Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2014, Article ID 587323, 10 pages
http://dx.doi.org/10.1155/2014/587323

Research Article

H_∞ ILC Design for Discrete Linear Systems with Packet Dropouts and Iteration-Varying Disturbances

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Received 22 February 2014; Accepted 22 May 2014; Published 9 June 2014

Academic Editor: Wenwu Yu

An H_∞ iterative learning controller is designed for networked systems with intermittent measurements and iteration-varying disturbances. By modeling the measurement dropout as a stochastic variable satisfying the Bernoulli random binary distribution, the design can be transformed into H_∞ control of a 2D stochastic system described by Roesser model. A sufficient condition for mean-square asymptotic stability and H_∞ disturbance attenuation performance for such 2D stochastic system is established by means of linear matrix inequality (LMI) technique, and formulas can be given for the control law design simultaneously. A numerical example is given to illustrate the effectiveness of the proposed results.

1. Introduction

Iterative learning control (ILC) has been extensively studied with significant progress in theory and widely applied in many fields [1–3]. Most of the reported results are based on an implicit assumption that the communication between the physical plant and controller is perfect; that is, the signals transmitted from the plant will arrive at the controller simultaneously and perfectly. However, in many practical situations, the systems may have intermittent measurements, especially in networked systems, which are becoming more and more popular for the reason that they have several advantages over traditional systems, such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability [4–6]. If network is introduced to ILC design, the data packet dropout phenomenon, which appears in a typical network environment, will naturally induce intermittent measurements from the plant to the controller.

There have already been a few results in this issue. In [7–9], some stability conditions for networked-based ILC systems are given. Key conclusions of these works are that the intermittent ILC systems can guarantee convergence even though there may be significant data dropout. In [10, 11], an optimal ILC controller is designed for linear intermittent systems. The proposed ILC schemes can compensate the packet dropout effectively in the iteration domain. In [12], an averaging ILC algorithm is proposed to overcome the random data dropout, and it is shown that such an ILC algorithm can perform well and achieve asymptotic convergence in ensemble average along the iteration axis.

H_∞ optimization is a powerful tool that can be used to design a robust controller or filter [13–15], which has been proved to be one of the most important strategies for disturbance attenuation. In [16], an algebraic H_∞ approach is introduced to design higher-order ILC for the plants that are subject to model uncertainties and iteration-varying disturbance. In [17], an H_∞ ILC design approach is proposed for linear systems with iteration-varying disturbances. In [18], an H_∞ ILC design is proposed for linear systems with intermittent measurement. A sufficient condition guaranteeing both exponentially mean-square stability of such ILC process and the desired H_∞ performance in the iteration domain is presented. However, these designs are all based on lifted system representation. It does not address the computational complexity of the lifted ILC design method that might hamper their real-life application [19]. Alternatively, H_∞ ILC design based on 2D system theory is an effective approach for linear systems. Recently, several H_∞ ILC methods have been proposed to cope with parameter uncertainties in ILC systems based on the results of H_∞ control for 2D system or repetitive system [20–25]. However, H_∞ ILC design based on 2D system and linear repetitive process are only considered for systems without intermittent measurements. To the best of our knowledge, the problem of intermittent ILC design has not been investigated in the framework of 2D system or linear repetitive process, which motivates the present study.

In this paper, the 2D design approach is developed to treat the problem of H_∞ ILC design with intermittent measurements and iteration-varying disturbance. For the ILC system to be stochastic instead of a deterministic one by considering intermittent measurement, a 2D stochastic Roesser system is established to describe the entire dynamics. To analyze the tracking performance of the 2D stochastic system, the definition of stochastic mean-square asymptotic stability is introduced. In this case, a sufficient condition can be established by means of LMI technique, and formulas can be given for the control law design simultaneously. Numerical example is also proposed to illustrate the effectiveness of theoretical results.

This paper is organized as follows. In Section 2, the mathematical description and design objectives of networked-based ILC system are presented, together with its transformation into an equivalent 2D stochastic Roesser system. In Section 3, a mean-square asymptotic stability condition for such 2D stochastic systems is derived, and an H_∞ ILC design approach can be given by means of LMI technique. The effectiveness of the proposed method is illustrated by a numerical example in Section 4. Finally, the conclusions are given in Section 5.

2. Problem Formulation

Consider the following linear discrete time system:

$$\begin{aligned} x(t+1,k) &= Ax(t,k) + Bu(t,k) + B_1 w(t,k) \\ y(t,k) &= Cx(t,k), \end{aligned} \tag{1}$$

where the subscript k denotes iteration and t denotes discrete time. $x(t,k)$, $u(t,k)$, $y(t,k)$, and $w(t,k)$ are state, input, output variables, and iteration-varying disturbances. A, B, C, B_1 are the system matrices with appropriate dimension. $x(0,k) = x_{0k}$ stands for the initial condition of the process in the kth cycle. The system is operated repeatedly in the iteration domain with a desired output $y_d(t)$, $t \in [0,T]$.

In this paper, the ILC law is given as

$$u(t,k+1) = u(t,k) + Ke(t+1,k), \tag{2}$$

where $e(t,k) = y_d(t) - y(t,k)$ is the tracking error and K is gain matrix to be designed.

Assume the ILC scheme (2) is implemented via a networked control system, where the data $e(t+1,k)$ is transferred from the remote plant to the ILC controller. In this process, the data $e(t+1,k)$ may be missed due to the network transmission failure. In this case, ILC law (2) can be described as

$$u(t,k+1) = u(t,k) + \alpha Ke(t+1,k), \tag{3}$$

where stochastic parameter α is a random Bernoulli variable taking the values of 0 and 1 with

$$\begin{aligned} \text{Prob}\{\alpha = 1\} &= E\{\alpha\} = \overline{\alpha}, \\ E\left\{\alpha^2\right\} &= \overline{\alpha}(1-\overline{\alpha}), \end{aligned} \tag{4}$$

in which $\overline{\alpha}$ satisfying $0 \le \overline{\alpha} \le 1$ is a known constant.

The design objective of this paper can be described as follows. For an initial condition x_{0k} and packet dropout satisfying (4), design an ILC law (3) such that the ILC system is stable, and the influence of the iteration-varying disturbances should be as small as possible.

The ILC systems (1) and (3) are essentially a 2D system with evolution along two independent axes: time t and iteration k. We can use the 2D analysis approach to ILC to derive an expression for the tracking error and the state error. Using (1) and (3), we can obtain

$$\begin{aligned} & e(t,k+1) - e(t,k) \\ &\quad = y(t,k) - y(t,k+1) \\ &\quad = CAx(t-1,k) + CBu(t-1,k) + CB_1 w(t-1,k) \\ &\qquad - CAx(t-1,k+1) - CBu(t-1,k+1) \\ &\qquad - CB_1 w(t-1,k+1) \\ &\quad = -CA\eta(t,k) - \alpha CBKe(t,k) - CB_1\widetilde{w}(t,k), \end{aligned} \tag{5}$$

where $\eta(t,k) = x(t-1,k+1) - x(t-1,k)$, $\widetilde{w}(t,k) = w(t-1,k+1) - w(t-1,k)$.

Next, from (1) and (3), the following can also be obtained:

$$\begin{aligned} & \eta(t+1,k) \\ &= x(t,k+1) - x(t,k) \\ &= Ax(t-1,k+1) + Bu(t-1,k+1) + B_1 w(t-1,k+1) \\ &\quad - Ax(t-1,k) - Bu(t-1,k) - B_1 w(t-1,k) \\ &= A\eta(t,k) + \alpha BKe(t,k) + B_1\widetilde{w}(t,k). \end{aligned} \tag{6}$$

Equations (5) and (6) can be rewritten as follows:

$$\begin{aligned} \begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} &= \begin{bmatrix} A & BK\alpha \\ -CA & I - CBK\alpha \end{bmatrix} \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix} \\ &\quad + \begin{bmatrix} B_1 \\ -CB_1 \end{bmatrix} \widetilde{w}(t,k). \end{aligned} \tag{7}$$

Denoting $\eta(t,k) = x^h(t,k)$, $e(t,k) = x^v(t,k)$; that is,

$$\begin{aligned} \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1) \end{bmatrix} &= \begin{bmatrix} A & BK\alpha \\ -CA & I - CBK\alpha \end{bmatrix} \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \\ &\quad + \begin{bmatrix} B_1 \\ -CB_1 \end{bmatrix} \widetilde{w}(t,k). \end{aligned} \tag{8}$$

We know that system (8) is a typical 2D Roesser system. Hence, the synthetic for ILC system under the control law (3) is equivalent to synthetic of Roesser's system in (8). Notice that the introduction of the stochastic variable α renders the 2D system to be stochastic instead of a deterministic one. Before proceeding further, we need to introduce the following definition of stochastic stability for the 2D Roesser system (8), which will be essential for our derivation.

Definition 1 (stochastic stability [26]). The 2D stochastic system (8) is said to be mean-square asymptotically stable if for every bounded initial condition $x^h(i,0)$, $x^v(0,i)$, the following is satisfied:

$$\lim_{t+k\to\infty} E\left\{\|x(t,k)\|^2\right\} = 0. \tag{9}$$

Definition 2 (H_∞ performance). Given a scalar $\gamma > 0$, the 2D stochastic system (8) is said to be mean-square asymptotically stable with an H_∞ disturbance attenuation level γ, if it is mean-square asymptotically stable and under zero boundary conditions, $\|x\|_E < \gamma\|\widetilde{w}\|_2$, for all $\widetilde{w} \in [0,\infty)$, where

$$\begin{aligned}\|x\|_E &= \sqrt{E\left\{\sum_{t=0}^{\infty}\sum_{k=0}^{\infty}\|x(t,k)\|^2\right\}},\\ \|\widetilde{w}\|_2 &= \sqrt{E\left\{\sum_{t=0}^{\infty}\sum_{k=0}^{\infty}\|\widetilde{w}(t,k)\|^2\right\}}.\end{aligned} \tag{10}$$

To this end, the problem to be addressed in this paper can be transformed as follows. Consider the system in (1) with packet dropouts described in (4). Given a real number $\gamma > 0$, design a controller in the form of (3) such that the 2D stochastic system (8) is mean-square asymptotically stable with an H_∞ disturbance attenuation level γ.

Remark 3. Since $\widetilde{w}(t,k) = w(t-1,k+1) - w(t-1,k)$, we can obtain that $\|\widetilde{w}\|_2 \le \sqrt{2}\|w\|_2$, and as a consequence, it follows that

$$\|x\|_E < \gamma\sqrt{2}\|w\|_2. \tag{11}$$

Therefore, the H_∞ objective of $\|x\|_E < \gamma\|w\|_2$ can be guaranteed by ensuring that the H_∞ performance of 2D system (8) fulfills $\|x\|_E < (\gamma/\sqrt{2})\|\widetilde{w}\|_2$.

3. Main Results

In this section, the stability analysis problem is first concerned. More specifically, we assume that the system matrices A, B, C, B_1 in (8) are known, and we study the condition under which the 2D system in (8) is mean-square asymptotically stable with a guaranteed H_∞ performance. Then, a feasible ILC controller gain matrix can be given based on the condition.

Define $\widetilde{\alpha} = \alpha - \overline{\alpha}$; it is obvious that

$$\begin{aligned}E\{\widetilde{\alpha}\} &= 0,\\ E\{\widetilde{\alpha}\widetilde{\alpha}\} &= \overline{\alpha}(1-\overline{\alpha});\end{aligned} \tag{12}$$

then the 2D system (8) can be rewritten as

$$\begin{aligned}\begin{bmatrix} x^h(t+1,k)\\ x^v(t,k+1)\end{bmatrix} &= \left(\begin{bmatrix} A & \overline{\alpha}BK\\ -CA & I-\overline{\alpha}CBK\end{bmatrix} + \widetilde{\alpha}\begin{bmatrix} 0 & BK\\ 0 & -CBK\end{bmatrix}\right)\\ &\quad\times\begin{bmatrix} x^h(t,k)\\ x^v(t,k)\end{bmatrix} + \begin{bmatrix} B_1\\ -CB_1\end{bmatrix}\widetilde{w}(t,k)\\ &= \left(\overline{A}_1 + \widetilde{\alpha}\overline{A}_2\right)\begin{bmatrix} x^h(t,k)\\ x^v(t,k)\end{bmatrix} + \overline{B}\widetilde{w}(t,k),\end{aligned} \tag{13}$$

where

$$\begin{gathered}\overline{A}_1 = \begin{bmatrix} A & \overline{\alpha}BK\\ -CA & I-\overline{\alpha}CBK\end{bmatrix},\qquad \overline{A}_2 = \begin{bmatrix} 0 & BK\\ 0 & -CBK\end{bmatrix},\\ \overline{B} = \begin{bmatrix} B_1\\ -CB_1\end{bmatrix}.\end{gathered} \tag{14}$$

Theorem 4. *Consider the 2D system* (13) *and suppose the matrices* $\overline{A}_1$, $\overline{A}_2$, $\overline{B}$ *are known. Then the system is mean-square asymptotically stable with a given* H_∞ *disturbance attenuation level* γ, *if there exists positive definite matrices* P_1, P_2 *satisfying*

$$\Xi_1^T P\Xi_1 + \theta^2\Xi_2^T P\Xi_2 + \Xi_3^T\Xi_3 + \Xi_4 < 0, \tag{15}$$

where

$$\begin{gathered}P \triangleq \operatorname{diag}\{P_1, P_2\} > 0,\\ \theta^2 = \overline{\alpha}(1-\overline{\alpha}),\\ \Xi_1 = \begin{bmatrix}\overline{A}_1 & \overline{B}\end{bmatrix},\qquad \Xi_2 = \begin{bmatrix}\overline{A}_2 & 0\end{bmatrix},\\ \Xi_3 = \begin{bmatrix} I & 0\end{bmatrix},\qquad \Xi_4 = \operatorname{diag}\left\{-P, -\gamma^2 I\right\}.\end{gathered} \tag{16}$$

Proof. We first prove the stochastic stability of 2D system (13) with zero disturbance $\widetilde{w}(t,k) = 0$. In this case, the system (13) becomes

$$\begin{bmatrix} x^h(t+1,k)\\ x^v(t,k+1)\end{bmatrix} = \left(\overline{A}_1 + \widetilde{\alpha}\overline{A}_2\right)\begin{bmatrix} x^h(t,k)\\ x^v(t,k)\end{bmatrix}, \tag{17}$$

and condition (15) is

$$\overline{A}_1^T P\overline{A}_1 + \theta^2\overline{A}_2^T P\overline{A}_2 - P < 0. \tag{18}$$

Define

$$\begin{aligned}W_1 &= E\left\{\begin{bmatrix} x^h(t+1,k)\\ x^v(t,k+1)\end{bmatrix}^T P\begin{bmatrix} x^h(t+1,k)\\ x^v(t,k+1)\end{bmatrix} \mid \widetilde{x}\right\},\\ W_2 &= \widetilde{x}^T P\widetilde{x},\end{aligned} \tag{19}$$

where $\widetilde{x} = \begin{bmatrix} x^h(t,k)\\ x^v(t,k)\end{bmatrix}$.

Consider the following index:

$$J \triangleq W_1 - W_2. \tag{20}$$

Substituting (17) into the index, we can obtain

$$\begin{aligned} J &= E\left\{\begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix}^T P \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix} \mid \tilde{x}\right\} - \tilde{x}^T P\tilde{x} \\ &= E\left\{\tilde{x}^T\left(\overline{A}_1 + \tilde{\alpha}\overline{A}_2\right)^T P\left(\overline{A}_1 + \tilde{\alpha}\overline{A}_2\right)\tilde{x} \mid \tilde{x}\right\} - \tilde{x}^T P\tilde{x} \\ &= E\left\{\tilde{x}^T\left(\overline{A}_1^T P\overline{A}_1 + \tilde{\alpha}\overline{A}_1^T P\overline{A}_2 + \tilde{\alpha}\overline{A}_2^T P\overline{A}_1 \right.\right. \\ &\qquad \left.\left. + \tilde{\alpha}^2\overline{A}_2^T P\overline{A}_2\right)\tilde{x} \mid \tilde{x}\right\} - \tilde{x}^T P\tilde{x} \\ &= \tilde{x}^T\left(\overline{A}_1^T P\overline{A}_1 + \theta^2\overline{A}_2^T P\overline{A}_2\right)\tilde{x} - \tilde{x}^T P\tilde{x} \\ &= \tilde{x}^T\Psi\tilde{x}, \end{aligned} \tag{21}$$

where $\Psi = \overline{A}_1^T P\overline{A}_1 + \theta^2\overline{A}_2^T P\overline{A}_2 - P$.

Since $\Psi < 0$, it means that for all $\tilde{x} \neq 0$ we have

$$\frac{W_1 - W_2}{W_2} = -\frac{\tilde{x}^T(-\Psi)\tilde{x}}{\tilde{x}^T P\tilde{x}} \leq -\frac{\lambda_{\min}(-\Psi)}{\lambda_{\max}(P)} = \delta - 1, \tag{22}$$

where $\delta = 1 - \lambda_{\min}(-\Psi)/\lambda_{\max}(P)$.

Notice that $\lambda_{\min}(-\Psi)/\lambda_{\max}(P) > 0$; we have $\delta < 1$. From (22), it is also easy to see that $\delta \geq W_1/W_2 > 0$. Hence, $\delta \in (0,1)$ and it is independent of $\tilde{x}$. Thus, we obtain $W_1 \leq \delta W_2$, and taking expectation of both sides yields

$$\begin{aligned} &E\left\{\begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix}^T P \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix}\right\} \\ &\qquad \leq \delta E\left\{\begin{bmatrix} x^h(t,k) \\ x^v(t,k)\end{bmatrix}^T P \begin{bmatrix} x^h(t,k) \\ x^v(t,k)\end{bmatrix}\right\}; \end{aligned} \tag{23}$$

that is,

$$\begin{aligned} &E\left\{x^v(i+1,0)^T P_2 x^v(i+1,0)\right\} \\ &\quad = E\left\{x^v(i+1,0)^T P_2 x^v(i+1,0)\right\}, \\ &E\left\{x^h(i+1,0)^T P_1 x^h(i+1,0) + x^v(i,1)^T P_2 x^v(i,1)\right\} \\ &\quad \leq \delta E\left\{x^h(i,0)^T P_1 x^h(i,0) + x^v(i,0)^T P_2 x^v(i,0)\right\}, \\ &E\left\{x^h(i,1)^T P_1 x^h(i,1) + x^v(i-1,2)^T P_2 x^v(i-1,2)\right\} \\ &\quad \leq \delta E\left\{x^h(i-1,1)^T P_1 x^h(i-1,1) \right. \\ &\qquad \left. + x^v(i-1,1)^T P_2 x^v(i-1,1)\right\}, \\ &E\left\{x^h(i-1,2)^T P_1 x^h(i-1,2) \right. \\ &\qquad \left. + x^v(i-2,3)^T P_2 x^v(i-2,3)\right\} \\ &\quad \leq \delta E\left\{x^h(i-2,2)^T P_1 x^h(i-2,2) \right. \\ &\qquad \left. + x^v(i-2,2)^T P_2 x^v(i-2,2)\right\}, \\ &\vdots \\ &E\left\{x^h(1,i)^T P_1 x^h(1,i) + x^v(0,i+1)^T P_2 x^v(0,i+1)\right\} \\ &\quad \leq \delta E\left\{x^h(0,i)^T P_1 x^h(0,i) + x^v(0,i)^T P_2 x^v(0,i)\right\}, \\ &E\left\{x^h(0,i+1)^T P_1 x^h(0,i+1)\right\} \\ &\quad = \left\{x^h(0,i+1)^T P_1 x^h(0,i+1)\right\}. \end{aligned} \tag{24}$$

Adding both sides of the inequality system (24) yields

$$\begin{aligned} &E\left\{\sum_{j=0}^{i+1}\left[x^h(i+1-j,j)^T P_1 x^h(i+1-j,j) \right.\right. \\ &\qquad \left.\left. + x^v(i+1-j,j)^T P_2 x^v(i+1-j,j)\right]\right\} \\ &\leq \delta E\left\{\sum_{j=0}^{i}\left[x^h(i-j,j)^T P_1 x^h(i-j,j) \right.\right. \\ &\qquad \left.\left. + x^v(i-j,j)^T P_2 x^v(i-j,j)\right]\right\} \\ &\quad + E\left\{x^v(i+1,0)^T P_2 x^v(i+1,0)\right\} \\ &\quad + E\left\{x^h(i+1,0)^T P_1 x^h(i+1,0)\right\}. \end{aligned} \tag{25}$$

Using this relationship iteratively, we can obtain

$$\begin{aligned} &E\left\{\sum_{j=0}^{i+1}\left[x^h(i+1-j,j)^T P_1 x^h(i+1-j,j) \right.\right. \\ &\qquad \left.\left. + x^v(i+1-j,j)^T P_2 x^v(i+1-j,j)\right]\right\} \end{aligned}$$

$$\begin{aligned}&\le \delta^{i+1}E\left\{x^h(0,0)^TP_1x^h(0,0)+x^v(0,0)^TP_2x^v(0,0)\right\}\\&\quad+E\left\{\sum_{j=0}^{i}\delta^j\left[x^v(i+1-j,0)^TP_2x^v(i+1-j,0)\right.\right.\\&\qquad\left.\left.+x^h(0,i+1-j)^TP_1x^h(0,i+1-j)\right]\right\}\\&=E\left\{\sum_{j=0}^{i+1}\delta^j\left[x^v(i+1-j,0)^TP_2x^v(i+1-j,0)\right.\right.\\&\qquad\left.\left.+x^h(0,i+1-j)^TP_1x^h(0,i+1-j)\right]\right\}\end{aligned} \tag{26}$$

which implies

$$\begin{aligned}&E\left\{\sum_{j=0}^{i+1}\|x(i+1-j,j)\|^2\right\}\\&\le\kappa\sum_{j=0}^{i+1}\delta^jE\left\{\|x^v(i+1-j,0)\|^2+\|x^h(0,i+1-j)\|^2\right\},\end{aligned} \tag{27}$$

where

$$\kappa:=\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}. \tag{28}$$

Now, denote $\chi_i:=\sum_{j=0}^{i}\|x(i-j,j)\|^2$; then upon inequality (27) we have

$$\begin{aligned}E\{\chi_0\}&\le\kappa E\left\{\|x^v(0,0)\|^2+\|x^h(0,0)\|^2\right\},\\E\{\chi_1\}&\le\kappa\left[\delta E\left\{\|x^v(0,0)\|^2+\|x^h(0,0)\|^2\right\}\right.\\&\qquad\left.+E\left\{\|x^v(1,0)\|^2+\|x^h(0,1)\|^2\right\}\right],\\E\{\chi_2\}&\le\kappa\left[\delta^2E\left\{\|x^v(0,0)\|^2+\|x^h(0,0)\|^2\right\}\right.\\&\qquad+\delta E\left\{\|x^v(1,0)\|^2+\|x^h(0,1)\|^2\right\}\\&\qquad\left.+E\left\{\|x^v(2,0)\|^2+\|x^h(0,2)\|^2\right\}\right]\\&\vdots\\E\{\chi_N\}&\le\kappa\left[\delta^NE\left\{\|x^v(0,0)\|^2+\|x^h(0,0)\|^2\right\}\right.\\&\qquad+\delta^{N-1}E\left\{\|x^v(1,0)\|^2+\|x^h(0,1)\|^2\right\}\\&\qquad\left.+\cdots+E\left\{\|x^v(N,0)\|^2+\|x^h(0,N)\|^2\right\}\right].\end{aligned} \tag{29}$$

Adding both sides of the inequalities yields

$$\begin{aligned}&\sum_{i=0}^{N}E\{\chi_i\}\\&\le\kappa\left(1+\delta+\cdots+\delta^N\right)E\left\{\|x^v(0,0)\|^2+\|x^h(0,0)\|^2\right\}\\&\quad+\kappa\left(1+\delta+\cdots+\delta^{N-1}\right)E\left\{\|x^v(1,0)\|^2+\|x^h(0,1)\|^2\right\}\\&\quad+\cdots+\kappa E\left\{\|x^v(N,0)\|^2+\|x^h(0,N)\|^2\right\}\\&\le\kappa\left(1+\delta+\cdots+\delta^N\right)E\left\{\|x^v(0,0)\|^2+\|x^h(0,0)\|^2\right\}\\&\quad+\kappa\left(1+\delta+\cdots+\delta^{N}\right)E\left\{\|x^v(1,0)\|^2+\|x^h(0,1)\|^2\right\}\\&\quad+\cdots+\kappa\left(1+\delta+\cdots+\delta^N\right)E\left\{\|x^v(N,0)\|^2+\|x^h(0,N)\|^2\right\}\\&=\kappa\times\frac{1-\delta^N}{1-\delta}E\left\{\sum_{i=0}^{N}\left[\|x^v(i,0)\|^2+\|x^h(0,i)\|^2\right]\right\}.\end{aligned} \tag{30}$$

Since $x(0,k)=x_{0k}$ is satisfied for all k in system (1), then

$$x^h(0,i)=\eta(0,i)=x(0,i+1)-x(0,i)=0, \tag{31}$$

and $x^v(i,0)=e(i,0)=y_d(i)-Cx(i,0)$ is bounded; hence, the right side of inequality (30) is bounded, which means $\lim_{i\to\infty}E\{\chi_i\}=0$; that is, $\lim_{t+k\to\infty}E\{\|x(t,k)\|^2\}=0$. Then the 2D stochastic system (13) with $\widetilde{w}(t,k)=0$ is mean-square asymptotically stable.

Now, the H_∞ performance for the 2D stochastic system (13) will be established. Assume zero initial and boundary conditions; that is, $x^h(0,i)=0$, $x^v(i,0)=0$ for all i. In this case, the index J becomes

$$\begin{aligned}J&=E\left\{\begin{bmatrix}x^h(t+1,k)\\x^v(t,k+1)\end{bmatrix}^TP\begin{bmatrix}x^h(t+1,k)\\x^v(t,k+1)\end{bmatrix}\mid\widetilde{x}\right\}-\widetilde{x}^TP\widetilde{x}\\&=E\left\{\left[\left(\overline{A}_1+\overline{\alpha}\overline{A}_2\right)\widetilde{x}+\overline{B}\widetilde{w}\right]^TP\left[\left(\overline{A}_1+\overline{\alpha}\overline{A}_2\right)\widetilde{x}+\overline{B}\widetilde{w}\right]\mid\widetilde{x}\right\}\\&\quad-\widetilde{x}^TP\widetilde{x}\\&=\widetilde{x}^T\left(\overline{A}_1^TP\overline{A}_1+\theta^2\overline{A}_2^TP\overline{A}_2\right)\widetilde{x}+\widetilde{x}^T\overline{A}_1^TP\overline{B}\widetilde{w}+\widetilde{w}^T\overline{B}^TP\overline{A}_1\widetilde{x}\\&\quad+\widetilde{w}^T\overline{B}^TP\overline{B}\widetilde{w}-\widetilde{x}^TP\widetilde{x}\\&=\widetilde{x}^T\Psi\widetilde{x}+\widetilde{x}^T\overline{A}_1^TP\overline{B}\widetilde{w}+\widetilde{w}^T\overline{B}^TP\overline{A}_1\widetilde{x}+\widetilde{w}^T\overline{B}^TP\overline{B}\widetilde{w}.\end{aligned} \tag{32}$$

Define $\xi = \left[\tilde{x}^T \quad \widetilde{w}^T\right]^T$; another index is introduced as

$$\begin{aligned}\Pi &\triangleq \tilde{x}^T\tilde{x} - \gamma^2\widetilde{w}^T\widetilde{w} + J \\ &= \tilde{x}^T\tilde{x} - \gamma^2\widetilde{w}^T\widetilde{w} + \tilde{x}^T\Psi\tilde{x} + \tilde{x}^T\overline{A}_1^T P\overline{B}\widetilde{w} + \widetilde{w}^T\overline{B}^T P\overline{A}_1\tilde{x} \\ &\quad + \widetilde{w}^T\overline{B}^T P\overline{B}\widetilde{w} \\ &= \xi^T\Omega\xi,\end{aligned} \tag{33}$$

where

$$\begin{aligned}\Omega &= \begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I\end{bmatrix} + \begin{bmatrix} \overline{A}_1^T P\overline{A}_1 & \overline{A}_1^T P\overline{B} \\ \overline{B}^T P\overline{A}_1 & \overline{B}^T\overline{B}\end{bmatrix} \\ &\quad + \begin{bmatrix} \theta^2\overline{A}_2^T P\overline{A}_2 & 0 \\ 0 & 0\end{bmatrix} + \begin{bmatrix} -P & 0 \\ 0 & 0\end{bmatrix} \\ &= \Xi_1^T P\Xi_1 + \theta^2\Xi_2^T P\Xi_2 + \Xi_3^T\Xi_3 + \Xi_4.\end{aligned} \tag{34}$$

From condition (15), for any $\xi \neq 0$, we have $\Pi < 0$; that is,

$$\begin{aligned}&E\left\{\begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix}^T P \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix} \mid \tilde{x}\right\} \\ &\quad < \tilde{x}^T P\tilde{x} - \tilde{x}^T\tilde{x} + \gamma^2\widetilde{w}^T\widetilde{w}.\end{aligned} \tag{35}$$

Taking the expectation of both sides yields

$$\begin{aligned}&E\left\{\begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix}^T P \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1)\end{bmatrix} \mid \tilde{x}\right\} \\ &\quad < E\left\{\tilde{x}^T P\tilde{x} - \tilde{x}^T\tilde{x}\right\} + \gamma^2\widetilde{w}^T\widetilde{w}.\end{aligned} \tag{36}$$

Due to the relationship (36), it can be established that

$$\begin{aligned}&E\left\{x^v(i+1,0)^T P_2 x^v(i+1,0)\right\} \\ &\quad = E\left\{x^v(i+1,0)^T P_2 x^v(i+1,0)\right\}, \\ &E\left\{x^h(i+1,0)^T P_1 x^h(i+1,0) + x^v(i,1)^T P_2 x^v(i,1)\right\} \\ &\quad \le E\left\{x^h(i,0)^T P_1 x^h(i,0) + x^v(i,0)^T P_2 x^v(i,0)\right\} \\ &\qquad - E\left\{\tilde{x}(i,0)^T\tilde{x}(i,0)\right\} + \gamma^2\widetilde{w}(i,0)^T\widetilde{w}(i,0), \\ &E\left\{x^h(i,1)^T P_1 x^h(i,1) + x^v(i-1,2)^T P_2 x^v(i-1,2)\right\} \\ &\quad \le E\left\{x^h(i-1,1)^T P_1 x^h(i-1,1) \right. \\ &\qquad \left. + x^v(i-1,1)^T P_2 x^v(i-1,1)\right\} \\ &\qquad - E\left\{\tilde{x}(i-1,1)^T\tilde{x}(i-1,1)\right\} \\ &\qquad + \gamma^2\widetilde{w}(i-1,1)^T\widetilde{w}(i-1,1), \\ &E\left\{x^h(i-1,2)^T P_1 x^h(i-1,2) \right. \\ &\quad \left. + x^v(i-2,3)^T P_2 x^v(i-2,3)\right\} \\ &\quad \le E\left\{x^h(i-2,2)^T P_1 x^h(i-2,2) \right. \\ &\qquad \left. + x^v(i-2,2)^T P_2 x^v(i-2,2)\right\} \\ &\qquad - E\left\{\tilde{x}(i-2,2)^T\tilde{x}(i-2,2)\right\} \\ &\qquad + \gamma^2\widetilde{w}(i-2,2)^T\widetilde{w}(i-2,2), \\ &\vdots \\ &E\left\{x^h(1,i)^T P_1 x^h(1,i) + x^v(0,i+1)^T P_2 x^v(0,i+1)\right\} \\ &\quad \le E\left\{x^h(0,i)^T P_1 x^h(0,i) + x^v(0,i)^T P_2 x^v(0,i)\right\} \\ &\qquad - E\left\{\tilde{x}(0,i)^T\tilde{x}(0,i)\right\} + \gamma^2\widetilde{w}(0,i)^T\widetilde{w}(0,i), \\ &E\left\{x^h(0,i+1)^T P_1 x^h(0,i+1)\right\} \\ &\quad = \left\{x^h(0,i+1)^T P_1 x^h(0,i+1)\right\}.\end{aligned} \tag{37}$$

Adding both sides of the inequality system, we have

$$\begin{aligned}&E\left\{\sum_{j=0}^{i+1}\left[x^h(i+1-j,j)^T P_1 x^h(i+1-j,j) \right.\right. \\ &\qquad \left.\left. + x^v(i+1-j,j)^T P_2 x^v(i+1-j,j)\right]\right\} \\ &\quad < \left\{\sum_{j=0}^{i}\left[x^h(i-j,j)^T P_1 x^h(i-j,j) \right.\right. \\ &\qquad \left.\left. + x^v(i-j,j)^T P_2 x^v(i-j,j)\right]\right\} \\ &\qquad + E\left\{x^v(i+1,0)^T P_2 x^v(i+1,0)\right\} \\ &\qquad + E\left\{x^h(0,i+1)^T P_1 x^h(0,i+1)^T\right\} \\ &\qquad - E\left\{\sum_{j=0}^{i}\left[\tilde{x}(i-j,j)^T\tilde{x}(i-j,j)\right]\right\} \\ &\qquad + \gamma^2\sum_{j=0}^{i}\widetilde{w}(i-j,j)^T\widetilde{w}(i-j,j).\end{aligned} \tag{38}$$

Summing up both sides of these inequalities from $i = 0$ to $i = N$, we have

$$E\left\{\sum_{i=0}^{N}\sum_{j=0}^{i}\tilde{x}(i-j,j)^T\tilde{x}(i-j,j)\right\} < \gamma^2\sum_{i=0}^{N}\sum_{j=0}^{i}\widetilde{w}(i-j,j)^T\widetilde{w}(i-j,j) + \sum_{i=0}^{N}\left(E\left\{x^v(i+1,0)^TP_2x^v(i+1,0)\right\} + E\left\{x^h(0,i+1)^TP_1x^h(0,i+1)^T\right\}\right) - E\left\{\sum_{j=0}^{N}\left[x^h(N+1-j,j)^TP_1x^h(N+1-j,j) + x^v(N+1-j,j)^TP_2x^v(N+1-j,j)\right]\right\} + E\left\{x^v(0,0)^TP_2x^v(0,0)\right\} + E\left\{x^h(0,0)^TP_1x^h(0,0)^T\right\}; \quad (39)$$

that is,

$$E\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{i}\tilde{x}(i-j,j)^T\tilde{x}(i-j,j)\right\} < \gamma^2\sum_{i=0}^{\infty}\sum_{j=0}^{i}\widetilde{w}(i-j,j)^T\widetilde{w}(i-j,j) + \sum_{i=0}^{\infty}\left(E\left\{x^v(i,0)^TP_2x^v(i,0)\right\} + E\left\{x^h(0,i)^TP_1x^h(0,i)^T\right\}\right). \quad (40)$$

Considering the zero boundary conditions, (40) means

$$\|\tilde{x}\|_E < \gamma^2\|\widetilde{w}\|_2. \quad (41)$$

This completes the proof. □

Remark 5. Theorem 4 provides a sufficient condition for the mean-square asymptotic stability of 2D discrete stochastic systems with intermittent measurement. If the communication links existing between the plant and the controller are perfect, that is, there is no data dropout during their transmission, then $\overline{\alpha} = 1$ and $\theta = 0$. In this case, the condition in Theorem 4 becomes

$$\Xi_1^TP\Xi_1 + \Xi_3^T\Xi_3 + \Xi_4 < 0 \quad (42)$$

which also can be easily obtained by the results in [25] for 2D discrete deterministic system. From this point of view, Theorem 4 can be seen as an extension of 2D discrete deterministic systems with intermittent measurement.

Theorem 4 only gives a mean-square asymptotic stability condition for system in (13) where the matrixes A, B, C, B_1 and parameters $\overline{\alpha}$ are all known. However, our eventual purpose is to determine the controller matrix K. In the following, we will give an approach to solve the controller design problem for 2D systems with stochastic intermittent measurement.

The following well-known lemma is needed in the proof of our main result.

Lemma 6 (Schur complement [27]). *Assume W, L, V are given matrices with appropriate dimension, where W and V are positive definite symmetric matrices. Then*

$$L^TVL - W < 0 \quad (43)$$

if and only if

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0, \quad (44)$$

or

$$\begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \quad (45)$$

Based on the above lemma, we can give our main result.

Theorem 7. *The 2D discrete stochastic closed system in* (13) *is mean-square asymptotically stable with a given H_∞ performance γ, if there exist positive definite matrices Q_1, Q_2, and M such that the following LMI holds:*

$$\begin{bmatrix} I & & & & & & I & 0 & 0 \\ & I & & & & & 0 & I & 0 \\ & & -Q_1 & & & & AQ_1 & \overline{\alpha}BM & B_1 \\ & & & -Q_2 & & & -CAQ_1 & P_2 - \overline{\alpha}CBM & -CB_1 \\ & & & & -Q_1 & & 0 & \theta BM & 0 \\ & & & & & -Q_2 & 0 & -\theta CBM & 0 \\ & & & & & & -Q_1 & 0 & 0 \\ & & & & & & & -Q_2 & 0 \\ & & & & & & & & -\gamma^2 I \end{bmatrix} < 0. \quad (46)$$

Also, if this condition holds, a suitable gain matrix of ILC law (3) *is defined by*

$$K = MQ_2^{-1}. \quad (47)$$

Proof. The condition in Theorem 4 can be rewritten as

$$\begin{bmatrix} \Xi_3^T & \Xi_1^TP & \theta\Xi_2^TP \end{bmatrix}\begin{bmatrix} I & & \\ & P & \\ & & P \end{bmatrix}\begin{bmatrix} \Xi_3 \\ P\Xi_1 \\ \theta P\Xi_2 \end{bmatrix} + \Xi_4 < 0. \quad (48)$$

By applying Lemma 6, condition (48) is equivalent to the following LMI condition:

$$\begin{bmatrix} -I & & & \Xi_3 \\ & -P & & P\Xi_1 \\ & & -P & \theta P\Xi_2 \\ & & & \Xi_4 \end{bmatrix} < 0. \quad (49)$$

Substituting matrices $\overline{A}_1$, $\overline{A}_2$, $\overline{B}$ into the LMI condition, we obtain

$$\begin{bmatrix} I & & & & & & I & 0 & 0 \\ & I & & & & & 0 & I & 0 \\ & & -P_1 & & & & P_1 A & \overline{\alpha} P_1 BK & P_1 B_1 \\ & & & -P_2 & & & -P_2 CA & P_2 - \overline{\alpha} P_2 CBK & -P_2 CB_1 \\ & & & & -P_1 & & 0 & \theta P_1 BK & 0 \\ & & & & & -P_2 & 0 & -\theta P_2 CBK & 0 \\ & & & & & & -P_1 & 0 & 0 \\ & & & & & & & -P_2 & 0 \\ & & & & & & & & -\gamma^2 I \end{bmatrix} < 0. \quad (50)$$

Defining $Q_1 = P_1^{-1}$, $Q_2 = P_2^{-1}$, pro-, and postmultiplying $\mathrm{diag}(I, I, Q_1, Q_2, Q_1, Q_2, Q_1, Q_2, I)$ for LMI condition (50) gives

$$\begin{bmatrix} I & & & & & & I & 0 & 0 \\ & I & & & & & 0 & I & 0 \\ & & -Q_1 & & & & AQ_1 & \overline{\alpha} BKQ_2 & B_1 \\ & & & -Q_2 & & & -CAQ_1 & P_2 - \overline{\alpha} CBKQ_2 & -CB_1 \\ & & & & -Q_1 & & 0 & \theta BKQ_2 & 0 \\ & & & & & -Q_2 & 0 & -\theta CBKQ_2 & 0 \\ & & & & & & -Q_1 & 0 & 0 \\ & & & & & & & -Q_2 & 0 \\ & & & & & & & & -\gamma^2 I \end{bmatrix} < 0. \quad (51)$$

Set $KQ_2 = M$ to obtain the LMI of (46) and the proof is complete. □

Remark 8. Theorem 7 provides an LMI condition for the mean-square asymptotic stability and H_∞ performance of 2D stochastic system in (13) which can be solved by LMI Toolbox. Then by (47), we also can give a suitable ILC law. The feasibility of the proposed method will be illustrated by the example given in Section 4.

Remark 9. For a fixed γ, the feasibility of (47) is a suboptimal H_∞ ILC. When γ is not fixed, the minimization of γ that satisfies (47) can be searched. That is, the optimal performance index can be achieved. Thus, the optimal H_∞ ILC design problem is equivalent to the following convex programming problem:

$$\begin{aligned} &\min_{Q_1, Q_2, M} \quad \gamma^2 \\ &\text{s.t.} \qquad (46)\,. \end{aligned} \quad (52)$$

Remark 10. Even though we consider the ILC law (3) only uses the error information from the previous iteration in networked systems framework, the general tools used can be extended to other ILC laws such as containing state signal in [23–25]. All of these problems end up with the same 2D system formulation as in (13).

4. Numerical Example

In this section, an example is given to illustrate the proposed results. Consider the following SISO linear system:

$$\begin{aligned} x(k, t+1) &= \begin{bmatrix} 0.25 & 0.6 \\ 0.6 & 0 \end{bmatrix} x(k, t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k, t) + w(k, t), \\ y(k, t) &= \begin{bmatrix} 1 & -1.3 \end{bmatrix} x(k, t), \end{aligned} \quad (53)$$

where $w(k, t)$ is an iteration-varying disturbance with $|w(k, t)| < 0.02$. The desired repetitive reference trajectory is given as

$$y_d(t) = \begin{cases} 2 * (-1)^{\mathrm{round}(t/100)}, & 1 \le t < 500 \\ 2 \sin\left(\dfrac{\pi t}{100}\right) + 0.3 \cos\left(\dfrac{\pi t}{100}\right), & 500 \le t < 1000. \end{cases} \quad (54)$$

For the initial state, it is assumed that $x_1(0, k) = x_2(0, k) = 0$ for all k. To perform the simulation, we consider the intermittent measurements $\overline{\alpha} = 0.8$. Obviously, it means that there is 20 percent missing measurements. By applying Theorem 7 and solving the optimization problem (52), the minimum H_∞ disturbance attenuation level for the ILC system is based on feasibility of the corresponding LMIs. Meanwhile, we obtain $\gamma_{\mathrm{opt}} = 27.5$ and $K = 0.72$. Simulation

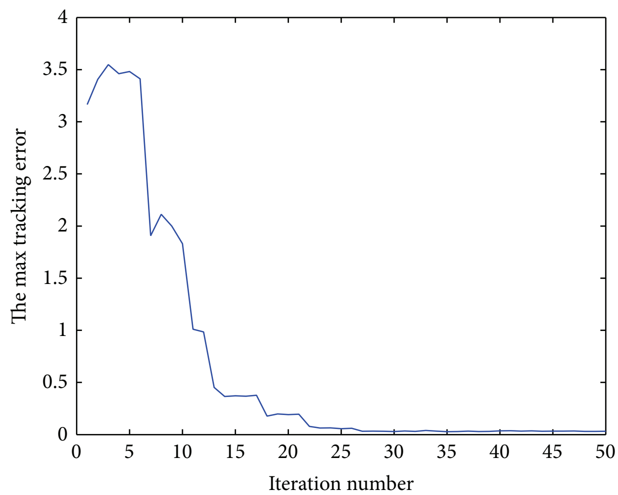

Figure 1: Max tracking error on iteration domain.

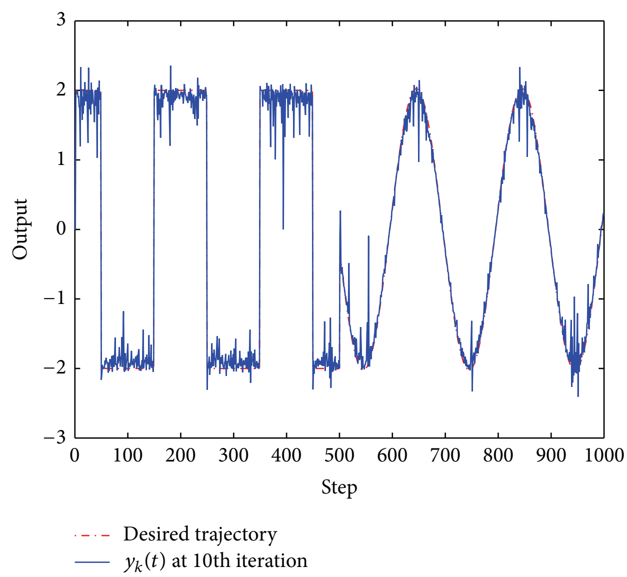

Figure 2: System output at 10th iteration.

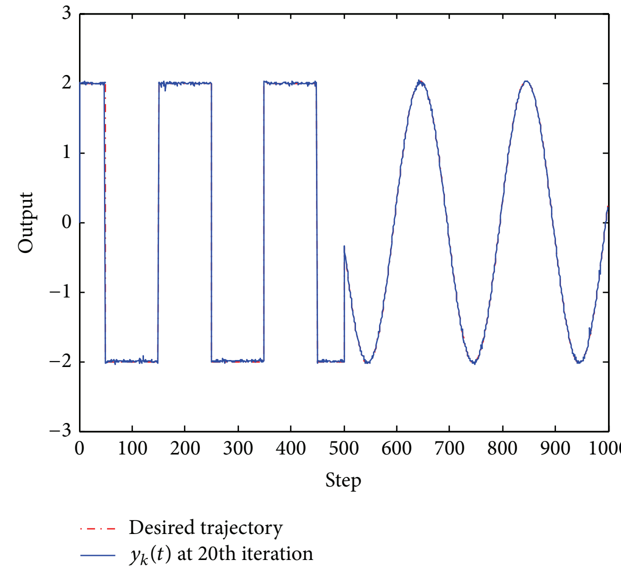

Figure 3: System output at 20th iteration.

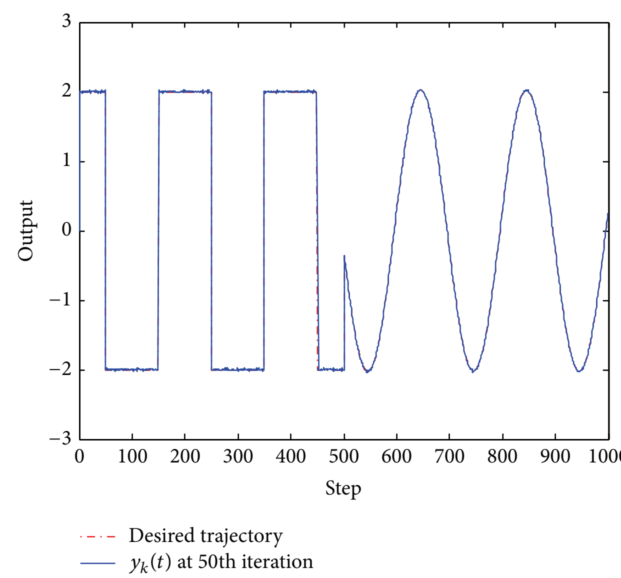

Figure 4: System output at 50th iteration.

results are shown in Figures 1, 2, 3, and 4, where the tracking error on iteration domain is plotted in Figure 1, and system outputs at 10th, 20th, and 50th iteration are plotted in Figures 2–4, respectively. It is observed that the tracking is worse and significant tracking errors exist in the start iteration due to the effect of significant measurement dropout. However, the tracking error can converge to zero after some iteration and the perfect tracking can be obtained. The ILC system is insensitive to intermittent measurement and iteration-varying disturbance.

5. Conclusions

In this paper, the problem of H_∞ ILC design for linear networked systems with intermittent measurements and iteration-varying disturbances has been investigated. A stochastic variable satisfying the Bernoulli random binary distribution is utilized to characterize the data missing phenomenon, and then the design of ILC law has been transformed into H_∞ control problem of a 2D stochastic system. A sufficient condition of mean-square asymptotic stability is established by means of LMI technique. An example is given to demonstrate the effectiveness and feasibility of the proposed design methods. This paper gives a systematic H_∞ design approach for stochastic ILC based on 2D system.

Conflict of Interests

There is no conflict of interests regarding the publication of the paper.

Acknowledgments

This work is supported by the Program of NSFC (no. 61203065 and no. 61340041), the program of Natural Science of Henan Provincial Education Department (12A510013), the program of Open Laboratory Foundation of Control Engineering Key Discipline of Henan Provincial High Education (KG 2011-10), the program of Key Young Teacher of Henan Polytechnic University, and the Doctoral Fund Program of Henan Polytechnic University (B2012-003).

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