal is shown in Figure 6. The simulation result of the switched system is shown in Figures 3–5. It can be seen from Figures 3–5 that the output errors between the original system and the reduced-order models obtained in this paper are smaller than that in [31].

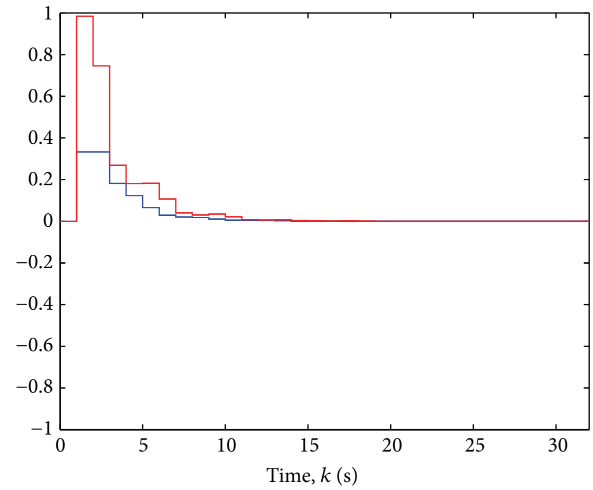

Figure 3: Output errors between the original system and the 3rd model.

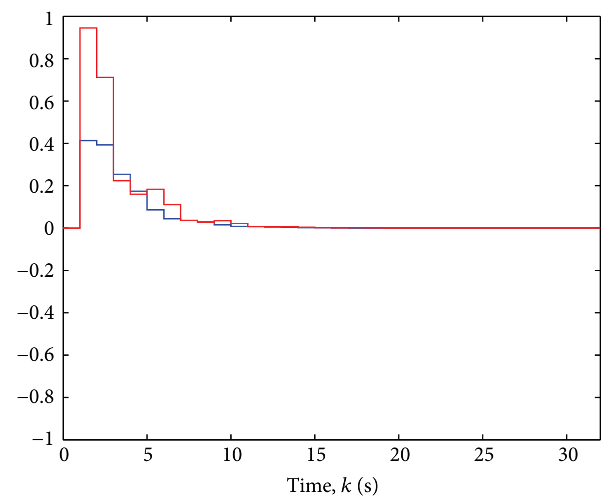

Figure 4: Output errors between the original system and the 2nd model.

6. Conclusions

The problem of exponential stability with H_∞ performance and H_∞ model reduction for a class of switched linear discrete-time systems with time-varying delay have been investigated in this paper. The switching law is given by ADT technique, such that even if one or more subsystem is unstable the overall switched system can still be exponentially stable. Sufficient conditions for the existence of the desired

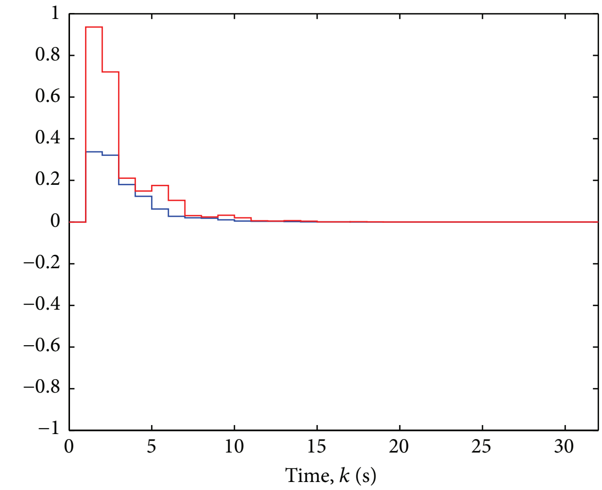

FIGURE 5: Output errors between the original system and the 1st model.

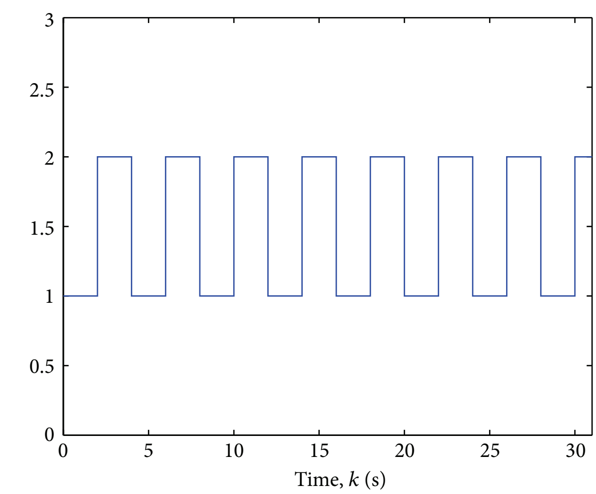

FIGURE 6: Switching law.

reduced-order model are derived and formulated in terms of strict LMIs. By solving the LMIs, the system of reduced-order model can be obtained, which also provides an H_∞ gain for the error system between the original system and the reduced-order model. Finally, numerical examples are provided to illustrate the effectiveness and less conservativeness of the proposed method. A potential extension of this method to nonlinear case deserves further research.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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