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Research Article

Crack Closure Effects on Fatigue Crack Propagation Rates: Application of a Proposed Theoretical Model

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Structural design taking into account fatigue damage requires a thorough knowledge of the behaviour of materials. In addition to the monotonic behaviour of the materials, it is also important to assess their cyclic response and fatigue crack propagation behaviour under constant and variable amplitude loading. Materials whenever subjected to fatigue cracking may exhibit mean stress effects as well as crack closure effects. In this paper, a theoretical model based on the same initial assumptions of the analytical models proposed by Hudak and Davidson and Ellyin is proposed to estimate the influence of the crack closure effects. This proposal based further on Walker's propagation law was applied to the P355NL1 steel using an inverse analysis (back-extrapolation) of experimental fatigue crack propagation results. Based on this proposed model it is possible to estimate the crack opening stress intensity factor, K_{op}, the relationship between $U = \Delta K_{eff}/\Delta K$ quantity and the stress intensity factor, the crack length, and the stress ratio. This allows the evaluation of the influence of the crack closure effects for different stress ratio levels, in the fatigue crack propagation rates. Finally, a good agreement is found between the proposed theoretical model and the analytical models presented in the literature.

1. Introduction

In the early seventies Elber [1, 2] introduced the crack closure and opening concepts. Initially Elber [1, 2] suggested that crack closure was associated with the effective stress intensity range, using U $(= \Delta K_{eff}/\Delta K)$ parameter which was assumed as a linear function of stress ratio, R, independent of K_{max} and ΔK. Later studies determined that the quantitative parameter U is dependent on K_{max}. After the proposal developed by Elber [1, 2], other proposals appeared, based on elastoplastic analytical concepts [3–7] or the near-threshold fatigue crack growth regimes [8–12]. All approaches are important contributions of the crack closure and opening effects, which occur in materials when subjected to constant amplitude loading, particularly in the near-threshold regime of the fatigue crack growth. All analytical techniques were supported by measurements of the crack-tip opening load. Other approaches have been implemented based on elastoplastic analysis, using finite element methods [13, 14].

This paper proposes a theoretical model based on Walker's propagation law [15] to estimate the quantitative parameter, U, important for the evaluation of the fatigue crack opening stress intensity factor, K_{op}, taking into account the stress ratio and crack closure and opening effects. The current study is applied to the P355NL1 steel using experimental fatigue crack propagation test results under constant amplitude loading [16]. The proposed model is applied using an inverse analysis of the experimental results. Various analytical models for crack closure are also compared in this study.

2. Overview of the Crack Closure and Opening Concepts and Existing Models

The crack closure and opening concept has been discussed in the scientific community since the first contributions by Elber [1, 2]. Elber suggested [1, 2] that crack closure and opening effects could be characterized in terms of the effective stress

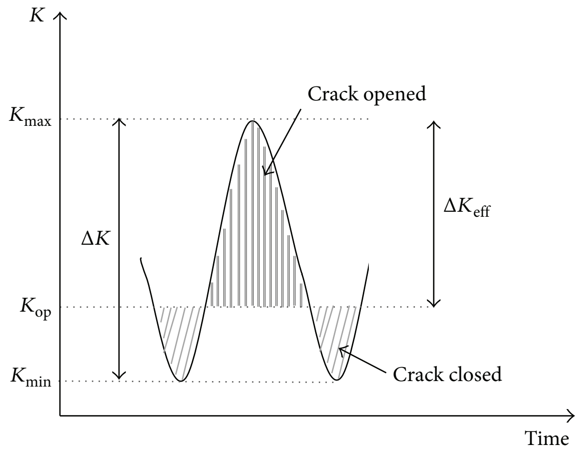

Figure 1: Definition of the effective and applied stress intensity factor ranges.

intensity factor range, which is normalized by the applied stress intensity factor range, resulting in the U ratio, with the following form:

$$U = \frac{\Delta K_{eff}}{\Delta K}, \tag{1}$$

where ΔK_{eff} is the effective stress intensity factor range and ΔK is the applied stress intensity factor range. Elber's formulation is supported by the following assumptions for the U parameter: (i) linear function of stress ratio (R) and (ii) being relatively independent of maximum stress intensity factor (K_{max}) or stress intensity factor range (ΔK). The effective stress intensity factor range is defined as

$$\Delta K_{eff} = K_{max} - K_{op}, \tag{2}$$

where K_{max} is the maximum stress intensity factor and K_{op} is the crack opening stress intensity factor. The applied stress intensity factor range, ΔK, is presented by

$$\Delta K = K_{max} - K_{min} \tag{3}$$

with K_{min} being the minimum stress intensity factor. Figure 1 illustrates the previous parameters definitions.

According to the approach of Elber [1, 2], the fatigue crack growth rate, da/dN, is a function of the effective stress intensity factor range, according to the following general expression:

$$\frac{da}{dN} = f\left(\Delta K_{eff}\right). \tag{4}$$

According to studies conducted by Elber for the 2024-T3 aluminum alloy, using stress ratios, R, between −0.1 and 0.7, U depends on R. The experimental results led to the following linear relationship:

$$U = 0.5 + 0.4R. \tag{5}$$

The quantitative parameter U of (5) can be substituted into (1) and using (2) results in the following relation for the K_{op}/K_{max} ratio:

$$\frac{K_{op}}{K_{max}} = 0.5 + 0.1R + 0.4R^2. \tag{6}$$

Subsequent studies using various fatigue loading variables clearly showed that U and K_{max} are related variables. Therefore, Elber [1, 2] approach appears to be inconsistent/incomplete.

An improved function of (5) was proposed by Schijve [9] with a more realistic behaviour to account for the crack closure and opening effects for $-1 \le R \le 1$:

$$U = 0.55 + 0.33R + 0.12R^2. \tag{7}$$

Therefore, it is possible to obtain a relation to the crack opening stress intensity factor as follows:

$$\frac{K_{op}}{K_{max}} = 0.45 + 0.22R + 0.21R^2 + 0.12R^3. \tag{8}$$

Another proposal was presented by ASTM [10, 11] together with a set of conditions:

$$\begin{gathered}\Delta K = K_{max} \quad \text{for } R < 0,\\ \Delta K = K_{max} - K_{min} \quad \text{for } R \ge 0,\\ U = 0.576 + 0.015R + 0.409R^2,\\ \frac{K_{op}}{K_{max}} = 0.424 + 0.561R - 0.394R^2 + 0.409R^3.\end{gathered} \tag{9}$$

Newman Jr. [3] proposed a general crack opening stress equation for constant amplitude loading, function of stress ratio, R, stress level, σ, and four nondimensional constants:

$$\begin{aligned}\frac{\sigma_{op}}{\sigma_{max}} &= A_0 + A_1R + A_2R^2 + A_3R^3 \quad \text{for } R \ge 0,\\ \frac{\sigma_{op}}{\sigma_{max}} &= A_0 + A_1R \quad \text{for } -1 \le R < 0,\end{aligned} \tag{10}$$

when $\sigma_{op} \ge \sigma_{max}$. The coefficients A_0, A_1, A_2, and A_3 are as follows:

$$\begin{aligned}A_0 &= \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)\left[\cos\left(\frac{\pi\sigma_{max}}{2\sigma_0}\right)\right]^{1/\alpha},\\ A_1 &= (0.415 - 0.071\alpha)\frac{\sigma_{max}}{\sigma_0},\\ A_2 &= 1 - A_0 - A_1 - A_3,\\ A_3 &= 2A_0 + A_1 - 1,\end{aligned} \tag{11}$$

where σ_0 is the flow stress (average between the uniaxial yield stress and uniaxial ultimate tensile strength of the material). For plane stress conditions, $\alpha = 1$, and for plane strain conditions, $\alpha = 3$. The normalized crack opening stress is obtained using the stress ratio for $\sigma_{max} = \sigma_0/3$.

This approach proposed by Newman Jr. [3] may be used to correlate fatigue crack growth rate data for other materials and thicknesses, under constant amplitude loading, once the proper constraint factor has been determined.

The crack closure effects may occur due to the surface roughness of the material in the presence of shear deformation at the crack tip [17, 18]. Microstructural studies [18] indicate that fracture surfaces demonstrate the possibility of crack propagation by the shear mechanism primarily in the near-threshold regime of ΔK [17]. In this fatigue regime, the crack closure effects have the greatest influence. Consequently, Hudak and Davidson [8] proposed a model defined by the following expression:

$$U = 1 - \frac{K_o}{K_{\max}} = 1 - \frac{K_o(1-R)}{\Delta K}, \quad (12)$$

where K_o is a constant related to the pure Mode I fatigue crack growth threshold.

The experimental results from fatigue crack propagation tests can be represented by the following generalized relations (see Figure 2):

$$U = \gamma\left(1 - \frac{K_o}{K_{\max}}\right), \quad K_{\max} \le K_L, \quad (13)$$

$$U = 1, \quad K_{\max} \ge K_L, \quad (14)$$

where K_L is the limiting $K_{\max}$ value above which no detectable closure occurs and γ is a parameter that can be obtained using the following expression:

$$\gamma = \left(1 - \frac{K_o}{K_L}\right). \quad (15)$$

The K_o and K_L constants are not material properties in the strict sense since they depend on measurements location and measurements sensitivity, resulting from the crack propagation tests aiming at evaluating the crack closure/opening effects. Note that in the limiting condition of K_o/K_L approaching zero (13) reduces to (12) thus describing the local measurements.

Ellyin [12] proposed an approach to define the effective stress intensity range, ΔK_{eff}, taking into account the stress ratio, R, and the threshold value of the stress intensity factor range, ΔK_{th}, with constant amplitude loading (see Figure 3). This approach is a modified version of proposal by Hudak and Davidson [8]. Generally, the crack opening stress intensity factor range, ΔK_{op}, is smaller than the threshold stress intensity factor range, ΔK_{th}; that is, $\Delta K_{\text{op}} < \Delta K_{\text{th}}$. Besides, the crack opening or closure stress intensities factors are not the same; that is, $\Delta K_{\text{cl}} < \Delta K_{\text{op}} < \Delta K_{\text{th}}$. Ellyin [12] proposed the following approach to obtain ΔK_{eff} (see Figure 3):

$$\Delta K_{\text{eff},0} = \left(\Delta K^2 - \Delta K_{\text{th}}^2\right)^{1/2} \approx \Delta K\left[1 - \frac{1}{2}\left(\frac{\Delta K_{\text{th}}}{\Delta K}\right)^2\right] > \Delta K - \Delta K_{\text{th}} \quad \text{for } R \approx 0,$$

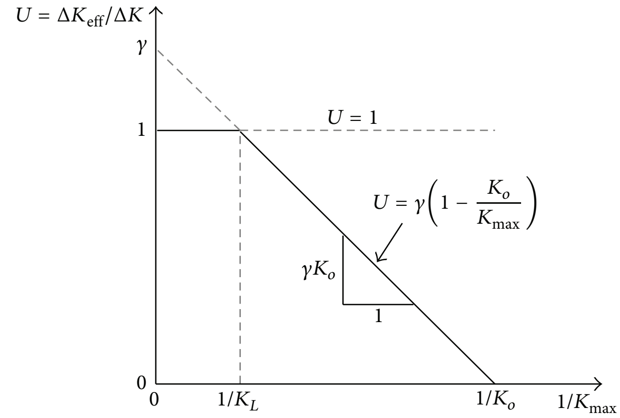

Figure 2: Functional relationship between U and $K_{\max}$, Hudak and Davidson [8].

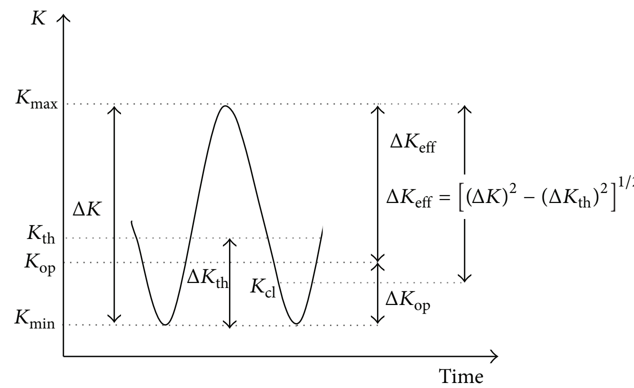

Figure 3: Definition of the effective stress intensity range according with Ellyin [12].

$$\Delta K_{\text{eff}} = \frac{\Delta K_{\text{eff},0}}{\left[1 - \left(\sigma_m/\sigma'_f\right)\right]} = \frac{\Delta K_{\text{eff},0}}{\left[1 - \left((1+R)\,\sigma_{\max}/2\sigma'_f\right)\right]} \quad \text{for } R \ne 0, \quad (16)$$

where σ_m is the mean stress, $\sigma_{\max}$ is the maximum stress, and σ'_f is the fatigue strength coefficient.

The crack closure and opening effects can be estimated using an elastoplastic analysis based on analytical [3, 19] or numerical [13, 14] approaches. Vormwald [4, 5] and Savaidis et al. [6, 7] estimated the crack closure and opening effects proposing the use of an analytical elastoplastic analysis developed by Neuber [19] and the modified crack closure and opening model suggested by Newman Jr. [3]. The numerical elastoplastic analysis, using the finite element method, is proposed by McClung and Sehitoglu [13] and Nakagaki and Atluri [14]. Based on finite element analysis performed by McClung and Sehitoglu [13], it has been shown that the results obtained for σ_{op} may deviate from the results obtained using the crack closure and opening model by Newman Jr. [3].

Nakagaki and Atluri [14] proposed an elastoplastic finite element procedure to account for arbitrary strain hardening material behaviour. In order to determine the crack closure and opening stresses, Nakagaki and Atluri [14] proposed the following procedure: (1) calculating the displacements of the nodes in the crack axis before the closure (and after opening); (2) determining by extrapolating to zero the restraining force at the respective nodal displacement in the corresponding node just after crack closure (and before the opening) by extrapolation against the load level. In all the cases studied, these two sets of extrapolated values for σ_{op} and σ_{cl} were found to correlate excellently.

3. Proposed Theoretical Model

In this paper, a theoretical model to obtain the effective stress intensity range, ΔK_{eff}, that takes into account the effects of the mean stress and the crack closure and opening effects is proposed. The starting point for the proposed theoretical model is the initial assumptions proposed by Elber [1, 2] and defined in (1) and (4). The basic fatigue crack propagation law in regime II was initially proposed by Paris and Erdogan [20], which has the following form:

$$\frac{da}{dN} = C(\Delta K)^m, \tag{17}$$

where C and m are material constants. However, this model does not include the other propagation regimes and does not take into account the effects of mean stress. Many other fatigue propagation models have been proposed in literature, with wider range of application and complexity, requiring a significant number of constants [21]. Nevertheless, the simple modification of the Paris relation in order to take into account the stress ratio effects is assumed, as proposed by Dowling [15]:

$$\frac{da}{dN} = C_w \left(\frac{\Delta K}{(1-R)^{1-\gamma}}\right)^{m_w} = C_w \left(\overline{\Delta K}\right)^{m_w}, \tag{18}$$

where C_w, m_w, and γ are constants. Besides, an extension of the Paris relation is adopted to account for crack propagation regime I, having the following configuration:

$$\frac{da}{dN} = C_w \left(\overline{\Delta K} - \Delta K_{th}\right)^{m_w}. \tag{19}$$

Based on the foregoing assumptions, the effective stress intensity factor range is given by the following expression:

$$\begin{aligned}\Delta K_{eff} &= \left(\overline{\Delta K} - \Delta K_{th}\right) = \left(\frac{\Delta K}{(1-R)^{1-\gamma}} - \Delta K_{th}\right)\\ &= \Delta K \left(\frac{1}{(1-R)^{1-\gamma}} - \frac{\Delta K_{th}}{\Delta K}\right).\end{aligned} \tag{20}$$

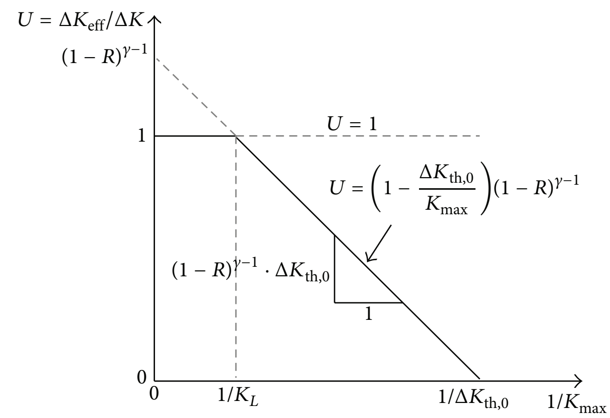

Figure 4: Functional relationship between U and K_{max} for proposed theoretical model.

Thus, the expression that allows obtaining the quantitative parameter, U, is given by the following formula:

$$\begin{aligned}U &= \frac{\Delta K_{eff}}{\Delta K} = \left(\frac{1}{(1-R)^{1-\gamma}} - \frac{\Delta K_{th}}{\Delta K}\right)\\ &= (1-R)^{\gamma-1} - \frac{\Delta K_{th}}{\Delta K}.\end{aligned} \tag{21}$$

If the material parameter, γ, is equal to 1, this means that the material is not influenced by the stress ratio. Thus, (21) is simplified and assumes the same form as (12) proposed by Hudak and Davidson [8] and is similar to the one proposed by Ellyin [12].

Equation (21) can be presented in another form:

$$U = (1-R)^{\gamma-1} - \frac{\Delta K_{th}}{(1-R)K_{max}}. \tag{22}$$

The threshold value of the stress intensity factor range can be described by a general equation, using the following form:

$$\Delta K_{th} = \Delta K_{th,0} \cdot f(R), \tag{23}$$

where ΔK_{th} is the threshold value of stress intensity range for a given stress ratio and $\Delta K_{th,0}$ is the threshold value of stress intensity range, for $R = 0$. Klesnil and Lukáš [22] proposed a relation between the threshold stress intensity factor range, ΔK_{th}, and the stress ratio, R, which is well known:

$$\Delta K_{th} = \Delta K_{th,0} \cdot (1-R)^{\gamma}. \tag{24}$$

The validity of (24) is for $R \geq 0$. Replacing this result in (22) results in

$$U = (1-R)^{\gamma-1} - \frac{\Delta K_{th,0} \cdot (1-R)^{\gamma}}{(1-R)K_{max}}. \tag{25}$$

Finally, it is possible to display (22) in a more simplified form (see Figure 4):

$$\begin{aligned}U &= \left(1 - \frac{\Delta K_{th,0}}{K_{max}}\right)(1-R)^{\gamma-1} \quad \text{for } K_{max} \leq K_L,\\ U &= 1 \quad \text{for } K_{max} \geq K_L,\end{aligned} \tag{26}$$

Table 1: Cyclic elastoplastic and strain-life properties of the P355NL1 steel, $R_\varepsilon = -1$ and $R_\varepsilon = 0$.

Material	σ'_f MPa	b —	ε'_f —	c —	K' MPa	n' —
P355NL1	1005.50	−0.1033	0.3678	−0.5475	948.35	0.1533

where K_L is the limiting K_{max}. For stress ratio, R, equal to 0 and material parameter, γ, equal to 1, (22) reduces to

$$U = 1 - \frac{\Delta K_{th,0}}{K_{max}}, \quad \text{for } R = 0,\ \gamma = 1. \qquad (27)$$

In summary, a theoretical model based on Walker's propagation law [15] was proposed, which relates the quantitative parameter U and ΔK (or K_{max}) and the stress ratio. The proposed model is based on the same initial assumptions of the analytical models proposed by Hudak and Davidson [8] and Ellyin [12].

4. Basic Fatigue Data of the Investigated P355NL1 Steel

The P355NL1 steel is a pressure vessel steel and is a normalized fine grain low alloy carbon steel. Its fatigue behaviour has been investigated [16, 23, 24]. A steel plate 5 mm thick was used to prepare specimens for experimental testing. This section presents the strain-life fatigue data and the fatigue crack propagation data obtained for the P355NL1 steel [16].

4.1. Strain-Life Behaviour. The strain-life behaviour of the P355NL1 steel was evaluated through fatigue tests of smooth specimens, carried out under strain controlled conditions, according to the ASTM E606 standard [25]. The Ramberg and Osgood [26] relation, as shown in the following equation, was fitted to the stabilized cyclic stress-strain data:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon^E}{2} + \frac{\Delta\varepsilon^P}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}, \qquad (28)$$

where K' and n' are the cyclic strain hardening coefficient and exponent, respectively; $\Delta\varepsilon^P$ is plastic strain range; and $\Delta\sigma$ is the stress range.

Low-cycle fatigue test results are very often represented using the relation between the strain amplitude and the number of reversals to failure, $2N_f$, usually assumed to correspond to the initiation of a macroscopic crack. Morrow [27] suggested a general equation, valid for low- and high-cycle fatigue regimes:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon^E}{2} + \frac{\Delta\varepsilon^P}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c, \qquad (29)$$

where ε'_f and c are, respectively, the fatigue ductility coefficient and fatigue ductility exponent; σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, and E is the Young modulus.

Table 2: Elastic and tensile properties of the P355NL1.

Material	E GPa	ν —	f_u MPa	f_y MPa
P355NL1	205.20	0.275	568.11	418.06

Alternatively to the Morrow relation, the Smith-Watson-Topper fatigue damage parameter [28] can be used, which shows the following form:

$$\sigma_{max} \cdot \frac{\Delta\varepsilon}{2} = \text{SWT} = \frac{\left(\sigma'_f\right)^2}{E}\left(2N_f\right)^{2b} + \sigma'_f \cdot \varepsilon'_f \cdot \left(2N_f\right)^{b+c}, \qquad (30)$$

where σ_{max} is the maximum stress of the cycle and SWT is the damage parameter. Both Morrow and Smith-Watson-Topper models are deterministic models and are used to represent the average fatigue behaviour of the bridge materials based on the available experimental data.

Two series of specimens were tested under distinct strain ratios, $R_\varepsilon = 0$ (19 specimens) and −1 (24 specimens). Figure 5 shows a plot of the experimental fatigue data in the form of strain-life and SWT-life curves, for the two strain ratios. The cyclic Ramberg-Osgood and Morrow strain-life parameters of the P355NL1 steel are summarized in Table 1, for the conjunction of both strain ratios [16, 23, 24]. The cyclic curve is shown in Figure 5(a), including the influence of the experimental results from both test series. This research adopted the values obtained by combining the results of the two test series together since the strain ratio effects are considered negligible (tests performed under strain controlled conditions). This could be explained by the cyclic mean stress relaxation phenomenon and also by a lower sensitivity of the material to the mean stress. In addition, Table 2 presents the elastic and monotonic tensile properties of this pressure vessel steel under investigation. The elastic and monotonic tensile properties are represented, respectively, by the Young modulus, E, Poisson ratio, ν, the ultimate tensile strength, f_u, and upper yield stress, f_y.

4.2. Fatigue Crack Propagation Rates. Fatigue crack growth rates of the P355NL1 pressure vessel steel were evaluated, for several stress ratios, using compact tension (CT) specimens, following the recommendations of the ASTM E647 standard [10]. The CT specimens of P355NL1 steel were manufactured with a width $W = 40$ mm and a thickness $B = 4.5$ mm [16]. All tests were performed in air, at room temperature, under a sinusoidal waveform at a maximum frequency of 20 Hz. The crack growth was measured on both faces of the specimens

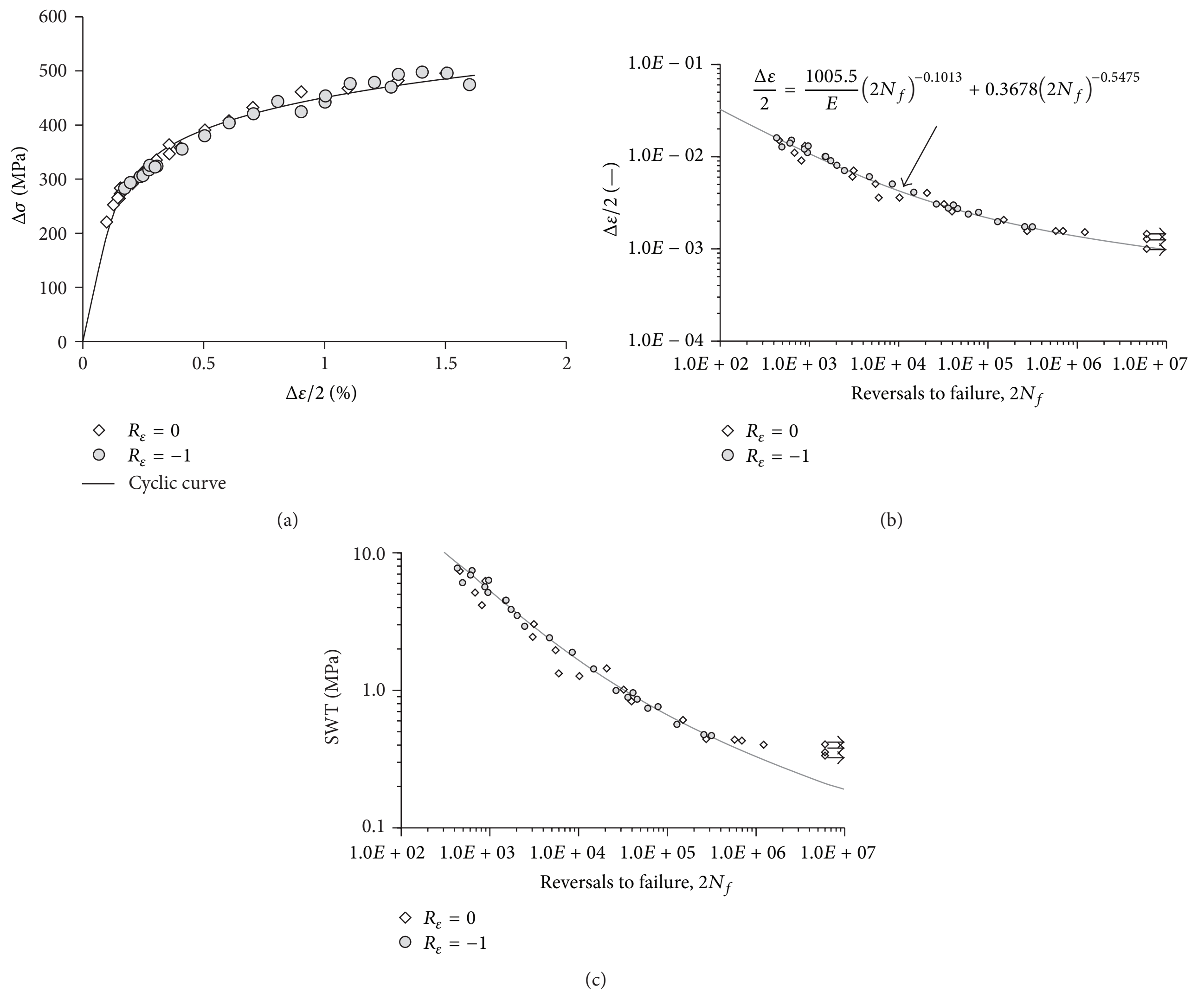

Figure 5: Fatigue data obtained for the P355NL1 steel, $R_\varepsilon = -1$ and $R_\varepsilon = 0$: (a) cyclic curve; (b) strain-life data; (c) SWT-life data.

by visual inspection, using two travelling microscopes with an accuracy of 0.001 mm.

Figures 6(a) and 6(b) represent the experimental data and the Paris law correlations for each tested stress ratio of the P355NL1 steel, $R = 0.0$, $R = 0.5$, and $R = 0.7$. The material constants of the Paris crack propagation relation are presented in Table 3. The crack propagation rates are only slightly influenced by the stress ratio. Higher stress ratios result in higher crack growth rates. The lines representing the Paris law, for $R = 0.0$ and $R = 0.5$, are approximately parallel to each other.

On the other hand, the line representing the Paris law for $R = 0.7$ converges to the other lines as the stress intensity factor range increases. In general, the stress ratio effects are more noticeable for lower ranges of the stress intensity factors. For higher stress intensity factor ranges, the stress ratio effect tends to vanish.

The stress ratio effects on fatigue crack growth rates may be attributed to the crack closure. Crack closure increases with the reduction in the stress ratio leading to lower fatigue crack growth rates. For higher stress ratios, such as $R = 0.7$, the crack closure is very likely negligible, which means that applied stress intensity factor range is fully effective. Also, the fatigue crack propagation lines for each stress ratio are not fully parallel to each other which means that crack closure also depends on the stress intensity factor range level.

Table 3: Material constants concerning the Paris crack propagation relation.

R	C^*	m	ΔK_{max}
—	—	—	$N{\cdot}mm^{-1.5}$
0.01	$7.1945E-15$	3.4993	1498
0.50	$6.2806E-15$	3.5548	1013
0.70	$2.0370E-13$	3.0031	687

* da/dN in mm/cycle and ΔK in $N{\cdot}mm^{-1.5}$.

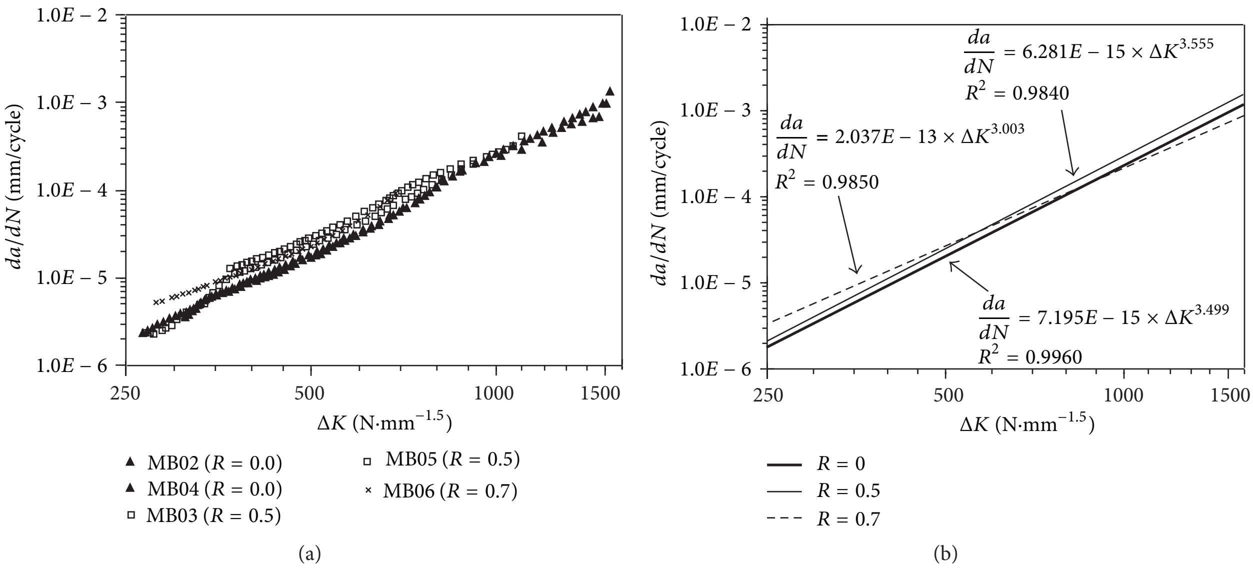

FIGURE 6: Fatigue crack propagation data obtained for the P355NL1 steel: (a) experimental data; (b) Paris correlations for each stress ratio, R.

TABLE 4: Walker constants for the fatigue crack propagation relation of the P355NL1 steel.

C^*	m	γ
7.1945E − 15	3.4993	0.92

$^*da/dN$ in mm/cycle and ΔK in N·mm$^{-1.5}$.

5. Application and Discussion of a Proposed Theoretical Model

The fatigue crack growth data presented in Figure 6 was correlated using the Walker relation (see (18)). The material constants of the Walker crack propagation relation, for the P355NL1 steel, are presented in Table 4. In addition, the Walker modified relation (see (19)) was used to model the fatigue crack propagation on region I and results are presented in Figure 7. Equation (19) requires the definition of the propagation threshold, which was estimated using the following linear relation:

$$\Delta K_{th} = 152 - 90.252 \cdot R, \quad (31)$$

where ΔK_{th} is the propagation threshold defined in N·mm$^{-1.5}$, R is the stress ratio, and constant value 152 corresponds to the propagation threshold for $R = 0$ (in N·mm$^{-1.5}$); the other constant (90.252) also shows the same units of the propagation threshold. The propagation threshold for $R = 0$ is very consistent with the threshold value for the mild steel with yield stress of 366 MPa. The crack propagation threshold ΔK_{th} is itself influenced by the stress ratio [29]. In [29], for a similar material to the P355NL1 steel, the relationship between ΔK_{th} and R is given, showing a linear relation between the crack propagation threshold and the stress ratio. Despite using two distinct crack propagation laws, they gave a continuous representation of the crack propagation data at the crack propagation I-II regimes transition. Figure 7 illustrates the fatigue propagation laws adopted in this investigation, for each tested stress ratio.

The proposed theoretical model is applied using an inverse analysis of the experimental fatigue crack propagation rates available for the P355NL1 steel. Using the inverse analysis of the proposed theoretical model, it is possible to obtain the relationship between the effective stress ratio, R_{eff}, and the applied stress intensity factor range, ΔK_{app}. R_{eff} accounts for minimum stress intensity factors higher than crack closure stress intensity factor. In this case, the effective stress intensity factor results from the relationship between the crack opening stress intensity factor, K_{op}, and the maximum stress intensity factor, K_{max}. Figure 8 shows that for stress ratios above 0.5 the crack closure and opening effects are not significant. For the stress ratio equal to 0 a significant influence on the effective stress ratio of the applied stress intensity factor range is observed. Such influence is more significant for the initial propagation phase of region II, decreasing significantly for higher applied stress intensity ranges. Thus, it is concluded that the crack closure and opening effects are more significant for the initial crack propagation phase, in region II, for higher levels of the stress intensity factor ranges along the crack length. Figure 9 shows the results obtained for the relationship between R_{eff} and R, using the inverse analysis of the proposed theoretical model. This figure helps to understand the influence of the crack closure and opening effects, as a function of stress ratios. The effective stress ratio, R_{eff}, shown in Figure 9, is the average of the values obtained as a function of ΔK, according to what is evidenced in Figure 8.

The proposed relationship between the quantitative parameter, U, and the maximum stress intensity, K_{max}, is also considered. Figure 10 shows the relationship between U and K_{max} for the studied stress ratios, 0, 0.5, and 0.7. The relationship between U and $(\Delta K_{th,0}/K_{max})$ for the studied stress ratios is shown in Figure 11. The proposed theoretical

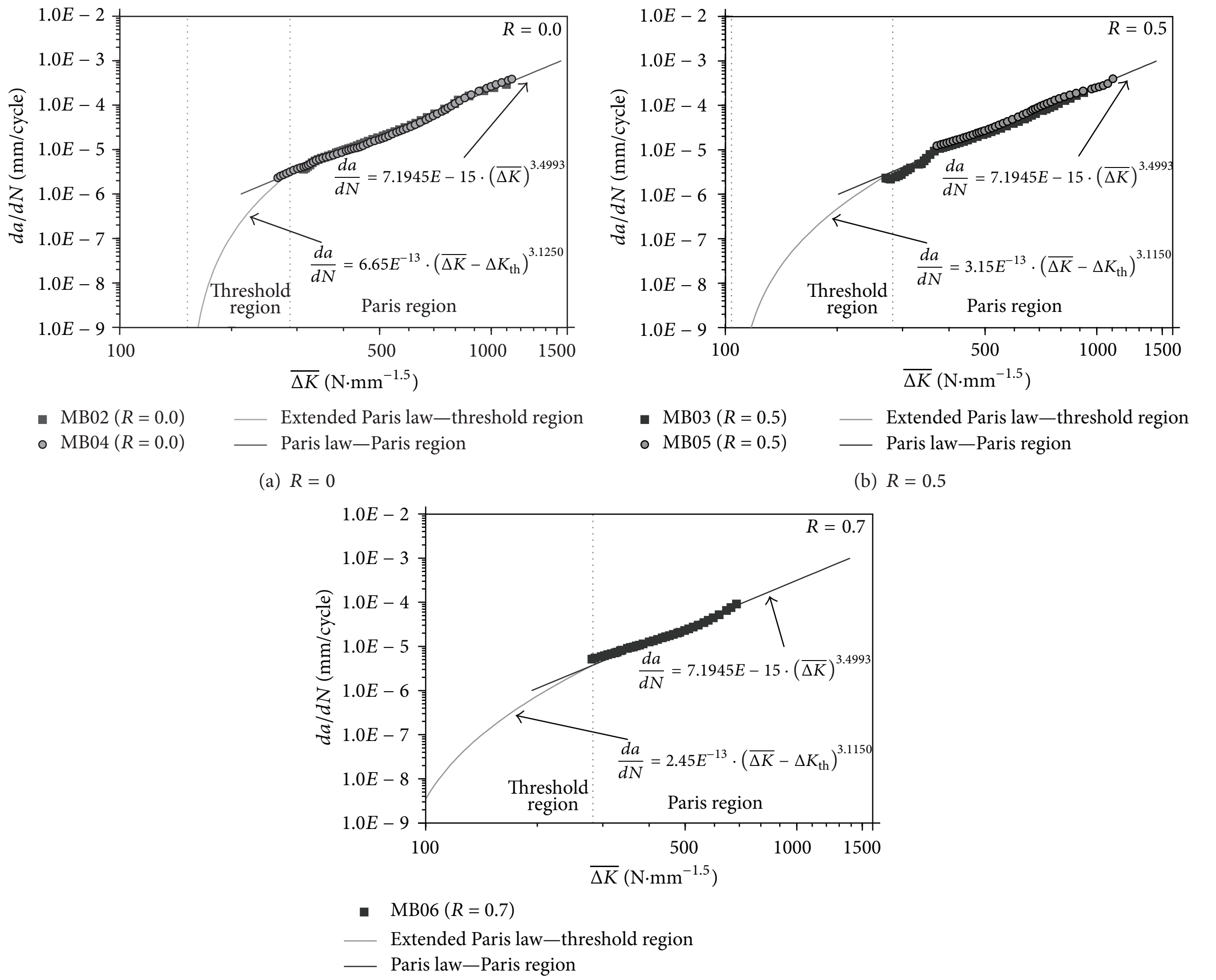

Figure 7: Fatigue crack growth data correlations for regimes I and II, using two relations.

model is able to describe the influence of the crack closure and opening effects on different stress ratios.

Figure 12 shows the results for the relationship between R_{eff} and R using various analytical models available in the literature. These model results are presented together with the results of the proposed theoretical model. Based on the analysis of Figure 12, it can be noted that the results obtained by applying the proposed theoretical model are between the results obtained using the Hudak and Davidson model (based on the same assumptions) and Savaidis model (with assumptions supported on plastic deformation of the material). The curves of Hudak and Davidson and Savaidis demonstrate being below the other models. Other models are based on direct polynomial relationships of orders 2 and 3, except Newman model that is based on a polynomial relationship of order 3 and uses properties of the plastic deformation of the material.

Figure 13 shows the fatigue crack propagation rates as a function of effective intensity range using the proposed method. The results obtained, using this theoretical model, are similar to results presented in the literature [3].

6. Conclusions

An interpretation of the crack closure and opening effects is possible, without resorting to an excessive computation time resulting from complex approaches.

Fatigue crack propagation tests allowed the estimation of the relationship between the effective stress ratios, R_{eff}, and the applied stress ratios, R, without the need for extensive experimental program.

The evaluation of the influence of the crack closure effects was possible using the inverse identification procedure applied with the proposed theoretical model. In this study, it was observed that, for high applied stress ratios, R, $0.5 \le R \le 0.7$, the crack closure effect has little significance. But for applied stress ratios, R, $0 \le R < 0.5$, there is a significant influence of the crack closure and opening effects.

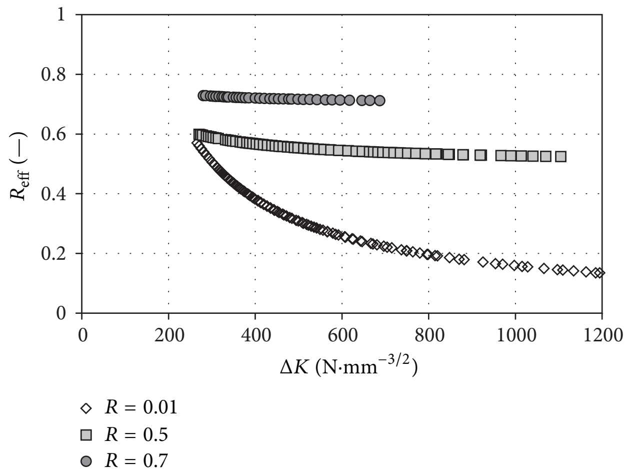

FIGURE 8: Relationship between effective stress ratio, R_{eff}, and applied stress intensity range, ΔK (P355NL1 steel).

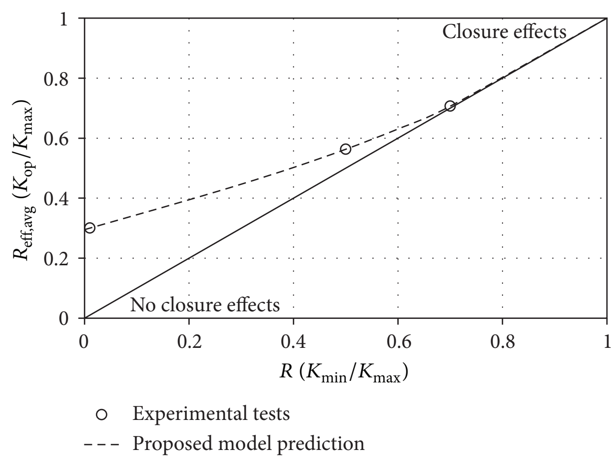

FIGURE 9: Effective stress ratio, R_{eff}, as a function of stress ratio (P355NL1 steel).

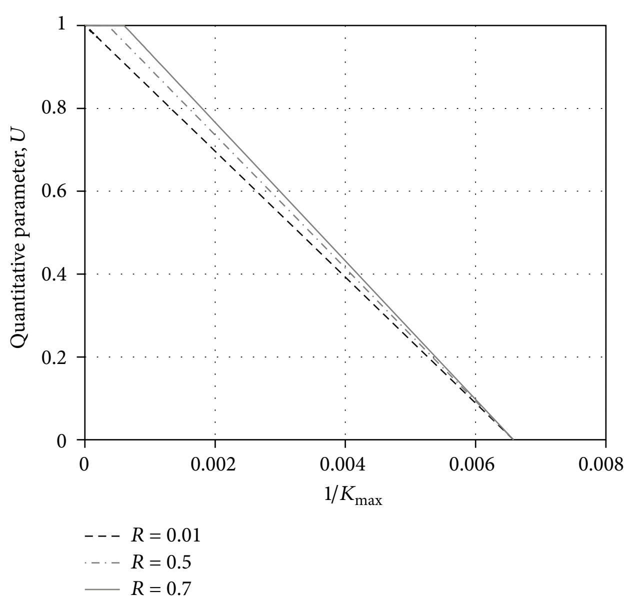

FIGURE 10: Influence of the stress ratio on U versus $1/K_{max}$ relationship (P355NL1 steel).

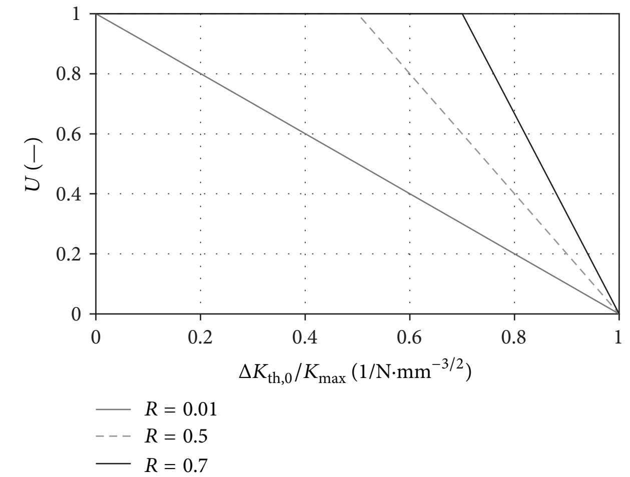

FIGURE 11: Influence of the stress ratio on U versus $\Delta K_{th,0}/K_{max}$ relationship (P355NL1 steel).

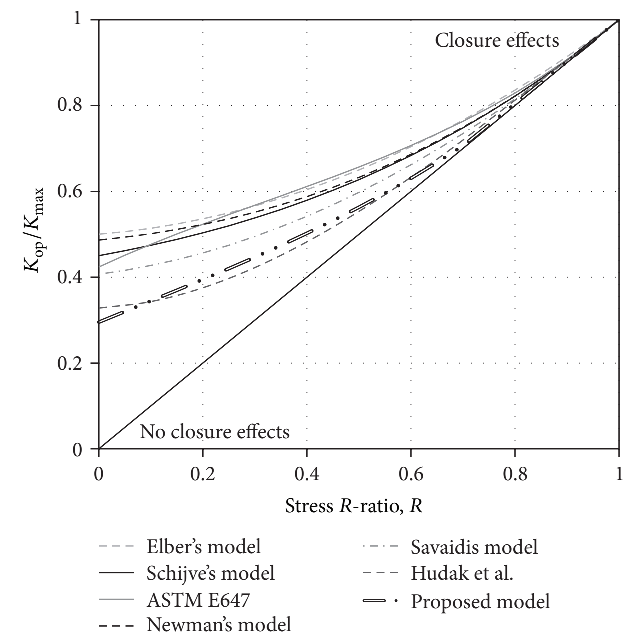

FIGURE 12: Relationship between $R_{eff} = K_{op}/K_{max}$ and stress ratio for several models (P355NL1 steel).

Note that, for applied stress ratios within $0 \leq R < 0.5$, the influence of the crack closure and opening effects decreases with increasing of the crack length or stress intensity factor range.

The results of the proposed theoretical model proved to be consistent when compared with other models proposed in the literature [1–12, 19]. It was possible to estimate the constants m and C of the modified Paris law (da/dN versus ΔK_{eff}). These parameters are important for fatigue life prediction studies of structural details based on fracture mechanics.

Other fatigue programs under constant and variable amplitude loading with other materials should be considered,

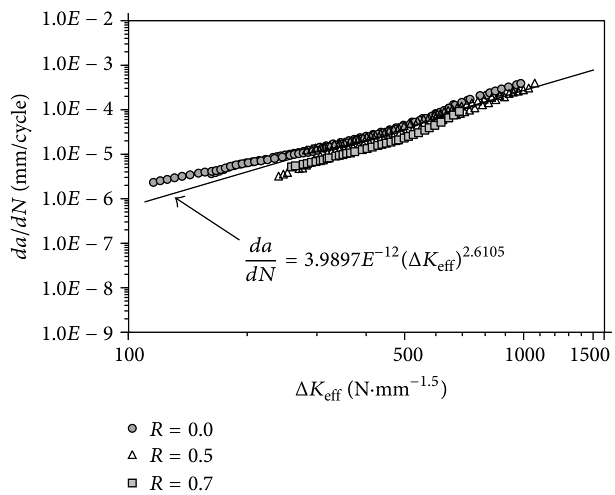

Figure 13: Correlation of fatigue crack growth rates using the crack closure analysis (P355NL1 steel).

through the use of full field optical techniques to experimentally estimate K_{op}, in order to confirm the results obtained in the inverse analysis of the proposed theoretical model.

Competing Interests

The authors declare that they have no competing interests regarding the publication of this paper.

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