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Research Article

H_∞ Static Output Tracking Control of Nonlinear Systems with One-Sided Lipschitz Condition

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This paper is concerned with H_∞ static output tracking control of nonlinear systems with one-sided Lipschitz condition. The dimensions of system model and reference model may be different. A static output feedback controller is designed to guarantee that the system output asymptotically tracks the reference output with H_∞ disturbance rejection level. A new sufficient condition is derived to obtain the static output feedback gain by linear matrix inequality (LMI), and no equality constraints can be needed. Finally, an example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Tracking control has been a hot point due to its wide applications. The main objective of tracking control is to minimize the error between the state (or output) of the plant and the state (or output) of a given reference model. So it involves two related problems, that is, state feedback tracking controller design [1–3] and output feedback tracking controller design [4–9]. Among them, the latter is paid much attention because of its attractive features such as low overheads of implementing control, the reliability of control systems, and widely practical applications where measurement of all the state variables is not possible. Furthermore, since the static output feedback case needs much lower costs than an observer-based approach, a few meaningful results about static output feedback tracking control have been presented [10–12].

From above results, it has been shown that the solution of the Riccati equation or LMI depends strongly on the Lipschitz constant, but when the Lipschitz constant becomes large, most of the existing results are infeasible. To enlarge the domain of attraction of nonlinear systems, the one-sided Lipschitz condition is proposed [13, 14, 14–21]. The one-sided Lipschitz constant is significantly smaller than the Lipschitz constant, which makes it much more suitable for estimating the influence of nonlinear part. One-sided Lipschitz condition is shown to be an extension of the Lipschitz condition and is less conservative. Recently, the problem of tracking control of nonlinear systems with one-sided Lipschitz condition has been presented [22]. In [22], the stabilization and signal tracking control for one-sided Lipschitz nonlinear differential inclusions are considered; a nonlinear state feedback tracking controller is designed. However, to the authors' knowledge, there are very few studies concerning static output tracking controller design of nonlinear systems with one-sided Lipschitz condition. These motivate our study.

This paper considers static output tracking control of nonlinear systems with one-sided Lipschitz condition. The dimensions of system model and reference model may be different. A design procedure of static output feedback controller is proposed to guarantee that the system output asymptotically tracks the reference output with H_∞ disturbance rejection level. A new sufficient condition is obtained by LMI, and no equality constraints can be needed. These will reduce the difficulty in solving output feedback gain. Finally, an example is given to illustrate the effectiveness of the proposed method.

Notations. R^n and $R^{n\times m}$ denote, respectively, the spaces of n-dimensional real numbers and $n \times m$ real matrices. Let M be a real symmetric matrix; $M > 0$ means M is positive definite. $\langle \cdot, \cdot \rangle$ is the inner product in R^n; that is, given x, $y \in R^n$, then

$\langle x, y\rangle = x^T y$, where x^T is the transpose of the column vector $x \in R^n$. I_n is an identity matrix with dimension n. $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. $*$ represents the omitted symmetric element of a matrix.

2. Problem Formulation

Consider the following nonlinear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \phi(x) + Dw(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector and $u(t) \in R^m$ is the control input. $y(t) \in R^p$ is the system output. $\phi(x)$ represents a nonlinear function that is continuous with respect to $x(t)$ and $\phi(0) = 0$. A, B, C, D are matrices with compatible dimensions and (A, B) is controllable. C is of full rank. $w(t)$ is a bounded disturbance.

The following concepts about Lipschitz property, the one-sided Lipschitz property, and quadratic inner-boundedness property for the nonlinear function $\phi(x)$ are introduced to further our study.

Definition 1 (see [22]). The nonlinear function ϕ is said to be locally Lipschitz in a region Q including the origin with respect to x, if there exists a constant $l > 0$ satisfying

$$\|\phi(x_1) - \phi(x_2)\| \le l\|x_1 - x_2\|, \quad \forall x_1, x_2 \in Q. \tag{2}$$

Definition 2 (see [22]). The nonlinear function ϕ is said to be one-sided Lipschitz, if there exists a constant $\rho \in R$ such that

$$\langle \phi(x_1) - \phi(x_2), x_1 - x_2\rangle \le \rho\|x_1 - x_2\|^2, \quad \forall x_1, x_2 \in Q, \tag{3}$$

where ρ is called the one-sided Lipschitz constant.

From Definitions 1 and 2, Lipschitz constant l must be positive; however, one-sided Lipschitz constant ρ can be positive, zero, or even negative. It is true that any Lipschitz function is also one-sided Lipschitz, not vice versa [15–17, 22].

Definition 3 (see [22]). The nonlinear function ϕ is called quadratic inner-bounded in the origin $\overline{Q}$, if there exists constants $\beta, \nu \in R$ such that

$$\Delta\phi^T\Delta\phi \le \beta\|x_1 - x_2\|^2 + \nu\langle x_1 - x_2, \Delta\phi\rangle, \quad \forall x_1, x_2 \in \overline{Q} \tag{4}$$

with $\Delta\phi = \phi(x_1) - \phi(x_2)$.

From the definition, the Lipschitz function is quadratically inner-bounded with $\beta > 0$ and $\nu = 0$, but the converse is not true. Note that ν is not necessarily positive. In fact, if ν is restricted to be positive, then it can be shown that ϕ must be Lipschitz.

Consider a reference model as follows [4]:

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + r(t), \\ y_m(t) &= C_m x_m(t), \end{aligned} \tag{5}$$

where $x_m(t) \in R^s$ is the reference state, $r(t)$ is a bounded reference input, and $y_m(t)$ is the reference output. A_m, C_m are matrices with compatible dimensions and A_m is a specific asymptotically stable matrix.

Let $z(t) = x(t) - Gx_m(t)$. From (1) and (5), we have

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t) - BHx_m \\ &\quad + \phi(z(t) + Gx_m(t)) + \overline{D}d(t), \\ e(t) &= y(t) - y_m(t) = Cz(t), \end{aligned} \tag{6}$$

where $\overline{D} = [D \;\; -G]$ and $d(t) = [w^T(t) \;\; r^T(t)]^T$; $G \in R^{n\times s}$ and $H \in R^{m\times s}$ satisfy

$$AG + BH = GA_m, \qquad CG = C_m. \tag{7}$$

Remark 4. In (7), the values of G, H can be obtained by the method in [8]. By using Kronecker product $\otimes$ of matrices and the vec$(\cdot)$ operation, (7) can be equivalent to

$$\Omega\Pi = \Lambda, \tag{8}$$

where

$$\begin{aligned} \Omega &= \begin{bmatrix} I_s \otimes A - A_m^T \otimes I_n & I_s \otimes B \\ I_s \otimes C & 0 \end{bmatrix}, \\ \Pi &= \begin{bmatrix} \operatorname{vec}(G) \\ \operatorname{vec}(H) \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} 0 \\ \operatorname{vec}(C_m^T) \end{bmatrix}. \end{aligned} \tag{9}$$

If and only if $\operatorname{rank}(\Omega, \Lambda) = \operatorname{rank}(\Omega)$, one of the solutions is

$$\Pi = \Omega^+\Lambda, \tag{10}$$

where Ω^+ denotes the Moore-Penrose inverse of Ω.

For (6), we design the following static output feedback controller

$$u(t) = Hx_m(t) + K(y(t) - y_m(t)), \tag{11}$$

where H is from (7) and K is a static output feedback gain and determined later.

From (5), (6), and (11), the closed-loop system can be written as

$$\begin{aligned} \dot{z}(t) &= \overline{A}z(t) + \phi(z(t) + Gx_m(t)) + \overline{D}d(t), \\ \dot{x}_m(t) &= A_m x_m(t) + Fd(t), \\ e(t) &= Cz(t), \end{aligned} \tag{12}$$

where $\overline{A} = A + BKC$ and $F = [0 \;\; I_s]$.

Note that the output matrix C is of full rank; without loss of generality, we assume that $C = [0 \;\; C_2]$, $C_2 \in R^{p\times p}$ is nonsingular.

The objective of this paper is to design a static output feedback controller (11) such that the closed-loop system (12) is asymptotically stable with an H_∞-norm bound γ; that is, the following conditions are achieved simultaneously.

(i) The closed-loop system (12) with $d(t) = 0$ is asymptotically stable.

(ii) The closed-loop system (12) has a given H_∞ disturbance rejection level. It is to make, under the zero-valued initial condition, the following inequality holds:

$$\|e(t)\|^2 < \eta^2 \|d(t)\|^2 \tag{13}$$

for any nonzero $w(t)$, where $\eta > 0$ is a prescribed scalar.

3. Main Results

In this section, a design algorithm is proposed to obtain the output feedback gain via LMI and the sufficient condition includes no equality constraints.

Theorem 5. *Given a constant $\eta > 0$. Suppose that the function $\phi(x)$ in the system (12) satisfies conditions (2) and (3) with constants ρ, β, and ν. The system (12) is asymptotically stable with an H_∞-norm bound η, if there exist matrices $P > 0, S > 0$, P_1, P_2, K and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following matrix inequality is true*

$$\Xi = \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 & P_1^T\overline{D} \\ * & -P_2 - P_2^T & 0 & P_2^T & P_2^T\overline{D} \\ * & * & \Xi_5 & \dfrac{\nu\varepsilon_2 - \varepsilon_1}{2}G^T & SF \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\eta^2 I \end{bmatrix} < 0, \tag{14}$$

where $\overline{A} = A + BKC$, $\Xi_1 = P_1^T\overline{A} + \overline{A}^T P_1 + (\varepsilon_1\rho + \varepsilon_2\beta)I + C^T C$, $\Xi_2 = P - P_1^T + \overline{A}^T P_2$, $\Xi_3 = (\varepsilon_1\rho + \varepsilon_2\beta)G$, $\Xi_4 = P_1^T + ((\nu\varepsilon_2 - \varepsilon_1)/2)I$, and $\Xi_5 = (\varepsilon_1\rho + \varepsilon_2\beta)G^T G + SA_m + A_m^T S$.

Proof. Consider the following Lyapunov functional candidate:

$$V(t) = z^T(t) P z(t) + x_m^T(t) S x_m(t). \tag{15}$$

Calculating the time derivative of $V(t)$ along the trajectory of system (12), we have

$$\dot{V}(t) = 2z^T(t) P \dot{z}(t) + x_m^T(t)\left(SA_m + A_m^T S\right) x_m(t) + 2x_m^T(t) SFd(t). \tag{16}$$

On the other hand, we have

$$2\left(z^T(t) P_1^T + \dot{z}^T(t) P_2^T\right) \times \left[-\dot{z}(t) + \overline{A}z(t) + \phi\left(z(t) + Gx_m(t)\right) + \overline{D}d(t)\right] = 0, \tag{17}$$

where P_1, P_2 are free-weighting matrices. In the following, $\phi(z(t) + Gx_m(t))$ can be shortened as ϕ for simplicity.

From (3), $\rho z^T(t)z(t) + 2\rho z^T(t)Gx_m(t) + \rho x_m^T(t)G^T Gx_m(t) - z^T(t)\phi + x_m^T(t)G^T\phi \geq 0$. Therefore, for any positive scalar ε_1, we have

$$\varepsilon_1 \begin{bmatrix} z(t) \\ x_m(t) \\ \phi \end{bmatrix}^T \begin{bmatrix} \rho & \rho G & -\dfrac{1}{2}I \\ * & \rho G^T G & -\dfrac{1}{2}G^T \\ * & * & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ x_m(t) \\ \phi \end{bmatrix} \geq 0. \tag{18}$$

From (4), similarly, for any positive scalar ε_2, we have

$$\varepsilon_2 \begin{bmatrix} z(t) \\ x_m(t) \\ \phi \end{bmatrix}^T \begin{bmatrix} \beta & \beta G & \dfrac{\nu}{2}I \\ * & \beta G^T G & \dfrac{\nu}{2}G^T \\ * & * & -I \end{bmatrix} \begin{bmatrix} z(t) \\ x_m(t) \\ \phi \end{bmatrix} \geq 0. \tag{19}$$

Then, adding (17) and the left-sided terms in (18) and (19) to time derivative of $V(t)$ yields

$$\dot{V}(t) + y_z^T(t) y_z(t) - \eta^2 d^T(t) d(t) \leq \xi^T \Xi \xi, \tag{20}$$

where $\xi = [z^T(t), \dot{z}^T(t), x_m^T(t), \phi^T, d^T(t)]^T$.

From (14), we can have

$$\dot{V}(t) + y_z^T(t) y_z(t) - \eta^2 d^T(t) d(t) \leq \xi^T \Xi \xi < 0, \quad (\text{for } \xi \neq 0). \tag{21}$$

Under the zero initial conditions, we have

$$\begin{aligned} \eta^2 \int_0^\infty d^T(t) d(t)\, dt &\geq V(\infty) - V(0) + \int_0^\infty y_z^T(t) y_z(t)\, dt \\ &\geq \int_0^\infty y_z^T(t) y_z(t)\, dt. \end{aligned} \tag{22}$$

That is, $\|y_z(t)\|^2 < \gamma^2\|d(t)\|^2$.

In fact, when $d(t) = 0$, in the same way, we easily obtain

$$\dot{V}(t) = \overline{\xi}^T \tilde{\Xi} \overline{\xi}, \tag{23}$$

where $\overline{\xi} = [z^T(t), \dot{z}^T(t), x_m^T(t), \phi^T]^T$ and

$$\tilde{\Xi} = \begin{bmatrix} \tilde{\Xi}_1 & \Xi_2 & \Xi_3 & \Xi_4 \\ * & -P_2 - P_2^T & 0 & P_2^T \\ * & * & \Xi_5 & \dfrac{\nu\varepsilon_2 - \varepsilon_1}{2}G^T \\ * & * & * & -\varepsilon_2 I \end{bmatrix}, \tag{24}$$

where $\tilde{\Xi}_1 = P_1^T\overline{A} + \overline{A}^T P_1 + (\varepsilon_1\rho + \varepsilon_2\beta)I$.

Noting that (14) implies $\tilde{\Xi} < 0$, we have

$$\dot{V}(t) = \overline{\xi}^T \tilde{\Xi} \overline{\xi} < 0, \quad \left(\text{for } \overline{\xi} \neq 0\right). \tag{25}$$

This implies that the system (12) with $d(t) = 0$ is asymptotically stable. Then the proof is completed. □

For (14), it is a nonlinear matrix inequality, there are no effective algorithms for solving P, S, P_1, P_2, K, and scalars ε_1, ε_2 simultaneously. The following theorem gives an approach to solve this problem.

Theorem 6. *Given a constant $\eta > 0$. Suppose that the function $\phi(x)$ in the system (12) satisfies conditions (2) and (3) with constants ρ, β, and ν. $M \in R^{p\times(n-p)}$ and $\omega \in R$ are priori selected tuning parameters. The system (12) is asymptotically stable with an H_∞-norm bound η, if there exist matrix $X > 0$, general matrix $X_1 = \begin{bmatrix} X_{11} & X_{12} \\ X_{22}M & \omega X_{22} \end{bmatrix}$ with $X_{22} \in R^{p\times p}$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following matrix inequality is true:*

$$\begin{bmatrix} \overline{\Xi}_1 & \overline{\Xi}_2 & \overline{\Xi}_3 & \overline{\Xi}_4 & \overline{D} & \sqrt{|\varepsilon_1\rho + \varepsilon_2\beta|}X_1^T & X_1^TC^T \\ * & -X_1 - X_1^T & 0 & I & \overline{D} & 0 & 0 \\ * & * & \Xi_5 & \frac{\nu\varepsilon_2 - \varepsilon_1}{2}G^T & SF & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -\eta^2 I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (26)$$

where $\overline{\Xi}_1 = AX_1 + X_1^TA^T + BL[M \ \ \omega I_p] + [M \ \ \omega I_p]^T L^T B^T$, $\overline{\Xi}_2 = X - X_1 + X_1^TA^T + [M \ \ \omega I_p]^T L^T B^T$, and $\overline{\Xi}_3 = (\varepsilon_1\rho + \varepsilon_2\beta)X_1^TG$, $\overline{\Xi}_4 = I + ((\nu\varepsilon_2 - \varepsilon_1)/2)X_1^T$. Furthermore, $K = L(C_2X_{22})^{-1}$.

Proof. The solution of (14) can be obtained by the following two-step procedures.

In the first step, $\varepsilon_1, \varepsilon_2$ can be given.

In the second step, for (14), choose $P_1 = P_2$ and denote $X_1 = P_1^{-1}$, $X = X_1^TPX_1$. Left- and right-multiplying both sides of (14) by $\operatorname{diag}\{X_1^T, X_1^T, I, I, I\}$ and $\operatorname{diag}\{X_1, X_1, I, I, I\}$, respectively, we obtain

$$\begin{bmatrix} \widehat{\Xi}_1 & \widehat{\Xi}_2 & \overline{\Xi}_3 & \overline{\Xi}_4 & \overline{D} \\ * & -X_1 - X_1^T & 0 & I & \overline{D} \\ * & * & \Xi_5 & \frac{\nu\varepsilon_2 - \varepsilon_1}{2}G^T & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\eta^2 I \end{bmatrix} < 0, \quad (27)$$

where $\widehat{\Xi}_1 = \overline{A}X_1 + X_1^T\overline{A}^T + (\varepsilon_1\rho + \varepsilon_2\beta)X_1^TX_1 + X_1^TC^TCX_1$, $\widehat{\Xi}_2 = X - X_1 + X_1^T\overline{A}^T$.

By considering the structure of X_1, we have

$$KCX_1 = F[0 \ \ C_2]X_1 = [KC_2X_{22}M \ \ \omega KC_2X_{22}]. \quad (28)$$

Define

$$L = FC_2X_{22}. \quad (29)$$

Then we have

$$KCX_1 = [LM \ \ \omega L] = L[M \ \ \omega I_p]. \quad (30)$$

Substituting (30) into (27) and using Schur's Lemma yield (26).

Because $\varepsilon_1, \varepsilon_2$ is given in advance, (26) is converted to a LMI. So we easily solve it by Matlab LMI toolbox [23]. The proof is completed. □

Remark 7. Theorem 6 provides a new solving method about static output feedback gain by LMI, when the parameters $\varepsilon_1, \varepsilon_2$ are given in advance. Moreover, no equality constraints are needed, so the difficulty in solving it can be reduced. It should be noted that the special form of matrix X_1 will bring some conservatism; however, thanks to this form, the output feedback gain can be easily obtained [24].

The procedure of the proposed H_∞ static output tracking control of nonlinear systems with one-sided Lipschitz condition is summarized as follows.

Step 1. Find the solutions of G and H from (10). If there exists no solution, then the reference model must be modified.

Step 2. Observe the structure of C in (12): if $C = [0 \ \ C_2]$, $C_2 \in R^{p\times p}$ is nonsingular, then the procedure directly goes to the next step, or there exists a state transformation $\overline{z}(t) = Tz(t)$ for (12) to make C satisfy this structure, where T is a nonsingular matrix.

Step 3. Choose the parameters $\varepsilon_1, \varepsilon_2, \eta > 0$, $M \in R^{p\times(n-p)}$, and $\omega \in R$ in Theorem 6.

Step 4. Calculate the matrix L and X_{22} in X_1 by solving LMI (26).

Step 5. Obtain the static output feedback gain $K = L(C_2X_{22})^{-1}$.

4. Discussion

Theorem 6 gives a new approach to solve the static output feedback gain; if we use traditional method including the equality constraints, the following theorem can be given.

Theorem 8. *Given a constant $\eta > 0$. Suppose that the function $\phi(x)$ in the system (12) satisfies conditions (2) and (3) with constants ρ, β, and ν. The system (12) is asymptotically stable with an H_∞-norm bound η, if there exist matrices $X > 0$, X_1, N, Y and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following matrix inequality is true:*

$$\begin{bmatrix} \underline{\Xi_1} & \underline{\Xi_2} & \Xi_3 & \Xi_4 & \overline{D} & \sqrt{|\varepsilon_1\rho+\varepsilon_2\beta|}\,X_1^T & X_1^TC^T \\ * & -X_1-X_1^T & 0 & I & \overline{D} & 0 & 0 \\ * & * & \Xi_5 & \dfrac{\nu\varepsilon_2-\varepsilon_1}{2}G^T & SF & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -\eta^2 I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (31)$$

$$NC = CX_1, \quad (32)$$

where $\underline{\Xi_1} = AX_1 + X_1^TA^T + BYC + C^TY^TB^T$, $\underline{\Xi_2} = X - X_1 + X_1^TA^T + C^TY^TB^T$, $\Xi_3 = (\varepsilon_1\rho+\varepsilon_2\beta)X_1^TG$, and $\Xi_4 = I + ((\nu\varepsilon_2 - \varepsilon_1)/2)X_1^T$. Furthermore, $K = YN^{-1}$.

Proof. The proofs can be easily obtained similar to the arguments in Theorem 6; the details are omitted. ☐

Remark 9. Noting that (31) can be converted to a LMI on condition that the parameters $\varepsilon_1, \varepsilon_2$ are also given in advance, the equality condition $NC = CX_1$ can be equivalently converted to trace $[(NC-CX_1)^T(NC-CX_1)] = 0$. Introduce the condition $(NC-CX_1)^T(NC-CX_1) \le \alpha I$ and Schur's complement gives

$$\begin{bmatrix} -\alpha I & (NC-CX_1)^T \\ * & -I \end{bmatrix} \le 0. \quad (33)$$

Hence, the static output tracking problem can be changed to a problem of finding a global solution of the following minimization problem:

$$\min \alpha \quad \text{subject to (31) and (33)}. \quad (34)$$

Remark 10. From Theorems 6 and 8, the sufficient conditions in Theorem 8 are more complex, which lead to a large amount of calculation. So the static output feedback gain can be conveniently obtained by the method in Theorem 6.

5. Example

Consider the systems (1) and (5) with $A = \begin{bmatrix} -3 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $w(t) = 0.1\cos(t)$, $A_m = -1$, $C_m = 2$, and $r(t) = 0.1\sin(t)$.

$\phi(x)$ is given by

$$\phi(x) = \begin{bmatrix} -x_1\left(x_1^2+x_2^2\right) \\ -x_2\left(x_1^2+x_2^2\right) \end{bmatrix}. \quad (35)$$

The above system model can be used to describe the motion of a moving object [22].

Let

$$\varpi = \min\left(\sqrt{\frac{-\nu}{4}}, \sqrt[4]{\beta+\frac{\nu^2}{4}}\right), \quad \nu < 0,\ \beta + \frac{\nu^2}{4} > 0. \quad (36)$$

According to [22], the quadratically inner-bounded property of $\phi(x)$ is verified in $\overline{Q}$, $\overline{Q} = \{x \in R^2 : \|x\| \le \varpi\}$. As the system is globally one-sided Lipschitz, that is, $Q = R^2$, $Q \cap \overline{Q} = \overline{Q}$. Note that the region $\overline{Q}$ can be made arbitrarily large by choosing appropriate values for β and ν.

We choose $\beta = -1$ and $\nu = -3$; then $\varpi = 0.8660$.

The initial value is $x(0) = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}^T \in \overline{Q}$ and $x_m(0) = -1$.

By solving (10), G and H can be obtained as

$$G = \begin{bmatrix} 0 & 2 \end{bmatrix}^T, \qquad H = -6. \quad (37)$$

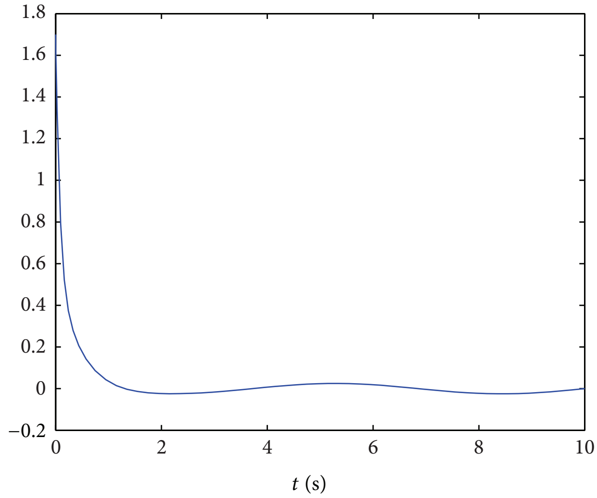

Figure 1: Error $e(t) : (= y(t) - y_m(t))$.

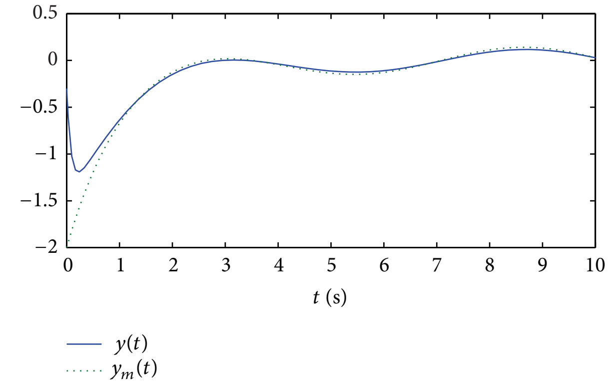

Figure 2: Output $y(t)$ and $y_m(t)$.

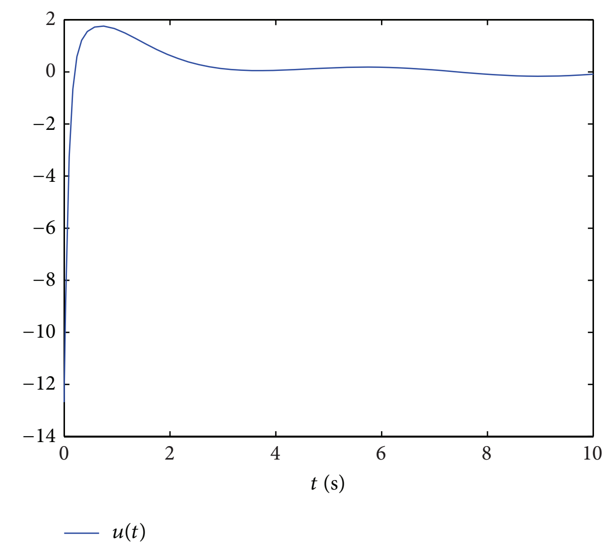

Figure 3: Control $u(t)$.

Given $\eta = 4.4711, \varepsilon_1 = 0.01, \varepsilon_2 = 1$, and $M = 0, \omega = 1$, then solving LMI (26) yields

$$X_1 = \begin{bmatrix} 1.8784 & -0.2931 \\ 0 & 1.5907 \end{bmatrix}, \qquad L = -17.4786. \tag{38}$$

Furthermore, noting that $C_2 = 1$, $X_{22} = 1.5907$, we obtain

$$K = L(C_2 X_{22})^{-1} = -10.9880. \tag{39}$$

Then the controller is designed as

$$u(t) = -6x_m(t) - 10.988(y(t) - y_m(t)). \tag{40}$$

The simulation results are shown in Figures 1, 2, and 3.

Figure 1 shows the time response of the error $e(t)$. The curves about system output $y(t)$ and reference output $y_m(t)$ are shown in Figure 2. Figure 3 shows the control input response curve. From the simulation results, it is concluded that the proposed method is effective.

Remark 11. By Theorem 8, we also can obtain the static output feedback gain, but both conditions (31) and (33) are calculated simultaneously. So the method by Theorem 6 is relatively simple.

6. Conclusion

In this paper, the problem of static output tracking control of nonlinear systems with one-sided Lipschitz condition has been investigated. A static output feedback controller is designed to guarantee that the system output asymptotically tracks the reference output with H_∞ disturbance rejection level. A sufficient condition is derived to obtain the static output feedback gain by linear matrix inequality (LMI), and no equality constraints can be needed. It should be noted that the special form of matrix X_1 will bring some conservatism, so further work is to study how to reduce it.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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