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Research Article

Master-Slave Synchronization of 4D Hyperchaotic Rabinovich Systems

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This paper is concerned with master-slave synchronization of 4D hyperchaotic Rabinovich systems. Compared with some existing papers, this paper has two contributions. The first contribution is that the nonlinear terms of error systems remained which inherit nonlinear features from master and slave 4D hyperchaotic Rabinovich systems, rather than discarding nonlinear features of original hyperchaotic Rabinovich systems and eliminating those nonlinear terms to derive linear error systems as the control methods in some existing papers. The second contribution is that the synchronization criteria of this paper are global rather than local synchronization results in some existing papers. In addition, those synchronization criteria and control methods for 4D hyperchaotic Rabinovich systems are extended to investigate the synchronization of 3D chaotic Rabinovich systems. The effectiveness of synchronization criteria is illustrated by three simulation examples.

1. Introduction

The classic hyperchaotic Rabinovich system was a system of 3D differential equations which was used to describe the plasma oscillation [1]. In [2], a 4D hyperchaotic Rabinovich system was introduced, which has been seen in wide applications in plasma oscillation, security communication, image encryption, and cell kinetics; see, for example, [2–4].

There exist various dynamical behaviors of 4D hyperchaotic Rabinovich systems. Synchronization is the typical dynamical behavior of chaotic systems [1, 5–31]. Master-slave synchronization of Rabinovich systems has been observed and attracted many researches' interests. In [32], some local synchronization criteria were derived for 3D Rabinovich systems by using linear feedback control and Routh-Hurwitz criteria. In [4, 13, 32], some synchronization criteria were derived for 3D or 4D Rabinovich systems by the control which eliminated all the nonlinear terms of the error system. However, the Rabinovich systems are nonlinear systems in which the nonlinear terms play an important role in the dynamical evolution of trajectories. The linear error systems can be derived by the control method of eliminating nonlinear terms in error systems. Thus, how to design controllers to remain nonlinear terms in error systems and how to use those controllers to derive global synchronization criteria are the main motivations of this paper.

In this paper, a master-slave scheme for 4D hyperchaotic Rabinovich systems is constructed. Some global master-slave synchronization criteria for 4D hyperchaotic Rabinovich systems are derived by using the designed controllers. The nonlinear features of error systems remained. Those control methods and synchronization criteria for 4D Rabinovich systems can be used to derive synchronization criteria for 3D Rabinovich systems. Three examples are used to illustrate the effectiveness of our results.

2. Preliminaries

Consider the following 4D Rabinovich system as a master system:

$$\dot{x}_1(t) = -ax_1(t) + hx_2(t) + x_2(t)x_3(t),$$
$$\dot{x}_2(t) = hx_1(t) - bx_2(t) - x_1(t)x_3(t) + x_4(t),$$

$$\begin{aligned}
\dot{x}_3(t) &= -dx_3(t) + x_1(t)x_2(t), \\
\dot{x}_4(t) &= -cx_2(t), \\
x_1(0) &= x_{1_0}, \\
x_2(0) &= x_{2_0}, \\
x_3(0) &= x_{3_0}, \\
x_4(0) &= x_{4_0},
\end{aligned} \tag{1}$$

where $(x_1(t), x_2(t), x_3(t), x_4(t))^T \in \mathbb{R}^4$ is the state variable and a, b, c, d, and h are four positive constants. When $h = 6.75$, $a = 4$, $b = 1$, $c = 2$, and $d = 1$, a hyperchaotic attractor can be observed [2].

Because the trajectories of a hyperchaotic system are bounded [2], one can assume that there exists a positive constant l such that

$$|x_2(t)| \leq l, \quad \forall t \geq 0, \tag{2}$$

where the bound l can be derived by observing the trajectory $x_2(t)$ of 4D master system when Matlab is used to plot the trajectory $x_2(t)$ of master system.

One can construct the following slave scheme associated with system (1):

$$\begin{aligned}
\dot{y}_1(t) &= -ay_1(t) + hy_2(t) + y_2(t)y_3(t) + u_1(t), \\
\dot{y}_2(t) &= hy_1(t) - by_2(t) + y_4(t) - y_1(t)y_3(t) \\
&\quad + u_2(t), \\
\dot{y}_3(t) &= -dy_3(t) + y_1(t)y_2(t) + u_3(t), \\
\dot{y}_4(t) &= -cy_2(t) + u_4(t), \\
y_1(0) &= y_{1_0}, \\
y_2(0) &= y_{2_0}, \\
y_3(0) &= y_{3_0}, \\
y_4(0) &= y_{4_0},
\end{aligned} \tag{3}$$

where $(y_1(t), y_2(t), y_3(t), y_4(t))^T \in \mathbb{R}^4$ is the state variable of slave system and $u_1(t)$, $u_2(t)$, $u_3(t)$, and $u_4(t)$ are the external controls.

Let $e_i(t) = x_i(t) - y_i(t)$ for $i = 1, 2, 3, 4$. Then, one can construct the following error system for schemes (1) and (3):

$$\begin{aligned}
\dot{e}_1(t) &= -ae_1(t) + he_2(t) \\
&\quad + (x_2(t)x_3(t) - y_2(t)y_3(t)) - u_1(t), \\
\dot{e}_2(t) &= he_1(t) - be_2(t) + e_4(t) \\
&\quad - (x_1(t)x_3(t) - y_1(t)y_3(t)) - u_2(t), \\
\dot{e}_3(t) &= -de_3(t) + (x_1(t)x_2(t) - y_1(t)y_2(t)) \\
&\quad - u_3(t), \\
\dot{e}_4(t) &= -ce_2(t) - u_4(t), \\
e_1(0) &= x_{1_0} - y_{1_0}, \\
e_2(0) &= x_{2_0} - y_{2_0}, \\
e_3(0) &= x_{3_0} - y_{3_0}, \\
e_4(0) &= x_{4_0} - y_{4_0}.
\end{aligned} \tag{4}$$

In this paper, we design $u_1(t) = k_1e_1(t) + k_4y_2^2(t)e_1(t)$, $u_2(t) = k_2e_2(t)$, $u_3(t) = k_3e_3(t)$, and $u_4(t) = k_5e_4(t)$. Then, the error system described by (4) can be rewritten as

$$\begin{aligned}
\dot{e}_1(t) &= -\left(a + k_1 + k_4y_2^2(t)\right)e_1(t) + he_2(t) \\
&\quad + (x_2(t)x_3(t) - y_2(t)y_3(t)), \\
\dot{e}_2(t) &= he_1(t) - (b + k_2)e_2(t) + e_4(t) \\
&\quad - (x_1(t)x_3(t) - y_1(t)y_3(t)), \\
\dot{e}_3(t) &= -(d + k_3)e_3(t) \\
&\quad + (x_1(t)x_2(t) - y_1(t)y_2(t)), \\
\dot{e}_4(t) &= -ce_2(t) - k_5e_4(t), \\
e_1(0) &= x_{1_0} - y_{1_0}, \\
e_2(0) &= x_{2_0} - y_{2_0}, \\
e_3(0) &= x_{3_0} - y_{3_0}, \\
e_4(0) &= x_{4_0} - y_{4_0}.
\end{aligned} \tag{5}$$

The main purpose of this paper is to design k_1, k_2, k_3, k_4, and k_5 to guarantee the global stability of the error system described by (5).

3. Main Results: Synchronization Criteria

3.1. Synchronization Criteria for 4D Hyperchaotic Rabinovich Systems. Now, we give some synchronization results for two 4D hyperchaotic Rabinovich systems described by (1) and (3).

Theorem 1. *If $k_5 > 0$ and k_1, k_2, k_3, and k_4 satisfy*

$$\begin{aligned}
&k_4 > \frac{1}{4(d + k_3)}, \\
&(a + k_1) > \frac{l^2}{4(d + k_3)} + \frac{h^2}{(b + k_2)}, \\
&l^2 < 4\frac{d + k_3}{k_4}\left(k_4 - \frac{1}{4(d + k_3)}\right)\left(a + k_1 - \frac{h^2}{b + k_2}\right),
\end{aligned} \tag{6}$$

then two 4D hyperchaotic Rabinovich systems described by (1) and (3) achieve global synchronization.

Proof. One can construct Lyapunov function

$$V(t) = \frac{e_1^2(t) + e_2^2(t) + e_3^2(t) + e_4^2(t)/c}{2}. \tag{7}$$

Calculating the derivative of $V(t)$ along with (5) gives

$$\begin{aligned}
\dot{V}(t) &= -\left(a + k_1 + k_4 y_2^2(t)\right) e_1^2(t) + 2he_1(t)e_2(t) \\
&\quad - (b + k_2) e_2^2(t) - (d + k_3) e_3^2(t) \\
&\quad + 2(x_2(t) + y_2(t)) e_1(t) e_3(t) - \frac{k_4}{c} e_4^2(t) \\
&\le -\left(\frac{h}{\sqrt{b + k_2}} e_1(t) - \sqrt{b + k_2} e_2(t)\right)^2 \\
&\quad + \frac{h^2}{b + k_2} e_1^2(t) \\
&\quad - \left(\frac{x_2(t) + y_2(t)}{2\sqrt{d + k_3}} e_1(t) - \sqrt{d + k_3} e_3(t)\right)^2 \\
&\quad + \frac{(x_2(t) + y_2(t))^2}{4(d + k_3)} e_1^2(t) - \frac{k_5}{c} e_4^2(t) \\
&\quad - \left(a + k_1 + k_4 y_2^2(t)\right) e_1^2(t).
\end{aligned} \tag{8}$$

It is easy to see that

$$\frac{h^2}{b + k_2} + \frac{(x_2(t) + y_2(t))^2}{4(d + k_3)} < a + k_1 + k_4 y_2^2(t) \tag{9}$$

and $e_i(t) \neq 0$ for $i = 1, 2, 3, 4$ can ensure $\dot{V}(t) < 0$.

The inequality described by (9) can be rearranged as

$$\widetilde{A} y_2^2(t) + \widetilde{B} y_2(t) + \widetilde{C} > 0 \tag{10}$$

with

$$\begin{aligned}
\widetilde{A} &= k_4 - \frac{1}{4(d + k_3)}, \\
\widetilde{B} &= -\frac{x_2(t)}{2(d + k_3)}, \\
\widetilde{C} &= -\frac{x_2^2(t)}{4(d + k_3)} - \frac{h^2}{(b + k_2)} + (a + k_1).
\end{aligned} \tag{11}$$

Solving (10), one can have

$$\begin{aligned}
\widetilde{A} &> 0, \\
\widetilde{C} &> 0, \\
\widetilde{B}^2 - 4\widetilde{A}\widetilde{C} &< 0;
\end{aligned} \tag{12}$$

that is,

$$\begin{aligned}
& k_4 > \frac{1}{4(d + k_3)}, \\
& (a + k_1) > \frac{x_2^2(t)}{4(d + k_3)} + \frac{h^2}{(b + k_2)}, \\
& x_2^2(t) \\
& \quad < 4\frac{d + k_3}{k_4}\left(k_4 - \frac{1}{4(d + k_3)}\right)\left(a + k_1 - \frac{h^2}{b + k_2}\right).
\end{aligned} \tag{13}$$

Due to the bound l of trajectory $x_2(t)$ in (2), one can get

$$\begin{aligned}
& (a + k_1) > \frac{l^2}{4(d + k_3)} + \frac{h^2}{(b + k_2)}, \\
& l^2 < 4\frac{d + k_3}{k_4}\left(k_4 - \frac{1}{4(d + k_3)}\right)\left(a + k_1 - \frac{h^2}{b + k_2}\right).
\end{aligned} \tag{14}$$

By virtue of LaSalle Invariant principle, one can derive that the trajectories of (5) will be convergent to the largest invariant set in $dV(t)/dt = 0$ when $t \to \infty$. One can also obtain that $\dot{V}(t) < 0$ for all $e_i(t) \neq 0$, $i = 1, 2, 3, 4$, which means the stability of the error system described by (5), that is, the synchronization of two hyperchaotic systems described by (1) and (3). This completes the proof. □

Remark 2. In [32], some synchronization criteria were derived for 3D Rabinovich systems by using linear feedback control and Routh-Hurwitz criteria. But those results were local, rather than global. The synchronization criterion in Theorem 1 of this paper is global, which is one contribution of this paper.

Remark 3. Rabinovich systems are nonlinear dynamical systems, in which nonlinear terms play an important role in the evolution of trajectories. In [13], some synchronization criteria were derived for 4D Rabinovich systems by the control which eliminated all the nonlinear terms of the error system. In [4, 32], some synchronization criteria were obtained for 3D Rabinovich systems by using the sliding mode controls which also eliminated the nonlinear terms of the error system. Although the linear error systems can be easily obtained after the nonlinear terms of error systems were eliminated and synchronization criteria for linear error systems can also be easily derived, the nonlinear features in the original 4D hyperchaotic systems were discarded. It should be pointed out that the synchronization criterion in Theorem 1 of this paper is global and the nonlinear terms of error systems remained which inherit the nonlinear features from master and slave 4D hyperchaotic Rabinovich systems by the control methods in this paper, which are the main contributions of this paper.

If $k_1 = 0$, one can have the following corollary.

Corollary 4. *If $k_5 > 0$, $k_1 = 0$, and k_2, k_3, k_4 satisfy*

$$\begin{aligned} k_4 &> \frac{1}{4(d+k_3)}, \\ a &> \frac{l^2}{4(d+k_3)} + \frac{h^2}{(b+k_2)}, \\ l^2 &< 4\frac{d+k_3}{k_4}\left(k_4 - \frac{1}{4(d+k_3)}\right)\left(a - \frac{h^2}{b+k_2}\right), \end{aligned} \tag{15}$$

then two 4D hyperchaotic Rabinovich systems described by (1) and (3) achieve global synchronization.

If $k_2 = 0$, one can derive the following corollary.

Corollary 5. *If $k_5 > 0$, $k_2 = 0$, and k_1, k_3, k_4 satisfy*

$$\begin{aligned} k_4 &> \frac{1}{4(d+k_3)}, \\ (a+k_1) &> \frac{l^2}{4(d+k_3)} + \frac{h^2}{b}, \\ l^2 &< 4\frac{d+k_3}{k_4}\left(k_4 - \frac{1}{4(d+k_3)}\right)\left(a + k_1 - \frac{h^2}{b}\right), \end{aligned} \tag{16}$$

then two 4D hyperchaotic Rabinovich systems described by (1) and (3) achieve global synchronization.

If $k_3 = 0$, one can obtain the following corollary.

Corollary 6. *If $k_5 > 0$, $k_3 = 0$, and k_1, k_2, k_4 satisfy*

$$\begin{aligned} k_4 &> \frac{1}{4d}, \\ (a+k_1) &> \frac{l^2}{4d} + \frac{h^2}{b+k_2}, \\ l^2 &< 4\frac{d}{k_4}\left(k_4 - \frac{1}{4d}\right)\left(a + k_1 - \frac{h^2}{b+k_2}\right), \end{aligned} \tag{17}$$

then two 4D hyperchaotic Rabinovich systems described by (1) and (3) achieve global synchronization.

If $k_1 = k_2 = k_3 = 0$, one can have the following corollary.

Corollary 7. *If $a > l^2/4d + h^2/b$, $k_5 > 0$, $k_1 = k_2 = k_3 = 0$, and k_4 satisfies*

$$\begin{aligned} \frac{1}{4d} &< k_4, \\ l^2 &< 4\frac{d}{k_4}\left(k_4 - \frac{1}{4d}\right)\left(a - \frac{h^2}{b}\right), \end{aligned} \tag{18}$$

then two 4D hyperchaotic Rabinovich systems described by (1) and (3) achieve global synchronization.

Remark 8. Corollary 7 is easier to be used than Theorem 1 and Corollaries 4, 5, and 6. But Corollary 7 is more conservative than those results.

3.2. An Application to Synchronization of 3D Chaotic Rabinovich Systems. Consider the following 3D Rabinovich system as a master system:

$$\begin{aligned} \dot{x}_1(t) &= -ax_1(t) + hx_2(t) + x_2(t)x_3(t), \\ \dot{x}_2(t) &= hx_1(t) - bx_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) &= -dx_3(t) + x_1(t)x_2(t), \\ x_1(0) &= x_{1_0}, \\ x_2(0) &= x_{2_0}, \\ x_3(0) &= x_{3_0}, \end{aligned} \tag{19}$$

where $(x_1(t), x_2(t), x_3(t))^T \in \mathbb{R}^3$ is the state variable and a, b, d, h are four positive constants. As the bound in (2), one can assume that there exists a constant l such that

$$|x_2(t)| \le l, \quad \forall t \ge 0. \tag{20}$$

One can construct the following slave scheme associated with system (19):

$$\begin{aligned} \dot{y}_1(t) &= -ay_1(t) + hy_2(t) + y_2(t)y_3(t) + u_1(t), \\ \dot{y}_2(t) &= hy_1(t) - by_2(t) - y_1(t)y_3(t) + u_2(t), \\ \dot{y}_3(t) &= -dy_3(t) + y_1(t)y_2(t) + u_3(t), \\ y_1(0) &= y_{1_0}, \\ y_2(0) &= y_{2_0}, \\ y_3(0) &= y_{3_0}, \end{aligned} \tag{21}$$

where $(y_1(t), y_2(t), y_3(t))^T \in \mathbb{R}^3$ is the state variable of slave system and $u_1(t)$, $u_2(t)$, and $u_3(t)$ are the external controls.

Let $e_i(t) = x_i(t) - y_i(t)$ for $i = 1, 2, 3$. Then, one may construct the following error system for schemes (19) and (21):

$$\begin{aligned} \dot{e}_1(t) &= -ae_1(t) + he_2(t) \\ &\quad + (x_2(t)x_3(t) - y_2(t)y_3(t)) - u_1(t), \\ \dot{e}_2(t) &= he_1(t) - be_2(t) \\ &\quad - (x_1(t)x_3(t) - y_1(t)y_3(t)) - u_2(t), \\ \dot{e}_3(t) &= -de_3(t) + (x_1(t)x_2(t) - y_1(t)y_2(t)) \\ &\quad - u_3(t), \\ e_1(0) &= x_{1_0} - y_{1_0}, \\ e_2(0) &= x_{2_0} - y_{2_0}, \\ e_3(0) &= x_{3_0} - y_{3_0}. \end{aligned} \tag{22}$$

In this paper, we choose $u_1(t) = k_1 e_1(t) + k_4 y_2(t)^2 e_1(t)$, $u_2(t) = k_2 e_2(t)$, and $u_3(t) = k_3 e_3(t)$. Thus, the 3D error system described by (22) can be rewritten as

$$\begin{aligned}
\dot{e}_1(t) &= -(a + k_1 + k_4) e_1(t) + h e_2(t) \\
&\quad + (x_2(t) x_3(t) - y_2(t) y_3(t)), \\
\dot{e}_2(t) &= h e_1(t) - (b + k_2) e_2(t) \\
&\quad - (x_1(t) x_3(t) - y_1(t) y_3(t)), \\
\dot{e}_3(t) &= -(d + k_3) e_3(t) \\
&\quad + (x_1(t) x_2(t) - y_1(t) y_2(t)), \\
e_1(0) &= x_{1_0} - y_{1_0}, \\
e_2(0) &= x_{2_0} - y_{2_0}, \\
e_3(0) &= x_{3_0} - y_{3_0}.
\end{aligned} \tag{23}$$

Constructing the Lyapunov function

$$V(t) = \frac{e_1^2(t) + e_2^2(t) + e_3^2(t)}{2} \tag{24}$$

and using the similar method in Theorem 1, one can have the following synchronization for 3D chaotic Rabinovich systems.

Theorem 9. *If k_1, k_2, k_3, k_4 satisfy*

$$\begin{aligned}
k_4 &> \frac{1}{4(d + k_3)}, \\
(a + k_1) &> \frac{l^2}{4(d + k_3)} + \frac{h^2}{(b + k_2)}, \\
l^2 &< 4\frac{d + k_3}{k_4}\left(k_4 - \frac{1}{4(d + k_3)}\right)\left(a + k_1 - \frac{h^2}{b + k_2}\right),
\end{aligned} \tag{25}$$

then two 3D chaotic Rabinovich systems described by (19) and (21) achieve global synchronization.

4. Three Illustrated Examples

Example 10. Consider the 4D hyperchaotic Rabinovich system described by (1) with $h = 6.75$, $a = 4$, $b = 1$, $c = 2$, and $d = 1$. The initial condition is $x_1(0) = 0.1, x_2(0) = 0.1, x_3(0) = 0, x_4(0) = 0$. Figures 1 and 2 demonstrate attractors of (1), in which the bound of $x_2(t)$ is 6.7, that is, $|x_2(t)| \le 6.7, \forall t \ge 0$.

Then, one can study slave Rabinovich system described by (3). The initial condition is $y_1(0) = 0.1$, $y_2(0) = 0.1$, $y_3(0) = -0.05$, and $y_4(0) = 0.1$. Defining $e_i(t) = x_i(t) - y_i(t)$ for $i = 1, 2, 3, 4$, one can derive error system (5), where the initial condition is $e_1(0) = x_1(t) - y_1(0) = 0$, $e_2(0) = x_2(t) - y_2(0) = 0$, $e_3(0) = x_3(t) - y_3(0) = 0.05$, and

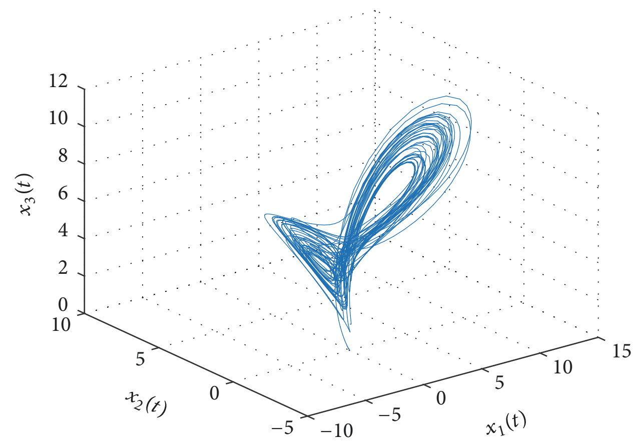

Figure 1: The attractor of (1) with $h = 6.75$, $a = 4$, $b = 1$, $c = 2$, and $d = 1$.

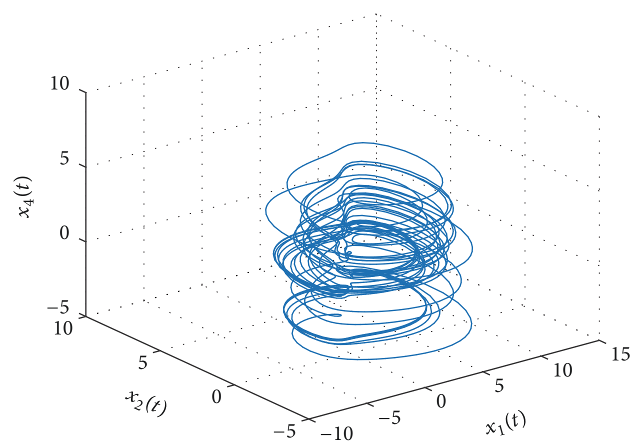

Figure 2: The attractor of (1) with $h = 6.75$, $a = 4$, $b = 1$, $c = 2$, and $d = 1$.

$e_4(0) = x_4(t) - y_4(0) = -0.1$. By using Theorem 1, one can derive

$$\begin{aligned}
k_4 &> \frac{1}{4(1 + k_3)}, \\
(4 + k_1) &> \frac{6.7^2}{4(1 + k_3)} + \frac{6.75^2}{(1 + k_2)}, \\
6.7^2 & \\
&< 4\frac{1 + k_3}{k_4}\left(k_4 - \frac{1}{4(1 + k_3)}\right)\left(4 + k_1 - \frac{6.75^2}{1 + k_2}\right).
\end{aligned} \tag{26}$$

If we choose $k_1 = 0.1$, $k_2 = 21.78125$, $k_3 = 4.61125$, and $k_5 = 1$, then $k_4 > 0.9356$. We choose $k_4 = 0.94$. Figure 3 illustrates the trajectories $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ for error system (5), which can clearly demonstrate the synchronization of hyperchaotic systems (1) and (3).

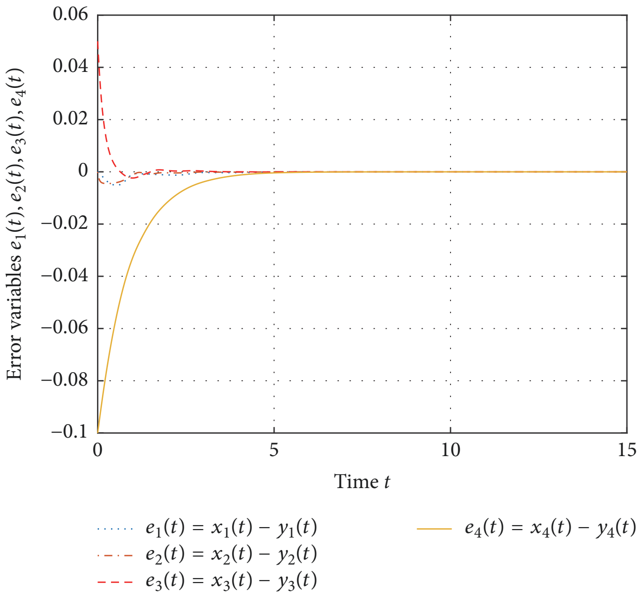

Figure 3: The trajectories of (5) with $k_1 = 0.1$, $k_2 = 21.78125$, $k_3 = 4.61125$, $k_4 = 0.94$, and $k_5 = 1$.

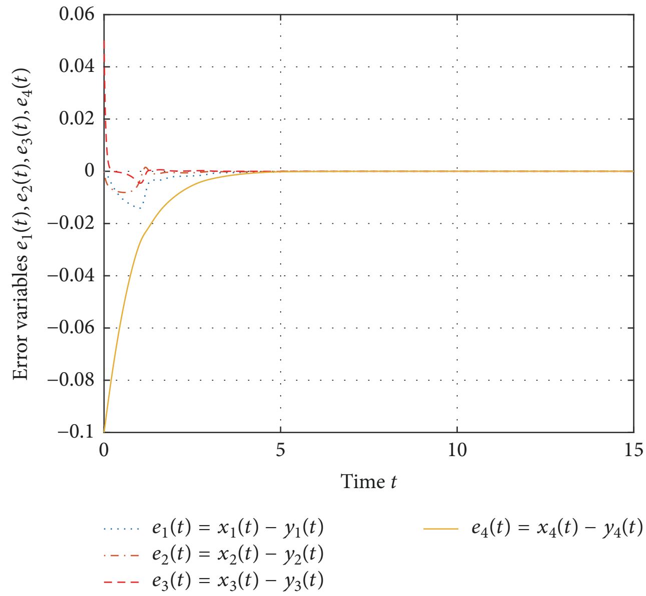

Figure 4: The trajectories of (5) with $k_1 = 0$, $k_2 = 14.1875$, $k_3 = 21.445$, $k_4 = 0.03$, and $k_5 = 1$.

If $k_1 = 0$ in (26), one can derive that

$$k_4 > \frac{1}{4(1+k_3)},$$
$$4 > \frac{6.7^2}{4(1+k_3)} + \frac{6.75^2}{(1+k_2)}, \quad (27)$$
$$6.7^2 < 4\frac{1+k_3}{k_4}\left(k_4 - \frac{1}{4(1+k_3)}\right)\left(4 - \frac{6.75^2}{1+k_2}\right).$$

After setting $k_2 = 14.1875$, $k_3 = 21.445$, and $k_5 = 1$, one can derive $k_4 > 1/44.89$ by Corollary 4. We choose $k_4 = 0.03$. Figure 4 reveals the trajectories $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ for error system (5), which can clearly illustrate the synchronization of hyperchaotic systems (1) and (3).

If $k_2 = 0$ in (26), one can obtain that

$$k_4 > \frac{1}{4(1+k_3)},$$
$$(4+k_1) > \frac{6.7^2}{4(1+k_3)} + 6.75^2, \quad (28)$$
$$6.7^2 < 4\frac{1+k_3}{k_4}\left(k_4 - \frac{1}{4(1+k_3)}\right)\left(4 + k_1 - 6.75^2\right).$$

Setting $k_1 = 43$, $k_3 = 10.2225$, and $k_5 = 1$, one can derive $k_4 > 0.07$ by Corollary 5. We choose $k_4 = 0.08$. Figure 5 gives the trajectories $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ for error system (5), which can clearly reveal the synchronization of hyperchaotic systems (1) and (3).

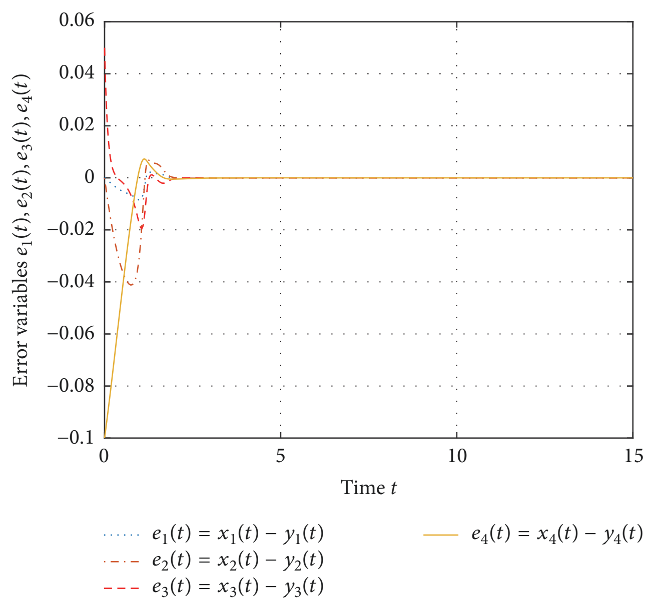

Figure 5: The trajectories of (5) with $k_1 = 43$, $k_2 = 0$, $k_3 = 10.2225$, $k_4 = 0.08$, and $k_5 = 1$.

If $k_3 = 0$ in (26), one can have

$$k_4 > \frac{1}{4},$$
$$(4+k_1) > \frac{6.7^2}{4} + \frac{6.75^2}{(1+k_2)}, \quad (29)$$
$$6.7^2 < 4\frac{1}{k_4}\left(k_4 - \frac{1}{4}\right)\left(4 + k_1 - \frac{6.75^2}{1+k_2}\right).$$

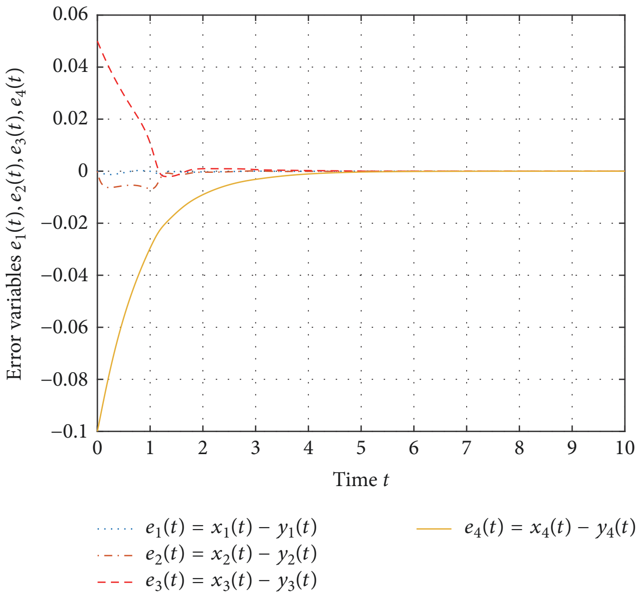

FIGURE 6: The trajectories of (5) with $k_1 = 19.2225$, $k_2 = 14.1875$, $k_3 = 0$, $k_4 = 3.86$, and $k_5 = 1$.

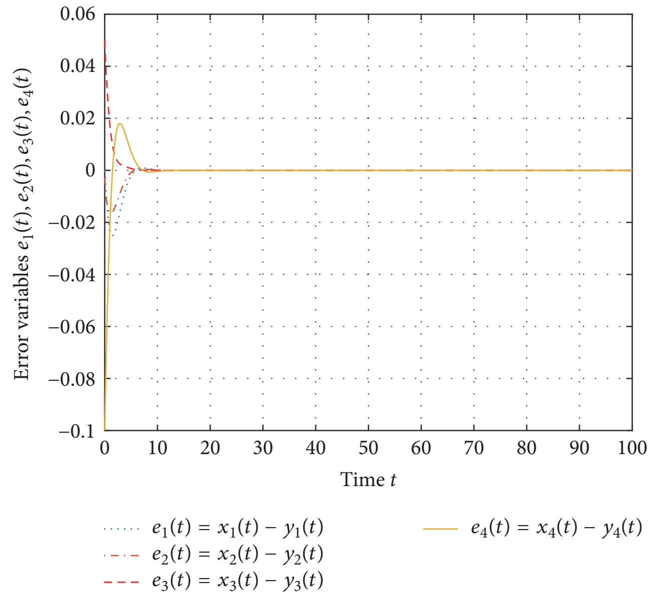

FIGURE 8: The trajectories of (5) with $k_1 = k_2 = k_3 = 0$, $k_4 = 0.26$, and $k_5 = 1$.

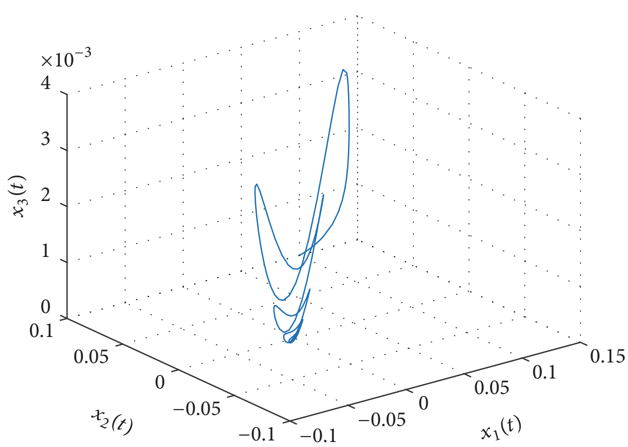

FIGURE 7: The trajectories of (19) with $h = 6.75$, $a = 4.3$, $b = 10.8$, $c = 2$, and $d = 1$.

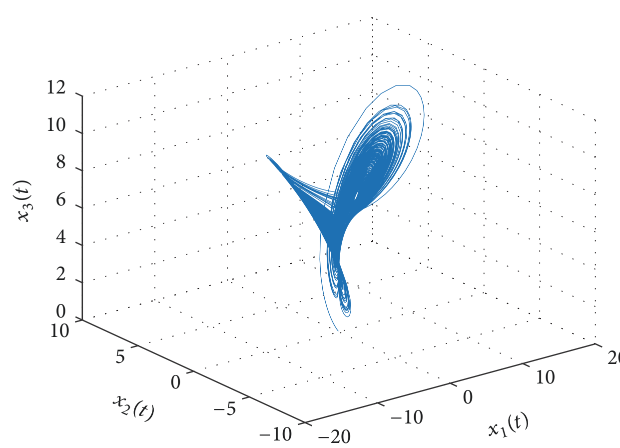

FIGURE 9: The trajectories of (19) with $h = 6.75$, $a = 4$, $b = 1$, and $d = 1$.

Setting $k_1 = 19.2225$, $k_2 = 14.1875$, and $k_5 = 1$, one can derive $k_4 > 3.85$ by Corollary 6. We choose $k_4 = 3.86$. Figure 6 gives the trajectories $e_1(t), e_2(t), e_3(t), e_4(t)$ for error system (5), which can clearly reveal the synchronization of hyperchaotic systems (1) and (3).

Remark 11. It is easy to see that Corollary 7 fails to make any conclusion because $4 < 6.7^2/4 + 6.75^2$ when $k_1 = k_2 = k_3 = 0$.

Example 12. Consider the 4D Rabinovich systems and the error system described by (1), (3), and (5) with $h = 6.75$, $a = 4.3$, $b = 10.8$, $c = 2$, and $d = 1$, respectively, where the initial conditions are the same as those in Example 10. Figure 7 implies that $|x_2(t)| \leq 0.1$ for $t \geq 0$. From Corollary 7, one can have $a = 4.3 > 0.1^2/4 + 6.75^2/10.8 = 4.213$, $k_4 > 0.2581$. We can choose $k_1 = k_2 = k_3 = 0$, $k_4 = 0.26$, and $k_5 = 1$. Figure 8 provides the trajectories $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ for error system (5), which can clearly illustrate the synchronization of Rabinovich systems (1) and (3).

Example 13. Consider the 3D hyperchaotic Rabinovich systems and the error system described by (19), (21), and (23) with $h = 6.75$, $a = 4$, $b = 1$, and $d = 1$, respectively, where the initial conditions are $x_1(0) = 0.1$, $x_2(0) = 0.1$, $x_3(0) = 0$, $y_1(0) = 0.1$, $y_2(0) = 0.1$, $y_3(0) = -0.05$, $e_1(0) = x_1(t) - y_1(0) = 0$, $e_2(0) = x_2(t) - y_2(0) = 0$, and $e_3(0) = x_3(t) - y_3(0) = 0.05$. Figure 9 implies that $|x_2(t)| \leq 6.1$ for $t \geq 0$.

Setting $k_1 = 14.1875$ and $k_2 = k_3 = 0$, one can have $k_4 > 0.26$ by Theorem 9. We can choose $k_1 = 14.1875$, $k_2 = k_3 = 0$, and $k_4 = 0.3$. Figure 10 gives the trajectories $e_1(t)$, $e_2(t)$,

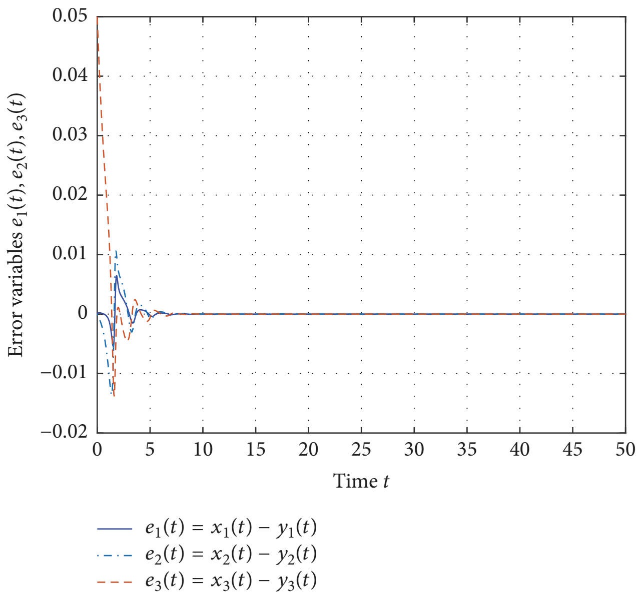

FIGURE 10: The trajectories of (23) with $k_1 = 14.1875$, $k_2 = k_3 = 0$, and $k_4 = 0.3$.

and $e_3(t)$ for error system (23), which can clearly illustrate the synchronization of chaotic systems (19) and (21).

5. Conclusions and Future Works

We have derived some global synchronization criteria for 4D hyperchaotic Rabinovich systems. We have kept the nonlinear terms of error systems. Those control methods and synchronization criteria for 4D hyperchaotic Rabinovich systems can be used to study the synchronization of 3D chaotic Rabinovich systems. We have used three examples to demonstrate the effectiveness our derived results. In this paper, we only consider the state feedback control. Our future research focus is to design the time-delayed controllers.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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