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Research Article

Global Stability of a Computer Virus Propagation Model with Two Kinds of Generic Nonlinear Probabilities

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Vaccination is one of the most effective measures for suppressing the spread of computer virus, and the bilinear incidence rate assumption for the majority of previous models, which is a good first approximation of the general incidence rate, is in disagreement with the reality. In this paper, a new dynamical model with two kinds of generic nonlinear probabilities (incidence rate and vaccination probability) is established. An exhaustive mathematical analysis of this model shows that (a) there are two equilibria, virus-free equilibrium and viral equilibrium, and (b) the virus-free (or viral) equilibrium is globally asymptotically stable when the basic reproduction number is less (or greater) than unity. The analysis of the basic reproduction number is also included. Additionally, some numerical examples are given to illustrate the main results, from which it can be seen that the generic nonlinear vaccination is helpful to strengthen computer security.

1. Introduction

Establishing rational dynamical models underlying the mechanism of the spread of computer virus is a significant issue in computer security, which can give important insights to defend against viral spread. Since the seminal work by Cohen [1] and Murray [2] as well as Kephart and White [3], various propagation models, such as susceptible-infected-susceptible (SIS) model [4, 5], susceptible-infected-recovered (SIR) model [6, 7], susceptible-infected-recovered-susceptible (SIRS) model [8–13], susceptible-exposed-infected-removed (SEIR) model [14, 15], susceptible-exposed-infected-quarantined-recovered-susceptible (SEIQRS) model [16], susceptible-antidotal-infectious-contaminated (SAIC) model [17], susceptible-antidotal-infected-removed (SAIR) model [18], susceptible-latent-breaking-susceptible (SLBS) model [19, 20], susceptible-latent-breaking-recovered-susceptible (SLBRS) model [21], susceptible-infected-countermeasure-susceptible (SICS) model [22], and susceptible-infected-external-susceptible (SIES) model [23–25], have been widely developed.

Vaccination (i.e., the measure that an uninfected computer has the newest-version antivirus software installed) plays an important role in repressing computer virus, by which a susceptible computer would have temporary immunity. The fact that a large number of susceptible computers are infected would enhance the probability that the user of a susceptible computer has his/her computer vaccinated, implying that vaccination probability is related to the number of infected computers. Indeed, Gan et al. [9, 11] recently investigated two SIRS models by incorporating a linear or nonlinear vaccination probability (i.e., the probability that a susceptible computer gets vaccinated is linear or nonlinear in the number of currently infected computers). Unfortunately, the bilinear incidence rate assumption for these two models, which is a good first approximation of the general incidence rate, is inconsistent with the actual conditions [10]. In reality, overcrowded infected computers and active protection measures would render this approximation to fail terribly. As a result, it is worthwhile to explore a dynamical model with generic nonlinear vaccination probability under more reasonable assumptions.

Having this idea in mind, in this paper, a new dynamical model of computer virus with generic nonlinear vaccination probability and nonlinear incidence rate is proposed. A detailed study of the model is provided. Specifically, the basic

reproduction number (i.e., the average number of secondary infections produced by a single infected computer during its life time), R_0, is determined, and the virus-free (or viral) equilibrium is shown to be globally asymptotically stable if $R_0 \leq 1$ (or $R_0 > 1$), implying that computer virus would tend to extinction or persist according to the value of R_0. The related analysis of R_0 is also conducted. Additionally, some numerical examples are examined to illustrate the main results, from which it can be seen that the generic nonlinear vaccination is helpful to suppress computer virus diffusion.

The organization of the rest of the paper is as follows. Section 2 formulates the new model. Section 3 proves the global stabilities of the virus-free and viral equilibria. A parameter analysis of the basic reproduction number is performed in Section 4. Finally, Section 5 summarizes this work.

2. Model Formulation

As usual, a computer is either internal (i.e., on the Internet) or external (i.e., outside the Internet). Moreover, an internal computer is assumed to be in one of three possible states: susceptible (i.e., uninfected but not immune), infected, and recovered (i.e., uninfected and immune).

Now, let us introduce some notations as follows, which will be adopted in the sequel:

$S(t)$: the average number of susceptible internal computers at time t,

$I(t)$: the average number of infected internal computers at time t,

$R(t)$: the average number of recovered internal computers at time t,

$N(t)$: the average number of internal computers at time t; that is, $N(t) = S(t) + I(t) + R(t)$.

For brevity, let S, I, R, and N represent $S(t)$, $I(t)$, $R(t)$, and $N(t)$, respectively.

The following fundamental assumptions of the new model are made.

(A1) External computers enter the Internet at rate $b > 0$, of which a fraction of $(1 - p)$ (resp., p) is susceptible (resp., immune), $0 \leq p \leq 1$.

(A2) Every internal computer leaves the Internet with probability per unit time $\mu > 0$.

(A3) Every susceptible internal computer is infected by infected internal computers with probability per unit time $\beta I/h(I)$, where $\beta > 0$; function $h(I)$ is continuously differentiable with $h(0) = 1$ and $h'(I) \geq 0$.

(A4) Due to treatment, every infected internal computer becomes recovered (resp., susceptible) with probability per unit time $\gamma_1 > 0$ (resp., $\gamma_2 > 0$).

(A5) Due to vaccination, every susceptible internal computer acquires temporary immunity with probability per unit time $\alpha_1 f(I)$, where $\alpha_1 > 0$; function $f(I)$ is continuously differentiable with $f(0) = 1$ and $f'(I) \geq 0$.

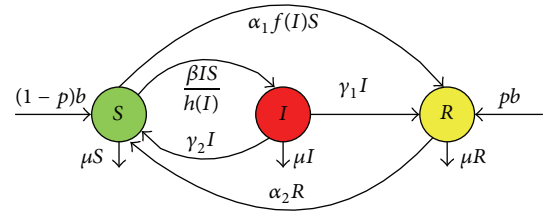


Figure 1: The transfer diagram of the new model.

(A6) Every recovered internal computer loses immunity with probability per unit time $\alpha_2 > 0$.

This collection of assumptions can be schematically shown in Figure 1, from which one can derive the differential system

$$\begin{aligned} \dot{S} &= (1-p)\,b - \mu S - \frac{\beta SI}{h(I)} - \alpha_1 f(I)\,S + \gamma_2 I + \alpha_2 R, \\ \dot{I} &= \frac{\beta SI}{h(I)} - \mu I - \gamma_1 I - \gamma_2 I, \\ \dot{R} &= pb - \mu R - \alpha_2 R + \alpha_1 f(I)\,S + \gamma_1 I, \end{aligned} \tag{1}$$

with initial condition $(S(0), I(0), R(0)) \in \mathbb{R}^3_+$.

3. Model Analysis

This section is devoted to study model (1) theoretically. The analysis of this model comprises the basic reproduction number, the existence of equilibria, and their global stabilities.

3.1. Basic Reproduction Number. Employing the next generation method (see [26]) to model (1), the basic reproduction number can be derived as

$$R_0 = \frac{\beta b\,(\mu + \alpha_2 - \mu p)}{\mu\,(\mu + \alpha_1 + \alpha_2)\,(\mu + \gamma_1 + \gamma_2)}. \tag{2}$$

3.2. Equilibria. Obviously, system (1) always has a unique virus-free equilibrium $E^0 = (S^0, 0, R^0)$, where

$$S^0 = \frac{b\,(\mu + \alpha_2 - \mu p)}{\mu\,(\mu + \alpha_1 + \alpha_2)}, \qquad R^0 = \frac{b\,(\alpha_1 + \mu p)}{\mu\,(\mu + \alpha_1 + \alpha_2)}. \tag{3}$$

Next, let us examine the existence of viral equilibria. The following result is obtained.

Lemma 1. *System* (1) *has a unique viral equilibrium* $E^* = (S^*, I^*, R^*)$ *if* $R_0 > 1$.

Proof. All viral equilibria of system (1) are determined by the following system of equations:

$$\begin{aligned}(1-p)b-\mu S-\frac{\beta SI}{h(I)}-\alpha_1 f(I)S+\gamma_2 I+\alpha_2 R=0,\\ \frac{\beta SI}{h(I)}-\mu I-\gamma_1 I-\gamma_2 I=0,\\ pb-\mu R-\alpha_2 R+\alpha_1 f(I)S+\gamma_1 I=0,\end{aligned} \tag{4}$$

where $I \neq 0$. Simplifying, one can get

$$\begin{aligned}(1-p)b-\mu S-(\mu+\gamma_1)I-\alpha_1 f(I)S+\alpha_2 R=0,\\ S=\frac{\mu+\gamma_1+\gamma_2}{\beta}h(I),\\ R=\frac{b}{\mu}-S-I.\end{aligned} \tag{5}$$

Substituting the last two equations into the first equation, one can obtain

$$\begin{aligned}F(I):=pb+(\mu+\alpha_2)\left(I+\frac{\mu+\gamma_1+\gamma_2}{\beta}h(I)-\frac{b}{\mu}\right)+\gamma_1 I\\ +\alpha_1 f(I)\frac{\mu+\gamma_1+\gamma_2}{\beta}h(I)=0.\end{aligned} \tag{6}$$

Note that $h(0)=1$, $h'(I)\geq 0$, $f(0)=1$, $f'(I)\geq 0$, and $R_0>1$. Then,

$$\begin{aligned}F'(I)>0,\\ \lim_{I\to\infty}F(I)=+\infty,\\ F(0)=\frac{(\mu+\alpha_1+\alpha_2)(\mu+\gamma_1+\gamma_2)}{\beta}(1-R_0)<0.\end{aligned} \tag{7}$$

Thus, function $F(I)$ has a unique positive zero. The proof is complete. □

For convenience, in what follows we will investigate the following equivalent system of model (1). Consider

$$\begin{aligned}\dot{N}&=b-\mu N,\\ \dot{I}&=\frac{\beta(N-I-R)I}{h(I)}-(\mu+\gamma_1+\gamma_2)I,\\ \dot{R}&=pb-(\mu+\alpha_2)R+\alpha_1 f(I)(N-I-R)+\gamma_1 I,\end{aligned} \tag{8}$$

with initial condition $(N(0),I(0),R(0))\in\mathbb{R}^3_+$. Clearly, the set

$$\Omega=\left\{(N,I,R)\in\mathbb{R}^3_+ : R+I\leq N\leq\frac{b}{\mu}\right\} \tag{9}$$

is positively invariant for system (8).

3.3. Global Stability of the Virus-Free Equilibrium

Theorem 2. *E^0 is globally asymptotically stable if $R_0\leq 1$.*

Proof. Let $x=N-b/\mu$, $y=I$, and $z=R-R^0$; then, system (8) can be rewritten as

$$\begin{aligned}\dot{x}&=-\mu x,\\ \dot{y}&=\frac{\beta y}{h(y)}\left(x-y-z+S^0\right)-(\mu+\gamma_1+\gamma_2)y,\\ \dot{z}&=-(\mu+\alpha_2)z+\gamma_1 y\\ &\quad+\alpha_1 f(y)\left(x-y-z+S^0\right)-\alpha_1 S^0.\end{aligned} \tag{10}$$

Consider Lyapunov function:

$$\begin{aligned}V=\frac{\alpha_2}{8\mu}x^2+\frac{\gamma_1}{\beta}\int_0^y h(u)\,du+\frac{\alpha_1 S^0}{\beta}\int_0^y\frac{h(u)[f(u)-1]}{u}du\\ +\frac{\alpha_1}{\beta}\int_0^y f(u)h(u)\,du+\frac{1}{2}(x-z)^2.\end{aligned} \tag{11}$$

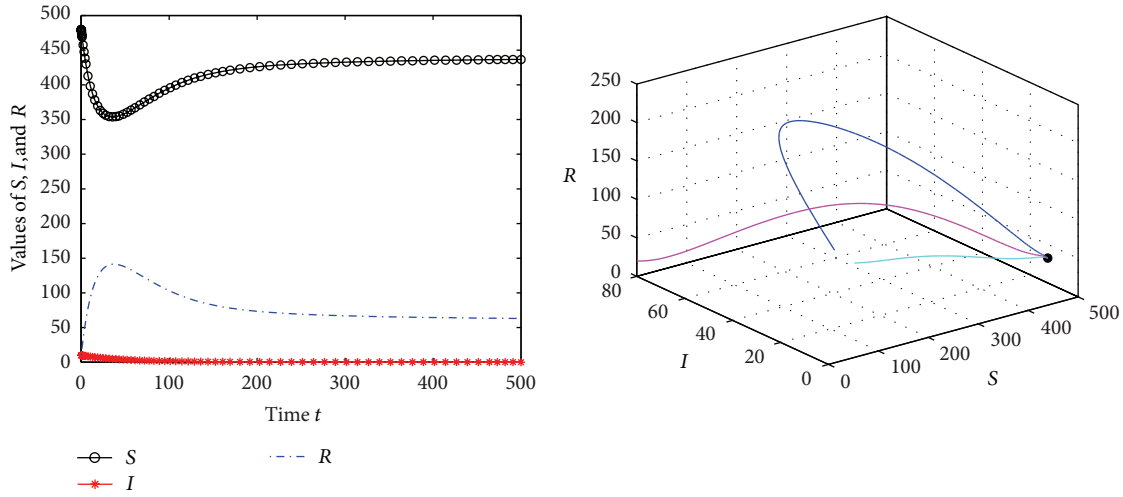
Then,

$$\begin{aligned}\dot{V}|_{(6)}&=\frac{\alpha_2}{4\mu}\dot{x}x+\frac{\gamma_1}{\beta}h(y)\dot{y}+\frac{\alpha_1 S^0}{\beta}\frac{h(y)[f(y)-1]}{y}\dot{y}\\ &\quad+\frac{\alpha_1}{\beta}f(y)h(y)\dot{y}+(x-z)(\dot{x}-\dot{z})\\ &=-\mu(x-z)^2-\alpha_2\left(\frac{x}{2}-z\right)^2\\ &\quad-\frac{\gamma_1(\mu+\gamma_1+\gamma_2)y}{\beta}[h(y)-R_0]\\ &\quad-\alpha_1 S^0[f(y)-1]y\\ &\quad-\frac{\alpha_1 S^0[f(y)-1](\mu+\gamma_1+\gamma_2)}{\beta}[h(y)-R_0]-\gamma_1 y^2\\ &\quad-\alpha_1 f(y)(x-y-z)^2\\ &\quad-\frac{\alpha_1 y f(y)(\mu+\gamma_1+\gamma_2)}{\beta}[h(y)-R_0]\\ &\leq 0.\end{aligned} \tag{12}$$

$\dot{V}|_{(6)}=0$ if and only if $(x,y,z)=(0,0,0)$; that is, $(S,I,R)=E^0$. Hence, the claimed result follows from the LaSalle Invariance Principle [27]. □

Remark 3. Theorem 2 implies that computer virus on the Internet would tend to extinction when the basic reproduction number is less than or equal to unity.

Example 4. Consider system (1) with $p=0.3$, $b=5$, $\mu=0.01$, and $\beta=0.00004$, $\alpha_1=0.002$, $\alpha_2=0.03$, $\gamma_1=0.008$,



(a) Time plots of S, I, and R

(b) Phase portrait for system (1)

FIGURE 2: An illustration of the dynamics of system (1) given in Example 4.

$\gamma_2 = 0.002$, $f(I) = I + 1$, and $h(I) = I^2 + 1$. Then, $R_0 = 0.881 < 1$. Figure 2(a) displays the time plots of S, I, and R for the initial condition $(S(0), I(0), R(0)) = (480, 10, 10)$, and Figure 2(b) shows the phase portrait for system (1), both consistent with Theorem 2 and Remark 3.

3.4. Global Stability of the Viral Equilibrium. Firstly, let us consider the following lemma.

Lemma 5. *Assume that $R_0 > 1$. For $y \in (-I^*, +\infty)$, let*

$$\begin{aligned}
V_1(y) &= \int_{I^*}^{y+I^*} \frac{h(u)(u-I^*)}{u}du,\\
V_2(y) &= \int_{I^*}^{y+I^*} \frac{h(u)f(u)(u-I^*)}{u}du,\\
V_3(y) &= \int_{I^*}^{y+I^*} \frac{h(u)[f(u)-f(I^*)]}{u}du,\\
V_4(y) &= y[h(y+I^*)-h(I^*)],\\
V_5(y) &= y[f(y+I^*)-f(I^*)],\\
V_6(y) &= [f(y+I^*)-f(I^*)][h(y+I^*)-h(I^*)].
\end{aligned} \tag{13}$$

Then, $V_i(y) \geq 0, i = 1, 2, \ldots, 6$. Moreover, $V_i(y) = 0$ if and only if $y = 0$.

Proof. Here, we will prove only the result of $V_1(y)$. The proofs of the remaining five functions are similar. It follows from $dV_1(y)/dy = h(y + I^*)y/(y + I^*)$ that $V_1(y)$ is decreasing in $(-I^*, 0]$ and is increasing in $(0, +\infty)$. As $V_1(0) = 0$, the claimed result follows. □

Now, let us explore the global stability of the viral equilibrium.

Theorem 6. *E^* is globally asymptotically stable if $R_0 > 1$.*

Proof. Let $x = N - b/\mu$, $y = I - I^*$, and $z = R - R^*$. Rewrite system (8) as

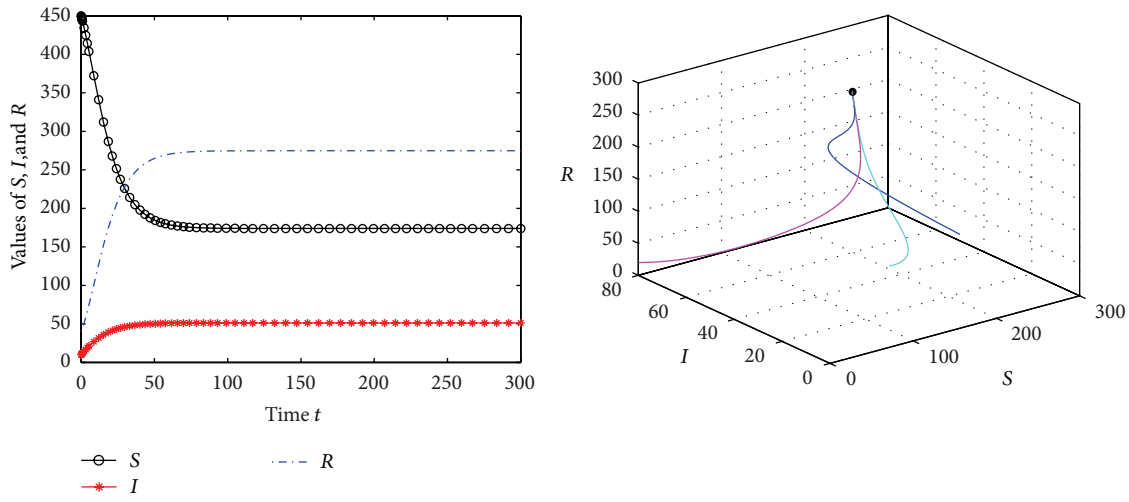
$$\begin{aligned}
\dot{x} &= -\mu x,\\
\dot{y} &= \frac{\beta(y+I^*)}{h(y+I^*)}\left\{x - y - z - \frac{S^*}{h(I^*)}[h(y+I^*)-h(I^*)]\right\},\\
\dot{z} &= -(\mu+\alpha_2)z + \gamma_1 y\\
&\quad + \alpha_1 f(y+I^*)(x-y-z+S^*)\\
&\quad - \alpha_1 f(I^*)S^*.
\end{aligned} \tag{14}$$

Consider Lyapunov function:

$$\begin{aligned}
V = \frac{\alpha_2}{8\mu}x^2 + \frac{\gamma_1}{\beta}V_1(y) + \frac{\alpha_1}{\beta}V_2(y)\\
+ \frac{\alpha_1 S^*}{\beta}V_3(y) + \frac{1}{2}(x-z)^2.
\end{aligned} \tag{15}$$

Then,

$$\begin{aligned}
\dot{V}|_{(8)} &= \frac{\alpha_2}{4\mu}\dot{x}x + \frac{\gamma_1}{\beta}\frac{dV_1(y)}{dy}\dot{y} + \frac{\alpha_1}{\beta}\frac{dV_2(y)}{dy}\dot{y}\\
&\quad + \frac{\alpha_1 S^*}{\beta}\frac{dV_3(y)}{dy}\dot{y} + (x-z)(\dot{x}-\dot{z})\\
&= -\mu(x-z)^2 - \alpha_2\left(\frac{x}{2}-z\right)^2\\
&\quad - \frac{\gamma_1 S^*}{h(I^*)}V_4(y) - \frac{\alpha_1 S^* f(y+I^*)}{h(I^*)}V_4(y)\\
&\quad - \alpha_1 f(y+I^*)(x-y-z)^2
\end{aligned}$$



(a) Time plots of S, I, and R

(b) Phase portrait for system (1)

Figure 3: An illustration of the dynamics of system (1) given in Example 8.

$$-\gamma_1 y^2 - \alpha_1 S^* V_5(y) - \frac{\alpha_1 S^{*2}}{h(I^*)} V_6(y) \leq 0. \tag{16}$$

$\dot{V}|_{(8)} = 0$ if and only if $(x, y, z) = (0, 0, 0)$; that is, $(S, I, R) = E^*$. Therefore, the claimed result follows from the LaSalle Invariance Principle [27]. □

Remark 7. Theorem 6 implies that computer virus on the Internet would tend to persist when the basic reproduction number is greater than unity.

Example 8. Consider system (1) with $p = 0.3, b = 5, \mu = 0.01$, $\beta = 0.006$, $\alpha_1 = 0.001$, $\alpha_2 = 0.03$, $\gamma_1 = 0.008$, $\gamma_2 = 0.002$, $f(I) = I + 1$, and $h(I) = I + 1$. Then, $R_0 = 135.3659 > 1$. Figure 3(a) demonstrates the time plots of S, I, and R for the initial condition $(S(0), I(0), R(0)) = (450, 10, 40)$, and Figure 3(b) exhibits the phase portrait for system (1), both in accordance with Theorem 6 and Remark 7.

4. Discussions

Remarks 3 and 7 tell us the fact that computer virus on the Internet would tend to extinction or persist according to the value of the basic reproduction number. So, the analysis of R_0 is performed.

Theorem 9. *Consider* (2). *Then,* $\partial R_0/\partial\beta > 0$, $\partial R_0/\partial\gamma_1 < 0$, $\partial R_0/\partial\gamma_2 < 0$, $\partial R_0/\partial\alpha_1 < 0$, $\partial R_0/\partial\alpha_2 > 0$, *and* $\partial R_0/\partial\mu < 0$.

Proof. It is easy to see that the first four inequalities are true. Consider

$$\frac{\partial R_0}{\partial \alpha_2} = \frac{\beta b(\alpha_1 + p\mu)}{\mu(\gamma_1 + \gamma_2 + \mu)(\alpha_1 + \alpha_2 + \mu)^2} > 0,$$

$$\frac{\partial R_0}{\partial \mu} = -\frac{\beta b(1-p)(\alpha_1 + \alpha_2 + \gamma_1 + \gamma_2 + 2\mu)}{(\alpha_1 + \alpha_2 + \mu)^2(\gamma_1 + \gamma_2 + \mu)^2} - \frac{\beta b \alpha_2 [(\alpha_1 + \alpha_2 + 2\mu)(\gamma_1 + \gamma_2 + \mu) + \mu(\mu + \alpha_1 + \alpha_2)]}{\mu^2(\alpha_1 + \alpha_2 + \mu)^2(\gamma_1 + \gamma_2 + \mu)^2} < 0. \tag{17}$$

The proof is complete. □

Furthermore, the influences of system parameters (i.e., $\alpha_1, \alpha_2, \gamma_1, \gamma_2, \beta$, and μ) on R_0 are illustrated in Figures 4, 5, and 6, in coherence with Theorem 9.

In addition, the following example indicates the effect of different incidence rates and vaccination probabilities on I (see Figure 7).

Example 10. Consider system (1) with $p = 0.3, b = 5, \mu = 0.01$, and $\beta = 0.006, \alpha_1 = 0.001, \alpha_2 = 0.03, \gamma_1 = 0.008$, and $\gamma_2 = 0.002$ and the initial condition $(S(0), I(0), R(0)) = (450, 10, 40)$.

5. Conclusions

This paper has studied the long-term behavior of computer virus in terms of a new propagation model with generic nonlinear incidence rate and nonlinear vaccination probability. An elaborate analysis of the model including the basic reproduction number, the existence of virus-free and viral equilibria, and their global stabilities has been conducted, from which it is found that computer virus on the Internet would tend to extinction or persist according to the value of the basic reproduction number. To illustrate

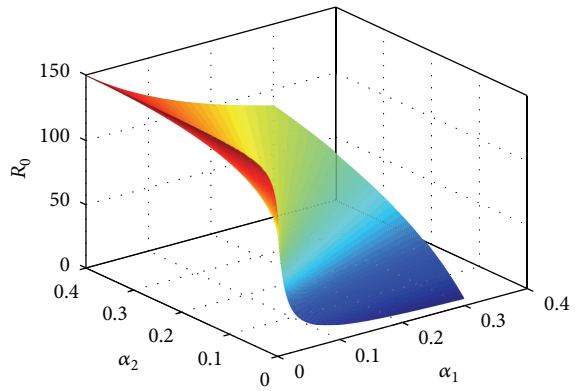


FIGURE 4: An illustration of the impact of α_1 and α_2 on R_0.

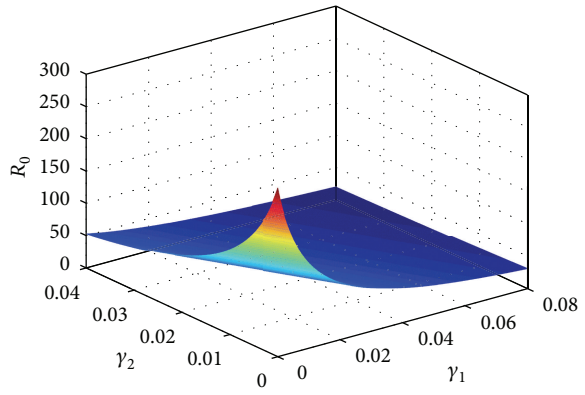


FIGURE 5: An illustration of the impact of γ_1 and γ_2 on R_0.

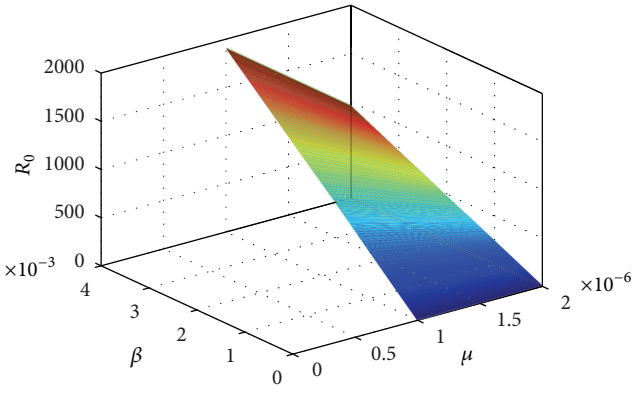


FIGURE 6: An illustration of the impact of β and μ on R_0.

the obtained main results, some numerical examples have been examined, from which it can be seen that the generic nonlinear vaccination is useful for the inhibition of viral spread.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

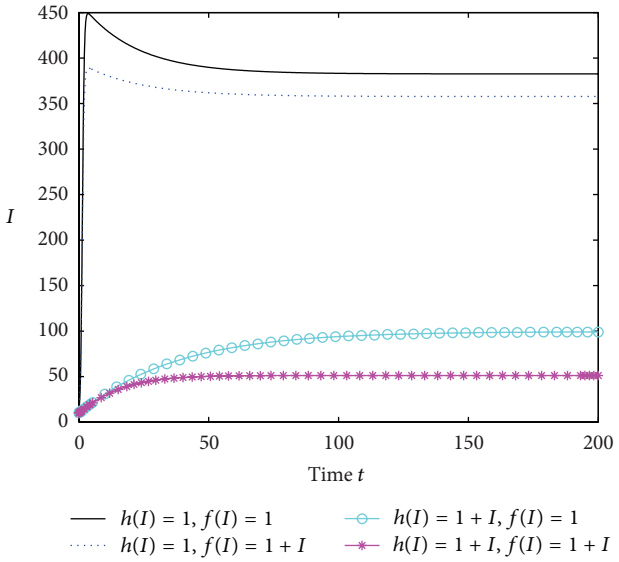


FIGURE 7: An illustration of the impact of different incidence rates and vaccination probabilities on I given in Example 10.

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