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Research Article

A Data-Driven Reliability Estimation Approach for Phased-Mission Systems

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We attempt to address the issues associated with reliability estimation for phased-mission systems (PMS) and present a novel data-driven approach to achieve reliability estimation for PMS using the condition monitoring information and degradation data of such system under dynamic operating scenario. In this sense, this paper differs from the existing methods only considering the static scenario without using the real-time information, which aims to estimate the reliability for a population but not for an individual. In the presented approach, to establish a linkage between the historical data and real-time information of the individual PMS, we adopt a stochastic filtering model to model the phase duration and obtain the updated estimation of the mission time by Bayesian law at each phase. At the meanwhile, the lifetime of PMS is estimated from degradation data, which are modeled by an adaptive Brownian motion. As such, the mission reliability can be real time obtained through the estimated distribution of the mission time in conjunction with the estimated lifetime distribution. We demonstrate the usefulness of the developed approach via a numerical example.

1. Introduction

1.1. Background. Many complex systems are designed to perform missions that consist of phases or stages in which deterioration and configuration of the system may change from phase to phase. These systems are called phased-mission systems in the literature. Formally, phased-mission system (PMS) is defined as the system subject to multiple, consecutive, and nonoverlapping phases of operation required to finish the final product or service [1, 2]. These systems were first introduced by [3] and a vast literature has accumulated since then. Most of real-world systems operate in phased missions where the reliability structure varies over consecutive time periods, known as phases. During each phase, the PMS has to accomplish a specified task. Thus, the system behavior can change from phase to phase. Particularly, a typical PMS which is frequently studied is represented by the on-board systems for the aided guide of aircraft, whose mission consists of takeoff, ascent, cruise, approach, and landing phases. Another example is NASA's Mars Exploration Rover Mission, which consists of many phases like vehicle launch, cruise, approach, entry, descent and landing to Mars, rover egress, and a number of surface operations that involve scientific data collection and transmission to earth. For mission success, all phases must be completed without failure. If the system cannot be repaired during the mission then it is known as a nonrepairable phased mission [4].

Reliability serves as an important measure for system design, operation, and maintenance and has been long recognized as a metric to quantify the performance of the engineering systems. Therefore, reliable and accurate estimates of the reliability of PMS are important for the maintenance and logistic support of such systems, which can lead to lifecycle cost reduction and avoiding catastrophic failures. In this paper we focus on the reliability estimation for PMS but with an emphasis on data-driven method as discussed later. Here the data mean the condition monitoring data obtained from the sensors.

1.2. The Literature Review. Reliability engineering research has developed many methods to analyze the reliability of the PMS, in which the fault tree analysis methods are mainstream. The earliest of these methods involved the direct manipulation of the fault trees. Esary and Ziehms [3] introduced a fault tree based method to transform a phased mission into an equivalent single phase mission. The transformed phase fault trees are then combined into a single fault tree, and standard fault tree methods are used to derive the system's reliability (see, e.g., [5–8]) However, these methods cause the size of the problem to become very large as the number of phases increases.

Recently, it is recognized that increasing the solution efficiency is particularly important for real-time analysis, where the timeliness of the analysis results is crucial [9]. Several papers addressed the issue of reducing the computational burden, including [10–13]. Even so, the fault tree based methods remain unsuitable for analyzing large systems within reasonable timeframes. This led to the adoption of the more efficient, powerful binary decision diagram (BDD) technique [4, 9].

Over the past decade, researchers have proposed a set of new algorithms based on BDD for fault tree analysis of a wide range of PMS. Zang et al. [14] proposed an algorithm for nonrepairable systems with general failure distribution. This work was the first to use the BDD method to analyze the reliability of phased-mission systems and marked a significant step forward by enabling large phased-mission systems to be analyzed. Xing and Dugan [15] analyze a more general class of systems which includes phased-mission systems with combinational phase requirement and imperfect coverage. Other important recent papers on generalized phased-mission systems including [16–19]. In a recent study, Çekyay and Özekici [20] analyzed the reliability of mission-based systems under a general setting by proposing three different reliability definitions. Çekyay and Özekici [21] further extended this line of research by analyzing the availability of mission-based systems under the maximal repair policy.

As observed in the literature, the current approaches are heavily dependent on the knowledge of the structure of the PMS to estimate the reliability of PMS. However, in practice, the structure of the mission system at hand is too complicated to determine and the complete knowledge is not always available. This leads to a great difficulty to apply these approaches for reliability estimation of a practical PMS. In addition, all focuses on a population of common type and there is no work directly establishing the link between the reliability and the historical data/real-time condition monitoring (CM) information of individual PMS in service. These approaches only consider the static scenario with an offline nature. Finally, most of previous works assume that the degradation of the PMS follows a finite state continuous/discrete-time Markov chain. This makes the lifetime estimation of the PMS depends only upon the current state. These limits drive our primary motivation to develop a novel reliability estimation approach for PMS.

1.3. The Proposed Approach. Due to the rapid development of information and sensing technologies, an abundance of data is now readily available in many real-world PMS. This profusion of process/product measurement data provides opportunities for effective reliability estimation through the full exploitation of the data-rich environment [22, 23]. To our best knowledge, there is no report on how to use such CM data to analyze the reliability of the PMS. Therefore, the primary purpose in the paper is to provide a useful answer to the above question.

In this paper we attempt to address the issues associated with reliability estimation for PMS and present a novel approach to achieve reliability estimation for PMS using the CM information and degradation data of such system under dynamic operating scenario. In order to establish a linkage between the historical data and real-time information of the individual PMS, we adopt a stochastic filtering model to model the phase duration and obtain the updated estimation of the mission time by the Bayesian law. At the meanwhile, the lifetime of PMS is estimated by the degradation data, which is modeled by an adaptive Brownian motion. As such, the mission reliability can be obtained through the estimated distribution of the mission time in conjunction with the estimated lifetime distribution. This is a new contribution but not documented before. We demonstrate the usefulness of the developed approach via a numerical example.

The remainder parts are organized as follows. Section 2 gives the problem description. In Section 3, we formulate the mission time estimation from the CM information. Section 4 formulates the degradation data-based lifetime estimation for the mission system. Section 5 discusses the mission reliability and presents the formulations. In Section 6, we provide a numerical study for illustration. Section 7 draws up the main conclusions and comments on the future research.

2. Problem Description and Assumptions

2.1. Problem Description. In this paper, we consider a multiphase mission process having N phases. Let X_n denote the duration of the nth phase, which is a random variable taking values in $\mathfrak{R}^+ = [0, +\infty)$. Further, we let a random variable T_M denote the total time of completing the mission. Thus, the random variable T_M can be represented as $T_M = \sum_{n=1}^{N} X_n$. If there are some linkages among X_n, $n = 1, \ldots, N$, such as the probability density function (PDF) $p_{X_n \mid X_1, k, X_{n-1}}(x_n \mid x_1, K, x_{n-1})$, for $2 \le n \le N$, then the PDF of the mission time T_M can be estimated from the historical data. However, this mechanism is aimed for the population of this type of mission systems. To achieve the aim for a specific system, we need to estimate the mission time at each phase using the CM information at the current time t_i, denoted by $\Phi_{i,n}$, which is related to the mission phase duration X_n. Here we represent the estimated PDF of the mission time as $p_{T_M \mid \Phi_{1:i}}(t_m \mid \Phi_{i,1})$, which shows the dependency of the estimated mission time on the CM information to date. Further, let a random variable T_d denote the lifetime of the mission system. To estimate the PDF of the lifetime from the observed degradation data to t_i, denoted by Y_i, we use the degradation modeling technique, in which the estimated PDF of T_d is represented as $f_{T_d \mid Y_i}(t_d \mid Y_i)$.

After obtaining the estimated $p_{T_M \mid \Phi_{1:i}}(t_m \mid \Phi_{i,1})$ and $f_{T_d \mid Y_i}(t_d \mid Y_i)$, our primary objective is to compute two kinds of the mission reliability. Here we specifically summarize the general formulations for these two cases as follows.

(i) Compute the probability that the mission can be successfully accomplished before a given time R without the system failure, formulated as $\Pr(T_M \le R \mid T_M \le T_d, \Phi_{i,n}, Y_i)$.

(ii) Compute the probability that the mission can be successfully accomplished before the system fails, formulated as $\Pr(T_d \ge T_M \mid \Phi_{i,n}, Y_i)$.

2.2. Assumptions

(1) No maintenance activities are involved during the process of carrying out a mission.

(2) The mission consists of a set of consecutive phases.

(3) For mission success, all phases must be completed.

(4) The phases of mission are sequential; that is, the order of the mission phase is deterministic.

(5) The durations of the different phases are dependent.

(6) The duration of every phase is random following a general distribution.

(7) The degradation process is independent of the mission process.

(8) Failure resulting from degradation will lead to a mission failure.

(9) The duration of the future phase is only dependent on the current and previous phases' duration; for example, $p_{X_2|X_1,\Phi_{i,1}}(x_2 \mid x_1, \Phi_{i,1}) = p_{X_2|X_1=x_1}(x_2 \mid x_1)$, and $p_{X_3|X_1,X_2,\Phi_{i,1}}(x_3 \mid x_1, x_2, \Phi_{i,1}) = p_{X_3|X_1,X_2}(x_3 \mid x_1, x_2)$.

Assumptions except (6) and (9) have already been widely adopted in the literature. Assumption (6) makes our focus on the random phase duration with a general distribution. Assumption (9) is used for model simplification but is also practical. For example, at the first phase, we only observe the CM information $\Phi_{i,1}$ which is related to the duration of the first phase. Therefore, given X_1 and $\Phi_{i,1}$, it is reasonable to assume that the duration X_2 of the second phase is only dependent on X_1. Following the same procedure, given X_1, X_2, and $\Phi_{i,1}$, the duration X_3 of the third phase is only dependent on X_1 and X_2, and so on.

3. Model Formulation for Mission Process to Estimate the Mission Time

Without loss of generality, we consider a three-phase mission process for illustration. In the following, we treat the model formation for the mission system phase by phase.

3.1. Model Formulation for the First Phase. Considering that the exact duration of the phase is unknown in its operation, but one thing we do know is that, over a monitoring interval of time, the duration is just an interval shorter at the end of the interval than at the beginning of the interval if nothing happened during that interval. In the meantime we may observe an increasing or decreasing trend of the monitored CM information $\phi_{i,1}$. Based on these observations, the problem can be formulated as follows with a simple and intuitive form. If we define $L_{i,1}$ as the remaining duration of the first phase at time t_i, the current monitoring check point, with the realization $l_{i,1}$, and the relationship between $L_{i,1}$ and $L_{i+1,1}$ can be described as $L_{i+1,1} = L_{i,1} - (t_{i+1} - t_i)$, if $L_{i+1,1} > t_{i+1} - t_i$. It is noted that $L_{0,1}$ is actually the duration of the first phase. Furthermore, the duration of the mission time is always positive and thus we use the transformation $Z_{i,1} = \ln L_{i,1}$ with the realization $z_{i,1}$ to guarantee $L_{i,1} > 0$. In order to estimate $L_{i,1}$ from $\phi_{i,1}$, we need to model the stochastic relationship between $l_{i,1}$ and $\phi_{i,1}$. To do so, we use a concept called a floating scale parameter to model the relationship between $z_{i,1}$ and $\phi_{i,1}$ which is modeled by a stochastic distribution in this paper [24–26]. The basic idea was to let the mean parameter of $\phi_{i,1}$ be a function of $z_{i,1}$, which enables an updating mechanism of the mean parameter.

Together with the above description, the relationship among $L_{i,1}$, $L_{i+1,1}$, $Z_{i,1}$, and $\phi_{i,1}$ can be described in [24] as follows:

$$
\begin{aligned}
L_{i+1,1} &= \begin{cases} L_{i,1} - (t_{i+1} - t_i) & \text{if } L_{i+1,1} > t_{i+1} - t_i, \\ \text{not defined} & \text{otherwise,} \end{cases} \\
Z_{i,1} &= \ln L_{i,1}, \\
\phi_{i,1} &= g_1(z_{i,1}) + \eta_{i,1},
\end{aligned}
\tag{1}
$$

where $g_1(z_{i,1})$ is a function to be determined, which describes the relationship between the mission process and the CM data relative to the duration of the phase, and $\eta_{i,1}$ is the normally distributed measurement error represented as $\eta_{i,1} \sim N(0, \sigma_1^2)$.

Therefore, the key for remaining time estimation is to formulate the relationship between $l_{i+1,1}$ and the condition monitoring history $\Phi_{i,1}$. By the classical stochastic filtering theory, it can be shown that this relationship can be established recursively as follows:

$$
\begin{aligned}
&p_{L_{i+1,1}|\Phi_{i,1}}(l_{i+1,1} \mid \Phi_{i,1}) \\
&\quad = \left(p(\phi_{i,1} \mid l_{i+1,1})\, p_{L_{i,1}|\Phi_{i-1,1}}(l_{i+1,1} + t_{i+1} - t_i \mid \Phi_{i-1,1})\right) \\
&\quad\quad \times \left(\int_0^\infty p(\phi_{i,1} \mid l_{i+1,1})\, p_{L_{i,1}|\Phi_{i-1,1}} \right. \\
&\quad\quad\quad \left. \times (l_{i+1,1} + t_{i+1} - t_i \mid \Phi_{i-1,1})\, dl_{i+1,1} \right)^{-1}.
\end{aligned}
\tag{2}
$$

In order to solve and formulate the above equation explicitly, we here develop a method using the extended Kalman filtering (EKF) technique based on the work in [24], in which the EKF technique was used to estimate the residual life. As aforementioned, the duration of the mission time must be positive. As such, we define $L_{i,1}$ as a log-normal random variable and thus $Z_{i,1} = \ln L_{i,1}$ as the unknown state of the model (1). After obtaining the CM information $\phi_{i,1}$ at t_i, we can use the EKF to estimate/update the conditional PDF of $Z_{i,1}$ and further the remaining duration $L_{i,1}$. We denote the updated and one-step predicted conditional PDF of $Z_{i,1}$ as $Z_{i,1} \mid \Phi_{i,1} \sim N(z_{i|i,1}, P_{i|i,1})$ and $Z_{i+1,1} \mid \Phi_{i,1} \sim N(z_{i+1|i,1}, P_{i+1|i,1})$, respectively, where the parameters $z_{i|i,1}$, $P_{i|i,1}$, $z_{i+1|i,1}$, and $P_{i+1|i,1}$ can be obtained by the EKF as follows. Specifically, the updating equation of the expectation of the state $Z_{i,1}$ can be formulated as

$$z_{i|i,1} = z_{i|i-1,1} + K_{i,1}\left[\phi_{i,1} - g_1\left(z_{i|i-1,1}\right)\right], \quad (3)$$

where $K_{i,1}$ is the Kalman gain function formulated by

$$K_{i,1} = \left[P_{i|i-1,1} g_1'\left(z_{i|i-1,1}\right)\right]\left[g_1'(z_{i|i-1,1})^2 P_{i|i-1,1} + \sigma_1^2\right]^{-1}, \quad (4)$$

where $g_1'(z_{i|i-1,1}) = dg_1(z_{i,1})/dz_{i,1}|_{z_{i,1}=z_{i|i-1,1}}$.

Correspondingly, the updating equation for the estimation variance can be obtained as

$$P_{i|i,1} = P_{i|i-1,1}\left(1 - K_{i,1} g_1'\left(z_{i|i-1,1}\right)\right). \quad (5)$$

When applying the above EKF methodology, we need to initiate the algorithm at the start of the mission phase using the parameters $z_{0|0,1}$ and $P_{0|0,1}$, which can be estimated from historical data. In addition, in the above updating equations, it is required to calculate the one-step estimation for the expectation $z_{i|i-1,1}$ and variance $P_{i|i-1,1}$. In the following, we present one method to obtain these quantities.

Considering that $Z_{i,1} \mid \Phi_{i,1} \sim N(z_{i|i,1}, P_{i|i,1})$ and $Z_{i,1} = \ln L_{i,1}$, we can obtain

$$E\left[L_{i,1} \mid \Phi_{i,1}\right] = e^{z_{i|i,1}+0.5P_{i|i,1}}, \quad (6)$$

with the associated variance

$$\operatorname{var}\left[L_{i,1} \mid \Phi_{i,1}\right] = \left(e^{P_{i|i,1}} - 1\right) e^{2z_{i|i,1}+P_{i|i,1}}. \quad (7)$$

The above two equations are implied from the relationship between the normal distribution and the log-normal distribution. Thus, we further have $L_{i,1} \mid \Phi_{i,1} \sim \log N(z_{i|i,1}, P_{i|i,1})$. Then, based on the first equation in (1), a one-step forecasting of the remaining mission phase duration from t_i to t_{i+1} is

$$E\left[L_{i+1,1} \middle| \Phi_{i,1}\right] = \begin{cases} E\left[L_{i,1} \middle| \Phi_{i,1}\right] - (t_{i+1} - t_i), & \text{if } E\left[L_{i,1} \middle| \Phi_{i,1}\right] > t_{i+1} - t_i \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Since the change in the established state equation is deterministic over the interval (t_i, t_{i+1}), the variance about the mean estimate is thus formulated as

$$\operatorname{var}\left[L_{i+1,1} \mid \Phi_{i,1}\right] = \operatorname{var}\left[L_{i,1} \mid \Phi_{i,1}\right]. \quad (9)$$

By reversing the relationships given in (6) and (7) and together with the previous results, $E[L_{i+1,1} \mid \Phi_{i,1}]$ can be transformed into $z_{i+1|i,1}$ for the next CM time as

$$\begin{aligned} z_{i+1|i,1} &= \ln\left[E\left(L_{i+1,1} \mid \Phi_{i,1}\right)\right] - 0.5\ln\left(1 + \frac{\operatorname{var}\left(L_{i+1,1} \mid \Phi_{i,1}\right)}{E(L_{i+1,1} \mid \Phi_{i,1})^2}\right) \\ &= \ln\left[E\left[L_{i,1} \mid \Phi_{i,1}\right] - (t_{i+1} - t_i)\right] - 0.5\ln\left(1 + \frac{\operatorname{var}\left(L_{i,1} \mid \Phi_{i,1}\right)}{\left[E\left(L_{i,1} \mid \Phi_{i,1}\right) - (t_{i+1} - t_i)\right]^2}\right) \\ &= \ln\left[e^{z_{i|i,1}+0.5P_{i|i,1}} - (t_{i+1} - t_i)\right] - 0.5\ln\left(1 + \frac{\left(e^{P_{i|i,1}} - 1\right) e^{2z_{i|i,1}+P_{i|i,1}}}{\left[e^{z_{i|i,1}+0.5P_{i|i,1}} - (t_{i+1} - t_i)\right]^2}\right). \end{aligned} \quad (10)$$

Furthermore, without any random variation in the prediction of the state, we have

$$P_{i+1|i,1} = P_{i|i,1}. \quad (11)$$

Using these results, the expectation $z_{i|i-1,1}$ and variance $P_{i|i-1,1}$ can be straightforwardly formulated as

$$\begin{aligned} z_{i|i-1,1} &= \ln\left[e^{z_{i-1|i-1,1}+0.5P_{i-1|i-1,1}} - (t_i - t_{i-1})\right] - 0.5\ln\left(1 + \frac{\left(e^{P_{i-1|i-1,1}} - 1\right) e^{2z_{i-1|i-1,1}+P_{i-1|i-1,1}}}{\left[e^{z_{i-1|i-1,1}+0.5P_{i-1|i-1,1}} - (t_i - t_{i-1})\right]^2}\right), \\ P_{i|i-1,1} &= P_{i-1|i-1,1}. \end{aligned} \quad (12)$$

From the above updating equation, the estimated PDF of the remaining duration of the first phase, $p_{L_{i,1}|\Phi_{i,1}}(l_{i,1} \mid \Phi_{i,1})$, can be formulated as

$$p\left(L_{i,1} \mid \Phi_{i,1}\right) = \frac{1}{l_{i,1}\sqrt{2\pi P_{i|i,1}}} e^{-(2P_{i|i,1})^{-1}(\ln l_{i,1} - z_{i|i,1})^2}. \quad (13)$$

According to the relationship between the duration of the first phase and its remaining duration, we have

$$X_1 \mid \Phi_{i,1} = L_{i,1} + t_i. \quad (14)$$

Then we directly estimate the distribution of X_1 according to the variable transformation as

$$p_{X_1|\Phi_{i,1}}\left(x_1 \mid \Phi_{i,1}\right) = p_{L_{i,1}|\Phi_{1:i}}\left(x_1 - t_i \mid \Phi_{i,1}\right), \quad (15)$$

where $p_{L_{i,1}|\Phi_{1:i}}(x_1 - t_i \mid \Phi_{i,1})$ can be calculated by (13). Therefore, the duration of the second stage conditional on the data up to t_i and X_1 can be computed as

$$\begin{aligned} p_{X_2|\Phi_{i,1}}\left(x_2 \mid \Phi_{i,1}\right) &= \int p_{X_2,X_1|\Phi_{i,1}}\left(x_1, x_2 \mid \Phi_{i,1}\right) dx_1 \\ &= \int p_{X_2|X_1,\Phi_{i,1}}\left(x_2 \mid x_1, \Phi_{i,1}\right) p_{X_1|\Phi_{i,1}}\left(x_1 \mid \Phi_{i,1}\right) dx_1 \\ &= \int p_{X_2|X_1}\left(x_2 \mid x_1\right) p_{X_1|\Phi_{i,1}}\left(x_1 \mid \Phi_{i,1}\right) dx_1 \\ &= \int p_{X_2|X_1}\left(x_2 \mid x_1\right) p_{L_{i,1}|\Phi_{1:i}}\left(x_1 - t_i \mid \Phi_{i,1}\right) dx_1. \end{aligned} \quad (16)$$

Similarly, the duration of the third phase conditional on the data up to t_i and X_1 can be computed as

$$p_{X_3|\Phi_{i,1}}(x_3 \mid \Phi_{i,1}) = \int p_{X_1,X_2,X_3|,\Phi_{i,1}}(x_1, x_2, x_3 \mid \Phi_{i,1})\, dx_1 dx_2. \tag{17}$$

Since $p_{X_1,X_2,X_3|\Phi_{i,1}}(x_1, x_2, x_3 \mid \Phi_{i,1}) = p_{X_3|X_1,X_2}(x_3 \mid x_1, x_2) p_{X_2|X_1}(x_2 \mid x_1) p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1})$, we have

$$\begin{aligned} p_{X_3|\Phi_{i,1}}(x_3 \mid \Phi_{i,1}) &= \int p_{X_3|X_1,X_2,\Phi_{i,1}}(x_3 \mid x_1, x_2, \Phi_{i,1})\, p_{X_2|X_1,\Phi_{i,1}} \\ &\quad \times (x_2 \mid x_1, \Phi_{i,1})\, p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1})\, dx_1 dx_2 \\ &= \int p_{X_3|X_2,X_1}(x_3 \mid x_2, x_1)\, p_{X_2|X_1} \\ &\quad \times (x_2 \mid x_1, \Phi_{i,1})\, p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1})\, dx_1 dx_2 \\ &= \int p_{X_3|X_2,X_1}(x_3 \mid x_2, x_1) \int p_{X_2|X_1}(x_2 \mid x_1)\, p_{L_{i,1}|\Phi_{i,1}} \\ &\quad \times (x_1 - t_i \mid \Phi_{i,1})\, dx_1 dx_2. \end{aligned} \tag{18}$$

Therefore, the distribution of the mission time T_m, denoted by $T_M \mid \Phi_{i,1} = (X_1 + X_2 + X_3) \mid \Phi_{i,1}$, can be calculated as

$$\begin{aligned} &\Pr(T_M \le t_m \mid \Phi_{i,1}) \\ &\quad = \Pr(X_1 + X_2 + X_3 \le t_m \mid \Phi_{i,1}) \\ &\quad = \Pr(0 < X_1 \le t_m, 0 < X_2 \le t_m \\ &\qquad - X_1, 0 < X_3 \le t_m - X_1 - X_2 \mid \Phi_{i,1}) \\ &\quad = \int_0^{t_m} \int_0^{t_m - x_1} \int_0^{t_m - x_1 - x_2} p_{X_1,X_2,X_3|\Phi_{i,1}} \\ &\qquad \times (x_1, x_2, x_3 \mid \Phi_{i,1})\, dx_1 dx_2 dx_3 \\ &\quad = \int_0^{t_m} \int_0^{t_m - x_1} \int_0^{t_m - x_1 - x_2} p_{X_3|X_1,X_2}(x_3 \mid x_1, x_2) \\ &\qquad \times p_{X_2|X_1}(x_2 \mid x_1) \\ &\qquad \times p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1})\, dx_1 dx_2 dx_3 \\ &\quad = \int_0^{t_m} p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1}) \\ &\qquad \times \int_0^{t_m - x_1} p_{X_2|X_1}(x_2 \mid x_1) \\ &\qquad \times \int_0^{t_m - x_1 - x_2} p_{X_3|X_1,X_2}(x_3 \mid x_1, x_2)\, dx_3 dx_2 dx_1 \\ &\quad = \int_0^{t_m} p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1}) \\ &\qquad \times \int_0^{t_m - x_1} p_{X_2|X_1}(x_2 \mid x_1) \\ &\qquad \times \Pr{}_{X_3|X_1,X_2}(t_m - x_1 - x_2 \mid x_1, x_2)\, dx_2 dx_1. \end{aligned} \tag{19}$$

Then, by differentiating $\Pr(T_M \le t_m \mid \Phi_{i,1})$ regarding to t_m, we have the PDF of the mission time as follows:

$$\begin{aligned} &p_{T_M|\Phi_{1:i}}(t_m \mid \Phi_{i,1}) \\ &\quad = \int_0^{t_m} p_{X_1|\Phi_{i,1}}(x_1 \mid \Phi_{i,1}) \\ &\qquad \times \int_0^{t_m - x_1} p_{X_2|X_1}(x_2 \mid x_1) \\ &\qquad \times p_{X_3|X_1,X_2}(t_m - x_1 - x_2 \mid x_1, x_2)\, dx_2 dx_1. \end{aligned} \tag{20}$$

From the above formulation, we can obtain the estimated mission time from the observed CM data associated with the duration of the first phase.

3.2. Model Formulation for the Second Phase. In this case, it is worth noting that the second phase duration is dependent on the termination time of the first phase. In a similar way to the case of the first phase, the relationship between $L_{i,2}$, $L_{i+1,2}$, $Z_{i,2}$, and $\phi_{i,2}$ can be described as

$$\begin{aligned} L_{i+1,2} &= \begin{cases} L_{i,2} - (t_{i+1} - t_i) & \text{if } L_{i+1,2} > t_{i+1} - t_i \\ \text{not defined} & \text{otherwise,} \end{cases} \\ Z_{i,2} &= \ln L_{i,2}, \\ \phi_{i,2} &= g_2(z_{i,2}) + \eta_{i,2}, \end{aligned} \tag{21}$$

where $g_2(z_{i,2})$ is a function to be determined and $\eta_{i,2}$ is the measurement error which is normally distributed as $\eta_{i,2} \sim N(0, \sigma_2^2)$.

After obtaining the CM information $\phi_{i,2}$ at t_i, we can use the EKF to estimate/update the conditional PDF of the remaining duration $L_{i,2}$ on the basis of $\Phi_{i,2}$. Define $Z_{i,2} \mid \Phi_{i,2} \sim N(z_{i|i,2}, P_{i|i,2})$ and $Z_{i+1,2} \mid \Phi_{i,2} \sim N(z_{i+1|i,2}, P_{i+1|i,2})$. As Section 3.1, the updating equation of the expectation of the state $Z_{i,2}$ can be formulated as

$$\begin{aligned} z_{i|i,2} &= z_{i|i-1,2} + K_{i,2}\left[\phi_{i,2} - g_2(z_{i|i-1,2})\right], \\ K_{i,2} &= \left[P_{i|i-1,2}\, g_2'(z_{i|i-1,2})\right]\left[g_2'(z_{i|i-1,2})^2 P_{i|i-1,2} + \sigma_2^2\right]^{-1}, \\ P_{i|i,2} &= P_{i|i-1,2}\left(1 - K_{i,2}\, g_2'(z_{i|i-1,2})\right), \end{aligned} \tag{22}$$

where $g_2'(z_{i|i-1,2}) = dg_2(z_{i,2})/dz_{i,2}|_{z_{i,2}=z_{i|i-1,2}}$.

Further, the one-step estimation for the expectation $z_{i|i-1,2}$ and variance $P_{i|i-1,2}$ can be formulated as follows:

$$\begin{aligned} z_{i|i-1,2} &= \ln\left[e^{z_{i-1|i-1,2} + 0.5 P_{i-1|i-1,2}} - (t_i - t_{i-1})\right] \\ &\quad - 0.5 \ln\left(1 + \frac{\left(e^{P_{i-1|i-1,2}} - 1\right) e^{2 z_{i-1|i-1,2} + P_{i-1|i-1,2}}}{\left[e^{z_{i-1|i-1,2} + 0.5 P_{i-1|i-1,2}} - (t_i - t_{i-1})\right]^2}\right), \\ P_{i|i-1,2} &= P_{i-1|i-1,2}. \end{aligned} \tag{23}$$

From the above updating equation, upon obtaining the CM information $\phi_{i,2}$ at t_i, the estimated PDF of the remaining duration of the second phase, $p_{L_{i,2}|\Phi_{i,2}}(l_{i,2} \mid \Phi_{i,2})$, can be formulated as

$$p\left(L_{i,2} \mid \Phi_{i,2}\right) = \frac{1}{l_{i,2}\sqrt{2\pi P_{i|i,2}}} e^{-(2P_{i|i,2})^{-1}(\ln l_{i,2} - z_{i|i,2})^2}. \tag{24}$$

According to the relationship between the duration of the second phase X_2 and its remaining duration $L_{i,2}$, we have

$$X_2 \mid \Phi_{i,2} = L_{i,2} + \left(t_i - x_1\right). \tag{25}$$

Here, x_1 is known since at this time the first phase of the mission process has been accomplished by our model setting of the sequential nature of the mission phase. This distinguishes the second phase from the first phase. Based on this fact, we directly have the distribution of X_2 according to the variable transformation as

$$p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) = p_{L_{i,2}|\Phi_{i,2}}\left(x_2 - \left(t_i - x_1\right) \mid \Phi_{i,2}\right), \tag{26}$$

where $p_{L_{i,2}|\Phi_{i,2}}(x_2 - (t_i - x_1) \mid \Phi_{i,2})$ can be calculated by (24).

Therefore, the duration of the third stage conditional on the data up to t_i can be estimated as

$$\begin{aligned} p_{X_3|\Phi_{i,2}}&\left(x_3 \mid \Phi_{i,2}\right) \\ &= \int p_{X_2,X_3|,\Phi_{i,2}}\left(x_3, x_2 \mid \Phi_{i,2}\right) dx_2 \\ &= \int p_{X_3|X_2,\Phi_{1:i}}\left(x_3 \mid x_2, \Phi_{i,2}\right) p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) dx_2 \\ &= \int p_{X_3|X_2}\left(x_3 \mid x_2\right) p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) dx_2 \\ &= \int p_{X_3|X_2}\left(x_3 \mid x_2\right) p_{L_{i,2}|\Phi_{i,2}}\left(x_2 - \left(t_i - x_1\right) \mid \Phi_{i,2}\right) dx_2. \end{aligned} \tag{27}$$

Accordingly, the distribution of the mission time T_m can be calculated by

$$\begin{aligned} \Pr&\left(T_M \le t_m \mid \Phi_{i,2}\right) \\ &= \Pr\left(x_1 + X_2 + X_3 \le t_m \mid \Phi_{i,2}\right) \\ &= \Pr\left(X_2 + X_3 \le t_m - x_1 \mid \Phi_{i,2}\right) \\ &= \Pr\left(0 < X_2 \le t_m - x_1, 0 < X_3 \le t_m - x_1 - X_2 \mid \Phi_{i,2}\right) \\ &= \int_0^{t_m - x_1} \int_0^{t_m - x_1 - x_2} p_{X_2,X_3|\Phi_{i,2}}\left(x_2, x_3 \mid \Phi_{i,2}\right) dx_3 dx_2. \end{aligned} \tag{28}$$

Since $p_{X_2,X_3|\Phi_{i,2}}(x_2, x_3 \mid \Phi_{i,2}) = p_{X_3|X_2}(x_3 \mid x_2) p_{X_2|\Phi_{i,2}}(x_2 \mid \Phi_{i,2})$, the above equation becomes

$$\begin{aligned} \Pr&\left(T_M \le t_m \mid \Phi_{i,2}\right) \\ &= \Pr\left(x_1 + X_2 + X_3 \le t_m \mid \Phi_{i,2}\right) \\ &= \Pr\left(X_2 + X_3 \le t_m - x_1 \mid \Phi_{i,2}\right) \\ &= \Pr\left(0 < X_2 \le t_m - x_1, 0 < X_3 \le t_m - x_1 - X_2 \mid \Phi_{i,2}\right) \\ &= \int_0^{t_m - x_1} \int_0^{t_m - x_1 - x_2} p_{X_2,X_3|\Phi_{i,2}}\left(x_2, x_3 \mid \Phi_{i,2}\right) dx_3 dx_2 \\ &= \int_0^{t_m - x_1} \int_0^{t_m - x_1 - x_2} p_{X_3|X_2}\left(x_3 \mid x_2\right) \\ &\qquad \times p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) dx_3 dx_2 \\ &= \int_0^{t_m - x_1} \left(p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) \right. \\ &\qquad \left. \times \int_0^{t_m - x_1 - x_2} p_{X_3|X_2}\left(x_3 \mid x_2\right) dx_3 \right) dx_2 \\ &= \int_0^{t_m - x_1} p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) \\ &\qquad \times \Pr_{X_3|X_2}\left(t_m - x_1 - x_2 \mid x_2\right) dx_2. \end{aligned} \tag{29}$$

Then, by differentiating $\Pr(T_M \le t_m \mid \Phi_{i,2})$ regarding to t_m, we have the PDF of the mission time as

$$\begin{aligned} p_{T_M|\Phi_{i,2}}&\left(t_m \mid \Phi_{i,2}\right) \\ &= \int_0^{t_m - x_1} p_{X_2|\Phi_{i,2}}\left(x_2 \mid \Phi_{i,2}\right) p_{X_3|X_2}\left(t_m - x_1 - x_2 \mid x_2\right) dx_2. \end{aligned} \tag{30}$$

3.3. Model Formulation for the Third Phase. In this case, the mission process is in the third phase and then in a similar way the relationship between $L_{i,3}$, $L_{i+1,3}$, $Z_{i,3}$, and $\phi_{i,3}$ can be similarly described as

$$\begin{gathered} L_{i+1,3} = \begin{cases} L_{i,3} - \left(t_{i+1} - t_i\right) & \text{if } L_{i+1,3} > t_{i+1} - t_i \\ \text{not defined} & \text{otherwise,} \end{cases} \\ Z_{i,3} = \ln L_{i,3}, \\ \phi_{i,3} = g_3\left(z_{i,3}\right) + \eta_{i,3}, \end{gathered} \tag{31}$$

where $g_3(z_{i,3})$ is a function to be determined and $\eta_{i,3}$ is the measurement error which is normally distributed as $\eta_{i,3} \sim N(0, \sigma_3^2)$.

After obtaining the CM information $\phi_{i,3}$ at t_i, the updating equation of the expectation of the state $Z_{i,3}$ can be formulated as

$$\begin{gathered} z_{i|i,3} = z_{i|i-1,3} + K_{i,3}\left[\phi_{i,3} - g_3\left(z_{i|i-1,3}\right)\right], \\ K_{i,3} = \left[P_{i|i-1,3} g_3'\left(z_{i|i-1,3}\right)\right]\left[g_3'(z_{i|i-1,3})^2 P_{i|i-1,3} + \sigma_3^2\right]^{-1}, \\ P_{i|i,3} = P_{i|i-1,3}\left(1 - K_{i,3} g_3'\left(z_{i|i-1,3}\right)\right), \end{gathered} \tag{32}$$

where $g_3'(z_{i|i-1,3}) = dg_3(z_{i,3})/dz_{i,3}|_{z_{i,3}=z_{i|i-1,3}}$.

Further, the one-step estimation for the expectation $z_{i|i-1,3}$ and variance $P_{i|i-1,3}$ can be formulated as

$$z_{i|i-1,3} = \ln\left[e^{z_{i-1|i-1,3}+0.5P_{i-1|i-1,3}} - (t_i - t_{i-1})\right] - 0.5\ln\left(1 + \frac{\left(e^{P_{i-1|i-1,3}} - 1\right)e^{2z_{i-1|i-1,3}+P_{i-1|i-1,3}}}{\left[e^{z_{i-1|i-1,3}+0.5P_{i-1|i-1,3}} - (t_i - t_{i-1})\right]^2}\right),$$
$$P_{i|i-1,3} = P_{i-1|i-1,3}. \tag{33}$$

From the above updating equation, upon obtaining the CM information $\phi_{i,3}$ at t_i, the estimated $p_{L_{i,3}|\Phi_{i,3}}(l_{i,3} \mid \Phi_{i,3})$ can be formulated as

$$p(L_{i,3} \mid \Phi_{i,3}) = \frac{1}{l_{i,3}\sqrt{2\pi P_{i|i,3}}} e^{-(2P_{i|i,3})^{-1}(\ln l_{i,3} - z_{i|i,3})^2}. \tag{34}$$

According to the relationship between X_3 and its remaining duration $L_{i,3}$, we have

$$X_3 \mid \Phi_{i,3} = L_{i,3} + (t_i - x_1 - x_2). \tag{35}$$

In this case, x_1 and x_2 are known since at this time the first and second phases of the mission process have been accomplished. Based on this fact, we directly have

$$p_{X_3|\Phi_{i,3}}(x_3 \mid \Phi_{i,3}) = p_{L_{i,3}|\Phi_{i,3}}(x_3 - (t_i - x_1 - x_2) \mid \Phi_{i,3}), \tag{36}$$

where $p_{L_{i,3}|\Phi_{i,3}}(x_3 - (t_i - x_1 - x_2) \mid \Phi_{i,3})$ can be calculated by (34).

Therefore, the distribution of the mission time T_m conditional on the related CM information $\Phi_{i,3}$, denoted by $T_M \mid \Phi_{i,3} = (X_1 + X_2 + X_3)|\Phi_{i,3}$, can be calculated by

$$\begin{aligned}\Pr(T_M \le t_m \mid \Phi_{i,3}) &= \Pr(x_1 + x_2 + X_3 \le t_m \mid \Phi_{i,3}) \\ &= \Pr(X_3 \le t_m - x_1 - x_2 \mid \Phi_{i,3}) \\ &= \int_0^{t_m - x_1 - x_2} p_{X_3|\Phi_{i,3}}(x_3 \mid \Phi_{i,3})\, dx_3 \\ &= \Pr_{X_3|\Phi_{i,3}}(t_m - x_1 - x_2 \mid \Phi_{i,3}).\end{aligned} \tag{37}$$

Then, differentiating $\Pr(T_M \le t_m \mid \Phi_{i,3})$ regarding to t_m yields

$$p_{T_M|\Phi_{1:i}}(t_m \mid \Phi_{i,3}) = p_{X_3|\Phi_{i,3}}(t_m - x_1 - x_2 \mid \Phi_{i,3}). \tag{38}$$

So far, we have completed the task of formulating the mission time distribution based on the related CM information.

4. Model Formulation for System Degradation Process to Estimate the Lifetime

In this paper, we use a Wiener process to model the degradation process of the mission system. Without loss of generality, we assume that the start reading of the degradation process is $Y(0) = 0$. Then, the evolution of the monitored variable over time can be described as

$$Y(t) = \lambda t + \sigma B(t). \tag{39}$$

This type of Wiener process-based model is a typical model used in the literature to characterize the evolving path of the degradation process [27–31]. Considering the potential for updating knowledge of the process, we model the degradation process over time since t_i as

$$Y(t) = y_i + \lambda(t - t_i) + \sigma B(t - t_i). \tag{40}$$

To incorporate the history of the observations, we consider an updating procedure for the drifting parameter λ by making λ evolve as $\lambda_i = \lambda_{i-1} + \eta$, where $\eta \sim N(0, Q)$. In order to establish the linkage between the drift parameter and the observation history up to date, the degradation equation can be reconstructed and taken to be a self-organizing state-space modelas

$$\begin{aligned}\lambda_i &= \lambda_{i-1} + \eta, \\ y_i &= y_{i-1} + \lambda_{i-1}(t_i - t_{i-1}) + \sigma\varepsilon_i,\end{aligned} \tag{41}$$

where $\eta \sim N(0, Q)$ and $\varepsilon_i \sim N(0, t_i - t_{i-1})$. The updated estimation of λ_i can be obtained from Algorithm 1.

Due to Gaussian's assumption and the principle of Bayesian filtering, we can obtain the PDF of λ_i at t_i as

$$f_{\lambda_i|Y_i}(\lambda_i \mid Y_i) = \frac{1}{\sqrt{2\pi P_{i|i}}} \exp\left[\frac{-\left(\lambda_i - \hat{\lambda}_i\right)^2}{2P_{i|i}}\right]. \tag{42}$$

Based on the threshold, the remaining useful life (RUL) modeling principle is presented as follows. When degradation $Y(t)$ modeled by (40) reaches a preset critical level w, the plant can be declared to fail. Therefore, it is natural to view the event of lifetime termination as the point that the degradation process $Y(t)$ exceeds the threshold level w for the first time. Therefore, using the concept of the first hitting time, we define the RUL S_i at time t_i as

$$S_i = \inf\{s_i : Y(s_i + t_i) \ge w \mid Y_i\}, \tag{43}$$

with the cumulative density function (cdf) $F_{S_i|Y_i}(s_i \mid Y_i)$ and the PDF $f_{S_i|Y_i}(s_i \mid Y_i)$.

Considering the adaptive mechanism introduced by the state-space model (41), we can predict the future degradation at t_i, represented by the PDF $f_{Y(s_i+t_i)|Y_i}(y \mid Y_i)$. This distribution is normal and can be written as $Y(s_i + t_i) \mid Y_i \sim N(y_i + \hat{\lambda}_i s_i, P_{i|i}s_i^2 + \sigma^2 s_i)$. Further, according to the standard theory of Wiener process, it is direct to obtain the PDF and CDF of the RUL at time t_i as follows:

$$f_{S_i|\lambda_i,Y_i}(t \mid \lambda_i, Y_i) = \frac{w - y_i}{\sqrt{2\pi s_i^3\sigma^2}} \exp\left(-\frac{(w - y_i - \lambda_i s_i)^2}{2\sigma^2 s_i}\right), \quad s_i > 0,$$

Step 1. Initialize $\widehat{\lambda}_0$, P_0.
Step 2. State estimation at time t_i

$$P_{i|i-1} = P_{i-1|i-1} + Q$$
$$K_i = (t_i - t_{i-1})^2 P_{i|i-1} + \sigma^2 (t_i - t_{i-1})$$
$$\widehat{\lambda}_i = \widehat{\lambda}_{i-1} + P_{i|i-1}(t_i - t_{i-1}) K_i^{-1} \left(y_i - y_{i-1} - \widehat{\lambda}_{i-1}(t_i - t_{i-1}) \right)$$

Step 3. Updating variance $P_{i|i} = P_{i|i-1} - P_{i|i-1}(t_i - t_{i-1})^2 K_i^{-1} P_{i|i-1}$

Algorithm 1: Kalman filtering algorithm.

$$F_{S_i|\lambda_i,Y_i}(t \mid \lambda_i, Y_i) = 1 - \Phi\left(\frac{w - y_i - \lambda_i s_i}{\sigma\sqrt{s_i}}\right) + \exp\left(\frac{2\lambda_i (w - y_i)}{\sigma^2}\right)\Phi\left(\frac{-(w - y_i) - \lambda_i s_i}{\sigma\sqrt{s_i}}\right). \tag{44}$$

As mentioned previously, the drift parameter is evolving as a random variable in model (41) with a distribution $f_{\lambda_i|Y_i}(\lambda_i \mid Y_i)$. To consider the impact of the adaptive mechanism on the estimated lifetime distribution, the PDF and CDF of the RUL conditional on the observations to date t_i can be, respectively, obtained by the total law of the probability as [31]

$$\begin{aligned} f_{S_i|Y_i}(s_i \mid Y_i) &= \frac{w - y_i}{\sqrt{2\pi s_i^3 (P_{i|i} s_i + \sigma^2)}} \exp\left(-\frac{(w - y_i - \widehat{\lambda}_i s_i)^2}{2 s_i (P_{i|i} s_i + \sigma^2)}\right), \quad s_i > 0, \\ F_{S_i|Y_i}(s_i \mid Y_i) &= 1 - \Phi\left(\frac{w - y_i - \widehat{\lambda}_i s_i}{\sqrt{P_{i|i} s_i^2 + \sigma^2 s_i}}\right) + \exp\left(\frac{2\widehat{\lambda}_i (w - y_i)}{\sigma^2} + \frac{2P_{i|i}(w - y_i)^2}{\sigma^4}\right) \\ &\quad \times \Phi\left(-\frac{2P_{i|i}(w - y_i) s_i + \sigma^2 (\widehat{\lambda}_i s_i + w - y_i)}{\sigma^2 \sqrt{P_{i|i} s_i^2 + \sigma^2 s_i}}\right). \end{aligned} \tag{45}$$

Accordingly, the PDF and CDF of the estimated lifetime of the PMS can be formulated as

$$\begin{aligned} f_{T_d|Y_i}(t_d \mid Y_i) &= \frac{w - y_i}{\sqrt{2\pi (t_d - t_i)^3 (P_{i|i}(t_d - t_i) + \sigma^2)}} \times \exp\left(-\frac{(w - y_i - \widehat{\lambda}_i (t_d - t_i))^2}{2(t_d - t_i)(P_{i|i}(t_d - t_i) + \sigma^2)}\right), \quad t_d > t_i, \end{aligned} \tag{46}$$

$$\begin{aligned} F_{T_d|Y_i}(t_d \mid Y_i) &= 1 - \Phi\left(\frac{w - y_i - \widehat{\lambda}_i (t_d - t_i)}{\sqrt{P_{i|i}(t_d - t_i)^2 + \sigma^2 (t_d - t_i)}}\right) + \exp\left(\frac{2\widehat{\lambda}_i (w - y_i)}{\sigma^2} + \frac{2P_{i|i}(w - y_i)^2}{\sigma^4}\right) \\ &\quad \times \Phi\left(-\frac{2P_{i|i}(w - y_i)(t_d - t_i) + \sigma^2 (\widehat{\lambda}_i (t_d - t_i) + w - y_i)}{\sigma^2 \sqrt{P_{i|i}(t_d - t_i)^2 + \sigma^2 (t_d - t_i)}}\right). \end{aligned} \tag{47}$$

From (47), we can also observe the dependency of the estimated lifetime of the system on the observation history up to t_i.

5. Reliability Estimation for PMS

After obtaining the estimated mission system lifetime distribution $f_{T_d|Y_i}(t_d \mid Y_i)$ and the mission time $p_{T_M|\Phi_{i,n}}(t_m \mid \Phi_{i,n})$, we can estimate the reliability of the mission process according to the two definitions of the mission reliability given in Section 2.1. Together with these analyses, the reliability of PMS at t_i under the nth phase can be, respectively, formulated as

$$\begin{aligned} \Pr(T_d \geq T_M \mid \Phi_{i,n}, Y_i) &= \int_{t_m > 0} p_{T_M|\Phi_{i,n}}(t_m \mid \Phi_{i,n}) \times \left(\int_{t_d \geq t_m} f_{T_d|Y_i}(t_d \mid Y_i)\, dt_d \right) dt_m, \end{aligned} \tag{48}$$

$$\begin{aligned} \Pr(T_M \leq R \mid T_M \leq T_d, \Phi_{i,n}, Y_i) &= \frac{\Pr(T_M \leq R, T_M \leq T_d \mid \Phi_{i,n}, Y_i)}{\Pr(T_M \leq T_d \mid \Phi_{i,n}, Y_i)} \\ &= \frac{\iint_{t_d \geq t_m, 0 < t_m \leq R} p_{T_m, T_d|\Phi_{i,n}, Y_i}(t_m, t_d \mid \Phi_{i,n}, Y_i)\, dt_d\, dt_m}{\Pr(T_M \leq T_d \mid \Phi_{i,n}, Y_i)} \end{aligned}$$

$$= \frac{\int_{0<t_m\le R} p_{T_M|\Phi_i}(t_m \mid \Phi_{i,n}) \left(\int_{t_d\ge t_m} f_{T_d|Y_i}(t_d \mid Y_i)\, dt_d\right) dt_m}{\Pr(T_M \le T_d \mid \Phi_{i,n}, Y_i)}$$
$$= \frac{\int_{0<t_m\le R} p_{T_M|\Phi_{i,n}}(t_m \mid \Phi_{i,n}) \left(\int_{t_d\ge t_m} f_{T_d|Y_i}(t_d \mid Y_i)\, dt_d\right) dt_m}{\int_{t_m>0} p_{T_M|\Phi_{i,n}}(t_m \mid \Phi_{i,n}) \left(\int_{t_d\ge t_m} f_{T_d|Y_i}(t_d \mid Y_i)\, dt_d\right) dt_m}, \quad (49)$$

where $f_{T_d|Y_i}(t_d \mid Y_i)$ and $p_{T_M|\Phi_{i,n}}(t_m \mid \Phi_{i,n})$ have been modeled in Sections 3 and 4.

From (48) and (49), we can observe that our approach for mission reliability estimation establishes a linkage between the historical data and real-time information of the individual PMS. The associated model parameters can be estimated based on the historical data by the maximum likelihood approach naturally and thus we do not specifically discuss this estimation issue to limit our scope.

6. Numerical Studies

In this section, we provide a numerical example to illustrate the implementation process and the performance of the presented approach in this paper.

Suppose that there is a PMS which is designed to complete the three-phase mission process. The phase durations are log normally distributed but correlated. For an individual PMS to conduct a particular mission process, there are some sensors to monitor the CM information related to the phase duration and the degradation data related to the lifetime of the PMS. The CM information is used to update the phase duration and the mission time, while the degradation data are used to estimate the lifetime of the PMS. Specifically, we consider the following relationship among the phase durations:

$$p_{X_1}(x_1) = \frac{1}{x_1\sqrt{2\pi\sigma_{x1}^2}} \exp\left(-\frac{(\ln x_1 - \mu_{x1})^2}{2\sigma_{x1}^2}\right), \quad (50)$$

$$p_{X_2|X_1}(x_2 \mid x_1) = \frac{1}{x_2\sqrt{2\pi\sigma_{x2}^2}} \exp\left(-\frac{(\ln x_2 + \mu_{x1} - \mu_{x2} - \ln x_1)^2}{2\sigma_{x2}^2}\right),$$

$$p_{X_3|X_1,X_1}(x_3 \mid x_1, x_2) = \frac{1}{x_3\sqrt{2\pi\sigma_{x3}^2}} \times \exp\left(-\frac{(\ln x_3 + \mu_{x1} + \mu_{x2} - \ln x_1 - \ln x_2 - \mu_{x3})^2}{2\sigma_{x3}^2}\right), \quad (51)$$

where μ_{xj}, σ_{xj}^2 and $j = 1,2,3$, are the parameters of the log-normal distributions. These distributions correspond, respectively, to the distributions of $L_{0,1}$, $L_{0,2}$ and $L_{0,3}$, in the filtering models.

In the presented approach, it is required to determine the functional forms of the CM information and the remaining phase duration, that is, $g_n(z_{i,n})$, $n = 1,2,3$. In this numerical study, we use the following functional forms of $g_n(z_{i,n})$:

$$g_n(z_{i,n}) = a_n + b_n \exp(-z_{i,n}), \quad n = 1,2,3. \quad (52)$$

The above is just an idea to model the relationship between $z_{i,n}$ and $\phi_{i,n}$. In order to generate the degradation data to estimate the lifetime of the PMS, we use the following discrete equation:

$$y_{i+1} = y_i + \lambda(t_{i+1} - t_i) + \sigma B(t_{i+1} - t_i), \quad (53)$$

where $B(t_{i+1} - t_i) \sim N(0, t_{i+1} - t_i)$.

Now, given (51), (52), and (53) and the parameters in these equations, we can simulate the required data for our modeling and reliability estimation. Table 1 shows the parameters used for the data simulation.

In Table 1, we also show the estimated parameters (indicated in the brackets) of our approach based on the multiple sample of the simulation data. It can be observed that the maximum likelihood estimation of these parameters can match the true parameters well. Figure 1 shows the particular simulation data of the CM information related to the phase duration (i.e., $\phi_{i,n}$) and the degradation data (i.e., y_i) under the above model settings and parameters specifications.

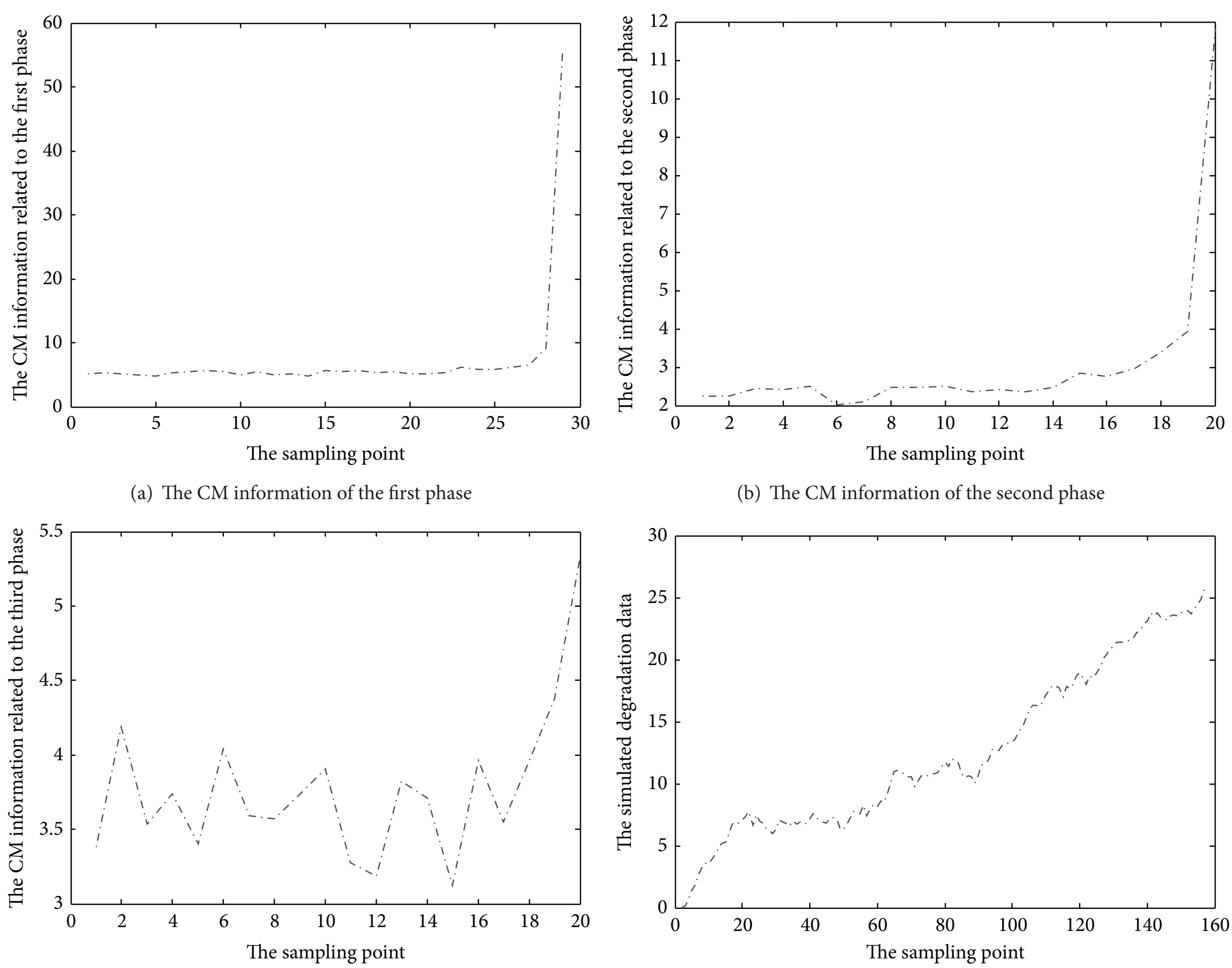
The data illustrated in Figure 1 are used to implement the presented reliability approach. These data consist of $\Phi_{i,n}$ and Y_i. Now, we use the approach developed in this paper and the model setting above to show the mission reliability results. When the mission process starts, we need to calculate the PDFs of the remaining phase durations and then update the PDF of the mission time at each phase. In estimating the remaining useful life of the RUL, we set the failure threshold as $w = 25$. Based on these estimated phase durations and RUL for the PMS, we can evaluate the mission reliability by its two different definitions. In order to shed light on the performance of our developed reliability approach, we consider the following two cases. The first case corresponds to a low degradation, while the second case corresponds to a fast degradation. In the degradation modeling of this paper, the degradation rate is represented by λ. In the RUL estimation, λ is adaptively updated by the Kalman filter, and thus the presented approach for reliability estimation can naturally characterize the varying mechanism of the degradation rate. In addition, the low degradation will lead to a small large lifetime, while the fast degradation leads to a small system lifetime. This can be controlled by the failure threshold as shown in (47). In this paper, we adjust the failure threshold to change the PMS lifetime. The reliability estimation results for these two cases are discussed as follows.

(i) Case 1. The degradation quantity is subtle and thus the lifetime of mission system is long enough.

In this case, because the degradation is subtle, the estimated RUL of the PMS is expected to be large. This is consistent with our intuition that, for a newly installed PMS, its lifetime is naturally large enough and may be designed

Table 1: The parameters used for simulation and the estimated parameters from the simulation data.

The 1st phase	$\mu_{x1} = 3, \sigma_{x1}^2 = 0.04$ ($\mu_{x1} = 2.8903, \sigma_{x1}^2 = 0.0342$)	$a_1 = 5, b_1 = 4$, and $\sigma_1 = 0.3$ ($a_1 = 4.8602, b_1 = 4.1032$, and $\sigma_1 = 0.2564$)
The 2nd phase	$\mu_{x2} = 2.5, \sigma_{x2}^2 = 0.06$ ($\mu_{x2} = 2.3625, \sigma_{x2}^2 = 0.0684$)	$a_2 = 2, b_2 = 3$, and $\sigma_2 = 0.2$ ($a_2 = 2.1286, b_2 = 2.9603$, and $\sigma_2 = 0.1532$)
The 3rd phase	$\mu_{x3} = 2.2, \sigma_{x3}^2 = 0.02$ ($\mu_{x3} = 2.0651, \sigma_{x3}^2 = 0.0245$)	$a_3 = 3.5, b_3 = 0.8$, and $\sigma_3 = 0.3$ ($a_3 = 3.2891, b_3 = 0.6459$, and $\sigma_3 = 0.3426$)
The degradation process	$\lambda = 0.2, \sigma = 0.4$ ($\lambda = 0.2105, \sigma = 0.4028$)	



(a) The CM information of the first phase

(b) The CM information of the second phase

(c) The CM information of the third phase

(d) The degradation data of the mission system

Figure 1: The simulated CM information related to the phase duration of the PMS and the simulated degradation data of the PMS.

to have the ability to perform many missions. In order to simulate this case, we set a large failure threshold, $w = 25$. The reason to do so is that a large threshold corresponds to a long lifetime of the PMS, and this can be verified by (47). Firstly, we calculate the mission reliability at each sampling point according to the definition that the mission can be successfully accomplished before the system fails. The result is show in Figure 2 by evaluating (48).

Figure 2 shows the evolving path of the mission reliability $\Pr(T_M \leq R \mid T_M \leq T_d, \Phi_{i,n}, Y_i)$ over t_i. As shown in Figure 2, the success probability of the phased-mission process will increase with the mission progressing. This can

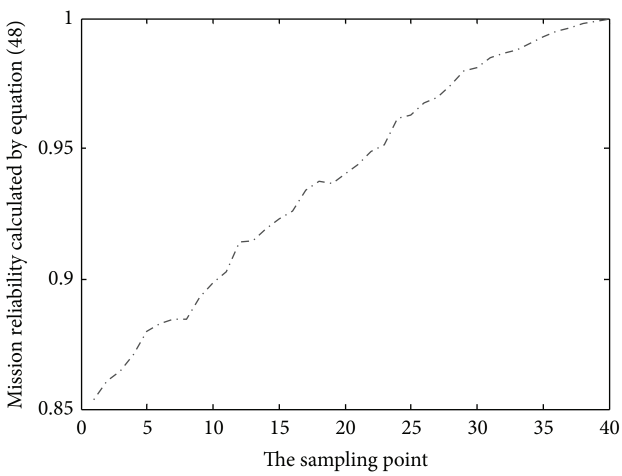


Figure 2: The evolving mission reliability $\Pr(T_M \leq R \mid T_M \leq T_d, \Phi_{i,n}, Y_i)$ over t_i that the mission can be successfully accomplished before a given time R without the system failure in Case 1.

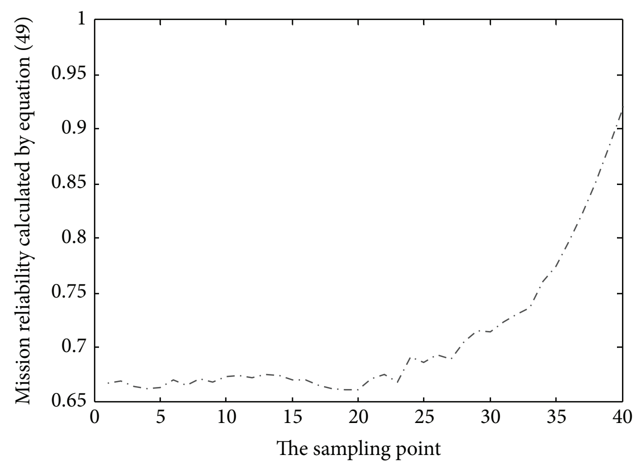


Figure 3: The evolving mission reliability $\Pr(T_d \geq T_M \mid \Phi_{i,n}, Y_i)$ over t_i that the mission can be successfully accomplished before the system fails in Case 1.

be well explained since with the mission process progressing the remaining mission time is less, but the reduced RUL of the PMS is not significant in contrast with its long lifetime in this case. Also, we can found that for the subtle degradation, the lifetime of the PMS will be large in stochastic sense. Therefore, the mission reliability will be maintained in a relatively high level. Figure 2 reflects this fact. Accordingly, the probability that the mission can be successfully accomplished before a given time R under condition that the system lifetime is longer than the total mission time at each phase can be obtained by evaluating (49). For illustration, we set $R = 50$ and the result is shown in Figure 3.

Figure 3 shows the evolving path of the mission reliability $\Pr(T_d \geq T_M \mid \Phi_{i,n}, Y_i)$ over t_i, which is calculated by (49). It is not surprising that this kind of the mission reliability also has an increasing trend, as illustrated in Figure 3. The reason for this is similar to the above result because the lifetime of the PMS in this case is set to be long enough compared with the mission time. Therefore, the reduction of the RUL of the PMS is not significant as the mission progressing. In addition, we can observe that the mission reliability in the early phase is relatively low. This is resulted from the fact that this kind of reliability is a ratio, as the denominator of (49) is relatively large in the early phase. However, as the mission progressing, the increase of the numerator is faster than the increase of the denominator. This leads to an increasing trend of this kind of mission reliability.

(ii) Case 2. The degradation is dramatic and thus the lifetime of mission system is small.

In this case, because the degradation is dramatic, the estimated RUL of the PMS is expected to be small. This is consistent with our intuition that, for an aged PMS, its lifetime naturally approaches the end and thus there is high probability that the mission will fail. In order to simulate this case, we set a small failure threshold, such as $w = 12$. The reason is that a small threshold corresponds to a short lifetime of the PMS. Similar to Case 1, we firstly calculate the mission reliability $\Pr(T_M \leq R \mid T_M \leq T_d, \Phi_{i,n}, Y_i)$. The result is illustrated in Figure 4.

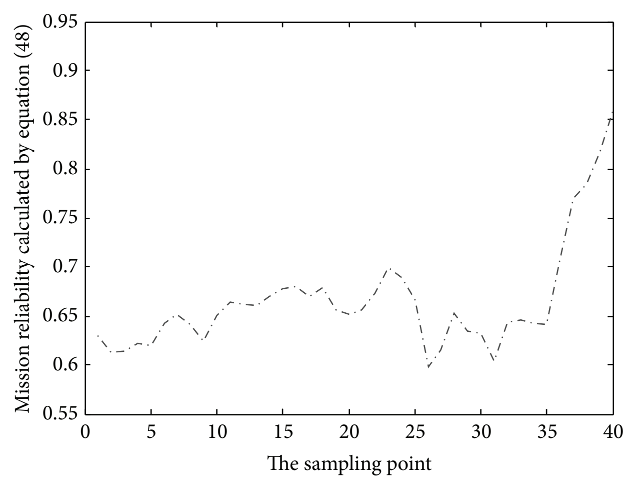


Figure 4: The evolving mission reliability $\Pr(T_M \leq R \mid T_M \leq T_d, \Phi_{i,n}, Y_i)$ over t_i that the mission can be successfully accomplished before a given time R without the system failure in Case 2.

Figure 4 shows the evolving path of the mission reliability $\Pr(T_M \leq R \mid T_M \leq T_d, \Phi_{i,n}, Y_i)$ over t_i, calculated by (48). Figure 4 shows that the mission reliability will be lower than the corresponding results in Case 1. Particularly, when the degradation is dramatic and the lifetime of mission system is small, the estimated mission reliability will fluctuate with the mission progressing to some extent though it still has certain increasing trend. These observations are largely resulted by the short lifetime of the PMS. In this case, with the mission progressing, the reduced RUL of the PMS is significant in contrast with the remaining mission time which is estimated from the CM information. Accordingly, $\Pr(T_d \geq T_M \mid \Phi_{i,n}, Y_i)$ can be obtained by evaluating (49). Similar to Case 1, the result is shown in Figure 5 with the setting $R = 50$.

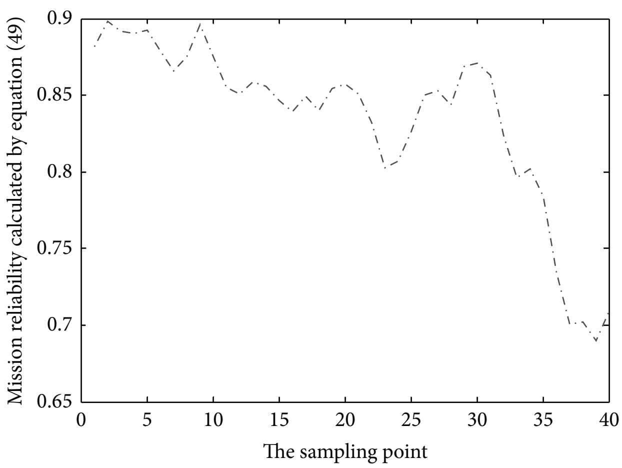


Figure 5: The evolving mission reliability $\Pr(T_d \geq T_M \mid \Phi_{i,n}, Y_i)$ over t_i that the mission can be successfully accomplished before the system fails in Case 2.

Figure 5 shows the evolving path of the mission reliability $\Pr(T_d \geq T_M \mid \Phi_{i,n}, Y_i)$ over t_i, which is calculated by (49). It is interesting to note that the mission reliability for the success in the required time will experience a decreasing trend. This differs clearly from the previous results. Similar to the first type of the mission reliability, when the degradation is dramatic, the estimated mission reliability of this type will fluctuate with the mission progressing. In this case, the mission reliability is a conditional probability as formulated in (49) and the denominator of this equation has an increasing trend as shown in Figure 4. In contrast with the result shown in Figure 3, the numerator of (49) is a probability to characterize the two events with the AND relationship: the event that the mission can be successfully accomplished before a given time R and the event that the system lifetime is longer than the total mission time. However, in this case, the lifetime of the PMS is small. Therefore, it is naturally expected that the increase of the numerator of (49) is not faster than that of the denominator. These observations finally result in the decreasing trend of the mission reliability of this type.

7. Conclusion

In this paper we attempt to address the issues associated with reliability estimation for PMS and present a novel approach to achieve reliability estimation for PMS using the condition monitoring information and degradation data of such system under dynamic operating scenario. In this sense, this paper contrasts sharply with the existing methods only considering the static scenario without using the real-time information, which aims to estimate the reliability for a population but not an individual. Specifically, to establish a linkage between the historical data and real-time information of the individual PMS, we adopt a stochastic filtering model to model the phase duration and obtain the updated estimation of the mission time by Bayesian filtering at each phase. At the meanwhile, the lifetime of PMS is estimated from the degradation data, which are modeled by an adaptive Brownian motion. As such, the mission reliability can be real time obtained through the estimated distribution of the mission time in conjunction with the estimated lifetime distribution. We demonstrate the implementation process and the usefulness of the developed approach via a numerical example.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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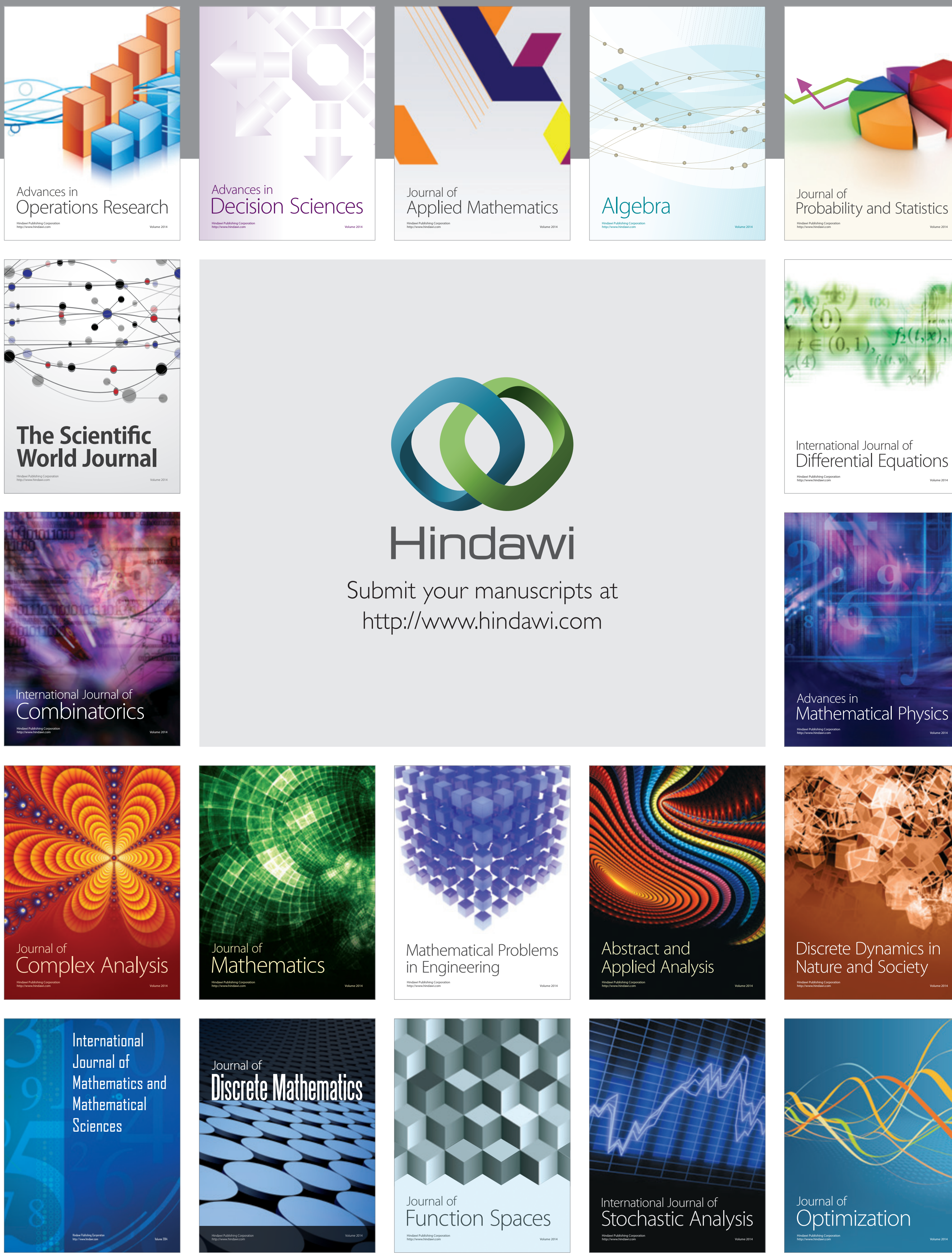
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