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*Research Article*

# Existence and Global Uniform Asymptotic Stability of Pseudo Almost Periodic Solutions for Cohen-Grossberg Neural Networks with Discrete and Distributed Delays

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This paper studies the existence and uniform asymptotic stability of pseudo almost periodic solutions to Cohen-Grossberg neural networks (CGNNs) with discrete and distributed delays by applying Schauder fixed point theorem and constructing a suitable Lyapunov functional. An example is given to show the effectiveness of the main results.

## 1. Introduction

Since the model of Cohen-Grossberg neural networks (CGNNs) was first proposed and studied by Cohen and Grossberg [1], it has been widely investigated because of the theoretical interest as well as the application considerations such as optimization, pattern recognition, automatic control, image processing, and associative memories. In recent years, there are many important results on dynamic behaviors of CGNNs. For instance, many sufficient conditions have been successively obtained to ensure the existence and stability of equilibrium point of CGNNs [1–10]. Some attractivity and asymptotic stability results have also been published [3, 11–14]. Many authors specially devote themselves to study the existence and global exponential stability of periodic or almost periodic solution to CGNNs [15–30]; for the other dynamic properties, see also the literatures [31, 32]. However, to the best of our knowledge, few authors have discussed the existence and the global uniform asymptotic stability of pseudo almost periodic solutions to CGNNs.

In this paper, we discuss the existence and the global uniform asymptotic stability of pseudo almost periodic solutions to the following CGNNs:

$$\begin{aligned} x_i'(t) &= -a_i(x_i(t)) \\ &\quad \times \left[ b_i(x_i(t)) - \sum_{j=1}^{m} c_{ij}(t) f_j(x_j(t)) \right. \\ &\quad - \sum_{j=1}^{m} d_{ij}(t) g_j(x_j(t-\tau_{ij}(t))), \\ &\quad \left. - \sum_{j=1}^{m} p_{ij}(t) \int_{-\infty}^{t} G_{ij}(t-s) h_j(x_j(s)) ds - I_i(t) \right], \\ &\qquad t \ge 0, \\ x_i(t) &= \Phi_i(t), \quad t < 0, \end{aligned} \tag{1}$$

where $c_{ij}(t)$, $d_{ij}(t)$, $p_{ij}(t)$, $I_i(t), \Phi_i(t) \in C(R,R)$, $\tau_{ij}(t) \in C(R,R^+)$ are pseudo almost periodic functions.

The organization of this paper is as follows. In Section 2, some basic definitions, marks, and lemmas are given. In Section 3, some results are given to ascertain the existence of pseudo almost periodic solution to the system (1) by applying Schauder fixed point theorem. In Section 4, the global uniform asymptotic stability of pseudo almost periodic solutions to the system (1) is obtained. In Section 5, an example is provided to demonstrate the effectiveness of the main results. In Section 6, the final conclusions are drawn.

## 2. Preliminaries

In this section, some basic definitions, lemmas, and assumptions are introduced.

*Definition 1* (see [33]). $f(t) \in BC(R,R)$ is said to be Bohr almost periodic if, for all $\epsilon > 0$, set

$$T(f,\epsilon) = \{|f(t+\tau) - f(t)| < \epsilon, \forall t \in R\} \tag{2}$$

is relatively dense. Namely, for any $\epsilon > 0$ there exists a number $l = l(\epsilon) > 0$ such that every interval $[a, a+l]$ contains at least one point of $\tau = \tau(\epsilon)$ such that $|f(t+\tau) - f(t)| < \epsilon$ for every $t \in R$. The collection of those functions is denoted by $AP(R,R^m)$. Define the class of functions $PAP_0(R,R^m)$ as follows:

$$PAP_0(R,R^m) = \left\{ f \in BC(R,R^m) \mid \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T} \|f(t)\|\,dt = 0 \right\}. \tag{3}$$

*Definition 2* (see [34]). A function $f \in BC(R,R^m)$ is called pseudo almost periodic if it can be expressed as

$$f = f_1 + f_0, \tag{4}$$

where $f_1 \in AP(R,R^m)$ and $f_0 \in PAP_0(R,R^m)$. The collection of such functions will be denoted by $PAP(R,R^m)$.

*Remark 3.* From the definitions above, we have $AP(R,R^m) \subset PAP(R,R^m)$.

**Lemma 4** (see [3]). *$PAP(R,R^m)$ is a Banach space with the norm $|\phi| = \sup_{t\in R}|\phi(t)|$.*

**Lemma 5** (see [19]). *If $f(t,u) \in C(R\times D, R^m)$, where $D$ is an open set in $R^m$ or $D = R^m$, $C(R\times D,R^m)$ denote continuous function class. Suppose $f \in PAP(R\times D)$ satisfies the Lipschitz condition*

$$|f(t,u) - f(t,v)| \le L|u-v|, \quad \forall t\in R,\ u,v \in D; \tag{5}$$

*if $\phi(t) \in PAP(R)$, then the composite function $f(t,\phi(t)) \in PAP(R)$. Suppose $f: R\times C \to R^m$; then the equation*

$$x'(t) = f(t, x_t) \tag{6}$$

*is called lagging-type almost periodic differential equation. The following system (7) is defined as the product systems of (6):*

$$x'(t) = f(t, x_t(t)), \qquad y'(t) = f(t, y_t(t)). \tag{7}$$

**Lemma 6.** *Suppose $\phi(t) \in PAP(R,R^m)$; then $\phi(t-\tau) \in PAP(R,R^m)$ for all $\tau \in R$.*

*Proof.* From Definition 2 of the $PAP$, we have $\phi = \phi_1 + \phi_0$, where $\phi_1 \in AP(R,R^m)$ and $\phi_0 \in PAP_0(R,R^m)$. Clearly $\phi(t-\tau) = \phi_1(t-\tau) + \phi_0(t-\tau)$; it is easy to know $\phi_1(t-\tau) \in AP(R,R^m)$ and

$$\begin{aligned} 0 &\le \frac{1}{2T}\int_{-T}^{T} |\phi_0(t-\tau)|\,dt = \frac{1}{2T}\int_{-(T+\tau)}^{T-\tau} |\phi_0(t)|\,dt \\ &\le \frac{T+\tau}{T}\cdot\frac{1}{2(T+\tau)}\int_{-(T+\tau)}^{T+\tau} |\phi_0(t)|\,dt. \end{aligned} \tag{8}$$

This indicates that $\phi_0(t-\tau) \in PAP_0(R,R^m)$. So $\phi(t-\tau) \in PAP(R,R^m)$. □

*Definition 7.* Assume that $x^*(t)$ is a pseudo almost periodic solution of system (1). By a translation transformation $y(t) = x(t) - x^*(t)$, system (1) is transformed into a new system. If the zero solution of new system is globally uniformly asymptotically stable, then the pseudo almost periodic solution of system (1) is said to be globally uniformly asymptotically stable. As for the uniform asymptotical stability, see [35].

**Lemma 8** (see [33]). *There is a continuous functional $V(t,\varphi,\psi)$ for $t \ge 0$, $\varphi,\psi \in C_H$, $C_H = \{\varphi : \varphi \in C, |\varphi| < H\}$, $|\varphi| = \sup_{\theta\in[-r,0]}|\varphi(\theta)|$ such that*

$(H'2.1)$ $u(|\varphi-\psi|) \le V(t,\varphi,\psi) \le v(|\varphi-\psi|)$;

$(H'2.2)$ $|V(t,\varphi_1,\psi_1) - V(t,\varphi_2,\psi_2)| \le k(|\varphi_1-\varphi_2| + |\psi_1-\psi_2|)$;

$(H'2.3)$ $V'_{(7)}(t,\varphi,\psi) \le -aV(t,\varphi,\psi)$,

*where $a$ is a positive constant and $u(s)$ and $v(s)$ are continuous nondecreasing functions; when $s \to 0$, $u(s) \to 0$, $k$ is a positive constant. At this time, if (7) has a bounded solution $x(t,\sigma,\varphi)$ such that $|x(t,\sigma,\varphi)| \le H_1$, where $t \ge \sigma \ge 0$, $H > H_1 > 0$, then (6) in $C_H$ has a unique almost periodic solution which is uniformly asymptotically stable.*

Throughout this paper, we make the following assumptions.

$(H2.1)$: Functions $a_i(u)$ are continuous bounded and there are positive constants $a_i^+, a_i^-$ such that

$$0 < a_i^- \le a_i(u) \le a_i^+, \quad \forall u \in R,\ i = 1,2,\dots,m. \tag{9}$$

$(H2.2)$: Functions $b_i(u) \in C(R,R)$ and there exist positive constants $b_i^-, b_i^+$ such that

$$b_i^- \le \frac{b_i(u) - b_i(v)}{u-v} \le b_i^+, \quad u \ne v, \qquad \forall u,v \in R,\ b_i(0) = 0. \tag{10}$$

$(H2.3)$: $c_{ij}(t)$, $d_{ij}(t)$, $p_{ij}(t)$, $I_i(t) \in C(R,R)$, $\tau_{ij}(t) \in C(R,R^+)$ are pseudo almost periodic functions:

$$\begin{aligned} &\sup_{t\in R} c_{ij}(t) = c_{ij}^+ > 0, \qquad \sup_{t\in R} d_{ij}(t) = d_{ij}^+ > 0, \\ &\sup_{t\in R} p_{ij}(t) = p_{ij}^+ > 0, \qquad \sup_{t\in R} I_i(t) = I_i^+ > 0, \end{aligned} \tag{11}$$

where $R^+ = [0,\infty)$, $i,j = 1,2,\dots,m$.

($H2.4$): Delay kernel functions $G_{ij} : [0,+\infty) \to [0,+\infty)$ are piecewise continuous and integrable

$$\int_0^{+\infty} G_{ij}(u)\,du = 1,$$
$$\int_0^{\infty} uG_{ij}(u)\,du < +\infty, \qquad (12)$$
$$i, j = 1, 2, \ldots, m.$$

($H2.5$): Functions $f_j(u), g_j(u), h_j(u) \in C(R,R)$ satisfy the Lipschitz condition; namely, there exist nonnegative constants $L_j^f$, $L_j^g$, and $L_j^h$ such that

$$\left|f_j(u) - f_j(v)\right| \le L_j^f |u - v|, \quad \forall u, v \in R, \ j = 1, 2, \ldots, m,$$
$$\left|g_j(u) - g_j(v)\right| \le L_j^g |u - v|, \quad \forall u, v \in R, \ j = 1, 2, \ldots, m,$$
$$\left|h_j(u) - h_j(v)\right| \le L_j^h |u - v|, \quad \forall u, v \in R, \ j = 1, 2, \ldots, m. \qquad (13)$$

## 3. The Existence of Pseudo Almost Periodic Solution

In this section, we study the existence of pseudo almost periodic solution to system (1).

It follows from ($H2.1$) that the antiderivative of $1/a_i(x_i)$ exists. Then we choose an antiderivative $F_i(x_i)$ of $1/a_i(x_i)$ that satisfies $F_i(0) = 0$. Clearly, $F_i'(x_i) = 1/a_i(x_i)$. Because $a_i(x_i) > 0$, $F_i(x_i)$ is increasing about $x_i$ and the inverse function $F_i^{-1}(x_i)$ of $F_i(x_i)$ is existential, continuous, and differential. Then $(F_i^{-1}(x_i))' = a_i(x_i)$. Denote $F_i'(x_i)x_i'(t) = x_i'(t)/a_i(x_i(t)) \doteq u_i'(t)$; we get $x_i(t) = F_i^{-1}(u_i(t))$. Substituting these equations into system (1), we get the following equivalent equation:

$$\begin{aligned} u_i'(t) = & -b_i\left(F_i^{-1}(u_i(t))\right) \\ & + \sum_{j=1}^m c_{ij}(t) f_j\left(F_j^{-1}\left(u_j(t)\right)\right) \\ & + \sum_{j=1}^m d_{ij}(t) g_j\left(F_j^{-1}\left(u_j\left(t - \tau_{ij}(t)\right)\right)\right) \\ & + \sum_{j=1}^m p_{ij}(t) \int_{-\infty}^t G_{ij}(t-s) h_j\left(F_j^{-1}\left(u_j(s)\right)\right) ds \\ & + I_i(t), \quad t \ge 0, \\ u_i(t) = & F_i^{-1}\left(\Phi_i(t)\right) = \varphi_i(t), \quad t < 0. \end{aligned} \qquad (14)$$

From (14), we get $b_i(F_i^{-1}(u_i(t))) = [b_i(F_i^{-1}(\theta_i u_i(t)))]' u_i(t) \doteq b_i^{\sim}(u_i(t))u_i(t)$, where $0 \le \theta_i \le 1$. Putting it into (14), we obtain

$$\begin{aligned} u_i'(t) = & -b_i^{\sim}(u_i(t))\, u_i(t) \\ & + \sum_{j=1}^m c_{ij}(t) f_j\left(F_j^{-1}\left(u_j(t)\right)\right) \\ & + \sum_{j=1}^m d_{ij}(t) g_j\left(F_j^{-1}\left(u_j\left(t - \tau_{ij}(t)\right)\right)\right) \\ & + \sum_{j=1}^m p_{ij}(t) \int_{-\infty}^t G_{ij}(t-s) h_j\left(F_j^{-1}\left(u_j(s)\right)\right) ds \\ & + I_i(t), \quad t \ge 0, \\ u_i(t) = & F_i^{-1}\left(\Phi_i(t)\right) = \varphi_i(t), \quad t < 0. \end{aligned} \qquad (15)$$

Thus, system (1) has at least one pseudo almost periodic solution if and only if the system (15) has at least one pseudo almost periodic solution. So we only consider the pseudo almost periodic solution of system (15). By Lagrange theorem, we have

$$\begin{aligned} \left|F_i^{-1}(u) - F_i^{-1}(v)\right| &= \left|\left[F_i^{-1}\left(v + \theta_i(u - v)\right)\right]'(u - v)\right| \\ &= \left|a_i\left(v + \theta_i(u - v)\right)\right| |u - v|. \end{aligned} \qquad (16)$$

Again by ($H2.1$), we get

$$a_i^- |u - v| \le \left|F_i^{-1}(u) - F_i^{-1}(v)\right| \le a_i^+ |u - v|. \qquad (17)$$

Combined with ($H2.2$), we have

($H3.6$): $b_i^- a_i^- \le b_i'(F_i^{-1}(\cdot)) \le b_i^+ a_i^+$.

In order to prove the main results, we give the following lemma.

**Lemma 9.** *Suppose that assumptions* ($H2.2$)–($H2.5$) *hold and if* $\phi(t) \in PAP(R, R^m)$, *then*

$$C_{ij} = \int_{-\infty}^t G_{ij}(t-s)\,\phi_j(s)\,ds \in PAP(R,R), \qquad i = 1, 2, \ldots, m. \qquad (18)$$

*Proof.* From Definition 2, we have $\phi_j = \phi_{j1} + \phi_{j0}$; then

$$\begin{aligned} C_{ij} &= \int_{-\infty}^t G_{ij}(t-s)\,\phi_{j1}(s)\,ds + \int_{-\infty}^t G_{ij}(t-s)\,\phi_{j0}(s)\,ds \\ &= C_{ij1} + C_{ij0}. \end{aligned} \qquad (19)$$

Firstly, we prove $C_{ij1} \in AP(R,R)$. For any $\epsilon > 0$, there exists a number $l = l(\epsilon) > 0$ such that every interval $[a, a + l]$

contains at least one point of $\tau=\tau(\epsilon)$ such that $|\phi_{j1}(t+\tau)-\phi_{j1}(t)|\le\epsilon$ for every $t\in R$. Therefore, from $(H2.2)$–$(H2.4)$, we obtain

$$\begin{aligned}&\left|C_{ij1}(t+\tau)-C_{ij1}(t)\right|\\&=\left|\int_{-\phi}^{t+\tau}G_{ij}(t+\tau-s)\phi_{j1}(s)\,ds-\int_{-\infty}^{t}G_{ij}(t-s)\phi_{j1}(s)\,ds\right|\\&\le\int_{-\infty}^{t}\left|G_{ij}(t-s)\right|\left|\varphi_{j1}(s+\tau)-\varphi_{j1}(s)\right|ds\\&\le\epsilon\int_{-\infty}^{t}\left|G_{ij}(t-s)\right|ds\\&\le\epsilon\end{aligned}\tag{20}$$

so that $C_{ij1}\in AP(R,R)$.

And then we show that $C_{ij0}\in PAP_0(R,R)$ because

$$\begin{aligned}&\lim_{T\to+\infty}\frac{1}{2T}\int_{-T}^{T}\left|C_{ij0}\right|dt\\&=\sup_{t\in R}\lim_{T\to+\infty}\frac{1}{2T}\int_{-T}^{T}\left|\int_{-\infty}^{t}G_{ij}(t-s)\phi_{j0}(s)\,ds\right|dt\\&\le\sup_{t\in R}\lim_{T\to+\infty}\frac{1}{2T}\int_{0}^{+\infty}\left|G_{ij}(u)\right|\int_{-(T+u)}^{T+u}\left|\phi_{j0}(v)\,dv\right|du\\&=0.\end{aligned}\tag{21}$$

Thus $C_{ij0}\in PAP_0(R,R)$. So $C_{ij}\in PAP(R,R)$. □

**Theorem 10.** *Suppose that* $(H2.1)$–$(H2.5)$ *and* $(H3.6)$ *hold; if*

$$\delta=\max_{1\le i\le m}\left\{\frac{a_i^+}{b_i^-a_i^-}\sum_{j=1}^{m}\left(L_j^fc_{ij}^++L_j^gd_{ij}^++L_j^hp_{ij}^+\right)\right\}<1,\tag{22}$$

*then the system* (1) *has at least one pseudo almost periodic solution.*

*Proof.* For all $z(t)=\phi(t)^T=(\phi_1(t),\dots,\phi_m(t))^T\in PAP(R,R^m)$, we define the nonlinear operator $T:z(t)\to T(z)(t)=z_{(\phi)^T}(t)=(x_\phi(t))^T$, where

$$\begin{aligned}x_{\phi_i}(t)=&\int_{-\infty}^{t}e^{-\int_s^t b_i^{\sim}(\phi_i(\tau))d\tau}\\&\times\left[\sum_{j=1}^{m}c_{ij}(s)f_j\left(F_j^{-1}\left(\phi_j(s)\right)\right)\right.\\&+\sum_{j=1}^{m}d_{ij}(s)g_j\left(F_j^{-1}\left(\phi_j\left(s-\tau_{ij}(s)\right)\right)\right)\\&+\sum_{j=1}^{m}p_{ij}(s)\int_{-\infty}^{t}G_{ij}(s-v)h_j\left(F_j^{-1}\left(\phi_j(v)\right)\right)dv\\&\left.+I_i(s)\vphantom{\sum}\right]ds.\end{aligned}\tag{23}$$

Now, we prove that

$$T:PAP(R,R^m)\longrightarrow PAP(R,R^m).\tag{24}$$

Let

$$\begin{aligned}E_{ij}=&\sum_{j=1}^{m}c_{ij}(s)f_j\left(F_j^{-1}\left(\phi_j(s)\right)\right)\\&+\sum_{j=1}^{m}d_{ij}(s)g_j\left(F_j^{-1}\left(\phi_j\left(s-\tau_{ij}(s)\right)\right)\right)\\&+\sum_{j=1}^{m}p_{ij}(s)\int_{-\infty}^{t}G_{ij}(s-v)h_j\left(F_j^{-1}\left(\phi_j(v)\right)\right)dv+I_i(s).\end{aligned}\tag{25}$$

For $z(t)\in PAP(R,R^m)$, conditions $(H2.2)$–$(H2.4)$, Lemmas 5, 6, and 9, and the composition theorem in [16], we will get $E_{ij}\in PAP(R,R)$, $\forall i,j=1,2,\dots,m$.

From Definition 2, we have $E_{ij}=E_{ij1}+E_{ij0}$, $\forall i,j=1,2,\dots,m$. Where $E_{ij1}\in AP(R,R)$ and $E_{ij0}\in PAP_0(R,R)$. Then

$$\begin{aligned}x_{\phi_i}(t)=&\int_{-\infty}^{t}e^{-\int_s^t b_i^{\sim}(\phi_i(\tau))d\tau}E_{ij1}(s)\,ds\\&+\int_{-\infty}^{t}e^{-\int_s^t b_i^{\sim}(\phi_i(\tau))d\tau}E_{ij0}(s)\,ds\\=&\,T_{ij1}+T_{ij0},\end{aligned}\tag{26}$$

where $T_{ij1}=\int_{-\infty}^{t}e^{-\int_s^t b_i^{\sim}(\phi_i(\tau))d\tau}E_{ij1}(s)ds$ and $T_{ij0}=\int_{-\infty}^{t}e^{-\int_s^t b_i^{\sim}(\phi_i(\tau))d\tau}E_{ij0}(s)ds$.

Because $E_{ij1}\in AP(R,R)$, for any $\epsilon>0$, there exists a number $l=l(\epsilon)>0$ such that every interval $[a,a+l]$ contains at least one point of $\delta=\delta(\epsilon)$ such that $\sup_{t\in R}|E_{ij1}(t+\delta)-E_{ij1}(t)|\le\epsilon$ for every $t\in R$ and $\forall i,j=1,2,\dots,m$. Hence, we obtain

$$\begin{aligned}&\left|T_{ij1}(t+\delta)-T_{ij1}(t)\right|\\&=\left|\int_{-\infty}^{t+\delta}e^{-\int_s^{t+\delta}b_i^{\sim}(\phi_i(\tau))d\tau}E_{ij1}(s)\,ds\right.\end{aligned}$$

$$-\int_{-\infty}^{t} e^{-\int_s^t b_i^-(\phi_i(\tau))d\tau} E_{ij1}(s)\,ds\Big|$$
$$\leq \int_{-\infty}^{t+\tau} \left| e^{-b_i^- a_i^-(t-s)} \mid E_{ij1}(s+\delta) - E_{ij1}(s) \right| ds$$
$$\leq \frac{\epsilon}{b_i^- a_i^-} \tag{27}$$

so that $T_{ij1} \in AP(R,R)$.

And because

$$\lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T}\left|\int_{-\infty}^{t} e^{-\int_s^t b_i^-(\phi_i(\tau))d\tau}E_{ij0}(s)\,ds\right|dt$$
$$\leq \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T}\left|\int_{-T}^{t} e^{-\int_s^t b_i^-(\phi_i(\tau))d\tau}E_{ij0}(s)\,ds\right|dt$$
$$+ \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T}\left|\int_{-\infty}^{-T} e^{-\int_s^t b_i^-(\phi_i(\tau))d\tau}E_{ij0}(s)\,ds\right|dt$$
$$\leq \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T}\left\|E_{ij0}(t)\right\|dt\int_{-T}^{t} e^{-b_i^- a_i^-(t-s)}ds$$
$$+ \lim_{T\to+\infty} \frac{\sup_{t\in R}\left|E_{ij0}(t)\right|}{2T}\int_{-T}^{T}dt\left(\int_{-\infty}^{-T}\left|e^{-b_i^- a_i^-(t-s)}\right|ds\right)$$
$$\leq \lim_{T\to+\infty} \frac{1}{2Tb_i^- a_i^-}\int_{-T}^{T}\left\|E_{ij0}(t)\right\|dt$$
$$+ \lim_{T\to+\infty} \frac{\sup_{t\in R}\left|E_{ij0}(t)\right|}{2T(b_i^- a_i^-)^2}\left(1-e^{-b_i^- a_i^-(2T)}\right)$$
$$= 0 + \lim_{T\to+\infty} \frac{\sup_{t\in R}\left|E_{ij0}(t)\right|}{2T(b_i^- a_i^-)^2}\left(1-e^{-b_i^- a_i^-(2T)}\right)$$
$$= 0, \tag{28}$$

thus $T_{ij0} \in PAP_0(R,R)$. So $\forall i,j = 1,2,\ldots,m$, $x_{\phi_i}(t) \in PAP(R,R)$. Therefore $z_{(\phi)^T}(t) \in PAP(R,R^m)$.

From Lemma 9, $X = PAP(R,R^m)$ is a Banach space. If

$$\delta < 1, \tag{29}$$

then there exists a sufficiently large $\beta \geq 1$ such that

$$\delta \leq 1 - \beta^{-1} I, \tag{30}$$

where

$$I = \max_{1\leq i\leq m}\left\{\frac{I_i^+}{b_i^- a_i^-}\right\}. \tag{31}$$

We choose a closed subset

$$B = \left\{z(t) = \phi(t)^T = \left(\phi_1(t),\ldots,\phi_m(t)\right)^T \in X : \|z\| \leq \beta\right\}. \tag{32}$$

Firstly, we prove that $T : B \to B$; that is, $TB \subset B$.

From (29)–(32) and for $\forall z \in B$, we get

$$\left|x_{\phi_i}(t)\right| \leq \max_{1\leq i\leq m}\left\{\frac{a_i^+}{b_i^- a_i^-}\sum_{j=1}^{n}\left(L_j^f c_{ij}^+ + L_j^g d_{ij}^+ + L_j^h p_{ij}^+\right)\right\}$$
$$\times \|z\| + \max_{1\leq i\leq m}\left\{\frac{I_i^+}{b_i^- a_i^-}\right\} \leq \beta\delta + I \leq \beta. \tag{33}$$

Secondly, we prove that the mapping $T$ is completely continuous.

By the continuity of the function $f_j, g_j, h_j$, for any $\varepsilon > 0$, there is $\gamma = \gamma(\varepsilon,\beta)$ such that

$$\left|f_j(u) - f_j(v)\right| \leq \frac{L_j^f\varepsilon}{\delta}, \qquad |u-v| \leq \gamma,$$
$$\forall u,v \in B,\ \ j = 1,2,\ldots,m,$$
$$\left|g_j(u) - g_j(v)\right| \leq \frac{L_j^g\varepsilon}{\delta}, \qquad |u-v| \leq \gamma,$$
$$\forall u,v \in B,\ \ j = 1,2,\ldots,m,$$
$$\left|h_j(u) - h_j(v)\right| \leq \frac{L_j^h\varepsilon}{\delta}, \qquad |u-v| \leq \gamma,$$
$$\forall u,v \in B,\ \ j = 1,2,\ldots,m. \tag{34}$$

Let $w(t) = (\psi_1(t),\ldots,\psi_m(t))$, $z,w \in B$, and $\|z-w\| \leq \gamma$; then $\|z\| \leq \gamma$, $\|w\| \leq \gamma$ and $\phi_j(t),\psi_j(t) \in C(R,B)$; then, for any $s \in R$, we get $|\phi_j(s) - \psi_1(s)| \leq \gamma$. So, we have

$$\left|f_j\left(F_j^{-1}\left(\phi_j(s)\right)\right) - f_j\left(F_j^{-1}\left(\psi_j(s)\right)\right)\right| \leq \frac{L_j^f\varepsilon}{\delta}, \quad |u-v| \leq \gamma,$$
$$\left|g_j\left(F_j^{-1}\left(\phi_j(s)\right)\right) - f_j\left(F_j^{-1}\left(\psi_j(s)\right)\right)\right| \leq \frac{L_j^g\varepsilon}{\delta}, \quad |u-v| \leq \gamma,$$
$$\left|h_j\left(F_j^{-1}\left(\phi_j(s)\right)\right) - f_j\left(F_j^{-1}\left(\psi_j(s)\right)\right)\right| \leq \frac{L_j^h\varepsilon}{\delta}. \quad |u-v| \leq \gamma. \tag{35}$$

Thus

$$\|T(z)(t) - T(w)(t)\|$$
$$\leq \max_{1\leq i\leq m}\left\{\frac{1}{b_i^- a_i^-}\sum_{j=1}^{m} a_i^+\left(L_j^f c_{ij}^+ + L_j^g d_{ij}^+ + L_j^h p_{ij}^+\right)\right\}\frac{\varepsilon}{\delta} \tag{36}$$
$$\leq \delta\frac{\varepsilon}{\delta} \leq \varepsilon.$$

Therefore, $T$ is continuous.

Thirdly, we show that $T$ is compact.

Let $S = \{z(t) \in X : \|z\| \leq K\}$, where $K > 0$ to be any constant. We denote $\rho = \max_{1\leq i\leq m}\{(a_i^+/b_i^- a_i^-)\sum_{j=1}^{m} K(L_j^f c_{ij}^+ + L_j^g d_{ij}^+ + L_j^h p_{ij}^+) + I_i^+\}$. Then we have

$$\|T(z)(t)\| = \sup_{t\in R}\max_{1\leq i\leq m}\left|x_{\phi_i}(t)\right| \leq \rho, \quad \forall z \in S. \tag{37}$$

Hence, $T$ is uniformly bounded. Then, from (23), we get

$$\begin{aligned}\left[\left|x_{\phi_i}(t)\right|\right]' &= -b_i^{\sim}\left(x_{\phi_i}(t)\right)x_{\phi_i}(t) \\ &\quad + \sum_{j=1}^{m} c_{ij}(t) f_j\left(F_j^{-1}\left(\phi_j(t)\right)\right) \\ &\quad + \sum_{j=1}^{m} d_{ij}(t) g_j\left(F_j^{-1}\left(\phi_j\left(t-\tau_{ij}(t)\right)\right)\right) \\ &\quad + \sum_{j=1}^{m} p_{ij}(t)\int_{-\infty}^{t} G_{ij}(t-s) h_j\left(F_j^{-1}\left(\phi_j(s)\right)\right) ds \\ &\quad + I_i(t) \\ &\le b_i^{+}\rho + \sum_{j=1}^{m} a_j^{+} K\left(L_j^{f} c_{ij}^{+} + L_j^{g} d_{ij}^{+} + L_j^{h} p_{ij}^{+}\right) + I_i^{+} \le L,\end{aligned} \tag{38}$$

where

$$L = \max_{1\le i\le m}\left\{b_i^{+}\rho + \sum_{j=1}^{m} a_j^{+} K\left(L_j^{f} c_{ij}^{+} + L_j^{g} d_{ij}^{+} + L_j^{h} p_{ij}^{+}\right) + I_i^{+}\right\}. \tag{39}$$

Therefore, $T$ is equicontinuous. By the Ascoli-Arzela theorem, the operator $T$ is compact; then it is completely continuous. By the Schauder fixed point theorem, the system (1) has at least one pseudo almost periodic solution. □

## 4. The Global Uniform Asymptotic Stability of Pseudo Almost Periodic Solution

In order to discuss the global uniform asymptotic stability of pseudo almost periodic solution to system (1), we give the following assumptions:

($H4.1$): delay functions $\tau_{ij}(t) \in C^1(R, R^{+})$ satisfy that $\dot{\tau}_{ij}(t) \le \tau_{ij}^{*} < 1$, $i, j = 1, 2, \ldots, m$;

($H4.2$): $N = \min_{1\le i\le m}\{N_i\} > 0$, where $N_i = b_i^{-} - \sum_{j=1}^{m} c_{ij}^{+} L_j^{f} - \sum_{j=1}^{m}(d_{ij}^{+} L_j^{g}/(1-\tau_{ij}^{*})) - \sum_{j=1}^{m} p_{ij}^{+} L_j^{h}$.

**Theorem 11.** *Assume that (H2.1)–(H2.5) and (H4.1)-(H4.2) hold; then the pseudo almost periodic solution of system* (1) *is globally uniformly asymptotically stable.*

*Proof.* The product system of the system (1) is

$$\begin{aligned}x_i'(t) &= -a_i\left(x_i(t)\right) \\ &\quad\times\left[b_i\left(x_i(t)\right) - \sum_{j=1}^{m} c_{ij}(t) f_j\left(x_j(t)\right)\right. \\ &\quad - \sum_{j=1}^{m} d_{ij}(t) g_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ &\quad \left.-\sum_{j=1}^{m} p_{ij}(t)\int_{-\infty}^{t} G_{ij}(t-s) h_j\left(x_j(s)\right) ds - I_i(t)\right], \\ y_i'(t) &= -a_i\left(y_i(t)\right) \\ &\quad\times\left[b_i\left(y_i(t)\right) - \sum_{j=1}^{m} c_{ij}(t) f_j\left(y_j(t)\right)\right. \\ &\quad - \sum_{j=1}^{m} d_{ij}(t) g_j\left(y_j\left(t-\tau_{ij}(t)\right)\right) \\ &\quad \left.-\sum_{j=1}^{m} p_{ij}(t)\int_{-\infty}^{t} G_{ij}(t-s) h_j\left(y_j(s)\right) ds - I_i(t)\right].\end{aligned} \tag{40}$$

In order to apply the conclusion of Lemma 8, we construct a Lyapunov functional about product system (40)

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{41}$$

where

$$\begin{aligned}V_1(t) &= \sum_{i=1}^{m}\left|\int_{y_i(t)}^{x_i(t)} \frac{1}{a_i(s)} ds\right|; \\ V_2(t) &= \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{d_{ij}^{+} L_j^{g}}{1-\tau_{ij}^{*}} \int_{t-\tau_{ij}(t)}^{t} \left|x_i(s) - y_i(s)\right| ds; \\ V_3(t) &= \sum_{i=1}^{m}\sum_{j=1}^{m} p_{ij}^{+} L_j^{h} \int_0^{+\infty} G_{ij}(s) \int_{t-s}^{t} \left|x_i(u) - y_i(u)\right| du\, ds.\end{aligned} \tag{42}$$

Let $X(t) = x(t)^T = (x_1(t), \ldots, x_m(t))^T$ and $Y(t) = y(t)^T = (y_1(t), \ldots, y_m(t))^T$. For product system $(X, Y)$, we receive

$$\begin{aligned}|X - Y| &\le V(t, X(t), Y(t)) \\ &\le \sum_{i=1}^{m}\left\{\frac{1}{a_i^{-}} + \sum_{j=1}^{m}\frac{d_{ij}^{+} L_j^{g} \tau_{ij}^{+}}{1-\tau_{ij}^{*}} + \sum_{j=1}^{n} p_{ij}^{+} L_i^{h} \int_0^{+\infty} G_{ij}(s) s\, ds\right\} \\ &\quad\times\left|x_i(t) - y_i(t)\right| \\ &\le M_i \sum_{i=1}^{m}\left|x_i(t) - y_i(t)\right| \\ &\le M|X - Y|,\end{aligned} \tag{43}$$

where $M_i = (1/a_i^{-}) + \sum_{j=1}^{m}(d_{ij}^{+} L_j^{g} \tau_{ij}^{+}/(1-\tau_{ij}^{*})) + \sum_{j=1}^{n} p_{ij}^{+} L_i^{h} \int_0^{+\infty} G_{ij}(s) s\, ds$ and $M = \max_{1\le i\le m}\{M_i\}$. Let

$u(s) = s$ and $v(s) = Ms$; we easily know it satisfies condition $(H'2.1)$ of Lemma 8. Then we obtain

$$\begin{aligned}
&|V(t,X,Y) - V(t,X^*,Y^*)| \\
&= \Bigg|\sum_{i=1}^{m}\left|\int_{y_i(t)}^{x_i(t)}\frac{1}{a_i(s)}ds\right| - \sum_{i=1}^{m}\left|\int_{y_i^*(t)}^{x_i^*(t)}\frac{1}{a_i(s)}ds\right| \\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{d_{ij}^+L_j^g}{1-\tau_{ij}^*}\int_{t-\tau_{ij}(t)}^{t}|x_i(s)-y_i(s)|\,ds \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{d_{ij}^+L_j^g}{1-\tau_{ij}^*}\int_{t-\tau_{ij}(t)}^{t}|x_i^*(s)-y_i^*(s)|\,ds \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{m}p_{ij}^+L_j^h\int_0^{+\infty}G_{ij}(s)\int_{t-s}^{t}|x_i(u)-y_i(u)|\,du\,ds \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{m}p_{ij}^+L_j^h\int_0^{+\infty}G_{ij}(s)\int_{t-s}^{t}|x_i^*(u)-y_i^*(u)|\,du\,ds\Bigg| \\
&\le \sum_{i=1}^{m}\frac{1}{a_i^-}|x_i(t)-x_i^*(t)| + \sum_{i=1}^{m}\frac{1}{a_i^-}|y_i(t)-y_i^*(t)| \\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{d_{ij}^+L_j^g}{1-\tau_{ij}^*} \\
&\quad \times \int_{t-\tau_{ij}(t)}^{t}\left(|x_i(s)-x_i^*(s)| + |y_i(s)-y_i^*(s)|\right)ds \\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{m}p_{ij}^+L_j^h\int_0^{+\infty}G_{ij}(s) \\
&\quad \times \int_{t-s}^{t}ds\left(|x_i(t)-x_i^*(t)| + |y_i(t)-y_i^*(t)|\right) \\
&\le M\left(|X-X^*| + |Y-Y^*|\right).
\end{aligned} \tag{44}$$

We also know that it satisfies condition $(H'2.2)$ of Lemma 8.

Calculating the upright derivative of $V_1(t)$, $V_2(t)$, and $V_3(t)$ along the system (40), respectively, and noting that $((1-\dot{\tau}_{ij}(t))/(1-\tau_{ij}^*)) > 1$, we have

$$\begin{aligned}
&D^+V_1(t)\big|_{(40)} \\
&= \sum_{i=1}^{m}\operatorname{Sgn}(x_i(t)-y_i(t))\left[\frac{x_i'(t)}{a_i(x_i(t))} - \frac{y_i'(t)}{a_i(y_i(t))}\right] \\
&\le \sum_{i=1}^{m}\Bigg\{-\frac{b_i(x_i(t))-b_i(y_i(t))}{x_i(t)-y_i(t)}|x_i(t)-y_i(t)| \\
&\quad + \sum_{j=1}^{m}c_{ij}^+L_j^f|x_j(t)-y_j(t)| \\
&\quad + \sum_{j=1}^{m}d_{ij}^+L_j^g\left|x_j(t-\tau_{ij}(t)) - y_j(t-\tau_{ij}(t))\right| \\
&\quad + \sum_{j=1}^{n}p_{ij}^+L_j^h\int_0^{+\infty}G_{ij}(s)|x_j(t-s)-y_j(t-s)|\,ds\Bigg\} \\
&\le \sum_{i=1}^{m}\Bigg\{-b_i^-|x_i(t)-y_i(t)| + \sum_{j=1}^{m}c_{ij}^+L_j^f|x_j(t)-y_j(t)| \\
&\quad + \sum_{j=1}^{m}d_{ij}^+L_j^g\left|x_j(t-\tau_{ij}(t)) - y_j(t-\tau_{ij}(t))\right| \\
&\quad + \sum_{j=1}^{n}p_{ij}^+L_j^h\int_0^{+\infty}G_{ij}(s)|x_j(t-s)-y_j(t-s)|\,ds\Bigg\}; \\
&D^+V_2(t)\big|_{(40)} \\
&= \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{d_{ij}^+L_j^g}{1-\tau_{ij}^*}|x_i(t)-y_i(t)| \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{d_{ij}^+L_j^g}{1-\tau_{ij}^*}\left|x_i(t-\tau_{ij}(t)) - y_i(t-\tau_{ij}(t))\right|; \\
&D^+V_3(t)\big|_{(40)} \\
&= \sum_{i=1}^{m}\sum_{j=1}^{m}p_{ij}^+L_j^h\int_0^{+\infty}G_{ij}(s)|x_i(t)-y_i(t)|\,ds \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{m}p_{ij}^+L_j^f\int_0^{+\infty}G_{ij}(s)|x_i(t-s)-y_i(t-s)|\,ds.
\end{aligned} \tag{45}$$

Combining (45) and assumptions $(H4.1)$ and $(H4.2)$, we get

$$\begin{aligned}
&D^+V(t)\big|_{(40)} \\
&\le \sum_{i=1}^{m}\left\{-b_i^- + \sum_{j=1}^{m}c_{ij}^+L_j^f + \sum_{j=1}^{m}\frac{d_{ij}^+L_j^g}{1-\tau_{ij}^*} + \sum_{j=1}^{m}p_{ij}^+L_j^h\right\} \\
&\quad \times |x_i(t)-y_i(t)| \\
&\le -N\sum_{j=1}^{m}|x_i(t)-y_i(t)| \\
&\le -aV(t).
\end{aligned} \tag{46}$$

From assumption $(H4.2)$, we have $a = N/M > 0$.

By Lemma 8, the pseudo almost periodic solutions of system (1) are globally uniformly asymptotically stable. This completes the proof. □

**Corollary 12.** *Consider the following periodic CGNNs systems:*

$$x_i'(t) = -a_i(x_i(t)) \times \left[ b_i(x_i(t)) - \sum_{j=1}^{m} c_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^{m} d_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) - \sum_{j=1}^{m} p_{ij}(t) \int_{-\infty}^{t} G_{ij}(t-s) f_j(x_j(s)) ds - I_i(t) \right], \quad t \ge 0,$$
$$x_i(t) = \Phi_i(t), \quad t < 0, \tag{47}$$

*where* $i = 1, 2, \ldots, m$, *and the following assumptions hold.*

(G4.1): *Functions* $a_i(u)$ *are continuous bounded and there are positive constants* $a_i^+, a_i^-$ *such that*

$$0 < a_i^- \le a_i(u) \le a_i^+, \quad \forall u \in R, \ i = 1, 2, \ldots, m. \tag{48}$$

(G4.2): *Functions* $b_i(u) \in C(R, R)$ *and there exist positive constants* $b_i^-, b_i^+$ *such that*

$$b_i^- \le \frac{b_i(u) - b_i(v)}{u - v} \le b_i^+, \quad u \ne v, \quad \forall u, v \in R, \ b_i(0) = 0. \tag{49}$$

(G4.3): $c_{ij}(t)$, $d_{ij}(t)$ *and* $p_{ij}(t)$, $I_i(t) \in C(R, R)$, $\tau_{ij}(t) \in C(R, R^+)$ *are all periodic functions, and*

$$\sup_{t\in R} c_{ij}(t) = c_{ij}^+ > 0, \qquad \sup_{t\in R} d_{ij}(t) = d_{ij}^+ > 0,$$
$$\sup_{t\in R} p_{ij}(t) = p_{ij}^+ > 0, \qquad \sup_{t\in R} I_i(t) = I_i^+ > 0, \tag{50}$$

*where* $R^+ = [0, \infty)$, $i, j = 1, 2, \ldots, m$.

(G4.4): *Delay kernel functions* $G_{ij} : [0, +\infty) \to [0, +\infty)$ *are piecewise continuous and integrable*

$$\int_0^{+\infty} G_{ij}(u) du = 1, \int_0^{\infty} u G_{ij}(u) du < +\infty, \quad i, j = 1, 2, \ldots, m. \tag{51}$$

(G4.5): *Functions* $f_j(u) \in C(R, R)$ *satisfy the Lipschitz condition; namely, there exist nonnegative constants* $L_j$ *such that*

$$|f_j(u) - f_j(v)| \le L_j |u - v|, \quad \forall u, v \in R, \ j = 1, 2, \ldots, m. \tag{52}$$

(G4.6): *If* $\delta = \max_{1\le i\le m}\{(a_i^+/b_i^- a_i^-) \sum_{j=1}^m L_j(c_{ij}^+ + d_{ij}^+ + p_{ij}^+)\} < 1$,

*then the system* (47) *has at least one periodic solution.*

**Corollary 13.** *Assume that (G4.1)–(G4.6) hold and suppose further that*

(G4.7): *delay functions* $\tau_{ij}(t) \in C^1(R, R^+)$ *satisfy that* $\dot{\tau}_{ij}(t) \le \tau_{ij}^* < 1$, $i, j = 1, 2, \ldots, m$.

(G4.8): $N = \min_{1\le i\le m}\{N_i\} > 0$, *where* $N_i = b_i^- - \sum_{j=1}^m c_{ij}^+ L_j - \sum_{j=1}^m (d_{ij}^+ L_j/(1-\tau_{ij}^*)) - \sum_{j=1}^m p_{ij}^+ L_j$; *then the periodic solution of system* (47) *is globally uniformly asymptotically stable.*

*Remark 14.* Recently, the global exponential stability of periodic or almost periodic solution to CGNNs is studied by many scholars (see [15–30]). However, few authors pay attention to the global uniform asymptotic stability. Corollaries 12 and 13 provide some new results.

## 5. An Example

An example is given to illustrate the feasibility of main results in this paper. Consider the following simple neural networks:

$$x_i'(t) = -a_i(x_i(t)) \times \left[ b_i(x_i(t)) - \sum_{j=1}^{2} c_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^{2} d_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) - \sum_{j=1}^{2} p_{ij}(t) \int_{-\infty}^{t} G_{ij}(t-s) h_j(x_j(s)) ds - I_i(t) \right], \quad t \ge 0, \ i = 1, 2, \tag{53}$$

where the initial functions $x_1(t) = 4 + \cos(\pi t)1$, $t < 0$, $x_2(t) = 5 + \sin(2t)$, $t < 0$. $a_i(x_i(t)) = 4 + \cos\pi t - e^{-|x_i(t)|}$, $b_i(x_i(t)) = 5 + \sin 2t - e^{-|x_i(t)|}$. Let

$$\begin{pmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{pmatrix} = \frac{1}{14} \begin{pmatrix} \cos t + e^{-t^4\cos^4 t} & \cos(\sqrt{2}t) + e^{-t^4\sin^4 t} \\ \cos(\sqrt{5}t) + e^{-t^4\cos^4 t} & \sin(2t) + e^{-t^2\cos^4 t} \end{pmatrix},$$

$$\begin{pmatrix} d_{11}(t) & d_{12}(t) \\ d_{21}(t) & d_{22}(t) \end{pmatrix} = \frac{1}{14}\begin{pmatrix} \sin t + e^{-t^2\cos^2 t} & \cos\left(\sqrt{4}t\right) + e^{-t^2\cos^4 t} \\ \sin\left(\sqrt{3}t\right) + e^{-t^4\cos^4 t} & \sin(2t) + e^{-t^4\cos^4 t} \end{pmatrix},$$

$$\begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} = \frac{1}{14}\begin{pmatrix} \cos\left(\sqrt{5}t\right) + e^{-t^4\cos^2 t} & \sin\left(\sqrt{3}t\right) + e^{-t^4\cos^4 t} \\ \cos\left(\sqrt{3}t\right) + e^{-t^2\cos^2 t} & \sin(2t) + e^{-t^2\cos^4 t} \end{pmatrix}. \quad (54)$$

$I_1(t) = I_2(t) = 2(\sin(\sqrt{3}t) + e^{-t^2\cos^4 t})$, $f_j(x_j) = g_j(x_j) = h_j(x_j) = ((|x+1| - |x-1|)/2)$, $G_{ij}(u) = e^{-u}$, $\tau_{ij}^* = 4/5$. Then, we have $a_i^+ = 5$, $a_i^- = 2$, $b_i^- = 3$, $c_{ij}^+ = d_{ij}^+ = p_{ij}^+ = 1/7$, $L_j^f = L_j^g = L_j^h = 1$, where $i, j = 1, 2$. Moreover

$$\delta = \max_{1\le i\le 2}\left\{\frac{a_i^+}{b_i^- a_i^-}\sum_{j=1}^{2}\left(L_j^f c_{ij}^+ + L_j^g d_{ij}^+ + L_j^h p_{ij}^+\right)\right\} = \frac{5}{7} < 1,$$

$$N_i = b_i^- - \sum_{j=1}^{2} c_{ij}^+ L_j^f - \sum_{j=1}^{2}\frac{d_{ij}^+ L_j^g}{1-\tau_{ij}^*} - \sum_{j=1}^{2} p_{ij}^+ L_j^h = 1 > 0, \quad i = 1, 2. \quad (55)$$

Thus, by Theorem 10, we know that system (53) has at least one pseudo almost periodic solution. It follows from Theorem 11 that the unique pseudo almost periodic solution of system (53) is globally uniformly asymptotically stable (Figure 1).

## 6. Conclusions

In this paper, the existence and uniform asymptotic stability of pseudo almost periodic solutions of system (1) is discussed. By applying Schauder fixed point theorem and constructing a suitable Lyapunov functional, some sufficient conditions are obtained to ensure the existence and uniform asymptotic stability of pseudo almost periodic solutions of system (1). The results have important leading significance in the design and applications of CGNNs. In addition, an example is given to demonstrate the effectiveness of main results.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

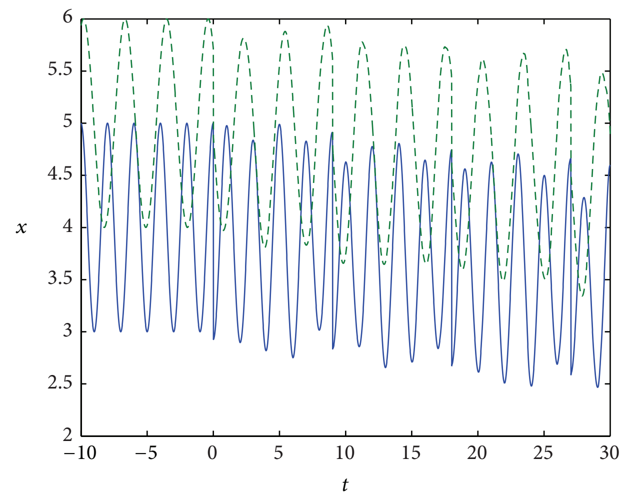


Figure 1: Simulated results of the solutions.

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