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Magnetohydrodynamic waves within the medium separated by the plane shock wave or rotational discontinuity

A. A. Lubchich and I. V. Despirak

Polar Geophysical Institute, Kola Science Centre, Russian Academy of Sciences, 184200 Apatity, Russia

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Abstract. Characteristics of small amplitude plane waves within the medium separated by the plane discontinuity into two half spaces are analysed. The approximation of the ideal one-fluid magnetohydrodynamics (MHD) is used. The discontinuities with the nonzero mass flux across them are mainly examined. These are fast or slow shock waves and rotational discontinuities. The dispersion equation for MHD waves within each of half space is obtained in the reference frame connected with the discontinuity surface. The solution of this equation permits one to determine the wave vectors versus the parameter c_p, which is the phase velocity of surface discontinuity oscillations. This value of c_p is common for all MHD waves and determined by an incident wave or by spontaneous oscillations of the discontinuity surface. The main purpose of the study is a detailed analysis of the dispersion equation solution. This analysis let us draw the following conclusions. (I) For a given c_p, ahead or behind a discontinuity at most, one diverging wave can transform to a surface wave damping when moving away from the discontinuity. The surface wave can be a fast one or, in rare cases, a slow, magnetoacoustic one. The entropy and Alfvén waves always remain in a usual homogeneous mode. (II) For certain values of c_p and parameters of the discontinuity behind the front of the fast shock wave, there can be four slow magnetoacoustic waves, satisfying the dispersion equation, and none of the fast magnetoacoustic waves. In this case, one of the four slow magnetoacoustic waves is incident on the fast shock wave from the side of a compressed medium. It is shown that its existence does not contradict the conditions of the evolutionarity of MHD shock waves. The four slow magnetoacoustic waves, satisfying the dispersion equation, can also exist from either side of a slow shock wave or rotational discontinuity. (III) The expressions determining the polarisation of the MHD waves are derived in the reference frame connected with the discontinuity surface. This form of presentation is much more convenient in investigating the interaction of small perturbations with MHD discontinuities. It is shown that the perturbations of the velocity and magnetic field associated with the surface magnetoacoustic wave have the elliptic polarisation. Usually the planes of polarisation for the perturbations of the velocity and magnetic field are not coincident with each other.

Correspondence to: A. A. Lubchich
(lubchich@pgi.kolasc.net.ru)

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1 Introduction

The problem of the interaction of small perturbations with plane magnetohydrodynamic (MHD) discontinuities through which there is a mass flux is of great scientific interest. The discontinuities can be both fast or slow shock waves, as well as rotational discontinuities. To our knowledge, the problem was formulated for the first time in 1959 (Kontorovich (1959)). But despite its rather long history, it is still far from a complete solution. In addition to its basic significance, the problem is also of considerable applied importance. Here we refer to only one example concerning solar-terrestrial coupling. Numerous fluctuations existing in the plasma and magnetic field of the solar wind enter into the Earth's magnetosphere, crossing the bow shock (which is a fast shock wave), magnetosheath, and magnetopause. The latter is typically presented either by a rotational or tangential discontinuity, depending on whether the magnetosphere is open or closed (e.g. Kwok and Lee (1984); Bauer et al. (2001)).

The problem of the interaction of small perturbations with plane discontinuity may be conventionally divided into two closely related problems. First is the theoretical investigation of the corrugated instability of a discontinuity. Under certain conditions the amplitude of a perturbation, spontaneously arising on the surface of a discontinuity, may start growing exponentially. Such growth may result in the violation of the initial conditions of flow. D'yakov (1954) and Kontorovich (1957) found the conditions of the corrugated instability of shock waves in the usual (non-magnetic) hydrodynamics. They showed that there are two types of instabilities: the absolute one and relative one. Under the absolute instability the amplitude of the ripples grows unrestrictively (certainly, within the linear approximation). Under the relative instability, the shock surface spontaneously emits an outcoming sound and entropy-vorticity waves. In this case the perturbations may exist for an arbitrarily long time without attenuation or amplification. The physical meaning of this type of instability remained mysterious for a long time. It became possible to succeed in achieving the progress only quite recently (Kuznetsov (1989)). Both types of instability can be realised only with a rather "exotic" shape of the shock adiabatic curves. This is the reason why the corrugated instability of hydrodynamic shock waves (to our knowledge) has not been observed experimentally. Generally, it is extremely difficult to carry out an analytical investigation of the corrugated instability of MHD discontinuity. The stability of MHD shock waves was analysed only in a special case of a parallel shock wave, when the external magnetic field is orthogonal to the front of the discontinuity. As shown by Gardner and Kruskal (1964), the conditions of absolute instability of the fast parallel MHD shock wave and a hydrodynamic shock wave are coincident. Pimenov (1982) investigated the possibility of spontaneous emission of MHD waves by the parallel shock. In particular, he has shown that in a sufficiently strong external magnetic field, the spontaneous emission of MHD waves by a fast parallel MHD shock wave may occur even in perfect gas. Lyubchich and Pudovkin (2003) investigated numerically the relative instability of a shock wave in perfect gas for the oblique magnetic field. The problem of absolute instability of a shock wave in the oblique magnetic field is still to be solved. We could not find articles analysing the corrugated stability of rotational discontinuity either.

The second problem is the investigation of the interaction of an MHD discontinuity with an incident wave of small amplitude. Lee (1982) studied analytically the passage of Alfvén waves through a coplanar rotational discontinuity. The chosen model of the coplanar rotational discontinuity (the tangential component of the magnetic field turns by 180°) strongly simplified the solution of the problem, but it was extremely idealised. Kwok and Lee (1984) performed detailed numerical calculations of transformation coefficients for MHD waves of different types and arbitrary rotational discontinuity. From their calculations it follows that sometimes the amplitudes of the refracted and reflected waves undergo strong amplification that could indicate a possible destabilising of the rotational discontinuity with respect to spontaneously emanating MHD waves. These results are often addressed in the explanation of the high level of turbulence in the magnetosheath, as well as in studying the correlation of fluctuations in the solar wind, magnetosheath and the Earth's magnetosphere (for example, Kessel et al. (2004)). Kwok and Lee (1984) considered only some of the possible orientations of the propagation vector of an incident wave, having limited the consideration by the analysis of relatively small incident angles. Unfortunately, the correctness of the numerical results of Kwok and Lee (1984) seems doubtful. From their calculations it follows that an Alfvén or magnetoacoustic wave incident on a rotational discontinuity gives rise to all theoretically possible emanating waves, including the entropy wave of nonzero amplitude. However, it contradicts basic physics. Indeed, all thermodynamic quantities, including both pressure P and density ρ, are continuous across a rotational discontinuity. As a consequence, the sound speed c_s should be continuous, too. Besides, it can be easily shown, that pressure and density variations, travelling with the MHD waves, do not undergo a jump on a rotational discontinuity, i.e. $\delta P_1 = \delta P_2$ and $\delta\rho_1 = \delta\rho_2$. As is known, the pressure variations are associated only with the magnetoacoustic waves ($\delta P = \delta P_{mag.w.}$). The density variations are associated with both magnetoacoustic waves and the entropy wave, i.e. $\delta\rho = \delta P_{mag.w.} / c_s^2 + \delta\rho_e$, where $\delta\rho_e$ is the amplitude of the entropy wave. The entropy wave is a wave incident on a rotational discontinuity in the upstream region (side 1) and a diverging wave in the downstream region (side 2). As a result, we obtain two boundary conditions:

$$\delta P_{mag.w.1} = \delta P_{mag.w.2}$$

$$\frac{\delta P_{mag.w.1}}{c_s^2} + \delta\rho_{e1}^{in} = \frac{\delta P_{mag.w.2}}{c_s^2} + \delta\rho_{e2}^{div},$$

from which it immediately follows that $\delta\rho_{e1}^{in} = \delta\rho_{e2}^{div}$. Thus, if there is an incident entropy wave, it passes through the rotational discontinuity without changing its amplitude, with the generation of diverging waves of other types, in principle, being possible. If there is no incident entropy wave, there will be no emanating entropy wave, that is $\delta\rho_{e2}^{div} = 0$. The results of Kwok and Lee (1984) are in conflict with this requirement. The authors have presented only numerical results. They did not considering some special cases for which an analytical solution is available. Therefore, it is difficult to specify where Kwok and Lee (1984) made a mistake. Lubchich and Pudovkin (1999) investigated the transmission of a fast magnetoacoustic wave through a rotational discontinuity. In contrast to the results of Kwok and Lee (1984), it has been found that the perturbations behind the front lead to a small amplification. Thereby, for the perturbation of solar-wind origin, the rotational discontinuity is not a significant barrier.

The investigation of the transformation of MHD waves on a shock wave is a complex problem because of intricate computations. Therefore, in its consideration the following two approaches are used. One consists of stating a general scheme of the solution without any particular calculations (Kontorovich (1959), McKenzie and Westphal (1970)). The other approach includes using some simplifying assumptions, allowing one to find an analytical solution. Sometimes a parallel shock wave is considered. The problem of the transmission of Alfvén waves through a shock wave was solved by McKenzie and Westphal (1969). They considered a special case when the wave vector of the incident wave, external magnetic field and normal to the shock lie in the same plane. In this case there are only two diverging Alfvén waves whose amplitudes were found. The problem was solved both for fast and slow shock waves. Scholer and Belcher (1971) studied the effect of the finite amplitude Alfvén waves on fast shock waves for the same orientation of vectors. The solution to the problem of the interaction of magnetoacoustic and entropy waves with a parallel shock wave was given by Westphal and McKenzie (1969). The approximation of a strong shock wave was also used when ahead of the shock the Mach and Alfvén numbers were much greater than unity. Hassam (1978) investigated analytically the incidence of Alfvén waves with the propagation vector parallel to the external magnetic field, on a strong shock wave. Pudovkin and Lyubchich (1989a) have calculated numerically the coefficients of transformation for arbitrarily directed Alfvén waves, incident upon a strong shock wave. Nearly everywhere (except for the paper by McKenzie and Westphal (1969)) the consideration was restricted by relatively small incident angles. Thus, in the case of a strong shock wave the incident angles considered make about 2/3 of the theoretically possible value.

Generally, the incidence of a wave on a shock results in arising shock surface oscillations (except for a special geometry of the problem considered in the paper of McKenzie and Westphal (1969)). The boundary conditions should be met on the perturbed (oscillating) surface of the shock wave. The characteristics of the medium in the local reference frame connected with the perturbed shock surface will differ from the corresponding characteristics in the reference frame connected with the stationary nonperturbed discontinuity. This difference, caused by transition from one reference frame to the other, will be proportional to the amplitude of the surface oscillations. The amplitude of the surface oscillations is one of the unknown quantities of the problem, along with the amplitudes of the waves diverging from the shock. In the above-mentioned works it is assumed that the transition between the reference frames is completely determined by two effects, that is, the additional velocity of the perturbed shock surface and the change in the direction of the normal to the perturbed shock surface. Lubchich and Pudovkin (2004) have shown that such a description is incomplete and, consequently, gives rise to erroneous results. In fact, it is necessary to take into account one more effect associated with the noninertiality of the local reference frame. The transition into the noninertial frame corresponds to the emergence of an inertial force field and additional pressure, applied on a perturbed shock surface. The physical ground of the inertial force appearance is nonideality of the medium inside the thin front of a real shock wave. This effect essentially influences the solutions of the problems on incident wave interaction with a shock wave and on the conditions of shock stability. Lubchich and Pudovkin (2004) considered a hydrodynamic problem. But the same effect should be taken into account in magnetic hydrodynamics, as well. If the conclusion of Lubchich and Pudovkin (2004) is correct, then the results of many of the above-mentioned works need to be revised. The exception is the work of McKenzie and Westphal (1969), where the shock surface oscillations do not emerge. The influence of additional pressure on the passage of the MHD waves through a perpendicular shock wave for the tangential component of the wave vectors being perpendicular to the external magnetic field, was studied by Lubchich and Pudovkin (1998).

The important interest represents the study of the interaction of the turbulence with a shock wave. Zank et al. (2002) have performed a self-consistent analysis of such an interaction by considering the influence of the turbulence on the mean characteristics of the medium. In particular, the mean shock speed was found to increase with increasing levels of upstream turbulence. Correspondingly, the efficiency of the upstream turbulence amplification by the shock decreased. The implication of this result is that the energy in upstream turbulent fluctuations, while being amplified at the shock, is also being converted into mean flow energy downstream. Hence, from the self-consistent analysis it follows that the study of the interaction of an incident perturbation with a shock wave in the linear approximation will tend to overestimate the levels of downstream turbulence. Unfortunately, the investigation has been limited by a hydrodynamic case, that is, the influence of a magnetic field was not considered.

From the above information it is clear that in the problem of the interaction of small perturbations with MHD discontinuities there are still many unsolved important points. Furthermore, some of the obtained solutions presumably need a revision. To meet this need, it seems useful to analyse in detail the properties of MHD waves in the half space limited by the front of the MHD discontinuity. Our paper presents such an analysis.

In the next section the various methods of the determination of the propagation direction for waves emanating from MHD discontinuity will be considered. After critical analysis a “well-behaved” method will be chosen.

In Sect. 3 within the chosen method we will analyse the properties of magnetoacoustic waves. In particular, we will define how many of the magnetoacoustic waves satisfying Snell’s law can become the surface waves which damp when moving away from the discontinuity. A mutual orientation of wave vectors of all possible modes from the two sides of a fast shock wave will be investigated in detail.

In Sect. 4 it will be shown that under certain conditions behind the front of a fast shock wave four homogeneous slow magnetoacoustic waves satisfying Snell’s law can propagate and any fast magnetoacoustic waves cannot propagate. We will state the favourable conditions for the realisation of such “exotic” cases. Also, it will be analyse as to how such a situation is related to the conditions of the evolutionarity of MHD shock waves.

Usually the problems of the interaction of small perturbations with discontinuities are treated in the reference frame connected to the plane discontinuity. On the contrary, the properties of MHD waves (their polarisation, dispersion relations, etc.) are determined in the reference frame in which plasma does not have any mass velocity. This complicates the solution to the problems. Therefore, in Sect. 5 we will obtain a set of equations for the amplitudes of the perturbations of speed, magnetic field, density and pressure in the MHD waves in the reference frame connected to the plane discontinuity. By using these equations we will determine the polarisation of the magnetoacoustic wave when it is a surface wave.

In the final section we will summarise the results obtained and discuss some of their implications.

2 The dispersion relation for MHD waves in the medium with a plane boundary

The equations of one-fluid ideal magnetohydrodynamics are used to describe the medium. It means that we neglect all dissipative effects (the viscosity and heat conductivity are infinitesimals and electroconductivity is infinitely large). We assume that the medium is divided by the plane MHD discontinuity into two half spaces. The discontinuity can be a fast or slow shock wave or rotational discontinuity. It means, that there is a nonzero mass flux through the surface. In this paper the property of MHD waves will be analysed in each of the half spaces.

We consider it necessary to keep in mind the general method of the solution for the problems of the interaction of small perturbations with MHD discontinuities. We will do this by choosing the investigation of the transformation of plane MHD waves on the plane discontinuity, as an example. This problem is usually investigated by the method of perturbations. The solution is accomplished in several stages.

1. The complete set of differential MHD equations determines all possible types of linear MHD waves and their characteristics in both semi-space.

2. The system of MHD boundary conditions, including the conservation laws of the mass flux, momentum flux, energy flux, as well as the conditions of the continuity of the normal magnetic field and tangential electric field, should be satisfied on the discontinuity. The unperturbed boundary conditions determine the possible types of the plane MHD discontinuities and the relation between the background quantities on both sides of the discontinuity. The first order perturbations in the boundary conditions yields the relation of the variations of various quantities (the magnetic field, fluid velocity, fluid density, etc.) on two sides of the discontinuity.

3. Variations of all quantities on the two sides of the discontinuity are expressed in terms of amplitudes and propagation angles of the incident wave and all theoretically possible emanating waves, including the amplitude of fluctuations of discontinuity speed.

4. The propagation angles of diverging waves are related to the direction of the propagation of an incident wave through Snell's law, following from the conditions of continuity of the wave frequencies (in the reference system connected to the unperturbed discontinuity) and tangential components of the wave vectors. Note that the conditions of continuity of these two quantities stem exclusively from the assumed smallness of the amplitudes of all waves and do not depend on a specific set of equations or a particular task (e.g. Landau and Lifshits (1986)).

5. The perturbed boundary conditions are expressed in terms of the amplitude of an incident wave which is assumed to be given, and the unknown amplitudes of emanating waves. After some algebra we obtain a system of linear non homogeneous equations. Having solved this system, the amplitudes of all diverging waves can be obtained.

In this section we consider methods which can be used to determine the propagation angles of diverging waves depending on the parameter c_p, which is the phase velocity of surface discontinuity oscillations. This parameter is set either by the incident wave or by spontaneously emerging fluctuations of the discontinuity surface. The reference frame connected to the unperturbed discontinuity surface is used in the analysis. The X-axis is chosen in the downstream direction and normally to the discontinuity surface. The Y-axis is directed along the component of the wave vectors tangential to the discontinuity. The Z-axis lies in the plane of discontinuity; the orthogonal right-hand coordinate system is used. In this coordinate system the propagation vectors of all waves to be investigated lie in the plane $Z=0$.

Usually one of the following two methods of determining the propagation direction for emanating MHD waves, depending on the property of the incident wave, is used, yet both of them have essential drawbacks. One is based on the graphic analysis of Friedrichs' diagram, expressing a dependence of phase velocity and group velocity of MHD waves on the polar angle between the wave vector and the external magnetic field (Kontorovich (1959), McKenzie and Westphal (1970), Westphal and McKenzie (1969)). The method has poor accuracy and is inconvenient for use in computations. Besides, it cannot be applied at a sufficiently large value of the tangential component of the wave vector when behind the discontinuity, the emanating fast magnetoacoustic wave is a surface wave. The other method is based on the Doppler effect (Lee (1982), Westphal and McKenzie (1969), Hassam (1978)). We describe it when it is applied to the transmission of MHD waves through a fast shock wave. The conditions of the continuity of the frequency ω and the tangential component of the wave vector k_y on the unperturbed shock may be written in the form

$$c_p \equiv \frac{\omega_{in}}{k_{in}\cdot\sin\lambda_{in}} = \frac{\omega_d}{k_d\cdot\sin\lambda_d}, \tag{1}$$

where the subscripts in and d refer to the incident and diverging waves, respectively; λ is the angle between the wave vector and X-axis. The value of c_p is the same for all waves. In particular, with such velocity, fluctuations on the discontinuity surface will propagate. Therefore, we name this parameter the phase velocity of surface discontinuity oscillations or the surface phase velocity. This equation can be rewritten as

$$\frac{u_{in}+V_{x1}\cdot\cos\lambda_{in}}{\sin\lambda_{in}}+V_{y1}=\frac{u_d+V_{x2}\cdot\cos\lambda_d}{\sin\lambda_d}+V_{y2}, \tag{2}$$

where the subscripts 1(2) refer to the quantities ahead (behind) of the discontinuity; V is the unperturbed fluid velocity and u the phase velocity of the wave. Equation (2) should be solved for the angle of refraction λ_d. At this point at least two difficulties arise. The mathematical one proceeds from a dependence of the phase velocity of diverging magnetoacoustic waves on the angle between the wave vector and the external magnetic field; while this angle, in turn, depends on the angle of refraction λ_d. Thereby, we obtain a transcendental equation, which can only be solved numerically, i.e. by iterations. This is not always convenient, even though the convergence of iterations is rather rapid (typically not more than ten iterations are required to achieve a sufficiently good accuracy). The physical difficulty consists in the necessity to initialise by hand, mostly based on intuition, the signs of the phase velocities of the incident and emanating waves. Based on our experience we know how easy it is to make a mistake here. Besides, physical interpretation of Eq. (2) is not so evident. Therefore, in analysing the transformation of MHD waves on a discontinuity, there is often an annoying misunderstanding. From Eq. (2) it is visible that the angles of refraction λ_d of all waves diverging from a discontinuity grow with the growth of the incident angle λ_{in}. The refraction angle of the fast magnetoacoustic wave grows more rapidly. At some incident angle, the wave vector of this emanating mode becomes parallel to the surface of the discontinuity. With further growth of λ_{in}, the wave vector of the fast magnetoacoustic mode becomes directed toward the discontinuity. And finally, starting with some incident angle, Eq. (2) can be satisfied only by a complex λ_d. The question arises as to when will the wave diverge from the discontinuity and how does one interpret a situation with a complex λ_d?

When investigating the spontaneous emission of the sound by a hydrodynamic shock wave, D'yakov (1954) considered the sound to be diverged from a shock if its wave vector was directed away from the discontinuity. Kontorovich (1957) showed that it is necessary to take into account a drag of the sound by the flow moving through a shock. The sound will remain an outgoing wave as far as the sum $V_{x2}+c_{s2}\cdot\cos\lambda_d$ remains positive. Here c_s is the sound speed. Physically, the situation is absolutely clear. If, for example, you are moving upstream a strong flow, you will move away from the object, instead of coming closer to it. With this effect included, Kontorovich (1957) corrected the conditions obtained by D'yakov (1954) of the spontaneous emission of the sound by a shock wave. As a result, the above problem did not arise in "usual" (non-magnetic) hydrodynamics. But in magnetohydrodynamics the situation is more complicated. In an anisotropic medium, such as the magnetohydrodynamic one, the directions of the group and phase velocities of the MHD waves are typically not coincident. Therefore, there can be a situation when the phase velocity is directed toward the discontinuity, while the group velocity is directed away from it, or vice versa. What does one do in this situation? For the first time such a question arose in crystal optics. Mandelshtam (1945) showed that the direction of wave propagation should be determined by the sign of the group velocity projection to a normal to the surface, since the group velocity is a physically meaningful characteristic of the wave propagation. Kontorovich (1959) formulated a similar principle for magnetic hydrodynamics. Westphal and McKenzie (1969) also took this point into account. They calculated the coefficients of transmission of MHD waves up to the incident angle, at which the group velocity of the fast magnetoacoustic wave, with the effect of the drift included, is parallel to the shock surface. They named the corresponding angle of incidence "a critical angle". Westphal and McKenzie (1969) did not consider larger incident angles, though there were no physical or mathematical reasons for such a restriction. Unfortunately, in more recent studies, a misunderstanding concerning the above problem arose again. For example, Hassam (1978), when investigating the passage of Alfvén waves through the fast shock wave, considered only those incident angles at which the wave vectors of all emanating modes are directed away from the discontinuity. He believed that even the real solutions of Eq. (2) at larger incident angles have no physical meaning. Zhuang and Russell (1982), in investigating the interaction of MHD waves with a strong MHD shock wave, discriminated the diverging modes by the sign of the X-component of the wave vector. Naturally, at those incident angles for which the wave vector of a refracted fast magnetoacoustic wave was directed toward the shock, their approach indicated a number of diverging waves by one less than that required by the conditions of evolutionarity. On this basis they made an erroneous conclusion that the fast shock wave is nonevolutionary against the perturbations incident upon a discontinuity at big angles. Whang et al. (1987) performed a similar study for fast shock waves of arbitrary intensity. They also came to an erroneous conclusion about non-evolutionarity of shock with respect to big incident angles of MHD waves. We keep in mind that the conditions of evolutionarity of shock waves have been obtained under the assumption that perturbations propagate along the normal to a discontinuity (Akhiezer et al. (1959), Syrovatskii (1959)). These conditions are

$$V_{x1} > V_{F1};\, V_{F2} > V_{x2} > V_{A2} \text{ for fast shock waves,}$$
$$V_{A1} > V_{x1} > V_{S1};\, V_{S2} > V_{x2}, \text{ for slow shock waves.} \tag{3}$$

Here V_F, V_A and V_S are the phase velocities of a fast magnetoacoustic wave, an Alfvén wave and a slow magnetoacoustic wave propagating along the normal to a shock. Kontorovich (1959) proved that the number of diverging waves at any c_p (1) is the same. Therefore, when conditions (3) are met, for normally incident perturbations, the shock wave will remain evolutionary against the perturbations incident at any angle.

To avoid such errors in the future, we are returning again to the problem of the determination of diverging angles for

MHD waves. The total set of MHD equations in each half spaces consists of the two Maxwell equations, equation of continuity, Euler equation, entropy equation and equation of state. These equations are

$$\mathrm{div}\ \boldsymbol{B}=0,$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=\mathrm{rot}\ [\boldsymbol{V}\times\boldsymbol{B}],$$

$$\frac{\partial \rho}{\partial t}+\mathrm{div}\ \rho\cdot\boldsymbol{V}=0,$$

$$\frac{\partial \boldsymbol{V}}{\partial t}+(\boldsymbol{V}\cdot\nabla)\cdot\boldsymbol{V}=-\frac{1}{\rho}\cdot\nabla P-\frac{1}{4\cdot\pi\cdot\rho}\cdot[\boldsymbol{B}\times\mathrm{rot}\ \boldsymbol{B}],$$

$$\frac{\partial S}{\partial t}+(\boldsymbol{V}\cdot\nabla)\,S=0,$$

$$P=P\,(\rho,S)\,. \tag{4}$$

Here B is the ambient magnetic field, ρ the fluid density, P the thermal pressure, and S the entropy. It is possible to specialise the form of the equation of state, using the approximation of perfect gas:

$$P=\rho^{\gamma}\cdot\exp\left(\frac{S}{c_V}\right). \tag{5}$$

Here c_V is the specific heat at constant volume; γ the ratio of the specific heats.

We shall present all quantities entered in Eqs. (4) as

$$A=A_0+\delta A\cdot\exp\left(\chi\cdot x+\mathrm{i}\cdot k_y\cdot y-\mathrm{i}\cdot\omega\cdot t\right). \tag{6}$$

Here A_0 is the background unperturbed value of quantity A; δA is its small variation travelling with a plane MHD wave. The MHD wave has a given frequency ω and tangential at the unperturbed surface component of the wave vector k_y. The component of the wave vector χ normal to the surface is an unknown quantity to be sought. It can be a real quantity in the case of a simple, homogeneous wave or a complex quantity in the case of a surface, inhomogeneous wave. As usual, i is an imaginary unit. We substitute the quantities of the form Eq. (6) into the set Eq. (4) of the MHD equations. After linearization we obtain a set of linear equations with respect to the small variations δA:

$$\chi\cdot\delta V_{Ax}+\mathrm{i}\cdot k_y\cdot\delta V_{Ay}=0,$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta V_{Ax}=\mathrm{i}\cdot k_y\cdot\left(V_{Ay}\cdot\delta V_x-V_{Ax}\cdot\delta V_y\right),$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta V_{Ay}=-\chi\cdot\left(V_{Ay}\cdot\delta V_x-V_{Ax}\cdot\delta V_y\right),$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta V_{Az}=\left(\chi\cdot V_{Ax}+\mathrm{i}\cdot k_y\cdot V_{Ay}\right)\cdot\delta V_z\text{-}V_{Az}\cdot$$

$$\left(\chi\cdot\delta V_x+\mathrm{i}\cdot k_y\cdot\delta V_y\right),$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta\rho+\rho\cdot\left(\chi\cdot\delta V_x+\mathrm{i}\cdot k_y\cdot\delta V_y\right)=0,$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta V_x=$$

$$-\frac{\chi\cdot\delta P}{\rho}-\left(\chi\cdot V_{Az}\cdot\delta V_{Az}+V_{Ay}\cdot\left(\chi\cdot\delta V_{Ay}-\mathrm{i}\cdot k_y\cdot\delta V_{Ax}\right)\right),$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta V_y=$$

$$-\frac{\mathrm{i}\cdot k_y\cdot\delta P}{\rho}-\left(\mathrm{i}\cdot k_y\cdot V_{Az}\cdot\delta V_{Az}-V_{Ax}\cdot\left(\chi\cdot\delta V_{Ay}-\mathrm{i}\cdot k_y\cdot\delta V_{Ax}\right)\right),$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta V_z=\left(\chi\cdot V_{Ax}+\mathrm{i}\cdot k_y\cdot V_{Ay}\right)\cdot\delta V_{Az},$$

$$-\mathrm{i}\cdot\omega_*\cdot\delta S=0,$$

$$\delta\rho=c_s^{-2}\cdot\delta P+\left(\frac{\partial\rho}{\partial S}\right)_P\cdot\delta S. \tag{7}$$

For convenience, we introduce a new quantity $\omega_*\equiv\omega+\mathrm{i}\cdot\chi\cdot V_x-k_y\cdot V_y$; hereinafter we omit index 0, denoting unperturbed quantities and a common exponential factor in perturbed quantities. Here $\boldsymbol{V}_A\equiv\frac{\boldsymbol{B}}{\sqrt{4\pi\rho}}$ is the Alfvénic speed and $\delta\boldsymbol{V}_A\equiv\frac{\delta\boldsymbol{B}}{\sqrt{4\pi\rho}}$.

We have written the linearized equation of state in a general form, without using the approximation of perfect gas. It is obvious that the first equation in set (7) is a linear combination of the two following equations. Therefore, it can be excluded from further consideration. As a result, we have a set of nine linear equations for nine small variations $\delta\boldsymbol{V}$, $\delta\boldsymbol{V}_A$, δP, $\delta\rho$, and δS. As the differential equations of initial set (4) were of the first order, the coefficients of the linearized set of Eqs. (7) are linear functions of the unknown quantity χ. A nontrivial solution of a set of linear homogeneous equations exists if its determinant equals zero. Since one of the nine equations (the linearized equation of state) does not depend on χ, the determinant can be written as an algebraic equation of the eighth-order for χ. Such a result was predicted by Westphal and McKenzie (1969). It is well known that the set of linearized MHD equations in the fluid's rest frame breaks up into two subsets determining Alfvén waves and magnetoacoustic waves. Therefore, we expect that our algebraic equation can be presented as a product of several factors, as well. This was also suggested by Kontorovich (1959). Indeed, after some algebra that is omitted here, we managed to write the condition of the determinant of the combined equations (7) to be zero in the form

$$\frac{\chi}{\mathrm{i}\cdot k_y}\cdot\left(\frac{\chi}{\mathrm{i}\cdot k_y}+\frac{V_y-c_p}{V_x}\right)\cdot\left(\frac{\chi}{\mathrm{i}\cdot k_y}+\frac{(V_y+V_{Ay})-c_p}{V_x+V_{Ax}}\right)\cdot$$

$$\left(\frac{\chi}{\mathrm{i}\cdot k_y}+\frac{(V_y-V_{Ay})-c_p}{V_x-V_{Ax}}\right)\cdot$$

$$\left(a_0\cdot\left(\frac{\chi}{\mathrm{i}\cdot k_y}\right)^4+a_1\cdot\left(\frac{\chi}{\mathrm{i}\cdot k_y}\right)^3+a_2\cdot\left(\frac{\chi}{\mathrm{i}\cdot k_y}\right)^2+a_3\cdot\left(\frac{\chi}{\mathrm{i}\cdot k_y}\right)+a_4\right)=0. \tag{8}$$

Here the coefficients a_0, a_1, a_2, a_3, and a_4 are given by

$$a_0=V_x^4-V_x^2\cdot\left(c_s^2+V_A^2\right)+V_{Ax}^2\cdot c_s^2,$$

$$a_1=2\cdot\left[V_x\cdot\left(c_p-V_y\right)\cdot\left(c_s^2+V_A^2-2\cdot V_x^2\right)+V_{Ax}\cdot V_{Ay}\cdot c_s^2\right],$$

$$a_2=-\left(V_x^2+\left(c_p-V_y\right)^2\right)\cdot\left(c_s^2+V_A^2\right)+c_s^2\cdot\left(V_{Ax}^2+V_{Ay}^2\right)$$
$$+6\cdot\left(c_p-V_y\right)^2\cdot V_x^2,$$
$$a_3=2\cdot\left[V_x\cdot\left(c_p-V_y\right)\cdot\left(c_s^2+V_A^2-2\cdot\left(c_p-V_y\right)^2\right)+V_{Ax}\cdot V_{Ay}\cdot c_s^2\right],$$
$$a_4=\left(c_p-V_y\right)^4-\left(c_p-V_y\right)^2\cdot\left(c_s^2+V_A^2\right)+c_s^2\cdot V_{Ay}^2. \tag{9}$$

Equation (8), with coefficients Eq. (9) describes the linear MHD waves from both sides of the plane MHD discontinuity of an arbitrary type. In particular, it can be a tangential discontinuity. In this case it is necessary to set $V_x=0$. In analysing the properties of MHD waves ahead (behind) of the discontinuity, one should take the parameters of the unperturbed medium in a corresponding half spaces. In deriving conditions Eqs. (6) and (7), the equation of state was not specified. Its particular form influences only the value of the sound speed in a medium. Thus, we obtain $c_s=\sqrt{\gamma\cdot\frac{P}{\rho}}$ in the case of perfect gas. Though Eqs. (8) and (9) were obtained for $k_y\neq0$, they can be applied when $k_y=0$, as well. In this case they describe the usual MHD waves propagating along the normal to the discontinuity. We have written Eq. (8) for $\frac{\chi}{\mathrm{i}\cdot k_y}$ which is ctg λ. The angle λ will be real for homogeneous MHD waves and complex for surface waves.

Next, we will analyse the solutions of algebraic Eq. (8). The first root $\chi=0$ does not represent any interest for the problem of the interaction of MHD waves with MHD discontinuities. It is easy to see from a substitution into Eq. (7), that the root $\chi=0$ describes either the trivial solution (the absence of perturbations), or one of MHD modes, whose wave vector is directed along the Y-axis. In the latter case the surface phase velocity c_p (in our problem it is an external parameter setting either the incident wave or "ripples" on the surface of the discontinuity) will be by coincidence equal to the phase velocity of any MHD wave propagating (in the sense of the direction of the wave vector) along the Y-axis. It can be both the Alfvén wave and fast, or a slow magnetoacoustic wave. But, in spite of this coincidence, this root will not contain any new information. It will be coincident with one of seven other roots of Eq. (8).

The second root of Eq. (8) corresponds to the entropy wave. This root is always real and equal to

$$\text{ctg }\lambda_e=\frac{c_p-V_y}{V_x}. \tag{10}$$

It is well known that the entropy wave drifts with the flow, that is, it has a zero frequency ω_0 in the fluid's rest frame. This wave transfers fluctuations of entropy and associated fluctuations of density

$$\delta\rho=\left(\frac{\partial\rho}{\partial S}\right)_P\cdot\delta S.$$

In the case of perfect gas the isobaric derivative of ρ with respect to entropy S is given by

$$\left(\frac{\partial\rho}{\partial S}\right)_P=-\frac{\rho}{\gamma\cdot c_V}.$$

Since the entropy wave drifts with the flow it does not emerge in the problem of the interaction of perturbations with tangential or contact discontinuities. In these cases it cannot be incident on the discontinuity or emanated from it. For a shock wave or rotational discontinuity, the entropy wave is incident from ahead of the discontinuity and diverged from behind it. In the problem of the interaction of perturbations with discontinuity, the waves having a positive frequency in the reference frame, connected with the discontinuity surface, are considered. Then the maximum possible angle of wave propagation is determined by the condition $\omega=0$. For the entropy wave it is equal to 90°, if the flux is normal to the surface, and it is smaller (larger) than 90°, if V_y is negative (positive).

We consider two more roots of Eq. (8). They are also always real and equal to

$$\text{ctg }\lambda_{A-}=\frac{c_p-\left(V_y-V_{Ay}\right)}{V_x-V_{Ax}}\ \text{and ctg }\lambda_{A+}=\frac{c_p-\left(V_y+V_{Ay}\right)}{V_x+V_{Ax}} \tag{11}$$

Substituting these roots into the set of Eqs. (7), we can make sure that they yield the directions of wave vectors of Alfvén waves. It is known that the vector of velocity fluctuations associated with the Alfvén wave is directed along the vector $\boldsymbol{k}\times\boldsymbol{B}$. Here λ_{A-} is the angle of propagation of the backward Alfvén wave, for which $\delta\boldsymbol{V}_A=\delta\boldsymbol{V}$, and λ_{A+} corresponds to the direction of the forward Alfvén wave, for which $\delta\boldsymbol{V}_A=-\delta\boldsymbol{V}$. Hence, it is possible to state that in the medium separated by a plane discontinuity into two half spaces, the Alfvén modes never transform into the surface waves. If the boundary is a rotational discontinuity, that is, in fact, a nonlinear Alfvén wave, one of the roots (11) cannot be determined for a nonzero frequency ω. Physically, it means that from each side of a rotational discontinuity there can only be one Alfvén wave with a given surface phase velocity c_p. It will be a wave propagating in the opposite direction to a rotational discontinuity. Ahead of the discontinuity, the Alfvén wave will be incident upon the surface and behind the discontinuity, it will be diverged from the front. (Note, that the "undetermined" root (11) corresponds to an infinite X-component of the wave vector. Here dissipation effects have to be taken into account. Roikhvarger and Syrovatskii (1974) have shown that in accounting the dissipation effects from each side of the rotational discontinuity, there will be one more dissipative Alfvén wave which is necessary for consideration at the investigation of the evolutionarity of the rotational discontinuity.)

In the problem of the refraction of MHD waves on a discontinuity the conditions relating the angles of propagation of the Alfvén waves and the entropy wave are more convenient compared to Eq. (11). It can be easily shown that these conditions are reduced to

$$\text{ctg }\lambda_{A\mp}=\text{ctg }\lambda_e-q_{\mp}, \tag{12}$$

where

$$q_{\mp}=\frac{V_{Ax}\cdot\text{ctg }\lambda_e+V_{Ay}}{V_{Ax}\mp V_x}.$$

If c_p is such that the wave vector of the entropy mode $\boldsymbol{k}_e$ is perpendicular to the external magnetic field, then $q_{\mp}=0$, and both Alfvén waves are focused across the magnetic field $\boldsymbol{B}$, too.

Now, it remains to analyse the properties of the roots of the last factor in Eq. (8). This factor is a polynomial of the fourth order that determines the angles of propagation of four magnetoacoustic waves. Note that the same polynomial was earlier obtained in a different way by Zhuang and Russell (1982). The analysis of this factor will be made in the next section.

3 The dispersion equation for magnetoacoustic waves in the medium with a plane boundary

Let's study the general properties of the roots of the algebraic equation

$$F(\varphi) \equiv a_0 \cdot \varphi^4 + a_1 \cdot \varphi^3 + a_2 \cdot \varphi^2 + a_3 \cdot \varphi + a_4 = 0, \tag{13}$$

where $\varphi = \frac{\chi}{i \cdot k_y} = ctg\lambda$. Here φ and λ can take both real and complex values. The coefficients a_0, a_1, a_2, a_3, and a_4 are defined by conditions Eq. (9) and are always real. The roots of Eq. (13) set the directions of the wave vectors of four magnetoacoustic waves from a chosen side of a discontinuity.

First of all, we note that the coefficient a_0 is a quadratic expression with respect to the quantity V_x^2. The equality $a_0=0$ is realised only if the quantity $|V_x|$ is equal to the phase velocity of a fast or slow magnetoacoustic mode, whose wave vector is directed along the X-axis. In this case Eq. (13) will evidently have only three roots. This can be easily understood, for here we are in the reference frame connected to the moving wave, in which the frequency of a wave is equal to zero and, hence, cannot take the value imposed by external conditions. Fast or slow shock waves of infinitesimal amplitude will be propagating at such speeds. But for any evolutionary discontinuity of nonzero amplitude we have $a_0 \neq 0$, and Eq. (13) has four roots. We note that in the medium ahead of a fast shock wave, behind a slow shock wave, and from each side of the tangential discontinuity, the inequality $a_0>0$ holds. In all other cases, the inequality $a_0<0$ is true.

Generally, an algebraic equation of the fourth order with real coefficients can have four real roots, or two real and two complex conjugate roots, or four complex roots. We will show that in the problem considered the situation with four complex roots is never realised.

Separately, we analyse the case when the wave vector $\boldsymbol{k}_e$ of the entropy wave, whose direction is determined by Eq. (10), is perpendicular to the vector $\boldsymbol{B}$ of the external magnetic field. In this case, as can be easily proved by a direct substitution, the quantity $\varphi=\varphi_e \equiv \text{ctg}\,\lambda_e$ is a double root of Eq. (13). Hence, in this case Eq. (13) will have at least two real roots. It is interesting to note that for $\boldsymbol{k}_e \perp \boldsymbol{B}$ the five MHD waves will be propagating in the coincident direction. They are the entropy wave, forward and backward Alfvén wave, and forward and backward slow magnetoacoustic wave. All these waves will have a zero frequency in the fluid's rest frame, that is, will simply drift with the moving flow. A similar situation is realised in the usual hydrodynamics for the propagation of an entropy-vorticity wave. The fact that under certain conditions five of the seven MHD waves are propagating in the coincident direction with an identical phase velocity can strongly simplify the problem of the interaction of MHD perturbations with shock waves. For example, Lubchich and Pudovkin (1998) treated this case and investigated the passage of MHD waves through a perpendicular shock wave when the external magnetic field was directed along the Z-axis. Under such an orientation of $\boldsymbol{B}$, the wave vector of any MHD wave from any side of the shock is perpendicular to the external magnetic field. It turned out that in this case, it is possible to obtain analytical expressions for the amplitudes of all diverging waves, depending on the type of incident wave, its direction of propagation, Mach number, and magnitude of external magnetic field.

Under any other direction of $\boldsymbol{k}_e$, we obtained

$$F(\varphi_e) = \frac{c_s^2}{\sin^2\lambda_e} \cdot \left(V_{Ax} \cdot \text{ctg}\,\lambda_e + V_{Ay}\right)^2 > 0.$$

Now let us determine the sign of $F(\varphi)$ at $\varphi=\varphi_{A\mp} \equiv \text{ctg}\,\lambda_{A\mp}$. Substituting Eq. (12) into Eq. (13), we obtain after some tiresome calculations the quadratic equation for ctg λ_e

$$F(\varphi_{A\mp}) = \frac{V_x^2 \cdot q_{\mp}^2}{(V_{Ax} \mp V_x)^2} \cdot$$

$$\left\{ -V_x^2 \cdot V_{A\tau}^2 \cdot ctg^2\lambda_e + 2 \cdot V_x \cdot V_{Ay} \cdot \left(V_x \cdot V_{Ax} \mp V_A^2\right) \cdot \right.$$

$$\left. ctg\lambda_e + \left[V_x^2 \cdot \left(V_{Ay}^2 - V_A^2\right) - V_A^2 \cdot \left(V_{Ax}^2 + V_{Ay}^2\right) \mp 2 \cdot V_x \cdot V_{Ax} \cdot V_A^2\right]\right\}$$

where $V_{A\tau}^2 = V_{Ay}^2 + V_{Az}^2$.
The discriminant of the expression in brackets is

$$\text{Disq} = -4 \cdot V_x^2 \cdot V_A^2 \cdot V_{Az}^2 \cdot (V_x \mp V_{Ax})^2 \leq 0.$$

One can see that $F(\varphi_{A\mp})$ does not change the sign, i.e. for any type of discontinuity and from any side of it, it is always $F(\varphi_{A\mp}) \leq 0$. It can be shown that under certain conditions the quantity φ_{A+} or φ_{A-} is a simple root of algebraic Eq. (13). Then they will refer to the magnetoacoustic mode, whose wave vector is collinear to the external magnetic field, and the phase velocity in the fluid's rest frame is equal to Alfvénic speed. In any case, $F(\varphi_e)$ and $F(\varphi_{A\mp})$ have different signs. Consequently, between φ_e and, for example, φ_{A+} there is at least one real root of algebraic Eq. (13). Then Eq. (13) must have one more real root.

We have shown that algebraic Eq. (13) always has at least two real roots. Consequently, at any value c_p from any side of any MHD discontinuity, there are at least two homogeneous magnetoacoustic waves having a given surface phase velocity. In the problem of the interaction of perturbations with the MHD discontinuity there can emerge only one surface wave from each side of the boundary.

To analyse the properties of the roots of Eq. (13) in more detail, it is necessary to specify the type of MHD discontinuity. For the readers not to become tired, we will restrict

Table 1. Characteristic values of propagation angles of MHD waves ahead of a fast shock wave and the corresponding signs of function F(φ).

The angle λ	0°	λ_{A1}	λ_e	λ_{A2}	180°
The sign of F(φ)	>0	≤0	≥ 0	≤0	>0

Table 2. Characteristic values of propagation angles of MHD waves behind a fast shock wave and the corresponding signs of function F(φ).

The angle λ	0°	λ_{A1}	λ_e	λ_{A2}	180°
The sign of F(φ)	<0	≤0	≥ 0	≤0	<0

the analysis to the fast shock wave. This type of discontinuity is of great importance, for example, in the study of solar-terrestrial coupling. Let us return to Eq. (12), that determines the angles of propagation of the two Alfvén waves. From any side of the fast shock wave the quantities q_+ and q_- evidently have different signs. Hence, the angle of propagation of the entropy wave λ_e lies between the angles λ_{A+} and λ_{A-}. Determining which of the angles λ_{A-} and λ_{A+} is smaller, depends on the orientation of the external magnetic field. Therefore, we shall denote λ_{A1}=min $(\lambda_{A-}, \lambda_{A+})$ and λ_{A2}=max $(\lambda_{A-}, \lambda_{A+})$.

As stated above, ahead of the fast shock wave the coefficient a_0 is always positive. Then at $\varphi=+\infty$, which corresponds to the angle of wave propagation $\lambda=0°$, and at $\varphi=-\infty$, which corresponds to the angle of wave propagation $\lambda=180°$, the sign of $F(\varphi)$ will be positive. In Table 1 the characteristic values of the angles of wave propagation and signs of function F(φ) corresponding to them are given.

As stated above, if $F(\varphi_e)=0$, then $\boldsymbol{k}_e\perp\boldsymbol{B}$ and the quantity $\lambda_{A1}=\lambda_e=\lambda_{A2}$ is the double root of Eq. (13). It can be shown that in this case $F'(\varphi_e)=0$ and

$$F''(\varphi_e)=\left\{-2\cdot V_x^2\cdot\left(c_s^2+V_A^2\right)+2\cdot c_s^2\cdot V_{Ax}^2\right\}\cdot\left(\varphi_e^2+1\right).$$

Taking into account conditions of evolution (3), it is obvious that $F''(\varphi_e)<0$ ahead of the fast shock wave. Hence, at $\varphi=\varphi_e$ the function $F(\varphi)$ has a maximum. As a consequence, Eq. (13) will have one real root in the intervals $\varphi<\varphi_e$ and $\varphi>\varphi_e$.

If $\boldsymbol{k}_e\cdot\boldsymbol{B}\neq0$, then $F(\varphi_e)>0$. If $F(\lambda_{A1})=0$, then $F(\lambda_{A2})<0$ and, on the contrary, if $F(\lambda_{A2})=0$, then $F(\lambda_{A1})<0$. If $F(\lambda_{A1})<0$ and $F(\lambda_{A2})<0$, then the function $F(\varphi)$ has changed the sign four times.

As a result, we come to the conclusion that ahead of the fast shock wave there always exist four real roots, corresponding to the four homogeneous magnetoacoustic waves incident on the discontinuity. Their propagation angles must lie in the intervals:

– (0°, λ_{A1}), for the backward fast magnetoacoustic wave,

– (λ_{A1}, λ_e), for the backward slow magnetoacoustic wave,

– (λ_e, λ_{A2}), for the forward slow magnetoacoustic wave,

– (λ_{A2}, 180°), for the forward fast magnetoacoustic wave.

The quartic Eq. (13) can be solved analytically, using Descartes-Euler-Cardano's algorithm or Ferrari's algorithm (Korn and Korn (1968)). But in practice it is easier to use numerical methods. For this case it is useful to specify the lower limit of the first interval and the upper limit of the fourth interval, for φ to always remain finitesimal. Using properties of roots of the algebraic equation (Korn and Korn (1968)), it is possible to replace 0° for arctg $(1+g)$ and 180° for 180°− arctg $(1+g)$, where g≡ $|a_0|^{-1}\cdot$max $(|a_1|, |a_2|, |a_3|, |a_4|)$.

Behind the front of the fast shock wave the coefficient a_0 is always negative. Hence, the function F(φ) takes negative values at $\varphi=\pm\infty$. The characteristic values of wave propagation angles and corresponding signs of the function F(φ) are given in Table 2. The function changes the sign at least twice. Therefore, behind the front of the fast shock wave there are always two real roots, corresponding to two slow magnetoacoustic waves. Their propagation angles must lie in the intervals:

- (λ_{A1}, λ_e), for the backward slow magnetoacoustic wave,
- (λ_e, λ_{A2}), for the forward slow magnetoacoustic wave.

Two other roots can be either real quantities or complex-conjugate quantities. If they are complex, the root with Re $\chi<0$ refers to a surface magnetoacoustic wave damping while moving off from the shock. Formally, this wave is considered to be diverging from the discontinuity. If these two roots are real, then both of them must lie in one of four intervals: (0°, λ_{A1}), (λ_{A1},λ_e), (λ_e, λ_{A2}) or (λ_{A2}, 180°). In the problem of the interaction of MHD waves with a fast shock wave, these two roots, as a rule, lie in the interval (λ_{A2}, 180°) and correspond to two fast magnetoacoustic waves. The greater root corresponds to the wave incident on the shock, while the smaller one refers to a wave emanating at the discontinuity.

The typical form of Snell's law for a fast shock wave is shown in Fig. 1. We have set all parameters of the flux in the uncompressed medium, that are needed: M_1=10; θ_1=0°; ψ_1=45°; α=30°; β_1=1.5; γ=5/3. Here $M={}^{V_x}\!/_{c_s}$ is the Mach number; θ is the angle between the plasma flow direction and normal to the shock (ahead of the shock wave it is equal to zero as a result of the choice of the reference frame); ψ is the angle between the direction of the external magnetic field and the X-axis; α is the angle between the tangential component of the magnetic field and the Y-axis; β is the ratio of the thermal pressure to magnetic pressure. For simplicity we use the approximation of perfect gas. The chosen parameters are typical, for example, for the solar wind ahead of the Earth's bow shock, which is a fast shock wave. Next, using the algorithm offered by Whang

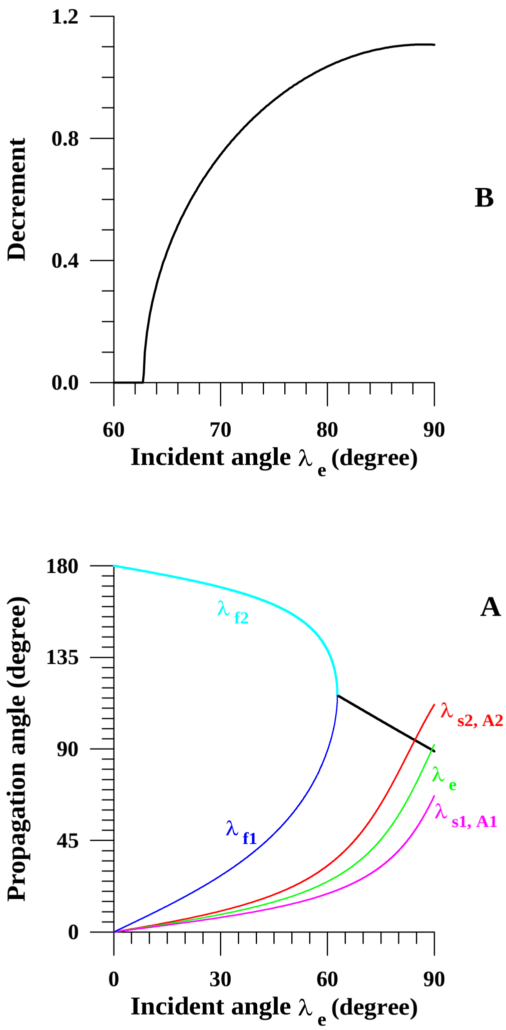

Fig. 1. Propagation angles of all theoretically possible MHD waves (A) and decrement of a surface fast magnetoacoustic wave (B) behind the front of a sufficiently strong, fast shock wave versus the incident angle of entropy wave λ_e. The entropy wave propagates in the uncompressed medium; the incident angle determines the surface phase velocity common for all waves propagating in the compressed medium. Behind the front of the fast shock wave there can be six diverging modes. They are the forward and backward slow magnetoacoustic waves (denoted by $s1$ and $s2$), forward and backward Alfvén waves (denoted by $A1$ and $A2$), entropy wave (denoted by e), and fast magnetoacoustic wave (denoted by $f1$). Besides, there is an incident fast magnetoacoustic wave (denoted by $f2$) shown with the bold line.

et al. (1987), we have calculated the parameters of flux in the compressed medium. The following values have been obtained: $M_2 \approx 0.47$; $\theta_2 \approx 2.5°$; $\psi_2 \approx 75°$; $\beta_2 \approx 22$; the density across the shock grows by a factor of ≈ 3.8, and plasma pressure grows by factor ≈ 119. Behind the shock the plasma pressure becomes much greater than the magnetic pressure. It is known (e.g. Hassam (1978)) that in such a medium the phase velocity of a slow magnetoacoustic wave is nearly equal to the phase velocity of Alfvén wave. Therefore, as one can see from Fig. 1, the angles of propagation of forward (backward) slow magnetoacoustic and Alfvén waves are practically coincident. The group and phase velocities of the fast magnetoacoustic wave have almost the same direction, and their values are close to the sound speed. Hence, the normal component of the group velocity of the fast magnetoacoustic wave is approximately equal to $c_{s2} \cdot \cos \lambda_f + V_{x2}$. For the wave diverging from the shock this sum is positive, while for the incident wave it is negative. For the angle of propagation of about 118° the sum of V_{x2} and the X-component of the group velocity, being the physical speed of wave propagation, is equal to zero; the wave propagates along the surface of the shock. We note that at such an angle of propagation the sum of V_{x2} and the normal component of the phase velocity is approximately equal to $0.04 \cdot V_{x2}$. This value is small, but nevertheless it is nonzero. At larger c_p the refracted fast magnetoacoustic wave is a surface wave. For this wave the real part of the complex angle of refraction is plotted in Fig. 1. The damping factor $\frac{|\mathrm{Re}\,\chi|}{k}$ of the surface wave, where $k = \sqrt{(\mathrm{Im}\,\chi)^2 + k_y^2}$, is also shown in Fig. 1. One can see that the surface wave damps rather rapidly when moving away from the front of shock - approximately by a factor of e on a wavelength.

In our numerical calculations we have also found some rare, exotic cases when the real roots of Eq. (13) lie in the interval $(\lambda_e, \lambda_{A2})$ or even in $(\lambda_{A1}, \lambda_e)$. In these cases they correspond to two more slow magnetoacoustic waves, with one of the slow magnetoacoustic waves being incident on the fast shock wave from the side of the compressed medium. Kontorovich (1959) suggested that behind the front of the fast shock wave with certain values of the surface phase velocity c_p there can be four slow magnetoacoustic waves, one of which is incident on the shock, while there is no fast magnetoacoustic wave. But he did not specify the conditions when this could be the case. In the following section we will discuss the conditions necessary to provide such exotic cases. It will also be shown that the existence under certain conditions of a slow magnetoacoustic wave incident on the fast shock wave from the compressed medium by no means contradicts the conditions (3) of shock waves evolutionarity.

4 Slow magnetoacoustic waves behind the front of a fast shock wave

Let us remember the Friedrichs diagram form, which expresses the dependence of phase and group velocity of MHD waves on a polar angle between the wave vector and external magnetic field. Its samples at different values of the β-parameter are shown in Fig. 2.

We should keep in mind that the phase velocity of magnetoacoustic waves is equal to

$$u^2 \equiv \left(\frac{\omega_0}{k}\right)^2 = \frac{1}{2} \cdot \left(V_A^2 + c_s^2 \pm \sqrt{\left(V_A^2 + c_s^2\right)^2 - 4 \cdot (\boldsymbol{V}_A \cdot \boldsymbol{i})^2 \cdot c_s^2} \right) \quad (14)$$

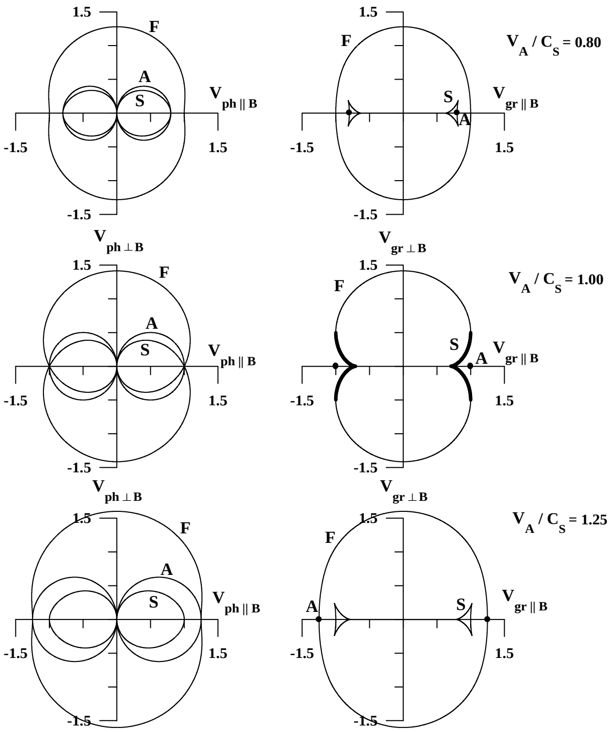

Fig. 2. Phase (left) and group (right) polar diagrams for MHD waves in three cases (from top to bottom): the sound speed in the medium is greater than, equal to or smaller than the Alfvén speed. The group and phase velocities shown on the diagrams are normalised by the sound speed.

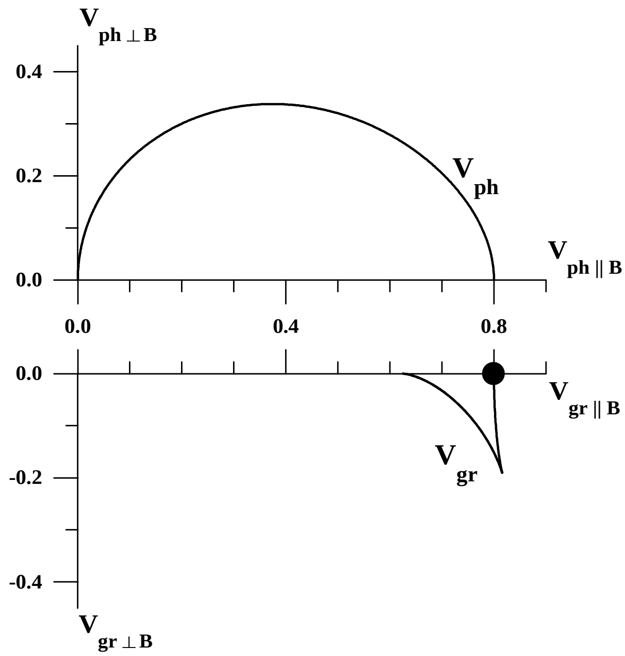

Fig. 3. Phase polar diagram for slow magnetoacoustic wave (upper plot) and the group polar diagram for a slow magnetoacoustic wave and an Alfvén wave (lower plot). The group velocity of the Alfvén wave is denoted by the symbol "•". It is assumed that $V_A = 0.8\cdot c_s$. In the plots all velocities are normalised by the sound speed.

Here $\boldsymbol{i}$ is a unit vector along the wave vector $\boldsymbol{k}$. The upper sign before the square root corresponds to a fast wave, while the lower one refers to a slow wave. Equation (14) defines the phase velocity u accurate to the sign. If $u>0$ the wave is the forward one. Otherwise, it is the backward one.

The group velocity of magnetoacoustic waves can be obtained by the differentiation of Eq. (14) over the wave vector $\boldsymbol{k}$ and written as

$$\boldsymbol{V}_{gr} \equiv \frac{\partial \omega_0}{\partial \boldsymbol{k}} = u\cdot\boldsymbol{i} \mp \frac{c_s^2\cdot(\boldsymbol{V}_A\cdot\boldsymbol{i})\cdot(\boldsymbol{V}_A\cdot\boldsymbol{j})}{u\cdot\sqrt{\left(V_A^2+c_s^2\right)^2-4\cdot(\boldsymbol{V}_A\cdot\boldsymbol{i})^2\cdot c_s^2}}\cdot\boldsymbol{j}. \qquad (15)$$

Here $\boldsymbol{j}$ is a unit vector directed along the vector $[\boldsymbol{k}\times[\boldsymbol{k}\times\boldsymbol{B}]]$. As in Eq. (14), the upper sign corresponds to the fast wave, while the lower sign refers to the slow one. One can see that the component of the group velocity normal to the wave front is coincident with the phase velocity. But there is also a component of the group velocity, which is aligned to the wave front.

Equations (12)–(13) take a simpler form for β=1, which was used, for example, by Kwok and Lee (1984) in the numerical calculations of MHD wave transmission coefficients through a rotational discontinuity. Unfortunately, at β=1 plasma has some peculiar properties, making it necessary to keep an extreme accuracy in the numerical calculations. In this case, as is clearly seen from Fig. 2, the vector of the group velocity does not tend to the direction along the external magnetic field for the angle between the wave vector and the field tending to zero. The angle between the group velocity and the external magnetic field remains nonzero; it is equal to $arctg\,(1/2)\approx 26°$. In this case, a phenomenon similar to the conic refraction in optics can be observed in plasma. A beam of magnetoacoustic waves directed along the magnetic field and incident on a plane boundary will turn into a cone with a half-angle of $\approx$26° (Kadomtsev (1988)). Alfvén (1981) points out that the topology of magnetic field in plasma may change radically due to many instabilities developing when β is close to unit.

But here we concentrate on another property of the Friedrichs diagram. From Fig. 2 it is clearly seen that the polar lines of group and phase velocity of slow magnetoacoustic waves are essentially different. Figure 3 is a close-up of the part of the group and phase polar line for slow magnetoacoustic waves, which is plotted for the angles between the wave vector and the external magnetic field lying in the interval from 0° to 90°. Firstly, one can see that the vectors $\boldsymbol{k}$ and $\boldsymbol{V}_{gr}$ lie in different quadrants. It is evident that the angle between them can be quite large. Secondly, it is seen that the group polar line has a break, in which the group velocity is maximum. Figure 4 shows a dependence of the angles θ_{ph} and θ_{gr} versus the β-parameter. Here θ_{gr} is the angle between the maximum group velocity and the external magnetic field; θ_{ph} is the angle between the wave vector $\boldsymbol{k}$, corresponding to this maximum value $V_{gr}\,\mathrm{max}$, and

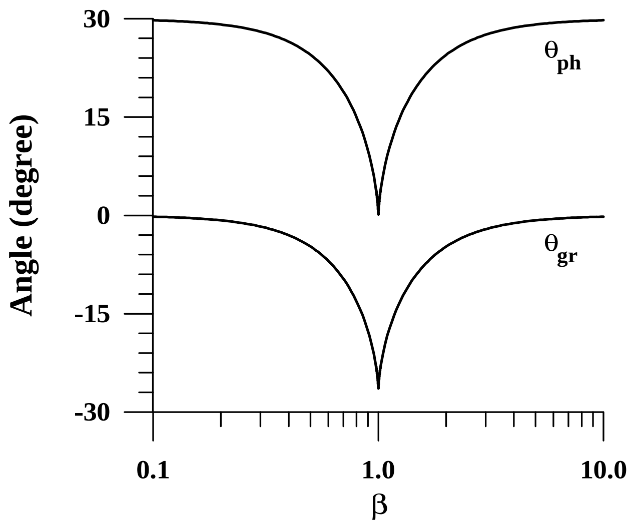

Fig. 4. Angle θ_{ph} (between the wave vector of slow magnetoacoustic mode and external magnetic field) at which the group velocity has a maximal value, and the angle θ_{gr} (between the maximal group velocity and external magnetic field) as a function of the β-parameter, which is shown in the logarithmic scale.

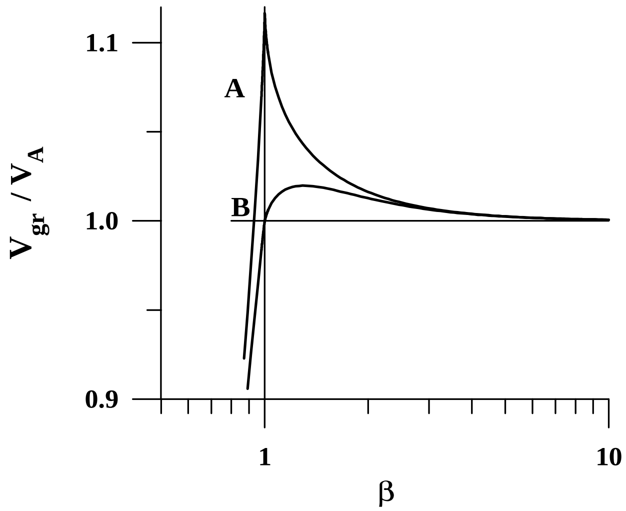

Fig. 5. Maximal group velocity of a slow magnetoacoustic wave (curve A) and its projection to the external magnetic field (curve B) versus the β-parameter, given in the logarithmic scale. The group velocity is normalised by the Alfvén speed.

$\boldsymbol{B}$. At any β the angle between V_{gr} max and the direction of the wave vector corresponding to the maximum group velocity remains almost constant and equal to ∼30°. As follows from Fig. 3, the maximum group velocity of a slow magnetoacoustic wave can exceed the Alfvénic speed. And more than that, even the projection of the group velocity on the direction of the external magnetic field can be greater than the Alfvénic speed. This especially true for the projections $\boldsymbol{V}_{gr}$ and $\boldsymbol{V}_A$ on a vector forming a large angle (close to 90°) with the external magnetic field. Figure 5 confirms these statements. It illustrates the dependence of the maximum group velocity of a slow magnetoacoustic wave and its projection on the direction of the external magnetic field versus β. It is seen, that the value of the group velocity is maximum for β=1. In this case it exceeds the Alfvénic speed by the factor of 1.12. The projection of the group velocity on the external magnetic field exceeds the Alfvénic speed at $\beta>1$. Its value reaches a maximum equal to 1.02 at $\beta\approx1.3$.

We have demonstrated that under certain conditions the physical speed of propagation of slow magnetoacoustic waves behind the front of a fast shock wave can exceed the lower limit given by the condition of evolutionarity of shock waves (3). Hence, it is theoretically possible for a slow magnetoacoustic wave to be incident on the fast shock wave from the side of the compressed medium. It is easy to formulate the conditions favourable for realisation of such a possibility. Firstly, the β-parameter behind the front of a shock wave should be close to unit. Secondly, it is desirable that the shock wave was be strong enough. Then the flow velocity behind the shock will be closer to the lower limit of evolutionarity (3).

Now we will analyse in detail the refraction of MHD waves on the fast shock waves that satisfy these conditions. If in front of a shock wave the parameters of the flow are M_1=7.2, θ_1=0°, ψ_1=15°, α=180°, β_1=0.1, and γ=5/3, then behind it these parameters become $M_2\approx0.72$, $\theta_2\approx39°$, $\psi_2\approx57°$, $\beta_2\approx1.08$. The density across the shock grows by a factor of ≈2.9, and thermal pressure grows by a factor of ≈35. Figure 6 shows the angles of propagation of MHD waves behind the front of such a shock wave versus the incident angle of the entropy wave that determines the surface phase velocity c_p. For the incident angles of the entropy wave smaller than ≈86°, Snell's law has a typical form. For the incident angle of the entropy wave ranging from 0° to 84.2° there are seven homogeneous MHD waves (six diverging waves and one incident wave). We list them in ascending order of propagation angle: a backward Alfvén wave, a backward slow magnetoacoustic wave, an entropy wave, a forward slow magnetoacoustic wave, a forward Alfvén wave, two forward fast magnetoacoustic waves (diverging from and incident on the shock). The following peculiarity of the shock waves can be easily shown: if in front of the shock the wave vector of the incident entropy, Alfvén or slow magnetoacoustic wave is orthogonal to the external magnetic field $\boldsymbol{B}_1$, then behind the shock the wave vector of five diverging modes (entropy, two Alfvén and two slow magnetoacoustic waves) is also orthogonal to the external magnetic field $\boldsymbol{B}_2$. In Fig. 6 we can see it at the incident angle of the entropy wave of 75°. At the incident angle of the entropy wave of about 84.2° (point b in Fig. 6) the angles of propagation of the two fast magnetoacoustic waves become equal to each other. We note that at point b the angle of propagation of fast magnetoacoustic waves slightly exceeds that of the forward Alfvén wave (123.6° and 123.0°, respectively). The emanating fast magnetoacoustic wave transforms to the surface wave at point b. On the segment bc the refracted fast magnetoacoustic mode is the surface wave damping off from the boundary. As far as its source is the front of the shock, formally the wave remains diverging from the boundary. On the segment cd instead of a surface mode there will be two usual homogeneous waves. There are two more forward slow magnetoacoustic waves s_3 and s_4, in addition to the previously existing forward s_2 and backward s_1 slow magnetoacoustic waves, with s_3 being incident on the shock, and s_4 diverging from it. Thus, in a certain range of surface phase velocities c_p there will be four slow magnetoacoustic waves and no fast waves. Here, there is no contradiction with the conditions of evolutionarity of a fast shock wave, since the number of waves diverging from the shock remains constant and equal to six. At point d the incident slow magnetoacoustic wave s_3 will be propagating at the same angle as the previously existing forward slow wave s_2. It is a point of transition of the diverging slow magnetoacoustic wave s_2 to the surface mode. Finally, on the segment dg there will be two diverging homogeneous slow magnetoacoustic waves, s_1 and s_4, along with the surface slow magnetoacoustic wave s_2 (formally this wave is diverging from the shock). With all theoretically possible incident angles of the entropy wave, behind the front of a fast shock wave there will be six diverging MHD waves. Therefore, the conditions of evolutionarity will always be met. Here we should note that Eq. (13) with coefficients Eq. (9) only determines the angles of propagation of magnetoacoustic waves, but does not stipulate the wave type. The latter can be determined in a different way. The magnetoacoustic wave propagating at some angle λ can only be one of the four types (forward fast, backward fast, forward slow or backward slow magnetoacoustic wave), that is, it can take only one of four phase velocities determined by Eq. (14): $+u_f$, $-u_f$, $+u_s$, or $-u_s$. Substituting all of them into Snell's law in form (1), we can determine the true phase velocity for the wave considered and hence its type. After that, from Eq. (15) its group velocity can be found. Then we calculate the projections of the phase and group velocities on the normal to the shock wave, and, finally, take into account a drag of the wave by the flow, that is, V_x+V_{phx} and V_x+V_{grx}. The results of the calculations are shown in Fig. 7. It is seen, that at points b, c, and d the X-component of the group velocity for the two waves (incident and diverging ones) vanishes. On segment ab there is a fast magnetoacoustic wave incident on shock, and on the segment cd there is an incident slow magnetoacoustic wave. For both of them $V_x+V_{grx}\leq 0$, that is, the physical speed of wave propagation is directed toward the shock. The phase velocity of the slow magnetoacoustic waves is always directed away from the shock, whereas the direction of phase velocity of the incident fast magnetoacoustic wave can be either. In Fig. 7 the phase and group velocities of the backward slow magnetoacoustic wave s_1 are not shown. For it $V_{x2}+V_{grx}\approx 1.5\cdot V_{x2}$, and $V_{x2}+V_{phx}\approx 0.9\cdot V_{x2}$.

In our numerical calculations we have revealed a more surprising situation. In front of a fast shock wave let the parameters of the flow be M_1=3.0, θ_1=0°, ψ_1=10°, α=0°, β_1=0.36, and γ=5/3. Then behind the front of the shock wave we have

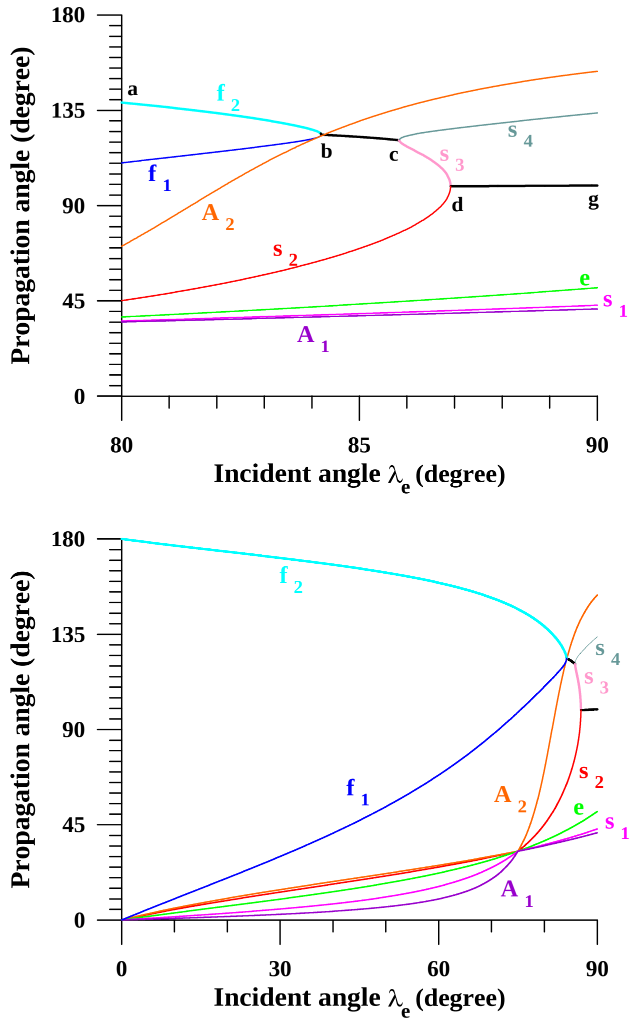

Fig. 6. Possible form of dependence of propagation angles of all theoretically possible MHD waves behind the front of the shock wave on the incident angle of entropy wave. In the top plot the range of incident angles from 80 up to 90° is shown in more detail.

$M_2\approx$0.84, $\theta_2\approx$36°, $\psi_2\approx$48°, $\beta_2\approx$0.97. The density and the thermal pressure grow across the fast shock wave by factors of $\approx$2.2 and $\approx$5.8, respectively. Figure 8 shows the angles of propagation of MHD waves behind the front of such a fast shock wave versus the incident angle of the entropy wave λ_e. For this case Fig. 9 shows the normal components of group and phase velocities of the backward slow magnetoacoustic waves versus the angle λ_e. On the length cd there is one forward and three backward slow magnetoacoustic waves. One of the backward slow magnetoacoustic waves (s_3) is incident on the shock surface. Here it is remarkable that the wave vector of the incident wave is directed away from the shock wave, that is, the angle of refraction $\lambda_{s3}<$90°! In Fig. 9 the velocities of the forward slow magnetoacoustic wave s_2 are not shown. For it $V_{x2}+V_{grx}\approx 1.6\cdot V_{x2}$, and $V_{x2}+V_{phx}\approx 0.9\cdot V_{x2}$.

We obtained the above-mentioned results using the numerical method of analysis. However, the identical results

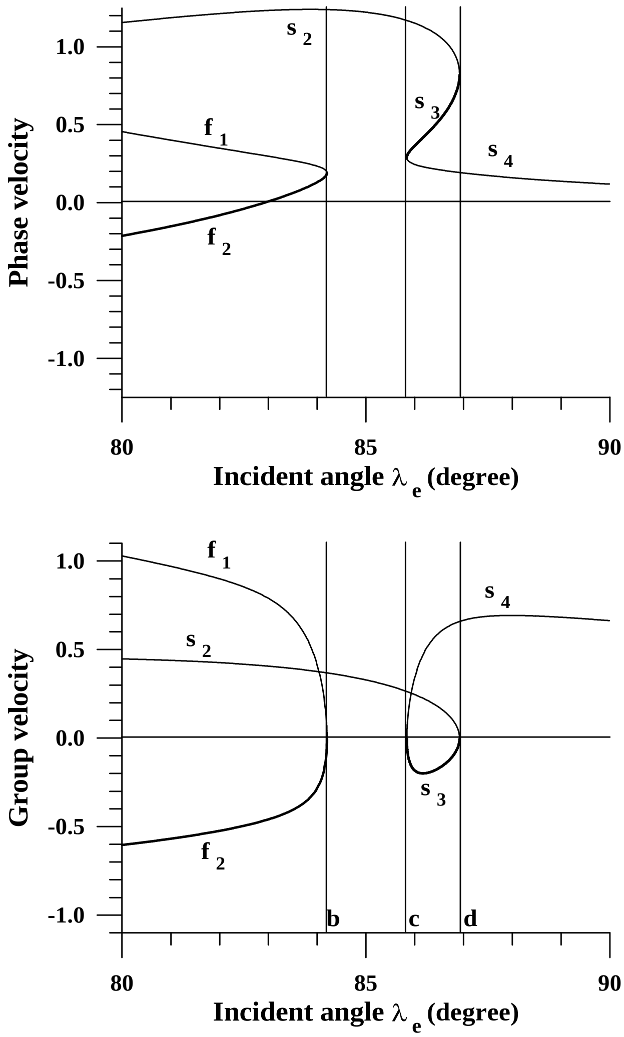

Fig. 7. X-components of the phase velocity (the upper plot) and group velocity (the lower plot) of magnetoacoustic waves as a function of the incident angle, changing from 80 up to 90°. All velocities are normalised by the normal component of the flow velocity V_{x2} behind the shock wave. Calculations are performed for the waves having the propagation angles shown in Fig. 6.

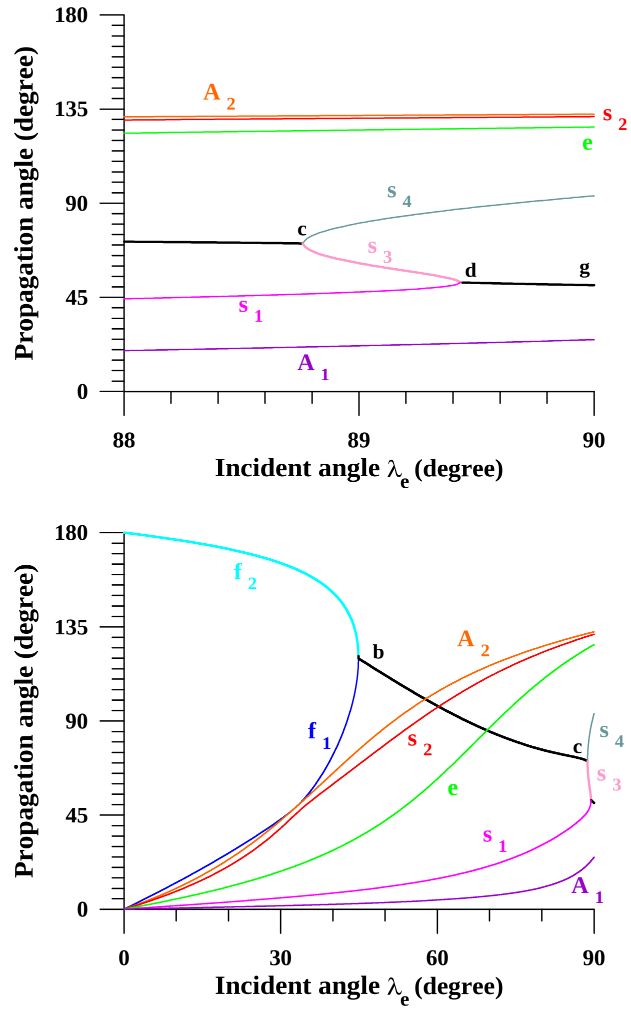

Fig. 8. Propagation angle of MHD waves versus the incident angle of entropy wave (the same as in Fig. 6 but for other characteristics of the fast shock wave).

can also be obtained with the graphic method offered by Kontorovich (1959). In Fig. 10 this method is applied for just the considered parameters of the medium (Figs. 8–9). Here the phase diagrams for the fast (F), slow (S) magnetoacoustic and Alfvén (A) waves behind the front of the shock wave are shown depending on the angle λ between the wave vector and shock normal. The phase velocity of all waves with a given value of c_p must lie on a circle (Ψ-circle in terminology of Kontorovich), passing through the extremity of the interval $[-V_{x2}, 0]$. Obviously, the centre of the circle is always at the point with the X-coordinate equal to $-V_{x2}/2$. Parameter c_p (or the incident angle and type of incident wave) sets the Y-coordinate of the centre. The polar angles of the cross points of the Ψ-circle and phase diagrams are propagation angles λ of corresponding waves. If the cross point lies in the lower semiplane, the corresponding wave is the backward one ($\omega_0<0$). At normal incidence, the Ψ-circle turns to a straight line, that is, the centre of the circle lies at the point with $Y=+\infty$. The Y-coordinate of the centre of the circle monotonously decreases with the growth of the incident angle λ_e. We perform the constructions for eight incident angles λ_e, corresponding to characteristic points in Figures 8-9. Plots A and B refer to the case of relatively small incident angles, when in the compressed medium there are two fast and two slow magnetoacoustic waves (the Ψ-circle crosses the phase diagrams F and S at two points). On the plot C the Ψ-circle is tangent to the diagram F. This case corresponds to the point b (Fig. 9) when the refracted wave transforms to the surface mode. On plots D (a point inside the segment bc) and H (point g) the Ψ-circle crosses the diagram S at two points and does not cross the diagram F. Here there is a surface wave and two refracted slow magnetoacoustic waves. On the plots E and G (points c and d) the Ψ-circle crosses twice and once is tangent to diagram S. Finally, on

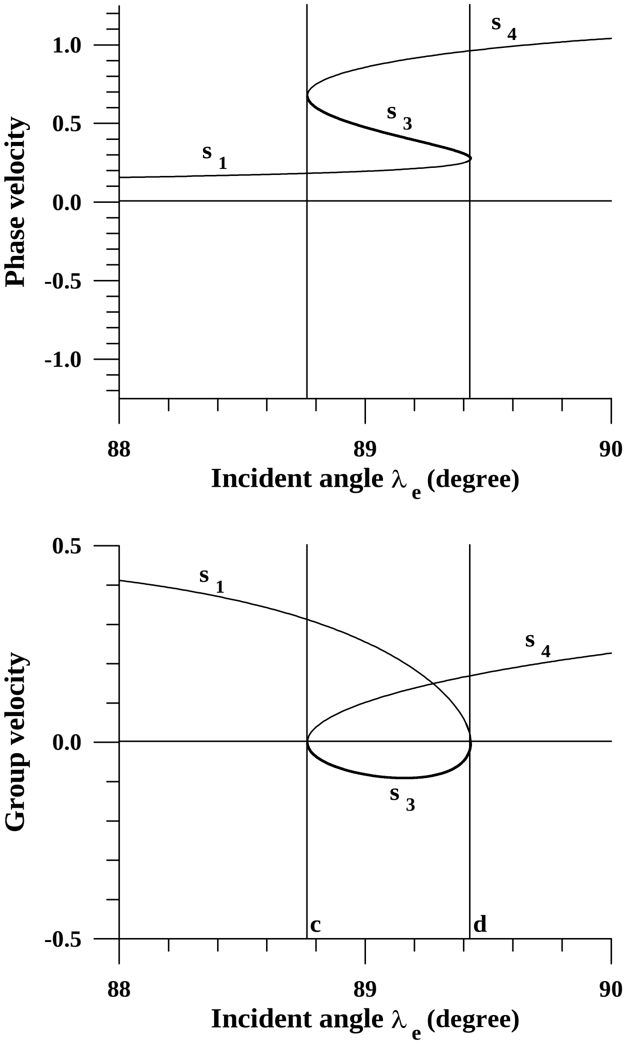

Fig. 9. X-components of the phase velocity and group velocity of the backward slow magnetoacoustic waves as a function of the incident angle of the entropy wave. All velocities are normalised by the normal component of the flow velocity V_{x2} behind the shock wave. Calculations are performed for the waves having the propagation angles shown in Fig. 8.

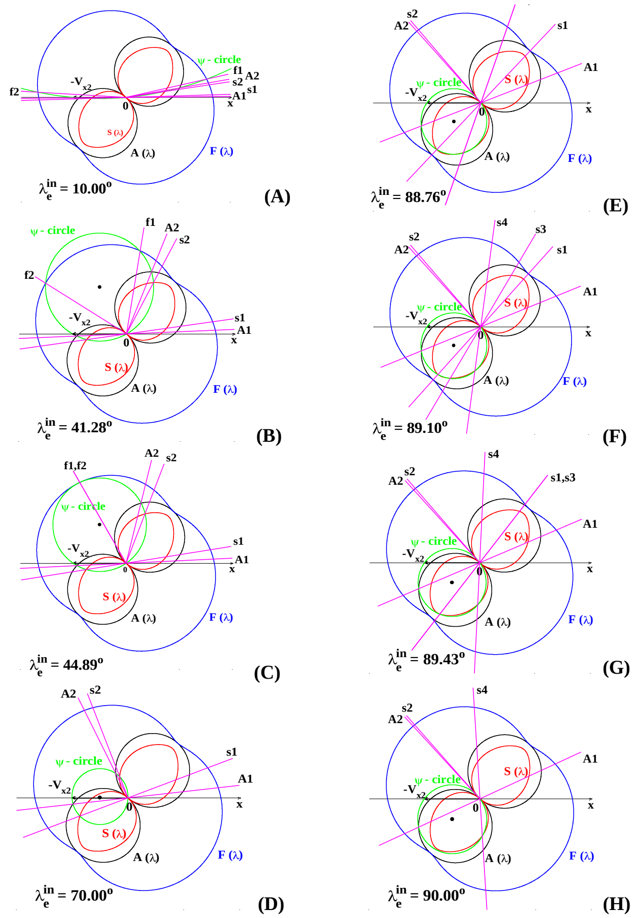

Fig. 10. Graphic analysis of refraction of MHD waves on a shock wave. The polar angles of the cross points of the Ψ-circle and phase diagrams for the fast (F), slow (S) magnetoacoustic and Alfvén (A) waves (the corresponding polar lines are shown on the plots) determine the propagation angles of the waves. Denotations are the same as in Figures 1, 6 and 8. The centre of the ψ-circle is denoted by the symbol "•" (not shown on the plot A).

the diagram F (a point inside the segment cd) the Ψ-circle crosses four times the phase diagram S which corresponds to four slow magnetoacoustic waves. We note that the graphic method also enables one to distinguish between incident and diverging waves (see Kontorovich (1959)). This confirms our conclusion that one of four slow magnetoacoustic waves will be incident on a shock wave, that is, the conditions of the evolutionarity will not be violated. The graphic method of the analysis is more transparent than either the analytical or numerical method. However, in our opinion, it has poor accuracy and is inconvenient for use in computations.

We have shown that under certain conditions behind the front of a fast shock wave there can propagate four homogeneous slow magnetoacoustic waves having an identical value of c_p, and any fast magnetoacoustic waves cannot propagate. It has been demonstrated that it does not at all contradict the condition of evolutionarity of the fast shock wave (3). Note, that a physically similar feature can be observed from two sides of a rotational discontinuity or a slow shock wave. For these two types of discontinuities it can be realised much easier than for a fast shock wave.

In the following section we will examine the polarisation of the surface magnetoacoustic waves.

5 Polarisation of the surface magnetoacoustic waves

We investigate the polarisation of magnetoacoustic waves described by dispersion Eq. (13) with coefficients Eq. (9). It is easy to show that for φ satisfying Eq. (13) the linearized set of MHD Eq. (7) will yield the following relations for the amplitudes of perturbations in the MHD wave

$$\delta V_{Ax} = \left(\varphi \cdot V_{Ay} - V_{Ax}\right) \cdot \left(-c_p + \varphi \cdot V_x + V_y\right) \cdot \delta A,$$

$$\delta V_{Ay}=-\varphi\cdot\left(\varphi\cdot V_{Ay}-V_{Ax}\right)\cdot\left(-c_p+\varphi\cdot V_x+V_y\right)\cdot\delta A,$$

$$\delta V_{Az}=-\left(\varphi^2+1\right)\cdot V_{Az}\cdot\left(-c_p+\varphi\cdot V_x+V_y\right)\cdot\delta A,$$

$$\delta V_x=\left[\varphi\cdot\left(-c_p+\varphi\cdot V_x+V_y\right)^2-\left(\varphi^2+1\right)\cdot\left(\varphi\cdot V_{Ax}+V_{Ay}\right)\cdot V_{Ax}\right]\cdot\delta A,$$

$$\delta V_y=\left[\left(-c_p+\varphi\cdot V_x+V_y\right)^2-\left(\varphi^2+1\right)\cdot\left(\varphi\cdot V_{Ax}+V_{Ay}\right)\cdot V_{Ay}\right]\cdot\delta A,$$

$$\delta V_z=-\left(\varphi^2+1\right)\cdot\left(\varphi\cdot V_{Ax}+V_{Ay}\right)\cdot V_{Az}\cdot\delta A,$$

$$\left(-c_p+\varphi\cdot V_x+V_y\right)\cdot\delta\rho=-\rho\cdot\left(\varphi\cdot\delta V_x+\delta V_y\right),$$

$$\delta P=c_S^2\cdot\delta\rho. \tag{16}$$

Here we have expressed the amplitudes of fluctuations in the components of the usual ($\delta\boldsymbol{V}$) and the magnetic ($\delta\boldsymbol{V}_A$) speeds through the common amplitude δA. The amplitudes of density and pressure fluctuations are expressed through the amplitudes of fluctuations in the X- and Y-components of the velocity.

If φ is real, Eqs. (16) define a usual linearly polarised magnetoacoustic wave. The amplitudes of the fluctuations of various values in the wave will be written in the reference frame connected to the unperturbed discontinuity. In solving the problems of the interaction of perturbations with a plane discontinuity, this presentation is much more convenient compared to the conventional one, in which the amplitudes of perturbations are given in the fluid's rest frame ($\boldsymbol{V}=0$). If φ has a nonzero imaginary part, Eqs. (16) determine the polarisation of a surface wave. The perturbations of the velocity and magnetic field in this wave have an elliptic polarisation, which is further demonstrated.

Let us write down the perturbations $\delta\boldsymbol{V}$ and $\delta\boldsymbol{V}_A$, determined by Eqs. (16), in the similar form

$$S_x=|S_x|\cdot\exp\mathrm{i}\cdot(\omega\cdot t+\alpha_x),$$

$$S_y=\left|S_y\right|\cdot\exp\mathrm{i}\cdot\left(\omega\cdot t+\alpha_y\right),$$

$$S_z=|S_z|\cdot\exp\mathrm{i}\cdot\omega\cdot t. \tag{17}$$

These expressions are written at a fixed spatial point. The initial time t_0 is chosen in such a way that the phase α_z is equal to zero. It is clear that the projection of $\boldsymbol{S}(t)$ on any of the three perpendicular planes $X-Y$, $X-Z$ and $Y-Z$ will be an ellipse. (Here we consider a circle or a line segment to be a special case of an ellipse.) But what will be the dependence $\boldsymbol{S}(t)$ in space?

Let $\boldsymbol{n}$ be the unit vector. In the polar coordinates it may be written as

$$\boldsymbol{n}=(\cos\eta\cdot\sin\vartheta,\ \sin\eta\cdot\sin\vartheta,\ \cos\vartheta).$$

Let us define the angles η and ϑ as follows:

$$\eta=\mathrm{arctg}\left(-\frac{|S_x|\cdot\sin\alpha_x}{\left|S_y\right|\cdot\sin\alpha_y}\right),$$

$$\vartheta=\mathrm{arctg}\left(\frac{|S_z|\cdot\sin\alpha_y}{|S_x|\cdot\cos\eta\cdot\sin\left(\alpha_x-\alpha_y\right)}\right).$$

The vector $\boldsymbol{n}$ is time independent and always orthogonal to the vector of perturbations $\boldsymbol{S}$. Therefore, the vector $\boldsymbol{S}$ always lies in the same plane. The angles η and ϑ have a simple physical sense; they define two successive turns of the coordinate system. The first turn is around the Z-axis by the angle η. After this turn the fluctuations along the new X'-axis and Z-axis will be in-phase. The second turn is around the new Y'-axis by the angle ϑ. After these two turns we obtain a new coordinate system $\bar{X}\bar{Y}\bar{Z}$. The $\bar{Z}$-axis will be directed along the vector $\boldsymbol{n}$. Now the vector of perturbations $\boldsymbol{S}(t)$ lies in the plane $\bar{X}-\bar{Y}$ and may be presented as

$$S_{\bar{x}}=|S_{\bar{x}}|\cdot\exp\mathrm{i}\cdot\omega\cdot t,$$

$$S_{\bar{y}}=\left|S_{\bar{y}}\right|\cdot\exp\mathrm{i}\cdot\left(\omega\cdot t+\alpha_{\bar{y}}\right),$$

$$S_{\bar{z}}=0.$$

It is known that a superposition of two sinusoidal oscillations shifted in phase and propagating in two mutually perpendicular directions yields an elliptic polarisation of the total movement. Thereby, we have shown that any vector quantity that can be presented in the form Eq. (17) has an elliptic polarisation. Certainly, this is also true for the perturbations of velocity and magnetic field in an inhomogeneous plane magnetoacoustic wave.

Note that usually the planes of polarisation for velocity and magnetic field perturbations are not coincident, except for the case when the external magnetic field has no Z-component, that is, it lies in the plane of wave propagation. If $B_z\neq0$, the angle between the planes of polarisation of $\delta\boldsymbol{B}$ and $\delta\boldsymbol{V}$ depends on χ and hence on the surface phase velocity c_p.

Let us consider in more detail the properties of the surface fast magnetoacoustic wave on the following two examples. First, we explore a surface wave behind the front of a rather strong fast shock wave. In front of the shock the parameters of flow are M_1=10, θ_1=0°, ψ_1=45°, α=30°, β_1=2, and γ=5/3. Behind the front we have $M_2\approx$0.46, $\theta_2\approx$2°, $\psi_2\approx$75°, and $\beta_2\approx$30. The density and thermal pressure increase across the fast shock wave by factors of $\approx$3.8 and $\approx$121, respectively. In the second example, the surface wave behind the front of a slow shock wave is considered. In this case ahead of the shock, M_1=0.5, and other parameters are the same, as in the first example. Behind the shock wave $M_2\approx$0.44, $\theta_2\approx$-22°, $\psi_2\approx$35°, and $\beta_2\approx$3. The density across the slow shock wave will increase by a factor of $\approx$1.1, and plasma pressure will increase by a factor of $\approx$1.2. On the slow shock wave the magnetic field decreases; therefore, $\psi_2<\psi_1$ and $\theta_2<\theta_1$.

In Fig. 11 we have plotted the polarisation for the perturbations of velocities $\delta\boldsymbol{V}_A$ and $\delta\boldsymbol{V}$ associated with the surface wave. The calculation results are given in three mutually perpendicular planes for both examples. The surface phase velocity c_p is set by the entropy wave incident at the angle of

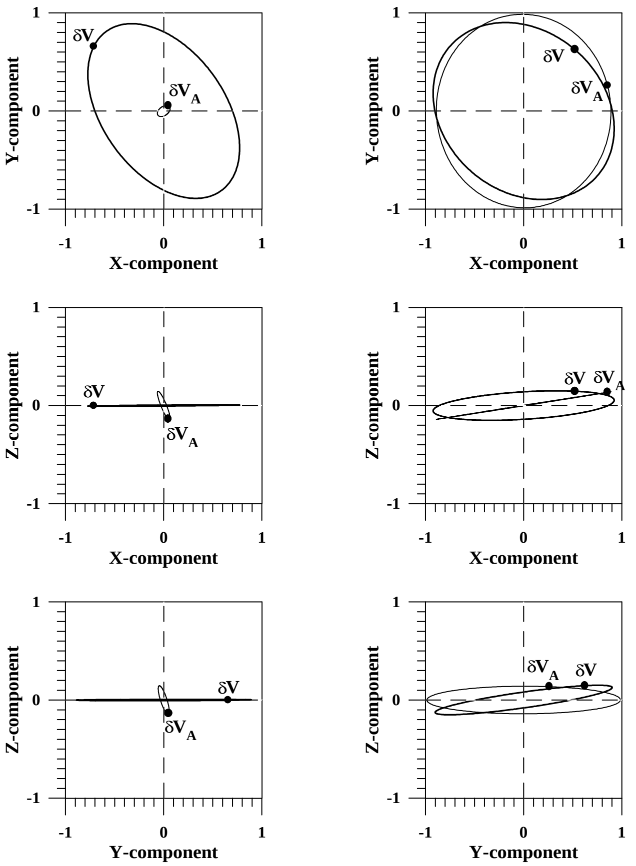

Fig. 11. Polarisation of perturbations $\delta \boldsymbol{V}$ and $\delta \boldsymbol{V}_A$ associated with the surface magnetoacoustic wave in three mutually perpendicular planes Z=0, Y=0, and X=0. Here the velocity $\delta \boldsymbol{V}$ is normalised by $|\delta \boldsymbol{V}_{\max}|$, that is, the maximal distance between the centre of the ellipse and a given point of the ellipse is equal to one. Relation (16) relates the value of $\delta \boldsymbol{V}$ to $\delta \boldsymbol{V}_A$. The left (right) plots show the polarisation of perturbations behind the front of a fast (slow) shock wave. The symbols "•" indicate the initial phases of perturbations. The vectors of polarisation rotate counterclockwise in the wave damping off from the discontinuity.

70°. A superposition of two movements yields an elliptic polarisation of $\delta \boldsymbol{V}$ in the surface wave. It is the movement along an ellipse in the plane of propagation Z=0, which is similar to the movement of water in a sea wave, and linearly polarised oscillations along an external magnetic field. The first component exceeds by far the second one behind the front of a sufficiently strong fast shock wave. Therefore, firstly, $|\delta \boldsymbol{V}| \gg |\delta \boldsymbol{V}_A|$ and, secondly, $\delta \boldsymbol{V}$ is polarised nearly in the plane Z=0. On the contrary, $|\delta \boldsymbol{V}|$ and $|\delta \boldsymbol{V}_A|$ have the same order behind the front of a slow shock wave. In Fig. 12 the direction of $\delta \boldsymbol{V}_{\max}$ versus the angle of incidence of the entropy wave is shown for both examples. The curves have a well pronounced jog at the critical incident angle when the refracted fast magnetoacoustic wave transforms to the surface mode. Behind the front of a sufficiently strong, fast shock wave, the characteristics of the refracted, fast magnetoacoustic wave are close to those of the sound wave. In this

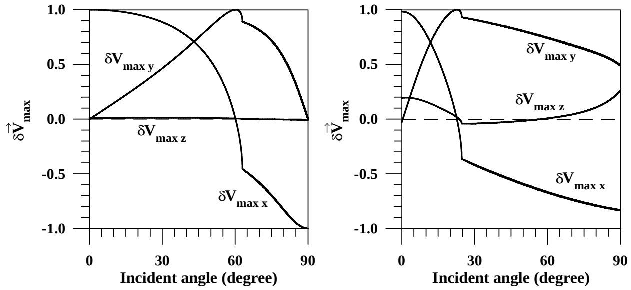

Fig. 12. Components of the unit vector along the vector of maximal perturbations of velocity $\delta \boldsymbol{V}_{\max}$ (for the surface wave it is the direction of the principal axis of the polarisation ellipse) versus the incident angle of the entropy wave setting the value c_p. The left (right) plots show the direction of the unit vector behind the front of the fast (slow) shock wave.

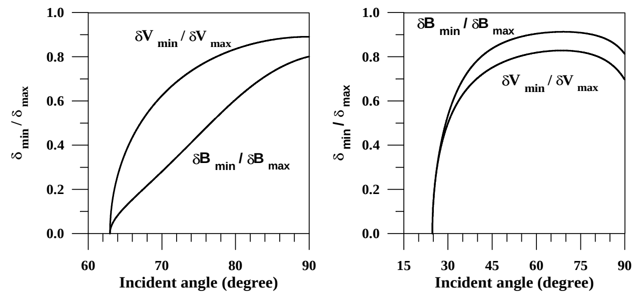

Fig. 13. Ratio of axis lengths for the polarisation ellipse of velocity and magnetic field perturbations, associated with the surface fast magnetoacoustic wave, versus the incident angle of the entropy wave setting the value c_p. The left (right) plots show this ratio behind the front of fast (slow) shock wave.

case we have $\delta V_z \approx 0$ for the refracted, fast magnetoacoustic wave. When such a refracted wave is homogeneous, $\delta \boldsymbol{V}_{\max}$ is directed almost along the wave vector. For an inhomogeneous surface wave, $\delta \boldsymbol{V}_{\max}$ is directed along the main axis of the polarisation ellipse. The angle ε between the X-axis and the main axis of the ellipse can be calculated by the formula

$$\mathrm{tg}\ 2\varepsilon = \frac{2 \cdot \mathrm{Im}\ \chi \cdot k_y}{|\chi|^2 - k_y^2}.$$

At the maximum possible angle of incidence of the entropy wave, at which c_p=0, the main axis of the polarisation ellipse coincides with the X-axis. Behind the front of the slow shock wave the behaviour of $\delta \boldsymbol{V}_{\max}$ is more complicated.

In Fig. 13 the ratios of the lengths of the small (δ_{min}) and large (δ_{max}) axes of the ellipse of polarisation for velocity and magnetic field perturbations are shown. In both cases these ratios can exceed 0.8, that is, at large angles of incidence of the entropy wave, the polarisation of velocity and magnetic field perturbations in the surface fast magnetoacoustic wave will insignificantly differ from the circular polarisation. The "stretching" of the ellipse is commonly estimated by the value of eccentricity e. It can be easily calculated by the formula

$$e=\sqrt{1-\frac{\delta_{\min}^2}{\delta_{\max}^2}}.$$

Note that in the second example the refracted, fast magnetoacoustic wave becomes the surface wave already at the angle of incidence of the entropy mode of about 25°.

In Figs. 11–13 the results of numerical calculations by Eq. (16) are illustrated. Certainly, for all characteristics of the ellipse of polarisation of velocity and magnetic field perturbations, including the direction of the main axes, eccentricity, etc., exact analytical expressions can be given. But unfortunately, they have an extremely bulky form. Therefore, we do not show them here.

6 Summary and discussion

We have considered the properties of plane MHD waves of small amplitude in the medium separated by a plane MHD discontinuity into two half spaces. The angles of propagation and the polarisation of MHD waves generated by the discontinuity are determined versus the phase velocity of surface discontinuity oscillations. The surface phase velocity is set either by an incident wave or spontaneously emerging ripples on the discontinuity surface. The method of the analysis is based on the solution of the dispersion equation for the MHD waves, written in the reference frame, connected with the unperturbed surface of the discontinuity. In our opinion, the proposed technique is more handy compared to other methods described earlier. The analysis of the obtained dispersion equation under a given surface phase velocity has shown that from the chosen side of the front there cannot exist more than one surface wave generated by discontinuity. If the surface wave has emerged, it can only be a fast or slow magnetoacoustic wave. The perturbations of velocity and magnetic field, associated with the surface wave, have an elliptic polarisation, and the planes of polarisation of velocity and magnetic field perturbations, as a rule, are not coincident. It has been shown that under certain conditions behind the front of an evolutionary fast shock wave there can be four slow magnetoacoustic waves, having identical surface phase velocity and no fast magnetoacoustic waves.

In our analysis we have used the MHD approximation, i.e. that of the continuous medium. Therefore, the results obtained refer to the case of collisional plasma. Space plasma is collisional, for example, in the atmospheres of the Sun and stars, in particular, in the solar chromosphere and corona. Thus, our research can be relevant to the analysis of solar flare generation mechanisms. Though the free path of solar wind particles is of the order of 1 AU, which, strictly speaking, means that the solar wind is a collisionless plasma, the MHD approximation is often used in studying the processes in the solar wind. For example, Wu et al. (2005) have used this approximation in the analysis of the evolution of slow shock waves and their interactions with other discontinuities in the heliosphere. The MHD approximation was utilised in most of the above-mentioned works by exploring the interaction of perturbations with MHD shock waves. The formal ground for the application of the MHD approximation to the solar wind is the suggestion that the role of the collisions is taken by the interaction of the solar wind particles with small-scale turbulence, which is always present in the interplanetary magnetic field. It has to be emphasised, however, that the use of the MHD approximation for the description of solar wind plasma rules out of consideration of many phenomena, for which solar wind plasma should be treated as an essentially collisionless medium and which are observed in the vicinity of interplanetary shock waves and the Earth's bow shock. The detailed analysis of these phenomena can be found in numerous works (see, for example, the review by Lembege et al. (2004) and references therein).

We use the one-liquid approach. It applies to waves with a frequency much less than the proton cyclotron frequency. Waves with a larger frequency are not the subject of our analysis. We shall note only that such "high"-frequency waves are often observed in the solar wind and therefore are intensively investigated by many authors. For example, Agim et al. (1995) investigated the generation and evolution of a spectrum of finite amplitude, right-hand elliptically polarised MHD waves with a frequency near the ion cyclotron frequency in the upstream region of interplanetary shocks. The waves are excited by an ion beam instability of a backstreaming ion beam formed by a specular reflection of a fraction of the incoming solar wind protons at the shock.

Regarding to the results obtained, two issues are of interest. The first issue concerns the evolutionarity of shock waves. For the first time the conditions of evolutionarity (a stability against a decay to several other discontinuities) of shock waves are obtained for the perturbations propagating along the normal to the shock (Akhiezer et al. (1959), Syrovatskii (1959)). The interaction of shock waves with the perturbations propagating at a certain angle to the front has been investigated by Kontorovich (1959) and Anderson (1963). They came to the conclusion that the shock wave, which is evolutionary to the perturbations propagating along the normal to the discontinuity, remains evolutionary for the perturbations propagating at any theoretically possible angle to the discontinuity. Later, a number of authors (e.g. Zhuang and Russell (1982), Whang et al. (1987)) questioned this conclusion. They argued that the shock wave satisfying the classical conditions of evolutionarity (3) turns non-evolutionary for the large angles of propagation, when the wave vector of a refracted, fast magnetoacoustic wave is directed toward the shock. The reason for such a conclusion is that they discriminated the waves, emanating from a shock, by the sign of the wave vector component normal to the surface. We consider such an approach to be physically incorrect. The selection of emanating waves should be performed by the sign of the normal component of the group velocity. Besides, it is necessary to take into account the drag of the wave by the flow. Our numerical results have confirmed that within the correct framework, the number of waves emanating from the evolutionary (according to (3)) shock wave will be unchanged for any value of the surface phase velocity or, otherwise, at any theoretically possible angle of emanation. The group velocity of diverging waves (with the drift included) will be directed away from the shock for homogeneous plane waves and parallel to the discontinuity for inhomogeneous (surface) waves. The number of diverging waves will be preserved, even in the specific case when for some values of surface phase speed behind the front of a fast shock wave there are four slow magnetoacoustic waves and no fast magnetoacoustic waves. One of the four slow magnetoacoustic waves in that case will be incident on the shock, the overall pattern being consistent with the conditions of evolutionarity (3). The normal component of the phase velocity of an emanating or incident wave can be of any sign.

The second issue concerns a possible transformation of refracted magnetoacoustic waves to the surface wave. Usually, in investigating the transformation of MHD waves on an MHD discontinuity, the solution of the problem is restricted by the angles of incidence at which all waves diverging from the discontinuity are homogeneous (e.g. Kwok and Lee (1984), Westphal and McKenzie (1969), Hassam (1978)). We think this limitation has neither a physical nor mathematical basis. Its artificial character can be illustrated by the following example. McKenzie and Westphal (1969), while investigating the passage of Alfvén waves through a fast shock wave considered a situation when the external magnetic field, wave vector and normal to the discontinuity, lay in the same plane. In this case the refracted magnetoacoustic waves did not emerge, and consideration included all theoretically possible angles of incidence. But if there was a small angle between the tangential components of wave vector and external magnetic field, the refracted magnetoacoustic waves would appear. Then, if the above-mentioned limitation were used, it would be necessary to restrict essentially the analysed range of incident angles (e.g. the upper limit of angles would change from 90° to ≈63°, for a strong fast shock wave). We argue that no problems emerge with the consideration of all theoretically possible angles of incidence.

We draw the readers' attention to an interesting point arising when a magnetoacoustic wave propagates at a large angle to a strong fast shock wave from the uncompressed medium. The incident wave transports the fluctuations of density and pressure, and has a considerable longitudinal component of velocity fluctuation. Behind the shock the fast magnetoacoustic wave will propagate as a surface wave, damping rather quickly off from the discontinuity, along with five diverging homogeneous waves (two Alfvén waves, two slow magnetoacoustic waves and an entropy wave). All homogeneous waves will have very small phase velocities of their own and, practically, the perturbations will be drifting with the flow. The whole wave will represent a transversely polarised entropy-vorticity wave, carrying the fluctuations of magnetic field, velocity, entropy, and density. The perturbations of pressure and temperature in such a wave are absent. As a result, at some distance from the discontinuity, where the surface wave has already damped, only the entropy-vorticity wave will be propagating, that is, there will be an almost total transformation of a mostly longitudinal incident wave to transverse diverging wave.

We have shown theoretically that the surface magnetoacoustic wave should possess an elliptic polarisation. The elliptically polarised MHD waves are sometimes observed in experiments. For example, Pudovkin and Lyubchich (1989b), using the King catalogue (1977), have shown that the long-period fluctuations of the magnetic field, accompanied by a rotation of the polarisation vector, sometimes occur in the solar wind in between the flare ejection and its bow shock.

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