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Research Article

Adaptive Asymptotical Synchronization for Stochastic Complex Networks with Time-Delay and Markovian Switching

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The problem of adaptive asymptotical synchronization is discussed for the stochastic complex dynamical networks with time-delay and Markovian switching. By applying the stochastic analysis approach and the M-matrix method for stochastic complex networks, several sufficient conditions to ensure adaptive asymptotical synchronization for stochastic complex networks are derived. Through the adaptive feedback control techniques, some suitable parameters update laws are obtained. Simulation result is provided to substantiate the effectiveness and characteristics of the proposed approach.

1. Introduction

As is known to all, complex networks widely exist in nature, such as brain structures, protein interactions, social networks, electrical power, and World Wide Web. Recently, the dynamical behaviors of complex networks have attracted ever-increasing research interest from a variety of communities such as mathematicians, computer scientists, and control engineers. As a result, a number of dynamic analysis issues have been extensively investigated for complex networks, such as synchronization, consensus, and flocking phenomenon, in which synchronization is one of the most important and has attracted special attention of researchers in different fields [1–13]. In [4], by using Lyapunov method and some properties of Kronecker product, a sufficient condition is proposed to ensure that the dynamics of the considered network globally exponentially synchronizes with the desired solution in the mean square sense. In particular, the proposed criteria for network synchronization are in terms of linear matrix inequalities (LMIs). In [13], a modified Lyapunov-Krasovskii functional is constructed by employing the more general decomposition approach; the novel delay-dependent synchronization conditions are derived in terms of LMIs, which can be easily solved by various convex optimization algorithms.

Meanwhile, the stability and synchronization of complex networks can be applied to secure communication systems [14], information science [15], and brain science [16], and so on. The synchronization of complex networks is to achieve the accordance of the states of the drive complex network and the response complex network in a moment. That is to say, the state of the error system can achieve zero eventually when the time approaches infinity. In particular, the adaptive synchronization for a complex network is such synchronization that the parameters of the drive complex network need to be estimated and the synchronization control law needs to be updated in real time when the complex network evolves. Furthermore, the stochastic complex dynamic network contains inherent time delay, which may cause instability or oscillation.

It should be pointed out that, up to now, the problem of adaptive asymptotical synchronization for stochastic complex networks with time-delay and Markovian switching has received very little research attention.

Summarizing the above discussions, the focus of this paper is on the adaptive asymptotical synchronization problem for stochastic delayed complex networks with Markovian switching. The main purpose of this paper is to establish stability criteria for testing whether the stochastic complex network is adaptive asymptotical synchronization. By using the stochastic analysis approach and the M-matrix method, several sufficient conditions to ensure adaptive synchronization for stochastic complex networks are derived. Via the adaptive feedback technique, some suitable parameters update laws are obtained. Moreover, a simulation example is provided to show the effectiveness of the proposed controller design scheme. The main novelty of our contribution is threefold: (1) adaptive asymptotical synchronization control is addressed for stochastic complex networks with time-delay and Markovian switching; (2) using the adaptive feedback control techniques, adaptive feedback controller is designed; (3) the M-matrix method of adaptive synchronization controller is given by employing a new nonnegative function.

The organization of this paper is as follows. In Section 2, the mathematical model of the stochastic complex networks with time-delay and Markovian switching is presented and some preliminaries are given. The main results of adaptive asymptotical synchronization are proved in Section 3. In Section 4, a simple example is given to demonstrate the effectiveness of the proposed results. Finally, the conclusions are presented in Section 5.

2. Problem Formulation and Preliminaries

The coupled complex networks can be called drive complex network and described as follows:

$$\dot{x}_l(t) = f(x_l(t)) + \sum_{p=1}^{N} a_{lp}\Theta x_p(t) + \sum_{p=1}^{N} b_{lp}\Theta x_p(t-\tau(t)), \quad l = 1, 2, \ldots, N, \tag{1}$$

where $t \geq 0$, $x_l(t) = [x_{l1}(t), x_{l2}(t), \ldots, x_{lN}(t)]^T \in \mathbb{R}^N$ is the state vector of the lth node, $f(x_l(t)) \in \mathbb{R}^N$ is a nonlinear vector-valued function, $\Theta = I_n = \operatorname{diag}\{1, 1, \ldots, 1\} \in \mathbb{R}^{N\times N}$ is an inner-coupling matrix, $A = (a_{lp})^{N\times N} \in \mathbb{R}^{N\times N}$ and $B = (b_{lp})^{N\times N} \in \mathbb{R}^{N\times N}$ are the connection weight and the delayed connection weight matrices, and a_{lp} and b_{lp} are the weight or coupling strength. If there exists a link from node l to p $(l \neq p)$, then $a_{lp} \neq 0$ and $b_{lp} \neq 0$. Otherwise, $a_{lp} = 0$ and $b_{lp} = 0$. $\tau(t)$ is the time-varying delay satisfying $0 < \tau(t) \leq \overline{\tau}$ and $\dot{\tau}(t) \leq \widehat{\tau} < 1$, where $\overline{\tau}$ and $\widehat{\tau}$ are constants.

Given a probability space $(\Omega, \mathcal{F}$, and $P)$, $\{r(t), t \geq 0\}$ is a homogeneous finite-state Markovian process with right continuous trajectories and taking values in finite set $S = \{1, 2, \ldots, N\}$ with the initial model $r(0) = r_0$. Let generator $\Gamma = (\gamma_{ij})_{N\times N}$, $i, j \in S$, be the transition rate matrix with transition probability

$$P\{r(t+\delta) = j \mid r(t) = i\} = \begin{cases} \gamma_{ij}\delta + o(\delta) & \text{if } i \neq j, \\ 1 + \gamma_{ii}\delta + o(\delta) & \text{if } i = j, \end{cases} \tag{2}$$

where $\delta > 0$ and $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, while

$$\gamma_{ii} = -\sum_{j=1, j\neq i}^{N} \gamma_{ij}. \tag{3}$$

Let $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T \in \mathbb{R}^{N\times N}$, $f(x(t)) = [f^T(x_1(t)), f^T(x_2(t)), \ldots, f^T(x_N(t))]^T$; the drive complex network (1) with Markovian switching can be rewritten as

$$\begin{aligned} dx(t) = {} & [f(x(t)) + A(r(t)) \otimes I_n x(t) \\ & + B(r(t)) \otimes I_n x(t-\tau(t))]\, dt, \end{aligned} \tag{4}$$

where, for the purpose of simplicity, we denote $r(t) = i$, $A(r(t)) = A^i$, $B(r(t)) = B^i$, and $x(t-\tau(t)) = x_\tau(t)$, respectively.

For the drive complex network (4), a response complex network is constructed in the following form:

$$\begin{aligned} dy(t) = {} & [f(y(t)) + A(r(t)) \otimes I_n y(t) \\ & + B(r(t)) \otimes I_n y_\tau(t) + U(t)]\, dt \\ & + \sigma(t, y(t) - x(t), y_\tau(t) - x_\tau(t))\, dw(t), \end{aligned} \tag{5}$$

where $y(t)$ is the state vector of the response complex network (5). $U(t) = (u_1(t), u_2(t), \ldots, u_N(t))^T \in R^N$ is a control input vector with the form of

$$\begin{aligned} U(t) &= K(t)(y(t) - x(t)) \\ &= \operatorname{diag}\{k_1(t), k_2(t), \ldots, k_n(t)\}(y(t) - x(t)), \end{aligned} \tag{6}$$

$\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_N(t))^T$ is an N-dimensional Brown moment defined on a complete probability space $(\Omega, \mathcal{F}$, and $P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ (i.e., $\mathcal{F}_t = \sigma\{\omega(s) : 0 \leq s \leq t\}$ is a σ-algebra) and is independent of the Markovian process $\{r(t)\}_{t\geq 0}$, and σ is the noise intensity matrix and can be regarded as a result from the occurrence of eternal random fluctuation and other probabilistic causes.

Let $e_l(t) = y_l(t) - x_l(t)$ and $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T \in \mathbb{R}^{n\times N}$. As a matter of convenience, we mark $e(t-\tau(t)) = e_\tau(t)$ and $\phi(e(t)) = f(y(t)) - f(x(t))$. From the complex networks (4) and (5), the error system is arranged as

$$\begin{aligned} de(t) = {} & [\phi(e(t)) + A(r(t)) \otimes I_n e(t) \\ & + B(r(t)) \otimes I_n e_\tau(t) + U(t)]\, dt \\ & + \sigma(t, e(t), e_\tau(t))\, dw(t). \end{aligned} \tag{7}$$

The main purpose of the rest of this paper is to set up a criterion of adaptive asymptotical synchronization for the system (4)–(7) via employing adaptive control and M-matrix methods. Next, we firstly introduce assumptions, definitions, and lemmas which will be used in the proofs of main results.

Assumption 1. The activation function $f(x(t))$ satisfies the Lipschitz condition. That is to say, there exists a constant $L > 0$ such that

$$|f(u) - f(v)| \leq L|u - v|, \quad \forall u, v \in R^n. \tag{8}$$

Assumption 2. The noise intensity matrix $\sigma(\cdot,\cdot,\cdot)$ satisfies the linear growth condition. That is to say, there exist positives H_1 and H_2, such that

$$\operatorname{trace}(\sigma(t,e,e_\tau))^T(\sigma(t,e,e_\tau)) \le H_1|e|^2 + H_2|e_\tau|^2. \tag{9}$$

Definition 3 (see [17]). The trivial solution $e(t,\zeta)$ of the error system (7) is said to be almost surely asymptotically stable if

$$P\left(\lim_{t\to\infty}|e(t;i,\zeta)| = 0\right) = 1 \tag{10}$$

for any $\zeta \in L^2_{\mathscr{L}_0}([-\tau,0];R^n)$.

The response system (5) and the drive system (4) are said to be asymptotically synchronized if the error system (7) is asymptotically stable.

Definition 4 (see [18]). Consider an n-dimensional stochastic delayed differential equation (SDDE, for short) with Markovian switching:

$$\begin{aligned} dx(t) &= \mathbb{f}(t,r(t),x(t),x_\tau(t))\,dt \\ &\quad + \mathbb{g}(t,r(t),x(t),x_\tau(t))\,d\omega(t) \end{aligned} \tag{11}$$

on $t\in[0,\infty)$ with the initial data given by

$$\{x(\theta): -\tau\le\theta\le 0\} = \xi \in L^2_{\mathscr{L}_0}([-\tau,0];R^n). \tag{12}$$

For $V\in C^{2,1}(R_+\times S\times R^n;R_+)$, define an operator $\mathscr{L}$ from $R_+\times S\times R^n$ to R by

$$\begin{aligned} &\mathscr{L}V(t,i,x(t),x_\tau(t)) \\ &\quad = V_t(t,i,x(t)) + V_x(t,i,x(t))\,\mathbb{f}(t,i,x(t),x_\tau(t)) \\ &\qquad + \left(\frac{1}{2}\right)\operatorname{trace}\Big(\mathbb{g}^T(t,i,x(t),x_\tau(t))\,V_{xx}(t,i,x(t)) \\ &\qquad\quad \cdot\,\mathbb{g}(t,i,x(t),x_\tau)(t)\Big) \\ &\qquad + \sum_{j=1}^{N}\gamma_{ij}V(t,j,x(t)), \end{aligned} \tag{13}$$

where

$$\begin{aligned} V_t(t,i,x(t)) &= \frac{\partial V(t,i,x(t))}{\partial t}, \\ V_x(t,i,x(t)) &= \left(\frac{\partial V(t,i,x(t))}{\partial x_1},\frac{\partial V(t,i,x(t))}{\partial x_2},\dots,\frac{\partial V(t,i,x(t))}{\partial x_n}\right), \\ V_{xx}(t,i,x(t)) &= \left(\frac{\partial^2 V(t,i,x(t))}{\partial x_j\partial x_k}\right)_{n\times n}. \end{aligned} \tag{14}$$

Lemma 5 (see [18]). *Let $x\in\mathbb{R}^n$ and $y\in\mathbb{R}^n$; then $x^Ty+y^Tx \le \epsilon x^Tx+\epsilon^{-1}y^Ty$, for any $\epsilon>0$.*

Lemma 6 (see [18]). *If $M=(m_{ij})_{n\times n}\in R^{n\times n}$ with $m_{ij}<0$ $(i\ne j)$, then the following statements are equivalent.*

(i) *M is a nonsingular M-matrix.*

(ii) *Every real eigenvalue of M is positive.*

(iii) *M is positive stable. That is, M^{-1} exists and $M^{-1}>0$ (i.e., $M^{-1}\ge 0$ and at least one element of M^{-1} is positive).*

Lemma 7 (see [17]). *Assume that there are functions $V\in C^{2,1}(R_+\times S\times R^n;R_+)$, $\psi\in L^1(R_+;R_+)$, and $w_1,w_2\in C(R^n;R_+)$ such that*

$$\begin{aligned} \mathscr{L}V(t,i,x,y) &\le \psi(t) - w_1(x) + w_2(y), \\ &\forall (t,i,x,y)\in R_+\times S\times R^n\times R^n, \end{aligned} \tag{15}$$

$$w_1(0)=w_2(0)=0,\quad w_1(x)>w_2(x)\quad \forall x\ne 0,\ y\ne 0, \tag{16}$$

$$\lim_{|x|\to\infty}\ \inf_{0\le t<\infty,i\in S} V(t,i,x)=\infty. \tag{17}$$

Then the solution of (11) is almost surely asymptotically stable.

3. Main Results

In this section, some criteria of adaptive asymptotical synchronization will be obtained for the system (4), (5), and (7).

Theorem 8. *Assume that $M := -\operatorname{diag}\{\underbrace{\theta,\theta,\dots,\theta}_{N}\}-\Gamma$ is a nonsingular M-matrix, where*

$$\begin{aligned} \theta &= 1+L^2+\alpha+\beta+H_1, \\ \alpha &= \lambda_{\max}\left(A^i\otimes I_n\right), \\ \beta &= \lambda_{\max}\left(\frac{1}{2}\left(B^i\otimes I_n\right)\left(B^i\otimes I_n\right)^T\right). \end{aligned} \tag{18}$$

Let $m>0$ and $\vec{m}=(\underbrace{m,m,\dots,m}_{N})^T$. That is to say, all elements of $M^{-1}\vec{m}$ are positive. According to Lemma 6, $(q_1,q_2,\dots,q_N)^T := M^{-1}\vec{m}\gg 0$. In addition, assume also that

$$(1+H_2)\,\overline{q} < -\left(\theta q_i+\sum_{v=1}^{N}\gamma_{iv}q_v\right),\quad \forall i\in S, \tag{19}$$

where $\overline{q}=\max_{i\in S}q_i$.

Under Assumptions 1 and 2, the response complex network (5) can be adaptively synchronized with the drive complex network (4), if the feedback gain $K(t)$ with the update law is chosen as

$$\dot{k}_j = -\nu_j q_i e_j^2. \tag{20}$$

Proof. Choose a nonnegative function candidate as

$$V(t,e) = q_i|e|^2+\sum_{j=1}^{n}\frac{1}{\nu_j}k_j^2. \tag{21}$$

The computation of $\mathscr{L}V(t,e)$ along with the solution of error system (7) and using (20) is

$$\begin{aligned}
&\mathscr{L}V(t,e)\\
&= V_t(t,e) + V_e(t,e)\\
&\quad\times\left[\phi(e(t)) + A^i\otimes I_n e(t) + B^i\otimes I_n e_\tau(t) + U(t)\right]\\
&\quad+\frac{1}{2}\operatorname{trace}\left(\sigma^T(t,e,e_\tau)V_{ee}(t,e)\sigma(t,e,e_\tau)\right)\\
&\quad+\sum_{\nu=1}^{N}\gamma_{i\nu}V(t,\nu,e)\\
&= 2\sum_{j=1}^{n}\frac{1}{\nu_j}k_j\dot{k}_j + 2q_i e^T\\
&\quad\times\left[\phi(e(t)) + A^i\otimes I_n e(t) + B^i\otimes I_n e_\tau(t) + Ke(t)\right]\\
&\quad+\frac{1}{2}\operatorname{trace}\left(\sigma^T(t,e,e_\tau)V_{ee}(t,e)\sigma(t,e,e_\tau)\right)\\
&\quad+\sum_{\nu=1}^{N}\gamma_{i\nu}q_\nu|e|^2\\
&= 2q_i e^T\left[\phi(e(t)) + A^i\otimes I_n e(t) + B^i\otimes I_n e_\tau(t)\right]\\
&\quad+q_i\operatorname{trace}\left(\sigma^T(t,e,e_\tau)\sigma(t,e,e_\tau)\right)+\sum_{\nu=1}^{N}\gamma_{i\nu}q_\nu|e|^2.
\end{aligned}\tag{22}$$

Now, according to Assumptions 1 and 2 together with Lemma 5, one obtains

$$\begin{aligned}
e^T\phi(e(t)) &\le \frac{1}{2}e^Te + \frac{1}{2}\phi^T(e)\phi(e) \le \frac{1}{2}\left(1+L^2\right)|e|^2,\\
e^TA^i\otimes I_n e &\le \alpha|e|^2,\\
e^TB^i\otimes I_n e_\tau &\le \frac{1}{2}e^T\left(B^i\otimes I_n\right)\left(B^i\otimes I_n\right)^Te + \frac{1}{2}e_\tau^Te_\tau\\
&\le \beta|e|^2 + \frac{1}{2}|e_\tau|^2,\\
\operatorname{trace}\left(\sigma^T(t,e,e_\tau)\sigma(t,e,e_\tau)\right) &\le H_1|e|^2 + H_2|e_\tau|^2.
\end{aligned}\tag{23}$$

Substituting (23) into (22), one gets

$$\begin{aligned}
&\mathscr{L}V(t,e)\\
&\le 2q_i\left[\frac{1}{2}\left(1+L^2\right)|e|^2 + \alpha|e|^2 + \left(\beta|e|^2 + \frac{1}{2}|e_\tau|^2\right)\right]\\
&\quad+q_i\left(H_1|e|^2 + H_2|e_\tau|^2\right) + \sum_{\nu=1}^{N}\gamma_{i\nu}q_\nu|e|^2\\
&= \left(\theta q_i + \sum_{\nu=1}^{N}\gamma_{i\nu}q_\nu\right)|e|^2 + (1+H_2)q_i|e_\tau|^2\\
&\le -m|e|^2 + (1+H_2)\overline{q}|e_\tau|^2,
\end{aligned}\tag{24}$$

where $m = -(\theta q_i + \sum_{\nu=1}^{N}\gamma_{i\nu}q_\nu)$ by $[q_1, q_2, \ldots, q_N]^T = M^{-1}\vec{m}$.

Let $\psi(t) = 0$, $\omega_1(e) = m|e|^2$, and $\omega_2(e_\tau) = (1+H_2)\overline{q}|e_\tau|^2$. Then inequality (24) holds such that inequality (15) holds. Consider $\omega_1(0) = 0$ and $\omega_2(0) = 0$ when $e = 0$ and $e_\tau = 0$, and inequality (19) implies $\omega_1(e) > \omega_2(e_\tau)$. So (16) holds. Moreover, (17) holds when $|e| \to \infty$ and $|e_\tau| \to \infty$. By Lemma 7, the error system (7) is adaptive almost surely asymptotically stable, and hence the noise-perturbed response complex network (5) can be adaptively almost surely asymptotically synchronized with the drive complex network (4). This completes the proof. □

Remark 9. For complex networks (1), the method in the paper can be used in some systems, such as multiagent systems [19–21] and wireless sensor networks [22], which are the next research topic for us.

Now, we are in a position to consider two cases of the complex networks (4)-(5), which have the following corollaries.

The Markovian switching is removed from the complex networks. That is to say, the drive complex network, the response complex network, and the error system can be represented, respectively, as follows:

$$\begin{aligned}
dx(t) &= \left[f(x(t)) + A\otimes I_n x(t) + B\otimes I_n x_\tau(t)\right]dt,\\
dy(t) &= \left[f(y(t)) + A\otimes I_n y(t)\right.\\
&\qquad\left.+\, B\otimes I_n y_\tau(t) + U(t)\right]dt\\
&\qquad+\sigma\left(t, y(t)-x(t), y_\tau(t)-x_\tau(t)\right)dw(t),\\
de&(t)\\
&= \left[\phi(e(t)) + A\otimes I_n e(t) + B\otimes I_n e_\tau(t) + U(t)\right]dt\\
&\quad+\sigma\left(t, e(t), e_\tau(t)\right)dw(t).
\end{aligned}\tag{25}$$

For this case, one can get the following result analogous to Theorem 8.

Corollary 10. *Assume that $M := -\operatorname{diag}\{\underbrace{\theta,\theta,\ldots,\theta}_{N}\} - \Gamma$ is a nonsingular M-matrix, where $\theta < 0$, $\theta = 1 + L^2 + \alpha + \beta + H_1$, and*

$$1 + H_2 < -\theta. \tag{26}$$

Under Assumptions 1 and 2, the noise-perturbed response complex network can be adaptively asymptotically synchronized with the drive complex network, if the feedback gain $K(t)$ of the controller (6) *with the update law is chosen as*

$$\dot{k}_j = -\nu_j e_j^2. \tag{27}$$

Proof. Choose the following nonnegative function:

$$V(t,e) = |e|^2 + \sum_{j=1}^{n}\frac{1}{\nu_j}k_j^2. \tag{28}$$

The remaining proof is similar to that of Theorem 8 and hence omitted. □

The noise-perturbation is removed from the response complex network (5); then the response complex network and the error system can be represented, respectively, as follows:

$$\begin{aligned} dy(t) = {} & [f(y(t)) + A(r(t)) \otimes I_n y(t) \\ & + B(r(t)) \otimes I_n y_\tau(t) + U(t)]\, dt, \\ de(t) = {} & [\phi(e(t)) + A(r(t)) \otimes I_n e(t) \\ & + B \otimes I_n e_\tau(t) + U(t)]\, dt, \end{aligned} \tag{29}$$

which can lead to the following results.

Corollary 11. *Assume that* $M := -\operatorname{diag}\{\underbrace{\theta, \theta, \dots, \theta}_{N}\} - \Gamma$ *is a nonsingular* M*-matrix, where* $\theta = 1 + L^2 + \alpha + \beta$ *and*

$$\overline{q} < -\left(\theta q_i + \sum_{v=1}^{N} \gamma_{iv} q_v\right), \quad \forall i \in S, \tag{30}$$

where $\overline{q} = \max_{i \in S} q_i$.

Under Assumptions 1 and 2, the noiseless-perturbed response complex network can be adaptively asymptotically synchronized with the drive complex network, if the feedback gain $K(t)$ *of the controller* (6) *with the update law is chosen as* (20).

Proof. The proof is similar to that of Theorem 8 and hence omitted. □

4. Illustrative Example

In this section, an illustrative example will be given to demonstrate the effectiveness of the proposed methods.

Example 1. The Lorenz system is described by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} a(x_2(t) - x_1(t)) \\ bx_1(t) - x_2(t) - x_1(t)x_3(t) \\ x_1(t)x_2(t) - cx_3(t) \end{bmatrix}, \tag{31}$$

where $a = 10$, $b = 28$, and $c = 10/3$.

According to Theorem 8, the complex networks (drive complex network and response complex network) with four nodes are described as follows:

$$\begin{aligned} \dot{x}_{l1} &= ax_{l2}(t) - ax_{l1}(t) + \sum_{p=1}^{4} a_{lp} x_{p1}(t) + \sum_{p=1}^{4} b_{lp} x_{p1}(t-\tau), \\ \dot{x}_{l2} &= bx_{l1}(t) - x_{l2}(t) - x_{l1}(t)x_{l3}(t) + \sum_{p=1}^{4} a_{lp} x_{p2}(t) \\ &\quad + \sum_{p=1}^{4} b_{lp} x_{p2}(t-\tau), \\ \dot{x}_{l3} &= x_{l1}(t)x_{l2}(t) - cx_{l3}(t) + \sum_{p=1}^{4} a_{lp} x_{p3}(t) \\ &\quad + \sum_{p=1}^{4} b_{lp} x_{p3}(t-\tau), \\ \dot{y}_{l1} &= ay_{l2}(t) - ay_{l1}(t) + \sum_{p=1}^{4} a_{lp} y_{p1}(t) + \sum_{p=1}^{4} b_{lp} y_{p1}(t-\tau) \\ &\quad + k_l [y_{l1} - x_{l1} + \tanh(y_{l1}) - \tanh(x_{l1})] \\ &\quad + 0.4 [y_{l1}(t) - x_{l1}(t)], \\ \dot{y}_{l2} &= by_{l1}(t) - y_{l2}(t) - y_{l1}(t)y_{l3}(t) + \sum_{p=1}^{4} a_{lp} y_{p2}(t) \\ &\quad + \sum_{p=1}^{4} b_{lp} y_{p2}(t-\tau) \\ &\quad + k_l [y_{l2} - x_{l2} + \tanh(y_{l2}) - \tanh(x_{l2})] \\ &\quad + 0.3 [y_{l2}(t) - x_{l2}(t)], \\ \dot{y}_{l3} &= y_{l1}(t)x_{l2}(t) - cy_{l3}(t) + \sum_{p=1}^{4} a_{lp} y_{p3}(t) \\ &\quad + \sum_{p=1}^{4} b_{lp} y_{p3}(t-\tau) \\ &\quad + k_l [y_{l3} - x_{l3} + \tanh(y_{l3}) - \tanh(x_{l3})] \\ &\quad + 0.3 [y_{l3}(t-\tau) - x_{l3}(t-\tau)]. \end{aligned} \tag{32}$$

In the simulation, let

$$\begin{gathered} A = \begin{bmatrix} -6 & 2 & 1 & 3 \\ 2 & -5 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 3 & 1 & 0 & -4 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -8 & 3 & 1 & 4 \\ 3 & -5 & 0 & 2 \\ 1 & 0 & -3 & 2 \\ 1 & 3 & 2 & -6 \end{bmatrix}, \\ B_2 = \begin{bmatrix} -2 & 1 & 1 & 0 \\ 2 & -3 & 0 & 1 \\ 1 & 0 & -2 & 1 \\ 0 & 2 & 2 & -4 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} -1.2 & 1.2 \\ 0.5 & -0.5 \end{bmatrix}, \\ \tau = 0.1. \end{gathered} \tag{33}$$

These parameters fully satisfy Assumptions 1 and 2 and condition (19). Therefore, it will prove the main result to be correct if the error system can be adaptively asymptotically synchronized satisfying Theorem 8.

To illustrate the effectiveness of the developed theory, we employ the nonnegative function to solve the solutions for stochastic complex networks and to simulate the dynamics of error system and the adaptive feedback gain. The simulation figures are shown in Figures 1, 2, 3, and 4.

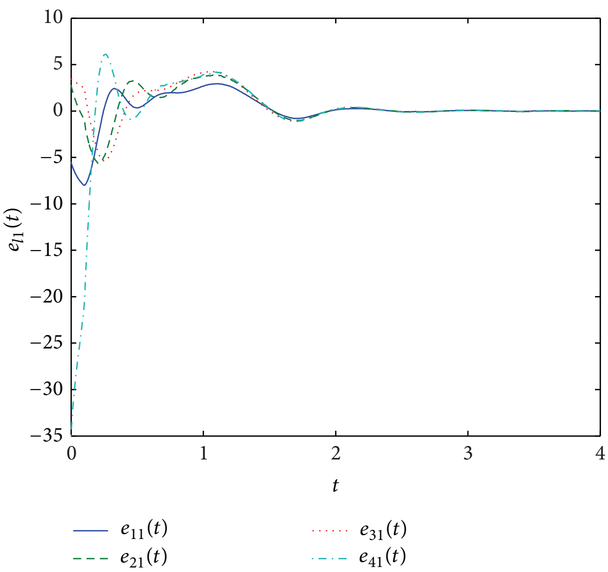

Figure 1: The error states of complex network e_{l1}.

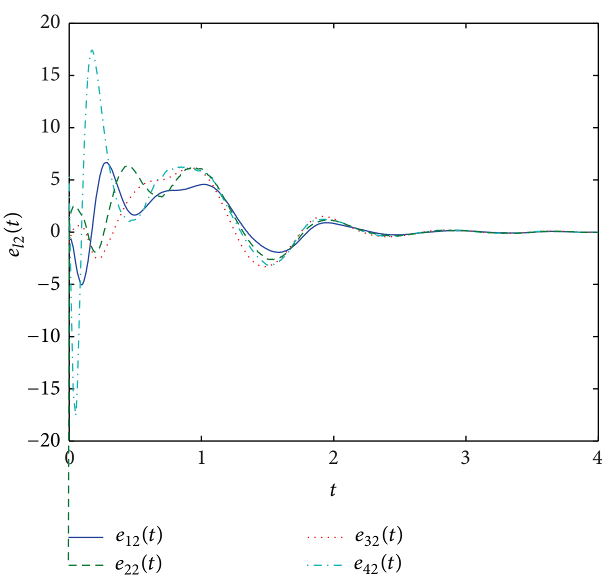

Figure 2: The error states of complex network e_{l2}.

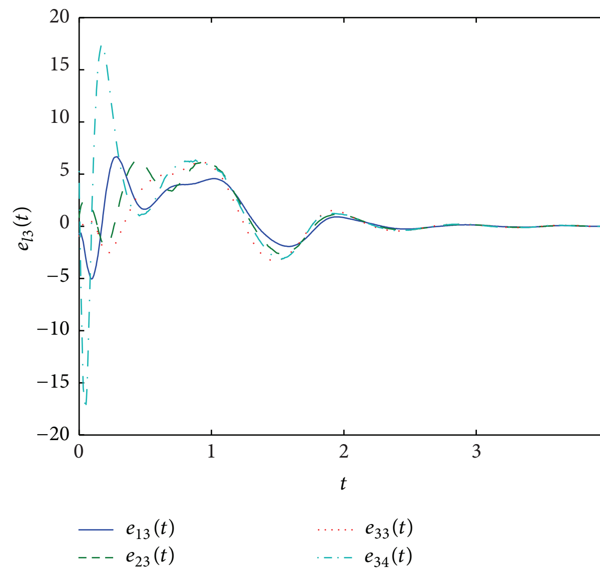

Figure 3: The error states of complex network e_{l3}.

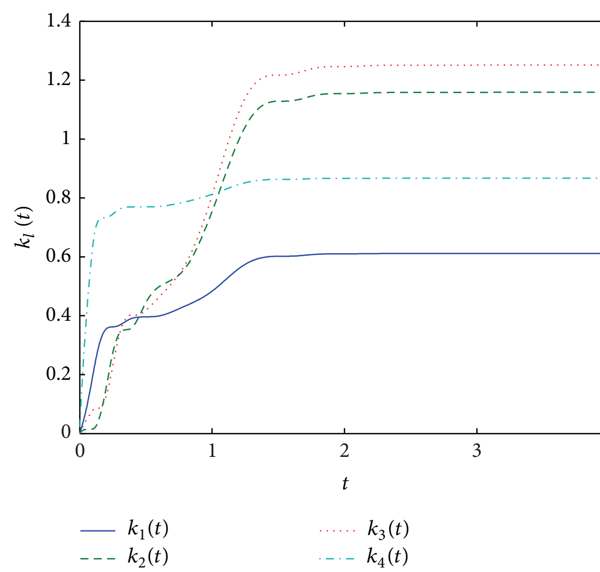

Figure 4: The feedback gain.

Among them, Figures 1–3 plot the error states of complex networks $e_{l1}(t)$, $e_{l2}(t)$, and $e_{l3}(t)$. Figure 4 depicts the adaptive feedback gain. From all these figures, one can find that the stochastic complex networks are adaptively asymptotically synchronized.

5. Conclusions

In this paper, we have investigated the adaptive synchronization problem for the stochastic complex networks with time-delay and Markovian switching. By combining the Lyapunov functional, stochastic analysis method, and M-matrix approach, some sufficient conditions have derived the above adaptive synchronization for the stochastic delayed complex networks. Through the adaptive control techniques, some suitable parameters update laws are obtained. Finally, an illustrative example has been used to demonstrate the effectiveness of the main results which are obtained in this paper.

Conflict of Interests

The authors declared that they have no conflict of interests to this work.

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